Subleading power corrections to the $B \to \gamma l \nu$ decay in the perturbative QCD approach

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The leptonic radiative decay $B \to \gamma \ell \nu$ is of great importance in the determination of B-meson wave functions, and evaluating the form factors $F_{V,A}$ is the essential problem on the study of this channel. We compute the next-to-leading power corrections to the form factors within the framework of PQCD approach, including the power suppressed hard kernel, the contribution from a complete set of three-particle B-meson wave functions up to twist-4 and two-particle off light-cone wave functions, the $1/m_b$ corrections in heavy quark effective theory (HQET), and the contribution from hadronic structure of photon. In spite of large theoretical uncertainties, the overall power suppressed contributions decreases about 50% of the leading power result. The λ_B dependence of the integrated branching ratio is reduced after including the subleading power contributions, thus the power corrections lead to more ambiguity in the determination of λ_B from $B \to \gamma \ell \nu$ decay.

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I. INTRODUCTION

k_T factorization theorem is an appropriate theoretical framework for exclusive B meson decays. By retaining parton transverse momenta k_T, the endpoint singularities which break collinear factorization are regularized. The perturbative QCD (PQCD) approach [1,2] based on the k_T factorization framework has been applied to various exclusive processes, especially semileptonic and nonleptonic B-meson decays, and other decay modes [3]. The resultant predictions are in agreement with most of the experimental data, and the most applaudable result is the CP violation in many nonleptonic B-meson decay channels [4]. The LHC-b and forthcoming Super-B factory experiments will accumulate more and more accurate data, which require more precise theoretical predictions. To achieve this target, both QCD radiative corrections and power corrections need to be considered. In PQCD approach, QCD radiative corrections are extensively studied in many processes, such as the pion transition form factor [5,6], the pion electromagnetic form factors [7–9], the $B \to \pi$ form factors [10,11] *et al.*, while the exploration on power corrections are very few. The motivation of this paper is to investigate the power corrections in the leptonic radiative decay mode $B \to \gamma \ell \nu$.

Most of the theoretical frameworks to study B meson decays are based on heavy quark expansion, and power corrections are important for finite b quark mass. While in the collinear factorization, the power suppressed contributions are in general nonfactorizable due to endpoint singularity, so they are often fitted by experimental data or estimated using nonperturbative methods. $1/m_b$ power corrections to $B \to \gamma \ell \nu$ were considered at tree level [12] where a symmetry-conserving form factor $\xi(E_\gamma)$ was introduced to parametrize the nonlocal power correction. An approach based on dispersion relations and quark-hadron duality was employed to study the power suppressed contributions in $B \to \gamma \ell \nu$ [13], where the "soft" two-particle correction to the $B \to \gamma$ form factors was computed at leading order. The one-loop corrections to this kind of subleading power contribution has been computed in [14], in addition the contribution from three-particle light-cone distribution amplitudes(LCDAs) was also considered at tree level. In a recent paper [15], using dispersion approach, the soft contribution of power-suppressed higher-twist corrections to the form factors that are due to higher Fock states of B-meson and to the transverse momentum (virtuality) of the light quark in the valence state was calculated, the results are found to be much smaller than that of twist-2 contribution. Based on the power counting in the soft-collinear effective

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theory (SCET [16,17]), the hadronic structure of photon can contribute at next-to-leading power, which was studied in [18,19]. The soft contribution and the contribution from the hadronic structure of photon are probably closely related, and it is interesting to uncover their relationship.

In the PQCD approach, the power suppressed local corrections to $B \to \gamma \ell \nu$ have been first studied in [20], and a more careful investigation of power corrections was performed in [21], in which three-particle B-meson wave functions, next-to-leading power (NLP) hard kernels, and long-distance vector meson dominance contribution are considered. In [21] the contribution from an incomplete set of three-particle B-meson LCDAs was estimated by power counting, but the detailed calculation is still absent. The long distance contribution is found to be cancelled by the radiative corrections, which makes the power correction very small. As a rough estimate, this conclusion needs to be checked by a more careful calculation. Our aim in this article is to make the following improvements: (1) The contribution from a complete set of higher twist B-meson wave functions, up to twist-4, will be investigated. The higher twist wave functions include both two-particle and three-particle Fock states, which are related by the equation of motion. (2) The contribution from the hadronic structure of photon will be calculated within PQCD framework. As the endpoint singularity appears in the collinear factorization is regularized by including the transverse momentum, this kind of contribution can be studied using factorization approach. (3) The $1/m_b$ corrections to the heavy-to-light current in HQET will be considered. Although the NLP contributions considered here are still far from a systematical study, but they can shed light on the correction arises from the power corrections, which makes great sense in the determination of the parameter λ_B.

This paper is organized as follows: In the next section we will present the analytic calculation of the decay amplitude of $B \to \gamma \ell \nu$, including both leading power (LP) and NLP contributions. The numerical analysis is given in the third section. Concluding discussions are presented in Sec. IV.

II. THE $B \to \gamma \ell \nu$ DECAY AMPLITUDE AT NEXT-TO-LEADING POWER

The radiative leptonic B-meson decay amplitude is given by

$$A(B \to \gamma \nu l) = \frac{G_F V_{ub}}{\sqrt{2}} \langle \gamma l \nu_l | \bar{l} \gamma^\nu (1-\gamma_5) \nu_l \bar{u} \gamma_\nu (1-\gamma_5) b | B \rangle. \tag{1}$$

At leading order in QED, the above amplitude can be written as

$$A(B \to \gamma \nu l) = \frac{G_F V_{ub}}{\sqrt{2}} (i g_{\mathrm{em}} \epsilon_\nu^*) [T^{\nu\mu}(p,q) \bar{l} \gamma_\mu (1-\gamma_5) \nu + Q_l f_B \bar{l} \gamma_\nu (1-\gamma_5) \nu], \tag{2}$$

where the momenta carried by photon, lepton-pair, and B-meson are p, q, and $p+q$ respectively. In the light-cone coordinate, $p_\mu = \frac{n\cdot p}{2} \bar{n}_\mu = E_\gamma \bar{n}_\mu$, $q_\mu = \frac{1}{2}(n \cdot q \bar{n}_\mu + \bar{n} \cdot q n_\mu)$, and $p_\mu + q_\mu = m_B v_\mu$. The hadronic tensor $T_{\nu\mu}$ reads

$$T_{\nu\mu}(p,q) = \int d^4 z e^{ip\cdot z} \langle 0 | \mathrm{T}[j_\nu^{\mathrm{em}}(z), \bar{u}(0) \gamma_\mu (1-\gamma_5) b(0)] \times |\bar{B}(p+q)\rangle, \tag{3}$$

with $j_\nu^{\mathrm{em}}(z) = \sum_q Q_q \bar{q}(z) \gamma_\nu q(z) + Q_\ell \bar{\ell}(z) \gamma_\nu \ell(z)$. Considering vector and axial vector current conservation, the decomposition of the hadronic matrix element reads

$$T_{\nu\mu}(p,q) = -iv\cdot p \epsilon_{\mu\nu\rho\sigma} n^\rho v^\sigma F_V(n\cdot p) + (g_{\mu\nu} v\cdot p - v_\nu p_\mu) F_A(n\cdot p) + g_{\mu\nu} f_B, \tag{4}$$

where the last term will cancel the contribution with photon radiated from the lepton. The differential decay rate of $B \to \gamma l \nu$ can be readily computed using the following formula

$$\frac{d\Gamma}{dE_\gamma}(B \to \gamma \ell \nu) = \frac{\alpha_{\mathrm{em}} G_F^2 |V_{ub}|^2}{6\pi^2} m_B E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right) \times [F_V^2(n\cdot p) + F_A^2(n\cdot p)]. \tag{5}$$

This equation indicates that the essential problem in the $B \to \gamma \ell \nu$ decays is to study the factorization of the form factors $F_{A,V}$. A systematical study on the power corrections for this process needs to analyze power suppressed SCET operators, which is rather complicated and we leave it for a future study. Alternatively, we follow [15] to expand the matrix element using HQET

$$\begin{aligned} T_{\nu\mu}(p) &= \sqrt{m_B} \int d^4 z e^{ip\cdot z} \langle 0 | \mathrm{T}[j_\nu^{\mathrm{em}}(z), \bar{u} \gamma_\mu (1-\gamma_5) h_v(0)] | \bar{B}(v) \rangle \\ &\quad + \frac{\sqrt{m_B}}{2m_b} \int d^4 z e^{ip\cdot z} \langle 0 | \mathrm{T}[j_\nu^{\mathrm{em}}(z), \\ &\quad \bar{u} \gamma_\mu (1-\gamma_5) i \not{D}_\perp h_v(0)] | \bar{B}(v) \rangle. \end{aligned} \tag{6}$$

In the first line, the power corrections arise from the light-cone expansion of quark propagator

$$\langle 0 | T\{q(z), \bar{q}(0)\} | 0 \rangle = \frac{i}{2\pi^2} \frac{\not{z}}{z^4} - \frac{i}{16\pi^2} \frac{1}{z^2} \int_0^1 du [\not{z} \sigma_{\alpha\beta} - 4iu z_\alpha \gamma_\beta] G^{\alpha\beta}(uz) + \cdots, \tag{7}$$

and the twist expansion of B-meson wave functions [22–27]

$$\langle 0|\bar{q}(z)W_z(n)^\dagger I_{n;z,0}W_0(n)\Gamma h_v(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\mathrm{Tr}\left\{\frac{1+\not{v}}{2}\left[2\Phi_B^+(t,z^2)+2z^2G^+(z^2) + \frac{\Phi_B^-(t,z^2)+z^2G^-(z^2)-\Phi_B^+(tz^2)-z^2G^+(z^2)}{t}\not{z}+\cdots\right]\gamma_5\Gamma\right\}. \quad (8)$$

The B-meson wave functions describe the distributions of the light parton in both the longitudinal direction denoted by $t = v\cdot z$ and the transverse direction denoted by z^2. In the above definition $z=(0,z^-,\mathbf{z}_T)$ is the coordinate of the anti-quark field $\bar{q}$, h_v is the b quark field in the HQET, and Γ represents a Dirac matrix. The Wilson line $W_z(n)$ is written as

$$W_z(n) = P\exp\left[-ig\int_0^\infty d\lambda n\cdot A(z+\lambda n)\right]. \quad (9)$$

The vertical link $I_{n;z,0}$ at infinity does not contribute in the covariant gauge [28]. Due to the light-cone divergences associated with the Wilson lines, the light-cone vector should be rotated to satisfy $n^2\neq 0$. The wave functions $\Phi_B^\pm(t,z^2)$ are of leading twist, and $G^\pm(z^2)$ are of higher twist. In addition, the definition of three-particle LCDAs are as follows

$$\begin{aligned}\langle 0|\bar{q}_{2\alpha}(z)G_{\mu\nu}(uz)b_\beta(0)|\bar{B}(v)\rangle = {}& \frac{\tilde{f}_B m_B}{4}\int_0^\infty d\omega\int_0^\infty d\xi\int\frac{d^2k_{1\perp}}{(2\pi)^2}\int\frac{d^2k_{2\perp}}{(2\pi)^2}e^{-i(k_1+uk_2)\cdot z}\\ &\times[(1+\not{v})\{(v_\mu\gamma_\nu - v_\nu\gamma_\mu)[\psi_A-\psi_V] - i\sigma_{\mu\nu}\psi_V - (\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)X_A\\ &+(\bar{n}_\mu\gamma_\nu-\bar{n}_\nu\gamma_\mu)(W+Y_A) + i\epsilon_{\mu\nu\rho\sigma}\bar{n}^\rho v^\sigma\gamma_5\tilde{X} - i\epsilon_{\mu\nu\rho\sigma}\bar{n}^\rho\gamma^\sigma\gamma_5\tilde{Y}\\ &-(\bar{n}_\mu v_\nu - \bar{n}_\nu v_\mu)\not{\bar{n}}W + (\bar{n}_\mu\gamma_\nu - \bar{n}_\nu\gamma_\mu)\not{\bar{n}}Z\}\gamma_5]_{\beta\alpha}(\omega,\xi,k_{1\perp},k_{2\perp}).\end{aligned} \quad (10)$$

The wave functions defined above do not have definite twist, but they are convenient in the calculation for their simple Lorentz structure.

For the second line of Eq. (6), although there already exists a suppressed factor $1/m_b$, higher-twist B-meson wave functions are still required as the power expansion in terms of $1/m_b$ is not equivalent to the twist expansion. In the following we will consider the contribution from leading-twist and higher-twist B-meson wave functions respectively in the first line of Eq. (6), and then evaluate the contribution from the second line Eq. (6). Furthermore, we will also investigate the contribution from the hadronic structure of photon at the last subsection.

A. Contribution from leading-twist B-meson wave functions

First we consider the LP result of $F_{V,A}$ and NLP corrections from leading-twist B-meson wave functions. From the definition in Eq. (8), the momentum space projector for B-meson twist-2 wave functions can be written by

$$M^B_{\alpha\beta} = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{\phi_B^+(\omega,k_\perp)\not{n}+\phi_B^-(\omega,k_\perp)\not{\bar{n}} - \int_0^\omega d\eta(\phi_B^-(\eta,k_\perp)-\phi_B^+(\eta,k_\perp))\gamma^\mu\frac{\partial}{\partial k_{\perp\mu}}\right\}\gamma_5\right]_{\alpha\beta}. \quad (11)$$

The leading power contribution is from Fig. 1(a), in which the light quark propagator can be decomposed as

$$\frac{i(\not{p}-\not{k})}{(p-k)^2} = -i\frac{E_\gamma\not{\bar{n}}}{2E_\gamma\omega+k_\perp^2} + i\frac{\omega\not{v}}{2E_\gamma\omega+k_\perp^2} - i\frac{\not{k}_\perp}{2E_\gamma\omega+k_\perp^2}, \quad (12)$$

where the first term is at leading power, and the other two terms are suppressed by $\lambda=\frac{\omega}{E_\gamma}$. Taking only the leading power contribution into account, the form factors $F_{V,A}$ can be written by

$$F_A^{\mathrm{LP}}(E_\gamma) = F_V^{\mathrm{LP}}(E_\gamma) = \frac{2}{3}\tilde{f}_B m_B\int_0^1 d\omega\int\frac{d^2k_\perp}{(2\pi)^2}\frac{\phi_B^+(\omega,k_\perp)}{2\omega E_\gamma+k_\perp^2}. \quad (13)$$

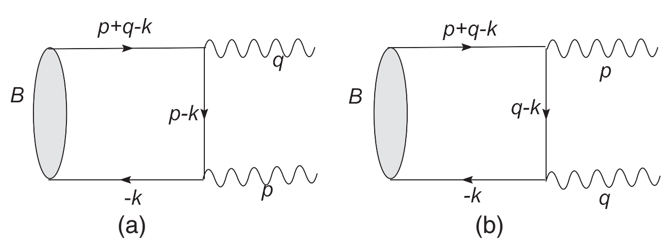

FIG. 1. Tree level diagrams with two-particle B meson wave functions.

According to [21], the mass dependence of the hadron state arises if the power suppressed operators $O_{1,2}$ are included

$$\langle 0|\bar{u}_\rho(z)h_\delta(0)|\bar{B}(Mv)\rangle_{\rm QCD} = \sum_{i=1,2}\langle 0|i\int d^4y T[\bar{u}_\rho(z)h_\delta(0)O_i(y)]|\bar{B}(v)\rangle. \quad (14)$$

where

$$O_1 = \frac{1}{m_b}\bar{h}(iD)^2h, \qquad O_2 = \frac{g}{2m_b}\bar{h}\sigma^{\mu\nu}G_{\mu\nu}h. \quad (15)$$

After considering the mass dependence of the hadronic state the momentum fraction of the soft quark inside the B meson can be defined by $x=\omega/m_B$, and Eq. (13) turns to

$$F_A^{\rm LP}(E_\gamma)=F_V^{\rm LP}(E_\gamma)=\frac{2}{3}\tilde{f}_Bm_B\int_0^1dx\int\frac{d^2k_\perp}{(2\pi)^2}\frac{\phi_B^+(x,k_\perp)}{2xm_BE_\gamma+k_\perp^2}. \quad (16)$$

The QCD correction to the B-meson wave functions and the leading-order (LO) hard kernel produces both the single and double logarithms $\ln^2\frac{k_\perp}{E_\gamma}$, $\ln\frac{k_\perp}{E_\gamma}$, $\ln^2 x$ and $\ln x$ respectively, which become large as $k_\perp \ll E_\gamma, x \ll 1$. These large logarithms need to be resummed, among them k_T resummation leads to Sudakov form factor, and threshold resummation (resumming $\ln^2 x$ and $\ln x$) leads to jet function. The k_T and threshold resummation improves the convergence of the perturbation series, and the resummation improved factorization formula can be rewritten by

$$\begin{aligned}F_A^{\rm LP}(E_\gamma) &= F_V^{\rm LP}(E_\gamma)\\ &= \frac{2}{3}\tilde{f}_Bm_B\int_0^1dx\int_0^\infty bdbK_0(\sqrt{2xE_\gamma m_B}b)\\ &\quad\times S_t(x)e^{-s_B(t)}\phi_B^+(x,b),\end{aligned} \quad (17)$$

where $s_B(t)$ is the Sudakov form factor and $S_t(x)$ is the jet function from the threshold resummation [1,2]. The threshold factor from the resummation of $\ln^2 x$ has been parametrized as

$$S_t(x,Q)=\frac{2^{1+c(Q^2)}\Gamma(\frac{3}{2}+c(Q^2))}{\sqrt{\pi}\Gamma(1+c(Q^2))}[x(1-x)]^{c(Q^2)}. \quad (18)$$

Both the hard kernel and the wave function have been transformed into the impact parameter space(b space) because it is more convenient to perform Sudakov resummation in b space. In the above equation the resummation of rapidity logarithms $\ln\frac{(n\cdot p)^2}{n^2}$, which will cause scheme dependence, is neglected. In [29] the joint resummation with respect to all the large logarithms is performed, and this effect will be considered in the future study.

The power suppressed amplitude includes the latter two terms in Eq. (12) and the contribution from Fig. 1(b). We note that the last term in Eq. (12), which is related to the transverse derivative in the B-meson wave function [the last term in Eq. (11)], vanishes in 4-dimension due to the Lorentz structure $\gamma_{\perp\mu}\not{\epsilon}^*_\perp\gamma^\mu_\perp$. The second term in Eq. (12) results in

$$\begin{aligned}F_A^{{\rm NLP1}a}(E_\gamma) &= -\frac{1}{3E_\gamma}\tilde{f}_Bm_B^2\int_0^1dxx\int_0^\infty bdbS_t(x)e^{-s_B(t)}\\ &\quad\times K_0(\sqrt{2xE_\gamma m_B}b)[\phi_B^-(x,b)+\phi_B^+(x,b)],\\ F_V^{{\rm NLP1}a}(E_\gamma) &= \frac{1}{3E_\gamma}\tilde{f}_Bm_B^2\int_0^1dxx\int_0^\infty bdbS_t(x)e^{-s_B(t)}\\ &\quad\times K_0(\sqrt{2xE_\gamma m_B}b)[\phi_B^-(x,b)-\phi_B^+(x,b)].\end{aligned} \quad (19)$$

The internal line in Fig. 1(b) is a heavy quark propagator, due to the basic idea of effective theory, it must be integrated out and leads to local contribution. In the diagrammatic approach, the propagator is proportional to $\frac{1}{2m_bE_\gamma+k_\perp^2}$, where $k_\perp^2$ in the denominator is obviously suppressed, and this term is identical to the collinear factorization result after $k_\perp$ is dropped

$$F_A^{{\rm NLP1}b}(E_\gamma)=-F_V^{{\rm NLP1}b}(E_\gamma)=\frac{f_Bm_B}{6m_bE_\gamma}. \quad (20)$$

Adding up the leading twist NLP contribution, we obtain

$$F_{A,V}^{\rm NLP1}(E_\gamma)=F_{A,V}^{{\rm NLP1}a}(E_\gamma)+F_{A,V}^{{\rm NLP1}b}(E_\gamma). \quad (21)$$

B. Contribution from higher-twist B-meson wave functions

Up to twist-4, the higher-twist B-meson wave functions include two-particle Fock state, i.e., $G^\pm(t,z^2)$, and three-particle Fock state defined in Eq. (10). According to twist expansion, the three-particle wave functions include one twist-3, $\phi_3=\psi_A-\psi_V$, and three twist-4, $\phi_4=\psi_A+\psi_V$, $\psi_4=\psi_A+X_A$, $\tilde{\psi}_4=\psi_V-\tilde{X}_A$, in which only two wave functions are independent [35]. We assume that all the wave functions have the factorized form, i.e., $G^\pm(t,z^2)=G_B^\pm(t)\tilde{\Sigma}(z^2)$, where $G_B^\pm(t)$ are B-meson LCDAs. The two-particle and three-particle LCDAs are related by the following equation of motion

$$\begin{aligned}2t^2G_B^+(t) &= -\frac{1}{2}\Phi_B^-(t)-\left(t\frac{d}{dt}-\frac{1}{2}+it\bar{\Lambda}\right)\Phi_B^+(t)\\ &\quad-t^2\int_0^1du\bar{u}\Psi_4(t,ut),\end{aligned} \quad (22)$$

and it is convenient to define

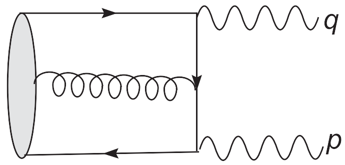

FIG. 2. Diagram of the contribution from three-particle B-meson wave functions.

$$2t^2\hat{G}_B^+(t) = -\frac{1}{2}\Phi_B^-(t) - \left(t\frac{d}{dt} - \frac{1}{2} + it\bar{\Lambda}\right)\Phi_B^+(t). \tag{23}$$

The contribution from three-particle Fock state is plotted in Fig. 2. Inserting Eqs. (10) and (7) into the correlation function $T_{\nu\mu}$, one can obtain the factorization formulas of contributions from three-particle B-meson wave functions. Combining the three-particle contribution with the contribution from $G^\pm(t, z^2)$, we have

$$\begin{aligned} F_A^{\mathrm{NLP2}a}(E_\gamma) &= F_V^{\mathrm{NLP2}a}(E_\gamma) \\ &= -\frac{4}{3} f_B m_B \int_0^\infty \frac{d\omega}{\sqrt{2E_\gamma\omega}} \\ &\quad \times \int_0^\infty b^2 db \hat{g}_B^+(\omega, b) K_1(\sqrt{2E_\gamma\omega}b), \end{aligned} \tag{24}$$

$$\begin{aligned} F_A^{\mathrm{NLP2}b}(E_\gamma) &= F_V^{\mathrm{NLP2}b}(E_\gamma) \\ &= \frac{1}{3\sqrt{2E_\gamma}} f_B m_B \int_0^\infty d\omega \int_0^\infty d\xi \\ &\quad \times \int_0^1 \frac{du}{\sqrt{\omega + u\xi}} \int_0^\infty b^2 db \\ &\quad \times K_1(\sqrt{2E_\gamma(\omega + u\xi)}b) \\ &\quad \times [\psi_4(\omega, \xi, b) - \tilde{\psi}_4(\omega, \xi, b)], \end{aligned} \tag{25}$$

where $\hat{G}^+(t, b) = \int_0^\infty d\omega e^{-i\omega t}\hat{g}^+(\omega, b)$. The total contribution from high twist wave functions is written by

$$F_{A,V}^{\mathrm{NLP2}}(E_\gamma) = F_{A,V}^{\mathrm{NLP2}a}(E_\gamma) + F_{A,V}^{\mathrm{NLP2}b}(E_\gamma). \tag{26}$$

C. Power suppressed contribution in HQET

To evaluate the $1/m_b$ correction in Eq. (6), one should take advantage of the formula

$$\begin{aligned} \bar{q}(z)\Gamma D_\rho h_v(0) &= \partial_\rho[\bar{q}(z)\Gamma h_v(0)] \\ &\quad + i\int_0^1 du\bar{u}\, \bar{q}(z) z^\lambda G_{\lambda\rho}(zu)\Gamma h_v(0) \\ &\quad + \left[\frac{\partial}{\partial z^\rho}\bar{q}(z)\right]\Gamma h_v(0). \end{aligned} \tag{27}$$

For the first term in the above equation, using the following relation

$$\begin{aligned} &\langle 0|\partial_\rho[\bar{q}(z)\Gamma h_v(0)]|\bar{B}(v)\rangle \\ &\quad = \frac{\partial}{\partial y^\rho} e^{-i\bar{\Lambda}v\cdot y}\langle 0|[\bar{q}(z)\Gamma h_v(0)]|\bar{B}(v)\rangle \\ &\quad = -i\bar{\Lambda}v_\rho\langle 0|[\bar{q}(z)\Gamma h_v(0)]|\bar{B}(v)\rangle, \end{aligned} \tag{28}$$

one can obtain

$$\begin{aligned} F_A^{\mathrm{NLP3}a}(E_\gamma) &= F_V^{\mathrm{NLP3}a}(E_\gamma) \\ &= \frac{\bar{\Lambda}}{3m_b} f_B m_B \int_0^1 dx \int_0^\infty b db K_0(\sqrt{2xE_\gamma m_B}b) \\ &\quad \times S_t(x) e^{-s_B(t)}\phi_B^+(x, b). \end{aligned} \tag{29}$$

The matrix element of the second term is related to the twist-3 three-particle wave function, following the same method with the above subsection, we have

$$\begin{aligned} F_A^{\mathrm{NLP3}b}(E_\gamma) &= F_V^{\mathrm{NLP3}b}(E_\gamma) \\ &= \frac{\sqrt{2E_\gamma}}{3m_b} f_B m_B \int_0^\infty d\omega \int_0^\infty d\xi \int_0^1 \frac{du}{\sqrt{\omega + u\xi}} \\ &\quad \times \int_0^\infty b^2 db K_1\left(\sqrt{2E_\gamma(\omega + u\xi)}b\right)\phi_3(\omega, \xi, b). \end{aligned} \tag{30}$$

The last term can be evaluated with integration by part, and the result reads

$$\begin{aligned} F_A^{\mathrm{NLP3}c}(E_\gamma) &= F_V^{\mathrm{NLP3}c}(E_\gamma) \\ &= \frac{f_B m_B}{3m_b}\int_0^1 dx \\ &\quad \times \int_0^\infty b db K_0(\sqrt{2xE_\gamma m_B}b)\Psi_B(x, b) \\ &\quad - \frac{2f_B m_B E_\gamma}{3m_b}\int_0^1 x dx \\ &\quad \times \int_0^\infty b db K_0(\sqrt{2xE_\gamma m_B}b)\phi_B^+(x, b), \end{aligned} \tag{31}$$

with $\Psi_B(\omega, b) = \int_0^\omega d\eta[\phi_B^-(\eta, b) - \phi_B^+(\eta, b)]$. Adding up all the above results we have

$$\begin{aligned} F_{A,V}^{\mathrm{NLP3}}(E_\gamma) &= F_{A,V}^{\mathrm{NLP3}a}(E_\gamma) + F_{A,V}^{\mathrm{NLP3}b}(E_\gamma) \\ &\quad + F_{A,V}^{\mathrm{NLP3}c}(E_\gamma). \end{aligned} \tag{32}$$

D. Contribution from hadronic structure of photon

To investigate the contribution of the hadronic structure of photon, it is essential to introduce the LCDAs of photon, which have been studied up to twist-4 level in [36]. In the present paper we will only consider the contribution of two-particle twist-2 and twist-3 LCDAs, which are defined below

$$
\begin{aligned}
\langle\gamma(p,\lambda)|\bar{q}(z)\sigma_{\alpha\beta}q(0)|0\rangle &= i g_{\mathrm{em}} Q_q \langle\bar{q}q\rangle (p_\beta\epsilon^*_\alpha - p_\alpha\epsilon^*_\beta) \\
&\quad\times\int_0^1 du e^{iup\cdot z}[\chi(\mu)\phi_\gamma(u,\mu)], \\
\langle\gamma(p,\lambda)|\bar{q}(z)\gamma_\alpha q(0)|0\rangle &= -g_{\mathrm{em}} Q_q f_{3\gamma}\epsilon^*_\alpha \\
&\quad\times\int_0^1 du e^{iup\cdot z}\psi_\gamma^{(v)}(u,\mu), \\
\langle\gamma(p,\lambda)|\bar{q}(z)\gamma_\alpha\gamma_5 q(0)|0\rangle &= \frac{1}{4} g_{\mathrm{em}} Q_q f_{3\gamma}\epsilon_{\alpha\beta\rho\sigma}p^\rho z^\sigma \epsilon^{*\beta} \\
&\quad\times\int_0^1 du e^{iup\cdot z}\psi_\gamma^{(a)}(u,\mu)
\end{aligned}
\tag{33}
$$

where $\phi_\gamma(u,\mu)$ is twist-2 LCDA, and $\psi_\gamma^{(a,v)}(u,\mu)$ are twist-3 LCDAs. The normalization constants of these LCDAs depend on the factorization scale, and the evolution behavior is written by

$$
\begin{aligned}
\chi(\mu) &= \left[\frac{\alpha(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{16}{33-2n_f}}\chi(\mu_0), \\
\langle\bar{q}q\rangle(\mu) &= \left[\frac{\alpha(\mu_0)}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}\langle\bar{q}q\rangle(\mu_0),
\end{aligned}
\tag{34}
$$

$$
f_{3\gamma}(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{23}{99-6n_f}} f_{3\gamma}(\mu_0). \tag{35}
$$

In the factorization formulas we will neglect the transverse momentum dependence of the wave functions, as the Sudakov effect for light $q\bar{q}$ state is significant, further suppression is not necessary. The momentum space projector for the two-particle LCDAs is written by (up to two-particle twist-3)

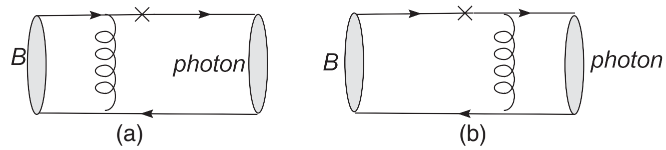

FIG. 3. Diagrams of the contribution from hadronic structure of photon.

$$
\begin{aligned}
M^\gamma_{\alpha\beta} = \frac{1}{4} g_{\mathrm{em}} Q_q \bigg\{ &-\langle\bar{q}q\rangle(\not\epsilon^*\not p)\chi(\mu)\phi_\gamma(u,\mu) \\
&- f_{3\gamma}(\not\epsilon^*)\psi_\gamma^{(v)}(u,\mu) \\
&- \frac{i}{8} f_{3\gamma}\epsilon_{\mu\nu\rho\sigma}(\gamma^\mu\gamma^5)\bar{n}^\rho\epsilon^{*\nu}\bigg[n^\sigma\frac{d}{du}\psi_\gamma^{(a)}(u,\mu) \\
&- 2E_\gamma\psi_\gamma^{(a)}(u,\mu)\frac{\partial}{\partial k_{\perp\sigma}}\bigg]\bigg\}_{\alpha\beta}.
\end{aligned}
\tag{36}
$$

The matrix element of $B\to\gamma$ transition can be calculated through the convolution formula

$$
\begin{aligned}
{}_{HS}\langle\gamma|\bar{q}\Gamma b|B\rangle = &\frac{4\pi\alpha_s C_F}{N_c}\int_0^1 dx\int_0^\infty b_1 db_1\int_0^1 du \\
&\times\int_0^\infty b_2 db_2 M^B_{\beta\rho}H^\Gamma_{\alpha\beta\rho\sigma}M^\gamma_{\sigma\alpha},
\end{aligned}
\tag{37}
$$

after evaluating the Feynman diagrams in Fig. 3, the results of the form factors $F_{V,A}$ read

$$
\begin{aligned}
F_A^{\mathrm{NLP4}a}(E_\gamma) = &-\frac{\pi\alpha_s C_F f_B m_B Q_u}{E_\gamma N_c}\int_0^1 dx\int_0^\infty b_1 db_1\int_0^1 du\int_0^\infty b_2 db_2 h_e^a(x,u,b_1,b_2) \\
&\times\bigg[2E_\gamma m_B\langle\bar{q}q\rangle\chi(\mu)\phi_\gamma(u,\mu)\phi_B^- - (2uE_\gamma\phi_B^+ + m_B(\phi_B^- + \phi_B^+))f_{3\gamma}\psi_\gamma^{(v)} \\
&+\frac{1}{2}\left(uE_\gamma\phi_B^+ + \frac{1}{2}m_B(\phi_B^- - \phi_B^+)\right)f_{3\gamma}\frac{d}{du}\psi_\gamma^{(a)}(u,\mu)\bigg],
\end{aligned}
\tag{38}
$$

$$
\begin{aligned}
F_V^{\mathrm{NLP4}a}(E_\gamma) = &-\frac{\pi\alpha_s C_F f_B m_B Q_u}{E_\gamma N_c}\int_0^1 dx\int_0^\infty b_1 db_1\int_0^1 du\int_0^\infty b_2 db_2 h_e^a(x,u,b_1,b_2) \\
&\times\bigg[2E_\gamma m_B\langle\bar{q}q\rangle\chi(\mu)\phi_\gamma(u,\mu)\phi_B^- + (2uE_\gamma\phi_B^+ + m_B(\phi_B^- - \phi_B^+))f_{3\gamma}\psi_\gamma^{(v)} \\
&-\frac{1}{2}\left(uE_\gamma\phi_B^+ + \frac{1}{2}m_B(\phi_B^- + \phi_B^+)\right)f_{3\gamma}\frac{d}{du}\psi_\gamma^{(a)}(u,\mu)\bigg],
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
F_A^{\mathrm{NLP4}b}(E_\gamma) = &\frac{\pi\alpha_s C_F f_B m_B Q_u}{E_\gamma N_c}\int_0^1 dx\int_0^\infty b_1 db_1\int_0^1 du\int_0^\infty b_2 db_2 h_e^b(x,u,b_1,b_2) \\
&\times\bigg[2E_\gamma\phi_B^+ f_{3\gamma}\psi_\gamma^{(v)} + \frac{1}{2}E_\gamma\phi_B^+ f_{3\gamma}\frac{d}{du}\psi_\gamma^{(a)}(u,\mu)\bigg],
\end{aligned}
\tag{40}
$$

$$F_V^{\mathrm{NLP4}b}(E_\gamma) = F_A^{\mathrm{NLP4}b}(E_\gamma), \tag{41}$$

with the hard functions

$$\begin{aligned} h_e^a(x,u,b_1,b_2) &= e^{-s_B(t)-s_\gamma(t)}[\theta(b_1-b_2)I_0(\sqrt{2uE_\gamma m_B}b_2)K_0(\sqrt{2uE_\gamma m_B}b_1) \\ &\quad + \theta(b_2-b_1)I_0(\sqrt{2uE_\gamma m_B}b_1)K_0(\sqrt{2uE_\gamma m_B}b_2)]K_0(\sqrt{2xum_B E_\gamma}b_1)S_t(u), \\ h_e^b(x,u,b_1,b_2) &= e^{-s_B(t)-s_\gamma(t)}[\theta(b_1-b_2)I_0(\sqrt{2xE_\gamma m_B}b_2)K_0(\sqrt{2xE_\gamma m_B}b_1) \\ &\quad + \theta(b_2-b_1)I_0(\sqrt{2xE_\gamma m_B}b_1)K_0(\sqrt{2xE_\gamma m_B}b_2)]K_0(\sqrt{2xum_B E_\gamma}b_1)S_t(u). \end{aligned} \tag{42}$$

Summing up the two diagrams, the form factors from photon hadronic structure can be written by

$$\begin{aligned} F_V^{\mathrm{NLP4}}(E_\gamma) &= F_V^{\mathrm{NLP4}a}(E_\gamma) + F_V^{\mathrm{NLP4}b}(E_\gamma), \\ F_A^{\mathrm{NLP4}}(E_\gamma) &= F_A^{\mathrm{NLP4}a}(E_\gamma) + F_A^{\mathrm{NLP4}b}(E_\gamma). \end{aligned} \tag{43}$$

In summary, combining all the NLP contributions together, we have

$$\begin{aligned} F_V^{\mathrm{NLP}}(E_\gamma) &= F_V^{\mathrm{NLP1}}(E_\gamma) + F_V^{\mathrm{NLP2}}(E_\gamma) \\ &\quad + F_V^{\mathrm{NLP3}}(E_\gamma) + F_V^{\mathrm{NLP4}}(E_\gamma), \\ F_A^{\mathrm{NLP}}(E_\gamma) &= F_A^{\mathrm{NLP1}}(E_\gamma) + F_A^{\mathrm{NLP2}}(E_\gamma) + F_A^{\mathrm{NLP3}}(E_\gamma) \\ &\quad + F_A^{\mathrm{NLP4}}(E_\gamma). \end{aligned} \tag{44}$$

Based on the calculations in above sections, several comments are as follows:

(i) All the results of the form factors are given at leading order. The radiative corrections are of great importance in the hard exclusive processes, and in the $B \to \gamma\ell\nu$ decay it can reduce the leading order amplitude by 20%–25% in collinear factorization [12,30–32]. In k_T factorization, the NLO corrections have been studied in [20], while the endpoint behavior in this study is under controversy, and a more comprehensive study is required, which is left for a future study.

(ii) For the contributions from higher twist B meson wave functions up to twist-4, it has been found that it is free from endpoint singularity in the collinear factorization, thus the endpoint region is not very important and the Sudakov form factor and jet function are not essential. In addition, there is no study on the k_T resummation effect for the higher twist wave functions so far, so the Sudakov factor is not considered here. If four-particle twist-5 and twist-6 wave functions are included, there does exist endpoint singularity [15], and the resummation effect must be considered.

(iii) Only two-particle twist-2 and twist-3 photon LCDAs are employed in the contribution from the hadronic structure of photon. In [19], the contributions from the full set of photon LCDAs up to twist-4 are studied using light-cone sum rules approach, and the results indicate that the contribution from two-particle twist-2 LCDA is dominant, and the contribution from higher twist and three-particle LCDAs is suppressed. Here we neglect higher twist photon LCDAs except for the two-particle twist-3 contribution which gives important contributions to the $B \to V$ form factors in the PQCD approach.

III. NUMERICAL ANALYSIS

The most important input parameters are wave functions of B meson and photon. We have assumed that the transverse momentum dependent B-meson wave function $\phi_B^\pm(x,k_T)$ possesses the factorized form

$$\phi_B^\pm(x,k_T) = \phi_B^\pm(x)\Sigma(k_T), \tag{45}$$

where the transverse part needs to be transformed into the impact parameter space through Fourier transform, and the wave functions turns to

$$\tilde{\phi}_B^\pm(x,b) = \phi_B^\pm(x)\tilde{\Sigma}(b). \tag{46}$$

For the transverse part the Gaussian model is usually adopt in the PQCD approach, i.e., $\tilde{\Sigma}(b) = e^{-\frac{1}{2}\omega_0^2 b^2}$, with $\omega_0 = \lambda_B$. For the longitudinal part, we employ the following models to check the model dependence of the form factors. The first one is a free parton model [33]

$$\begin{aligned} \phi_{\mathrm{B\,I}}^+(x) &= \frac{x}{2x_1^2}\theta(2x_1 - x), \\ \phi_{\mathrm{B\,I}}^-(x) &= \frac{2x_1 - x}{2x_1^2}\theta(2x_1 - x), \end{aligned} \tag{47}$$

where $x_1 = x_0 = \omega_0/m_B$. The second one is from the QCD sum rules with local duality approximation [34]

$$\phi^+_{\mathrm{B\,II}}(x) = \frac{3x}{4x_2^3}(2x_2 - x)\theta(2x_2 - x),$$
$$\phi^-_{\mathrm{B\,II}}(x) = \frac{1}{8x_2^3}\left[3(2x_2 - x)^2 + \frac{10(\lambda_E^2 - \lambda_H^2)}{3x_2^2 m_B^2}(3x^2 - 6xx_2 + 2x_2^2)\right]\theta(2x_2 - x), \tag{48}$$

where $x_2 = 3/2x_0$. In the phenomenological studies with PQCD approach, a more widely used model is as follows,

$$\phi^+_{\mathrm{B\,III}}(x) = \phi^-_{\mathrm{B\,III}}(x) = N_B x^2(1-x)^2 e^{-\frac{x^2}{2x_0^2}}, \tag{49}$$

where normalization constant N_B is determined by λ_B. For the model of two-particle twist-4 B-meson LCDA, following [15] we adopt

$$\hat{g}_B^+(\omega) = \frac{\omega^2}{2\lambda_B}\left(1 - \frac{\lambda_E^2 - \lambda_H^2}{36\lambda_B^2}\right)e^{-\frac{\omega}{\lambda_B}}, \tag{50}$$

where the parameters λ_E^2 and λ_H^2 which are related to the matrix elements of local quark-gluon operator can be estimated with QCD sum rules approach. The three-particle B wave function is also supposed to satisfy $\psi(\omega, \xi, b, ub) = \psi(\omega, \xi)\tilde{\Sigma}(b)$, and the exponential model of the longitudinal part is widely used

$$\phi_3(\omega, \xi) = \frac{\lambda_E^2 - \lambda_H^2}{6\lambda_B^5}\omega\xi^2 e^{-\frac{\omega+\xi}{\lambda_B}},$$
$$\psi_4(\omega, \xi) = \frac{\lambda_E^2}{3\lambda_B^4}\omega\xi e^{-\frac{\omega+\xi}{\lambda_B}},$$
$$\tilde{\psi}_4(\omega, \xi) = \frac{\lambda_H^2}{3\lambda_B^4}\omega\xi e^{-\frac{\omega+\xi}{\lambda_B}}. \tag{51}$$

The light-cone distribution amplitudes $\phi_\gamma(u)$, $\psi_\gamma^{(v,a)}(\omega, \xi)$ have been systematically studied in [36], and the expressions are quoted as follows. The two-particle twist-2 LCDA is expanded in terms of Gegenbauer polynomials,

$$\phi_\gamma(u, \mu) = 6u\bar{u}\left[1 + \sum_{n=2}^{\infty} b_n(\mu_0)C_n^{3/2}(u - \bar{u})\right], \tag{52}$$

and twist-3 LCDAs in conformal expansion read

$$\psi^{(v)}(\xi,\mu) = 5(3\xi^2 - 1) + \frac{3}{64}[15\omega_\gamma^V(\mu) - 5\omega_\gamma^A(\mu)](3 - 30\xi^2 + 35\xi^4),$$
$$\psi^{(a)}(\xi,\mu) = \frac{5}{2}(1-\xi^2)(5\xi^2 - 1)\left(1 + \frac{9}{16}\omega_\gamma^V(\mu) - \frac{3}{16}\omega_\gamma^A(\mu)\right). \tag{53}$$

In addition to the normalization constant Eqs. (34), (35), the scale dependence of the parameters in the LCDAs can be written as

$$b_2(\mu) = \left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\frac{8}{33-2n_f}} b_2(\mu_0),$$
$$\begin{pmatrix} \omega_\gamma^V(\mu) - \omega_\gamma^A(\mu) \\ \omega_\gamma^V(\mu) + \omega_\gamma^A(\mu) \end{pmatrix} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\Gamma_\omega/\beta_0} \begin{pmatrix} \omega_\gamma^V(\mu_0) - \omega_\gamma^A(\mu_0) \\ \omega_\gamma^V(\mu_0) + \omega_\gamma^A(\mu_0) \end{pmatrix}, \tag{54}$$

where the anomalous dimension matrix Γ_ω and β_0 is given by [36,37]

$$\Gamma_\omega = \begin{pmatrix} 3C_F - \frac{2}{3}C_A & \frac{2}{3}C_F - \frac{2}{3}C_A \\ \frac{5}{3}C_F - \frac{4}{3}C_A & \frac{1}{2}C_F + C_A \end{pmatrix},$$
$$\beta_0 = 11 - \frac{2}{3}n_f. \tag{55}$$

The value of the parameters used in the calculations are presented in Table I, among them the scale dependent parameters are given at $\mu_0 = 1.0$ GeV. These parameters should be run to the factorization scale t in numerical analysis.

Now we present the numerical results for the form factors $F_{V,A}$ and the branching ratio of $B \to \gamma\ell\nu$ decay. In the physical interesting photon energy region $1.5\ \mathrm{GeV} < E_\gamma < 2.6$ GeV, the leading power results of $F_{V,A}(E_\gamma)$ at tree level are plotted in Fig. 4, where all the parameters are fixed at the central values in Table I. At leading power $F_V = F_A$ due to the left-handedness of the standard model. The three curves are from the three models of leading twist B meson wave functions, and the difference between them is only about 3%–5%. In the following we

TABLE I. Numerical value of the parameters entering the calculations.

Parameter	$\omega_0(\lambda_B)$	x_0	$\chi(1\ \mathrm{GeV})$	$\langle\bar{q}q\rangle(1\ \mathrm{GeV})$
Value	0.35 ± 0.10 GeV	0.076 ± 0.015	$(3.15 \pm 0.03)\ \mathrm{GeV}^{-2}$	$-[(256^{+14}_{-16})\ \mathrm{MeV}]^3$
Parameter	$b_2(1\ \mathrm{GeV})$	$f_{3\gamma}(1\ \mathrm{GeV})$	$\omega_\gamma^V(1\ \mathrm{GeV})$	$\omega_\gamma^A(1\ \mathrm{GeV})$
Value	0.07 ± 0.07	$-(4 \pm 2) \times 10^{-3}\ \mathrm{GeV}^2$	3.8 ± 1.8	-2.1 ± 1.0
Parameter	N_B	λ_E^2	λ_H^2	f_B
Value	3417	$0.06 \pm 0.04\ \mathrm{GeV}^{-2}$	$0.12 \pm 0.05\ \mathrm{GeV}^{-2}$	0.19 ± 0.02 GeV

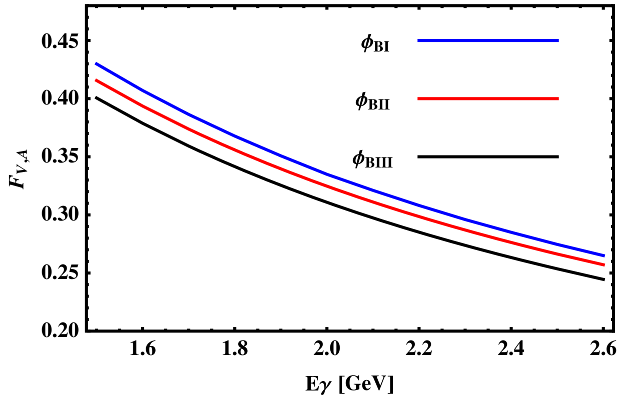

FIG. 4. The leading power contribution to the form factors $F_{V,A}$, where blue, red, and black curves are corresponding to the wave functions $\phi_{\rm B\,I}$, $\phi_{\rm B\,II}$, and $\phi_{\rm B\,III}$ respectively.

set the model $\phi^{\pm}_{\rm B\,III}$ as default, which approaches zero at endpoint region. $\phi^{-}_{\rm B\,I,II}$ does not vanish when $x = 0$, and it will lead to too large endpoint contribution when entering the factorization formula. Compared with the result of leading order $F_{V,A}$ in collinear factorization, the PQCD result is relatively smaller due to the inclusion of transverse momentum in the denominator of the propagators as well as suppression from k_T resummation and threshold resummation.

The NLP contribution to the form factors are presented in Fig. 5. Among various kinds of contributions, the one from hadronic structure of photon is most important. It decreases the leading power contribution by about 20% for the symmetric form factor $(F_V + F_A)/2$, and this result is consistent with the predictions from light-cone sum rules [19]. It can only give rise to a minor contribution to the symmetry breaking part $(F_V - F_A)/2$ as the leading twist photon LCDA provides identical result for F_V and F_A, and the symmetry breaking effect comes only from higher twist photon LCDA. The contribution from higher twist B-meson wave functions, including both two-particle and three-particle Fock states, also decreases the leading power contribution by about 20%, and it keeps the symmetry between F_V and F_A. The contribution from three particle B-meson wave functions is much smaller than that from higher twist two-particle wave function, which is consistent with the rough estimate in [21]. The power suppressed hard kernel can also give rise to sizeable corrections as the suppression factors ω/E_γ is not very small when E_γ is not large. It is the main source of symmetry breaking part $(F_V - F_A)/2$. The $1/m_b$ suppression term from HQET is negligible due to the cancellation between different part in Eq. (32). The different pieces of the NLP corrections considered in this paper are all sizable except for the $1/m_b$ suppression term from HQET, furthermore, their effects are all negative. The overall NLP correction is then significant, it decreases the LP result by about 50%. This result indicates the extraordinarily importance of power corrections in this channel.

Now we present the uncertainties from the various parameters in Table I. If we fix $E_\gamma = 2.0$ GeV and $\lambda_B = 0.35$ GeV, then the form factors with uncertainty are obtained as (in the unit of GeV)

$$F_V(2\ \mathrm{GeV}) = 0.169 + \begin{pmatrix} +0.003 \\ -0.003 \end{pmatrix}_{\lambda_E^2-\lambda_H^2} + \begin{pmatrix} +0.020 \\ -0.020 \end{pmatrix}_{f_{3\gamma}} + \begin{pmatrix} +0.003 \\ -0.003 \end{pmatrix}_{b_2} + \begin{pmatrix} +0.011 \\ -0.011 \end{pmatrix}_{\omega_\gamma^V} + \begin{pmatrix} +0.002 \\ -0.002 \end{pmatrix}_{\omega_\gamma^A} + \begin{pmatrix} +0.018 \\ -0.018 \end{pmatrix}_{f_B} + \begin{pmatrix} +0.006 \\ -0.006 \end{pmatrix}_{\langle\bar{q}q\rangle} + \begin{pmatrix} +0.009 \\ -0.012 \end{pmatrix}_{S_t}, \quad (56)$$

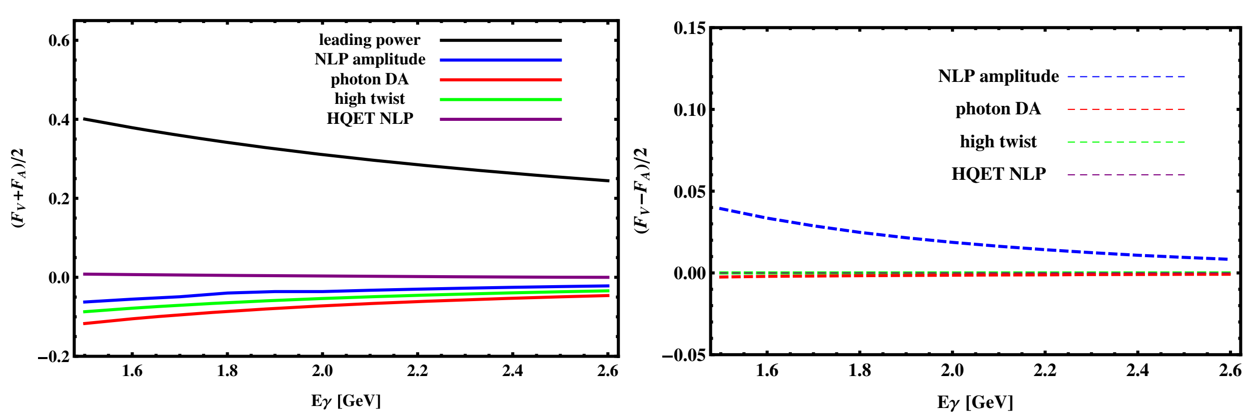

FIG. 5. The next-to-leading power contribution to the form factors $F_{V,A}$. The left(right) panel denotes the photon momentum dependence of $(F_V + F_A)/2((F_V - F_A)/2)$ respectively.

$$F_A(2\text{ GeV}) = 0.135 + \begin{pmatrix} +0.003 \\ -0.003 \end{pmatrix}_{\lambda_E^2-\lambda_H^2} + \begin{pmatrix} +0.018 \\ -0.019 \end{pmatrix}_{f_{3\gamma}} + \begin{pmatrix} +0.003 \\ -0.003 \end{pmatrix}_{b_2} + \begin{pmatrix} +0.008 \\ -0.008 \end{pmatrix}_{\omega_\gamma^V} + \begin{pmatrix} +0.001 \\ -0.002 \end{pmatrix}_{\omega_\gamma^A} + \begin{pmatrix} +0.015 \\ -0.014 \end{pmatrix}_{f_B} + \begin{pmatrix} +0.006 \\ -0.006 \end{pmatrix}_{\langle\bar{q}q\rangle} + \begin{pmatrix} +0.008 \\ -0.011 \end{pmatrix}_{S_t}, \quad (57)$$

where the important sources of the uncertainties include the parameters $f_{3\gamma}$ and ω_V in the distribution amplitude of photon, the decay constant of B meson, and the parameter c in the threshold resummation. For simplicity, $c(Q^2)$ has been fixed as a constant and varies in the region [0.45, 0.65]. Due to the variation regions of the twist-2 parameters $\chi(\mu_0)$ and $\langle\bar{q}q\rangle$ are very small, the uncertainties from them are not important. The E_γ dependence of the form factors with uncertainties is plotted in Fig. 6, where the errors are added in quadrature, and the overall uncertainty is expressed in the shaded region. Here the form factor F_A is not shown for its uncertainty region is overlapped with F_V, instead, the uncertainty region of the symmetry breaking effect $(F_V - F_A)/2$ is presented. The uncertainty region of F_V is large because the parameters in the B meson and photon wave functions are not well determined, and they should be constrained by more preciously measured physical quantities such as $B \to \pi$ transition form factors.

Having the theoretical predictions of the form factors $F_{V,A}$ at hands, we proceed to discuss the theory constraints on the first inverse moment λ_B using integrated branching ratios of $B \to \gamma\ell\nu$. The lower limit of integral should be a photon-energy cut to get rid of the soft photon radiation. The integrated branching fractions with the phase-space cut on the photon energy read

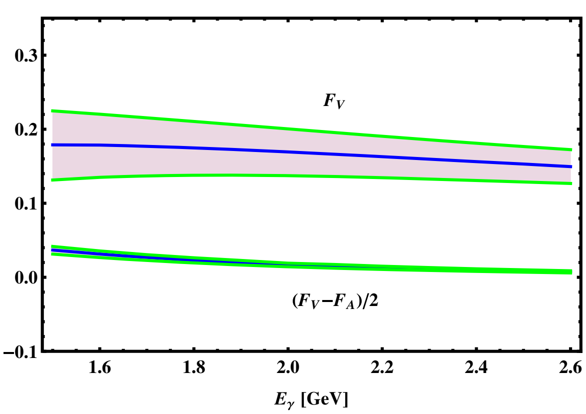

FIG. 6. The form factors with uncertainty.

$$\mathcal{BR}(B \to \gamma\ell\nu, E_\gamma \geq E_{\text{cut}}) = \tau_B \int_{E_{\text{cut}}}^{m_B/2} dE_\gamma \frac{d\Gamma(B \to \gamma\ell\nu)}{dE_\gamma}, \quad (58)$$

where τ_B indicates the lifetime of the B-meson. Our predictions for the partial branching ratios of $B \to \gamma\ell\nu$ decay including power suppressed contributions are displayed in Fig. 7. The variation range of the first inverse moment λ_B is $[0.25, 0.45]$ GeV. It can be observed that the integrated branching fractions $\mathcal{BR}(B \to \gamma\ell\nu, E_\gamma \geq E_{\text{cut}})$ grow with the decrease of λ_B, but the slope becomes small when λ_B is getting large, in addition, the theoretical uncertainty is big. This λ_B dependence behavior makes it more difficult to precisely determine the parameter λ_B. Recently, Belle collaboration reported their improved measurement of the branching ratio of $B \to \gamma\ell\nu$ with the energy cut $E_\gamma > 1$ GeV [38], the measured branching ratio is given by

$$\mathcal{BR}(B \to \gamma\ell\nu, E_\gamma \geq 1.0\text{ GeV}) = (1.4 \pm 1.0 \pm 0.4) \times 10^{-6}, \quad (59)$$

and a Bayesian upper limit of $\mathcal{BR}(B \to \gamma\ell\nu, E_\gamma \geq 1.0\,\text{GeV}) < 3.0 \times 10^{-6}$ is determined at 90% confidence level. Furthermore, the predictions and uncertainties of partial decay rate in Ref. [15] extrapolated to $E_\gamma > 1$ GeV are used to determine λ_B. While if our result is employed, the uncertainty of λ_B determined from $B \to \gamma\ell\nu$ decay should be larger. Thus a more systematic study of the NLP corrections to this channel is of great importance. On the experimental side, it is meaningful to measure the branching fraction with the phase-space cut on the photon energy larger than 1.5 GeV, which is helpful to reduce model dependence.

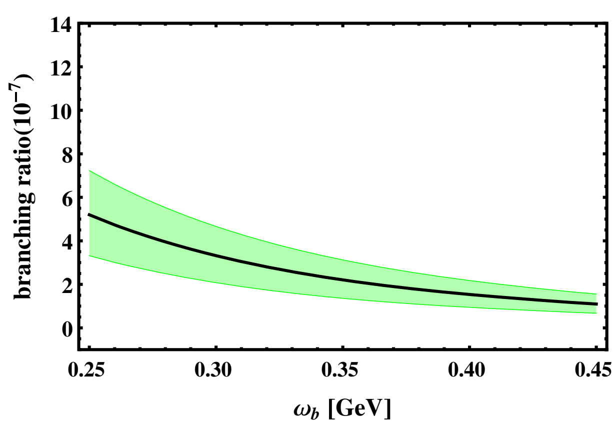

FIG. 7. Dependence of the partial branching fractions $\mathcal{BR}(B \to \gamma\ell\nu, E_\gamma \geq E_{\text{cut}})$ on the first inverse moment $\lambda_B(\mu_0)$ for $E_{\text{cut}} = 1.5$ GeV (blue band) and $E_{\text{cut}} = 2.0$ GeV (green band).

IV. CONCLUSION AND DISCUSSION

The leptonic radiative decay $B \to \gamma \ell \nu$ is believed to be an ideal channel to determine the B meson wave functions, especially the first inverse moment λ_B, which is an important input in the semileptonic and nonleptonic B meson decays. In the study of $B \to \gamma \ell \nu$ decay, the key problem is to investigate the form factors $F_{V,A}(E_\gamma)$. We computed next-to-leading power corrections to the form factors within the framework of PQCD approach, including the power suppressed hard kernel, the contribution from a complete set of three-particle B-meson wave functions up to twist-4 and two-particle off light-cone wave functions, the $1/m_b$ corrections in HQET, and the contribution from the hadronic structure of photon taking advantage of two-particle twist-2 and twist-3 photon LCDAs. In the study of power corrections, PQCD approach has its unique advantage because it is free from endpoint singularity through keeping transverse momentum of parton. Numerically, both the contribution from the higher twist B meson wave functions and the hadronic structure of photon can reduce the leading power result by about 20%, and the power suppressed hard kernel decrease the leading power amplitude over 10%. The overall result is about 50% smaller than leading power result, under the condition that the QCD radiative corrections are not considered. Within the parameter space in this paper, the power correction is so important that one can hardly using the leading power result to reasonably determine the B-meson wave function. After including the power corrections, the integrated branching ratio of $B \to \gamma \ell \nu$ grows with decreasing λ_B, but the rate of change is smaller than the leading power case, in addition to the large theoretical uncertainty, it is difficult to precisely determine λ_B only employing this processes. We should point out that our study is far from a systematic investigation, and more efforts need to be made to uncover the influence of the power corrections. With more and more precise measurements of $B \to \gamma \ell \nu$ decay, the parameters in B meson wave functions must be better constrained.

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