n-$\bar{n}$ oscillations and the neutron lifetime

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Neutron-antineutron oscillations are considered in the light of recently proposed particle models, which claim to resolve the neutron lifetime anomaly, indicating the existence of baryon violating $\Delta B = 1$ interactions. Possible constraints are derived coming from the nonobservation of neutron-antineutron oscillations, which can take place if the dark matter particle produced in neutron decay happens to be a Majorana fermion. It is shown that this can be realized in a simple minimal supersymmetric Standard Model (MSSM) extension where only the baryon number violating term $u^c d^c d^c$ is included while all other R-parity violating terms are prevented to avoid rapid proton decay. It is demonstrated how this scenario can be implemented in a string motivated Grand Unified Theories broken to MSSM by fluxes.

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I. MOTIVATION AND FACTS

Neutrons, together with protons and electrons, are the fundamental constituents of atomic matter and their properties have been studied for almost a century. A free neutron, in particular, disintegrates to a proton, an electron and its corresponding antineutrino, according to the well-known β-decay process $n \to p + e^- + \bar{\nu}_e$. Notwithstanding those well-known facts, the precise lifetime of the neutron remains a riddle wrapped up in an enigma. The problem lies in the fact that the two distinct techniques employed to measure the lifetime end up in a glaring discrepancy [1]. More specifically, in one method a certain number of neutrons are collected in a container [2] (known as a “bottle”), where, after a certain time duration (comparable to the neutron lifetime), several of them decay. The remaining fraction of them can be used to determine the lifetime, which is found to be $\tau_n \approx 879.6 \pm 0.6$ sec. In an alternative way of measuring the lifetime named [3,4] “beam,” a neutron beam with known intensity is directed to an electromagnetic trap. Counting the emerging protons within a certain time interval, it is found that their numbers are consistent with a neutron lifetime of $\tau_n = 888.0 \pm 2.0$ sec. These two measurements display a 4.0σ discrepancy which cannot be attributed to statistical uncertainties. An explanation of this difference of the two measurements could be that other decay channels contribute to the total lifetime in the “beam” case. In the context of the minimal Standard Model, however, there are no available couplings and particles that lead to such a channel and could thus account for this difference. According to a recent proposal [5] the discrepancy could be interpreted if neutrons have a decay channel to a dark matter (DM) candidate particle χ with a branching ratio $\sim 1\%$ and a mass comparable to the neutron’s mass. The simplest possibility is realized with the neutron decay to a two particle final state consisting of a DM fermion χ and a monochromatic photon, $n \to \chi + \gamma$. Operators describing this type of decay, however, violate baryon number. At the microscopic level, the description of the above decay requires the existence of a color scalar field with the quantum numbers of a Standard Model (SM) color triplet, $D = (3,1)_{-1/3}$, with mass $M_D \geq$ 1 TeV and the couplings

$$\mathcal{L}_{D\chi} \supset \lambda_q \bar{u^c}_L d_R D + \lambda_\chi \bar{D} \bar{\chi}\, d_R + m_\chi \bar{\chi}\chi. \tag{1}$$

Two basic assumptions have been made for this scenario to work. Firstly, it is assumed that other baryon violating couplings of the new color triplet, $D = (3,1)_{-1/3}$, are substantially suppressed. Indeed, a color triplet introduces other baryon nonconserving couplings similar to the R-parity violating ($\not{R}$) ones of the supersymmetric theories. Unless their couplings are unnaturally tiny, they lead to fast proton decay at unacceptable rates. In the context of SM, there are no obvious symmetries which prevent their appearance while leaving the terms (1) intact. Secondly, it is assumed that the DM fermion χ is a Dirac particle. Since χ is a neutral field, however, it could be likewise a Majorana particle and, in such a case, might contribute to n-$\bar{n}$ oscillations.

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In this paper, we reconsider the interpretation of the neutron lifetime discrepancy described above, in the context of minimal supersymmetric Standard Model (MSSM) extensions and in particular SUSY and string motivated Grand Unified Theories (GUTs). There are many good reasons to implement the above scenario in this context. We firstly remark that the kind of scalar particle introduced to realize the processes has the quantum numbers of a down quark color triplet. Thus, in the context of MSSM, this could be the scalar component $\tilde{d}_j^c$ of a down quark supermultiplet. There are good chances that the supersymmetry breaking scale is around the TeV scale and the sparticle spectrum may be accessible either at the LHC or its upgrades. Thus, taking into account the recent bounds of LHC experiments, its mass $m_{\tilde{d}_j^c}$ could be around the TeV scale which is adequate to interpret the neutron lifetime discrepancy. Notice, however, that in the MSSM context, terms such as (1) appear together with other baryon and lepton number violating interactions giving rise to fast proton decay, and therefore, they are forbidden by R-symmetry. There are examples of grand unified theories with string origin, however, equipped with symmetries and novel symmetry breaking mechanisms where it is possible to end up with a Lagrangian only with the desired $\not{R}$-coupling and all the others forbidden. Thus, in the presence only of the trilinear coupling shown in (1) which can account for the discrepancy, the only possible baryon violating processes are neutron-antineutron oscillations. Our aim in the present work is to investigate under what conditions the issue of neutron lifetime is solved. In particular, we will examine whether the strength of the couplings and the mass scale required to interpret the discrepancy are consistent with the bounds on n-$\bar{n}$ oscillations.

The layout of the paper is as follows. In Sec. II we present a short overview of gauge invariant baryon and lepton number violating symmetries in the context of GUTs with an emphasis on R-parity violating supersymmetry. In Sec. III we summarize the essential formalism related to neutron-antineutron oscillations and in Sec. IV we present the main results, including bounds of the relevant baryon violating couplings in the TeV scale extracted from the current limits of n-$\bar{n}$ oscillations. Some concluding remarks and a short discussion are presented in Sec. V. Finally, for the reader's convenience, some detailed formulas entering our calculations are given in the Appendix.

II. A BRIEF OVERVIEW OF R-PARITY IN FLUXED GUTs

In the nonsupersymmetric Standard Model, at the renormalizable level, baryon (B) and lepton (L) numbers are conserved quantum numbers, due to accidental global symmetries. This fact is consistent with the observed stability of the proton and the absence of lepton decays (such as $\beta\beta$-decay) which violate B and L. Introducing new colored particles which imply additional interactions, however, this is no longer true.

In the supersymmetric Lagrangian of the Standard Model symmetry, in principle, one could write down gauge invariant terms which violate B and L numbers. In superfield notation these are

$$\mathcal{W}_{\not{R}} \supset \lambda_{ijk} Q_i d_j^c \ell_k + \lambda'_{ijk} \ell_i \ell_j e_k^c + \lambda''_{ijk} u_i^c d_j^c d_k^c + \lambda_{h\ell} h_u \ell_j. \tag{2}$$

If all these couplings were present, for natural values of Yukawas $\lambda_{ijk} \sim \mathcal{O}(10^{-1})$, violation of B and L would occur at unacceptable rates. As is well known, in the minimum supersymmetric Standard Model the adoption of R-symmetry prevents all these terms.

Without the existence of R-symmetry or other possible discrete and $U(1)$ factors, these terms are also present in GUTs. In the minimal $SU(5)$ for example, the most common B and L violating terms arise from the coupling

$$10_f \cdot \bar{5}_f \cdot \bar{5}_f \to Q d^c \ell + e^c \ell \ell + u^c d^c d^c. \tag{3}$$

In a wide class of string motivated GUTs there are cases where some of the terms in (3) are absent in a natural way. In a particular class of such models, where the breaking of the gauge symmetry occurs due to fluxes which are switched on along the dimensions of the compact manifold, we may have for example the following SM decomposition:

$$10_f \cdot \bar{5}_f \cdot \bar{5}_f \to u^c d^c d^c + \text{nothing else}, \tag{4}$$

which is just the operator required to mediate n-$\bar{n}$ oscillations. The absence of the remaining couplings in (2) ensures that the proton remains stable, or its decay occurs at higher orders in perturbation theory and therefore its decay rate is highly suppressed and undetectable from present day experiments.

To be more precise, focusing in $SU(5)$ as a prototype unified theory, the flux mechanism works as follows [6]: assuming that $SU(5)$ chirality has been obtained by fluxes associated with Abelian factors embedded together with $SU(5)$ into a higher symmetry, another flux is introduced along the hypercharge generator $U(1)_Y$ to break $SU(5)_{\mathrm{GUT}}$ [6]. It turns out that this is also responsible for the splitting of the $SU(5)$ representations. If some integers M, N represent these two kinds of fluxes piercing certain "matter curves" of the compact manifold hosting the 10-plets and 5-plets, the following splittings of the corresponding representations occur:

TABLE I. Induced MSSM matter content from fluxed $SU(5)$ representations.

10-plets	Flux units	10 content	5-plets	Flux units	$\bar{5}$ content
10_1	$M_{10}=1, N=0$	(Q, u^c, e^c)	$\bar{5}_1$	$M_5=+1, N=0$	(d^c, ℓ)
10_2	$M_{10}=0, N=+1$	$(-, \bar{u}^c, e^c)$	$\bar{5}_2$	$M_5=0, N=+1$	$(-, \ell)$
10_3	$M_{10}=0, N=-1$	$(-, u^c, \bar{e}^c)$	$\bar{5}_3$	$M_5=0, N=-1$	$(-, \bar{\ell})$
10_4	$M_{10}=1, N=+1$	$(Q, -, 2e^c)$	$\bar{5}_4$	$M_5=+1, N=+1$	$(d^c, 2\ell)$
10_5	$M_{10}=1, N=-1$	$(Q, 2u^c, -)$	$\bar{5}_5$	$M_5=+1, N=-1$	$(d^c, -)$

TABLE II. $SU(5)$-fluxed representations with incomplete MSSM content, and $\not{R}$-processes emerging from the trilinear coupling $10_i\bar{5}_j\bar{5}_j$ for selected combinations of the multiplets given in Table I.

$SU(5)$-term	MSSM content	$\not{R}$-operator(s)	Dominant process
$10_1\bar{5}_1\bar{5}_1$	$(Q, u^c, e^c)(d^c, \ell)^2$	All	Proton decay
$10_1\bar{5}_4\bar{5}_4$	$(Q, u^c, e^c)(d^c, 2\ell)^2$	All	Proton decay
$10_1\bar{5}_3\bar{5}_3$	$(Q, u^c, e^c)(-, \bar{\ell})^2$	None	None
$10_1\bar{5}_2\bar{5}_2$	$(Q, u^c, e^c)(-, \ell)^2$	$\ell\ell e^c$	$\ell_{e,\mu,\tau}$ violation
$10_1\bar{5}_5\bar{5}_5$	$(Q, u^c, e^c)(d^c, -)^2$	$u^c d^c d^c$	n-$\bar{n}$ oscillation
$10_3\bar{5}_5\bar{5}_5$	$(-, u^c, \bar{e}^c)(d^c, -)^2$	$u^c d^c d^c$	n-$\bar{n}$ oscillation

$$\#\underline{\mathbf{10}} - \#\underline{\overline{\mathbf{10}}} \Rightarrow \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} = M_{10} \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} = M_{10} - N \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} = M_{10} + N \end{cases} \tag{5}$$

$$\#\underline{\mathbf{5}} - \#\underline{\bar{\mathbf{5}}} \Rightarrow \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{+\frac{1}{3}}} = M_5 \\ n_{(1,2)_{+\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} = M_5 + N\cdot \end{cases} \tag{6}$$

The integers M_{10}, M_5, N are related to specific choices of the fluxes, and may take any positive or negative value, leading to a different number of SM representations. Hence, there is a variety of possibilities which can be fixed only if certain string boundary conditions have been chosen. In order to exemplify the effect of these choices, here we assume only a few arbitrary cases where the integers M, N take the lower possible values $\pm 1, 0$[1] for the $SU(5)$ representations. Substituting these numbers in (5) and (6) we obtain a variety of possibilities, and some of them are shown in Table I.

Hence, we end up with incomplete $SU(5)$ representations. Some examples of R-parity violating operators formed by trilinear terms involving the above incomplete representations are shown in Table II. (For a comprehensive analysis and a complete list of possibilities see [7].) We observe that the couplings $10_1 \cdot \bar{5}_5 \cdot \bar{5}_5$ and $10_3 \cdot \bar{5}_5 \cdot \bar{5}_5$ in the last two lines of this table give exactly the required R-violating trilinear coupling, while all the other couplings are absent. This is just the case that will be considered in the subsequent analysis.

[1]Of course, larger M, N values are also possible. They may imply different numbers of SM representations on matter curves but will not lead to new types of splittings [7] other than those of Table I.

III. NEUTRON-ANTINEUTRON OSCILLATION FORMALISM

In this section we will briefly present the main features of the n-$\bar{n}$ oscillations mainly to establish notation and put the recently baryon violating scenario, proposed for the extra exotic channel of neutron decay to a light dark matter, in a broader perspective. In this context additional processes entering n-$\bar{n}$ oscillations at tree level or at the one-loop level are presented.

A. Neutron and antineutron bound wave functions

We will consider the neutron as a bound state of three quarks (antiquarks) for the neutron (antineutron), in a color singlet s-state in momentum space. The orbital part is of the form

$$\Psi_{\mathbf{P}0s0s}(\mathbf{Q}, \xi, \eta) = \sqrt{3\sqrt{3}}(2\pi)^{3/2}\delta(\sqrt{3}\mathbf{Q} - \mathbf{P})\phi(\xi)\phi(\eta), \tag{7}$$

where $\mathbf{P}$ is the hadron momentum and

$$\mathbf{Q} = \frac{1}{\sqrt{3}}(\mathbf{p_1} + \mathbf{p_2} + \mathbf{p_3}), \qquad \eta = \frac{\mathbf{1}}{\sqrt{\mathbf{6}}}(\mathbf{p_1} + \mathbf{p_2} - \mathbf{2p_3}),$$
$$\xi = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}(\mathbf{p_1} - \mathbf{p_2}), \tag{8}$$

with $\mathbf{p_i}, \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}$ the quark momenta. The functions $\phi(\xi), \phi(\eta)$ are assumed to be $0s$ harmonic oscillator wave functions. These functions are assumed to be normalized in the usual way:

$$\begin{aligned}&\langle\Psi_{P,0s,0s}|\Psi_{P'0s0s}\rangle\\&=(2\pi)^3(3\sqrt{3})\int d^3\mathbf{Q}\delta(\sqrt{3}\mathbf{Q}-\mathbf{P})\delta(\sqrt{3}\mathbf{Q}-\mathbf{P}')\\&\int d^3\xi|\phi(\xi)|^2\int d^3\eta|\phi(\eta)|^2\\&=(2\pi)^3\delta(\mathbf{P}-\mathbf{P}')\cdot\end{aligned}$$

B. Neutron-antineutron transition mediated by dark matter Majorana fermion

A dark matter colorless Majorana fermion of mass m_χ emitted from a neutron of momentum P and absorbed by an antineutron of momentum P' can lead to n-$\bar{n}$ oscillations. This process is exhibited in Fig. 1(a). In a previous study [5] this did not happen, since the mediating fermion was assumed to be a Dirac-like particle, but there is no reason to restrict in this choice. In fact there exists the possibility of this particle being a Majorana-like fermion in which case neutron-antineutron oscillations become possible.

The orbital matrix element takes the form

$$\begin{aligned}ME=&(2\pi)^3(3\sqrt{3})\left(\frac{g_q g_\chi}{m_D^2}\right)^2\frac{1}{m_\chi}\int d^3\mathbf{Q}\\&\int d^3\mathbf{Q}'\delta(\sqrt{3}\mathbf{Q}-\mathbf{P})\delta(\sqrt{3}\mathbf{Q}'-\mathbf{P}')\delta(\sqrt{3}(\mathbf{Q}-\mathbf{Q}')\\&\int d^3\xi\int d^3\eta\int d^3\xi'\int d^3\eta'\phi(\xi)\phi(\eta)\phi(\xi')\phi(\eta')\cdot\end{aligned}\tag{9}$$

Using (9) the matrix element can be written as follows:

$$\begin{aligned}&ME=(2\pi)^3\delta(\mathbf{P}-\mathbf{P}')\mathcal{M}_{\mathrm{orbital}},\\&\mathcal{M}_{\mathrm{orbital}}=\frac{1}{3\sqrt{3}}\left(\frac{g_q g_\chi}{m_D^2}\right)^2\frac{1}{m_\chi}\int d^3\xi\int d^3\eta\int d^3\xi'\\&\qquad\int d^3\eta'\phi(\xi)\phi(\eta)\phi(\xi')\phi(\eta')\cdot\end{aligned}\tag{10}$$

Now the $0s$ wave function is

$$\phi(\mathbf{x})=\left(\frac{b_N}{\sqrt{\pi}}\right)^{3/2}e^{-\frac{b_N^2x^2}{2}},\qquad \mathbf{x}=\xi,\eta,\xi',\eta'.$$

Thus, performing the Gaussian integral, we get

$$\begin{aligned}I&=\int d^3\mathbf{x}\phi(\mathbf{x})=\left(\frac{b_N}{\sqrt{\pi}}\right)^{3/2}4\pi\int_0^\infty dx x^2e^{-\frac{b_N^2x^2}{2}}\\&=2\sqrt{2}\left(\frac{\sqrt{\pi}}{b_N}\right)^{3/2},\end{aligned}$$

and the orbital part becomes

$$\mathcal{M}_{\mathrm{orb}}=\frac{64}{3\sqrt{3}}\pi^3\left(\frac{\lambda_q\lambda_\chi}{m_D^2}\right)^2\frac{1}{m_\chi}\left(\frac{1}{b_N}\right)^6\cdot\tag{11}$$

It is instructive to compare this with the probability for finding the quark at the origin inside the nucleon:

$$|\psi(0)|^2=\frac{1}{\pi\sqrt{\pi}}\frac{1}{b_N^3}.$$

Then

$$\mathcal{M}_{\mathrm{orb}}=\frac{64}{3\sqrt{3}}\pi^6\left(\frac{\lambda_q\lambda_\chi}{m_D^2}\right)^2\frac{1}{m_\chi}|\psi(0)|^4\cdot\tag{12}$$

The color factor is quite simple since it involves the same hadron. It takes the form

$$\sum_\alpha(ud)_S(0,1)_{-\alpha}(-1)^{\phi_\alpha}d(1,0)\alpha=\sqrt{3}(0,0),\tag{13}$$

where $(ud)_S(0,1)_{-\alpha}$ is the flavor symmetric color antisymmetric two quark state; ϕ_α the conjugation phase; and (0, 0) is the color singlet hadronic state. Thus

$$\mathcal{M}_{\mathrm{colour}}=3,\tag{14}$$

$$\mathcal{M}_{\mathrm{DM}}=\frac{64}{\sqrt{3}}\pi^3\frac{1}{2}\left(\frac{\lambda_q\lambda_\chi}{m_D^2}\right)^2\frac{1}{m_\chi}\left(\frac{1}{b_N}\right)^6\cdot\tag{15}$$

The factor of $\frac{1}{2}$ came from chirality since the propagating fermion is only left handed.

In the case of supersymmetry induced oscillation, Fig. 1(b), we find an analogous expression. The orbital part is similar to the previous one with the obvious modifications $m_D\to m_{\tilde{d}}$, $\lambda_q\lambda_\chi\to c_{bd}\lambda''_{ub\tilde{d}}g/2$. Here, c_{bd} is the flavor violating mixing between the scalars $\tilde{d}$ and $\tilde{b^c}$, which induces baryon violation. Thus

$$\mathcal{M}_{\mathrm{orbital}}=\frac{64}{3\sqrt{3}}\pi^3\left(c_{b\tilde{d}}\frac{\lambda''_{ud\tilde{b}}g/2}{m_{\tilde{d}}^2}\right)^2\frac{1}{m_{\tilde{W}_3}}\left(\frac{1}{b_N}\right)^6\cdot\tag{16}$$

Including the color helicity factors we set

$$\mathcal{M}_{\mathrm{SUSYDM}}=\frac{64}{\sqrt{3}}\pi^3\frac{1}{2}\left(c_{b\tilde{d}}\frac{\lambda''_{ud\tilde{b}}g/2}{m_{\tilde{d}}^2}\right)^2\frac{1}{m_{\tilde{W}_3}}\left(\frac{1}{b_N}\right)^6\cdot\tag{17}$$

Estimates of the final expression will be given in the subsequent section. For the time being we mention that the parameters involved in (21) should respect bounds coming from other rare processes. In several MSSM extensions experimental bounds on B_d^0 mixing constrain the factor c_{bd} although in a model-dependent way. Soft supersymmetry breaking terms, for example, contribute to the parameter $\delta\approx\frac{\mu\langle H\rangle}{m_{\tilde{d}}^2}V_d\lambda_dV_d^\dagger$.

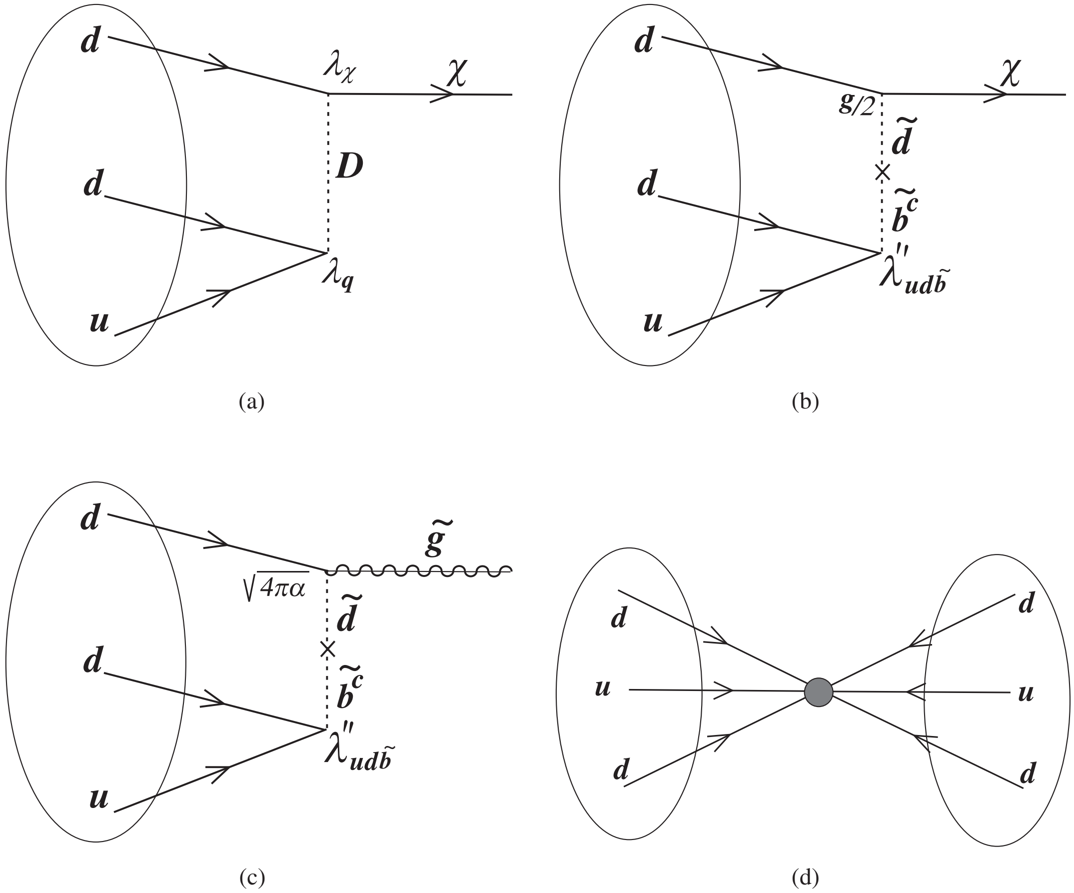

FIG. 1. (a) A dark matter colorless Majorana fermion emitted from a neutron. (b) A dark matter particle χ emitted in the context of R-parity violating supersymmetry, e.g., the gaugino $\tilde{W}_3$ with coupling $g/2$, $\tilde{B}$ with coupling $g'/2$, or any of the two Higgsinos with more complicated couplings. (c) A gluino, emitted from a neutron in R-parity violating supersymmetry. The emitted Majorana fermion propagates and it can get absorbed by an antineutron leading to n-$\bar{n}$ oscillations. (d) Such an oscillation can also be induced by a box diagram leading to a contact interaction.

C. Additional neutron-antineutron mechanisms at tree level

n-$\bar{n}$ oscillations with gluino exchange take place at tree level; see Fig. 2. This is directly comparable with n-decay process through DM particle χ. However, because of the color antisymmetry, the coupling $u^c d^c \tilde{d}^c$ cannot be realized directly and it requires mass insertion; thus a suppression factor emerges due to assumed mixing between $\tilde{b}^c$, $\tilde{s}^c$ and

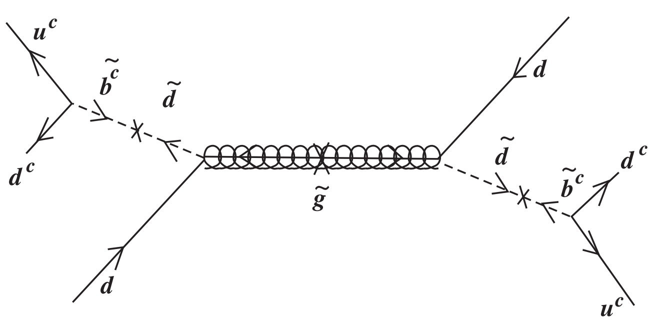

FIG. 2. n-$\bar{n}$ oscillations with gluino exchange take place at tree level.

their left components $\tilde{b}$, $\tilde{s}$. (This requirement is beyond the minimal flavor violation scenario which assumes a diagonal mass matrix.[2])

It is known that a dinucleon decays to two Kaons, $NN \to KK$, imposing stringent constraints on the coupling $\lambda''_{ud\tilde{s}}$. Hence, we will focus only on $\lambda''_{ud\tilde{b}}$ which becomes more relevant for neutron-antineutron oscillations.

The gluino exchange diagram of Fig. 1(c) (see also Fig. 2), differs from that of Fig. 1(b) in the sense that the gluino is a color octet and interacts strongly. Thus

$$\mathcal{M}_{\mathrm{orbital}} = \frac{64}{3\sqrt{3}}\pi^3 \left(c_{b\tilde{d}} \frac{\lambda''_{ud\tilde{b}}\sqrt{4\pi\alpha_s}}{m_{\tilde{d}}^2} \right)^2 \frac{1}{m_{\tilde{g}}} \left(\frac{1}{b_N}\right)^6 \cdot \quad (18)$$

The color factor is a bit more complicated. We encounter the combination

$$\sum_{\alpha,\beta,\gamma} (ud)_S (01)_\alpha d_\beta \tilde{g}_\gamma c_{\alpha,\beta,\gamma}, \quad (19)$$

[2] See for example [8,9] and references therein.

with a similar combination on the other hadron. The states are specified as follows:

$$\alpha = 1 \Leftrightarrow |(0,1) - 2,0,0\rangle, \qquad \alpha = 2 \Leftrightarrow |(0,1)1,\frac{1}{2},-\frac{1}{2}\rangle, \qquad \alpha = 3 \Leftrightarrow |(0,1)1,\frac{1}{2},\frac{1}{2}\rangle$$

$$\beta = 1 \Leftrightarrow |(1,0)2,0,0\rangle, \qquad \beta = 2 \Leftrightarrow |(1,0) - 1,\frac{1}{2},\frac{1}{2}\rangle, \qquad \beta = 3 \Leftrightarrow |(1,0)1,\frac{1}{2},-\frac{1}{2}\rangle$$

$$\gamma = 1 \Leftrightarrow |(1,1)3,\frac{1}{2},\frac{1}{2}\rangle, \qquad \gamma = 2 \Leftrightarrow |(1,1)3,\frac{1}{2},-\frac{1}{2}\rangle, \qquad \gamma = 3 \Leftrightarrow |(1,1)0,1,1\rangle, \qquad \gamma = 4 \Leftrightarrow |(1,1)0,1,0\rangle$$

$$\gamma = 5 \Leftrightarrow |(1,1)0,1,-1\rangle, \qquad \gamma = 6 \Leftrightarrow |(1,1)0,0,0\rangle, \qquad \gamma = 7 \Leftrightarrow |(1,1) - 3,\frac{1}{2},\frac{1}{2}\rangle, \qquad \gamma = 8 \Leftrightarrow |(1,1) - 3,\frac{1}{2},\frac{1}{2}\rangle,$$

in the standard SU(3) labeling of the states [10] $|(\lambda,\mu)\epsilon,\Lambda,\Lambda_0\rangle$.

The symmetry coefficients allowed by SU(3) can be easily calculated from the tables involving the reduction $(01)\otimes(10)\rightarrow(11)$; see Table 1 of Ref. [11]. The obtained results are presented in Table III. Expanding the hadronic states in terms of an antisymmetric pair of quarks and a single quark we find

$$\mathcal{M}_{\text{colour}} = -\frac{1}{\sqrt{3}}\sum_{\alpha,\beta,\gamma} 3(c_{\alpha,\beta,\gamma})^2\left(-\frac{1}{\sqrt{3}}\right) = 8, \tag{20}$$

and thus, we get

$$\mathcal{M}_{\text{gluino}} = \frac{512}{3\sqrt{3}}\pi^3\frac{1}{2}\left(c_{b\tilde{d}}\frac{\lambda''_{ud\tilde{b}}\sqrt{4\pi\alpha_s}}{m^2_{\tilde{d}}}\right)^6. \tag{21}$$

Estimates for the final expression will be given in the subsequent section. For the time being we mention that the parameters involved in (21) should respect bounds coming from other rare processes. In several MSSM extensions experimental bounds on B^0_d mixing constrain the factor $c_{b\tilde{d}}$ although in a model-dependent way. Soft supersymmetry breaking terms proportional to the trilinear parameter A and the μ-term contribute to $c_{b\tilde{d}}$ [see Eq. (35) of the Appendix]. The μ parameter generates a contribution proportional to the parameter $\delta_{ij}\approx\frac{\mu\langle H\rangle}{m^2_{\tilde{d}}}(V_d\lambda_d V^\dagger_d)_{ij}$ and a similar one comes for A.[3] These imply a value $c_{b\tilde{d}}\sim 10^{-3}-10^{-4}(\frac{1\ \text{TeV}}{m_{\tilde{d}}})^2$ comparable to experimental bounds, if $m_{\tilde{d}}$ is in the TeV range.

TABLE III. The nonvanishing coefficients $c_{\alpha,\beta,\gamma}$ allowed by the SU(3) symmetry. For notation see text.

α	β	γ	$c_{\alpha,\beta,\gamma}$
3	1	1	1
2	1	2	1
3	2	3	1
2	1	2	1
3	3	4	$\frac{1}{\sqrt{2}}$
2	2	5	$-\frac{1}{\sqrt{2}}$
1	1	6	$\frac{\sqrt{2}}{\sqrt{3}}$
3	3	6	$-\frac{1}{\sqrt{6}}$
2	2	6	$\frac{1}{\sqrt{6}}$
1	2	8	1
1	3	8	1

D. Neutron-antineutron transition mediated by box diagrams

In this case there is no need to have flavor off-diagonal baryon violating interactions, a mixing between the scalars $\tilde{b}$ and $\tilde{b}^c$ is adequate. The generation mixing can be induced as in the Standard Model via the wino and the W-boson in a box diagram. In this case the interaction between the neutron and antineutron does not take the simple form found above at tree level. Since, however, the $\tilde{b}$-scalars are quite heavy, it leads to a contact interaction; see Fig. 1(d). Since no color particle propagates between the two hadrons the color factor is 3 and the orbital part can be written in the form

$$\mathcal{M}_{\text{orbital}} = \frac{64}{3\sqrt{3}}\pi^3(c_{b\tilde{b}}\lambda''_{ud\tilde{b}})^2\frac{1}{m^4_{\tilde{d}}}s_{\text{box}}\frac{1}{m_{\tilde{W}}}\left(\frac{1}{b_N}\right)^6, \tag{22}$$

where s_{box} is dependent on the masses of the particles circulating in the loop, namely the W-boson, the top quark, the wino and the $\tilde{u}$ scalars. Thus

$$\mathcal{M}_{\text{box}} = \frac{64}{3\sqrt{3}}\pi^3\frac{3}{2}(c_{b\tilde{b}}\lambda''_{ud\tilde{b}})^2\frac{1}{m^4_{\tilde{d}}}g^4 s_{\text{box}}\frac{1}{m_{\tilde{W}}}\left(\frac{1}{b_N}\right)^6, \tag{23}$$

where $g^2 = 4\sqrt{2}G_F m^2_W \approx 0.4$ and s_{box} will be evaluated in the Appendix.

IV. NEUTRON-ANTINEUTRON OSCILLATION RESULTS

Combining the two cases, namely the nonsupersymmetric dark matter and the corresponding supersymmetric

[3] See for example [12] and [13] as well as references therein.

processes, we find that the transition amplitude takes the form

$$\mathcal{M} = m_n\kappa, \kappa = \frac{64}{3\sqrt{3}}\pi^3\frac{1}{2}\left[3\left(\left(\frac{\lambda_q\lambda_\chi}{b_N^2 2m_D^2}\right)^2\frac{1}{b_N^2 m_n m_\chi}\right) + 3\left(\frac{c_{b\tilde{d}}\lambda''_{ud\tilde{b}}g/2}{b_N^2 m_{\tilde{d}}^2}\right)^2\frac{1}{b_N^2 m_n m_{\tilde{W}_3}} + 8\left(\frac{c_{b\tilde{d}}\lambda''_{ud\tilde{b}}\sqrt{4\pi\alpha_s}}{b_N^2 m_{\tilde{d}}^2}\right)^2\frac{1}{b_N^2 m_n m_{\tilde{g}}} + 3\left(\frac{c_{b\tilde{b}}\lambda''_{ud\tilde{b}}}{b_N^2 m_{\tilde{d}}^2}\right)^2\frac{g^4 s_{\text{box}}}{b_N^2 m_n m_{\tilde{W}}} + \cdots\right] \tag{24}$$

where $s_{\text{box}} \approx 3.0\times 10^{-6}$; see the Appendix. Due to this factor as well as the small mixing $c_{b\tilde{b}}$, the parameter $\lambda''_{ud\tilde{b}}$ need not be extremely small. Notice also that a graph involving the bino will give a contribution similar to the second term. Analogous graphs involving Higgsinos are also possible, but they provide no new insights and will not be elaborated.

It is now natural to assume that the mass of the propagating scalar is the same in all models. If constraints come from other experiments we will compensate by adjusting the relevant couplings. Then we can take the scale of the masses to be of the order 1 TeV.[4] Another parameter to be determined is the nucleon size parameter which is usually taken to be 0.8 fm. This is related to the nucleon wave function at the origin:

$$\psi(0)^2 = \frac{1}{\pi\sqrt{\pi}}\frac{1}{b_N^3}.$$

In Ref. [5] the value of $\psi(0)^2 = 0.014\ \text{GeV}^3$ was adopted taking into account effects arising from lattice gauge calculations [15]. This leads to a value of about 0.5 fm. We will adopt this value in the present calculation. Thus we can write κ in the form

$$\kappa = \kappa_0\kappa_1, \qquad \kappa_0 = 4.0\times 10^{-15}, \tag{25}$$

with

$$\kappa_1 = \left[3(\lambda_q\lambda_\chi)^2\frac{m_n}{m_\chi} + 3(c_{b\tilde{d}}\lambda''_{ud\tilde{b}}g/2)^2\frac{m_n}{m_{\tilde{W}_3}} + 8(c_{b\tilde{d}}\lambda''_{ud\tilde{b}}\sqrt{4\pi\alpha_s})^2\frac{m_n}{m_{\tilde{g}}} + 3(c_{b\tilde{b}}\lambda''_{ud\tilde{b}})^2 g^4 s_{\text{box}}\frac{m_n}{m_{\tilde{W}}}\right]. \tag{26}$$

[4]Recent results from LHC experiments [14] considering simplified models with only first- and second-generation squarks give a lower squark mass bound $m_{\tilde{q}} \sim 1.5$ TeV. Implementing this bound will result in a small suppression of (24); this, however, will not alter the conclusion of our analysis.

The n-$\bar{n}$ mixing matrix becomes

$$m_n\begin{pmatrix}1 & \kappa\\ \kappa & 1\end{pmatrix},$$

which leads to complete mixing with energies $E_1 = m_n(1+\kappa)$, $E_2 = m_n(1-\kappa)$. Thus the neutron-antineutron oscillation probability in vacuum becomes

$$P(n\leftrightarrow\bar{n}) = \frac{1}{2}|e^{iE_1 t} - e^{-iE_2 t}|^2 = \sin^2(m_n\kappa t). \tag{27}$$

In other words the oscillation time is

$$\tau = \frac{1}{m_n\kappa} \approx \frac{7\times 10^{-24}}{\kappa}\ \text{s} \approx \frac{1.8\times 10^{-9}}{\kappa_1}\ \text{s}. \tag{28}$$

In the presence of matter the diagonal elements of the matrix are not the same, since the neutron and the antineutron interact differently with any surrounding magnetic field or matter. A tiny magnetic field of the order of 10^{-10} T can lead to an energy difference of $\sim 10^{-26}m_n$. In current experiments the magnetic fields are limited [16] to 10^{-8} T, which leads to $\delta \approx 10^{-24}$. Thus the oscillation probability becomes [16]

$$P(n\leftrightarrow\bar{n}) = \frac{(\kappa/\delta)^2}{1+(\kappa/\delta)^2}\sin^2(m_n\sqrt{\delta^2+\kappa^2}t)e^{-\lambda t} \approx \frac{\kappa^2}{\delta^2}\sin^2(m_n\sqrt{\delta^2+\kappa^2}t)e^{-\lambda t}, \tag{29}$$

where $\lambda = 1/\tau_n$ with τ_n the neutron lifetime, as, e.g., measured in the "bottle" experiment mentioned in the Introduction. The nonobservation of neutrino oscillations implies $\kappa \ll \delta$, $\kappa < 10^{-9}\delta = 10^{-33}$, $\tau > 10^8$ s.

In the dark matter mediated process the value of $\kappa_1 = 3(6.7\times 10^{-6})^2 \approx 1.3\times 10^{-10}$ was employed [5]. Thus

$$\tau \approx 1.3\times 10^1\ \text{s}, \qquad \kappa \approx 5\times 10^{-25}. \tag{30}$$

This is in conflict with the neutron oscillation data.

In the context of the R-parity violating supersymmetry model [8] we will try to extract a limit on the value of $c_{b\tilde{d}}\lambda_{ud\tilde{b}}$ in the case of the tree diagrams and the $c_{b\tilde{b}}\lambda_{ud\tilde{b}}$ in the case of the box diagram from the nonobservation of n-$\bar{n}$ oscillation, i.e., to solve the relations

$$\left[3(c_{b\tilde{d}}\lambda_{ud\tilde{b}}g/2)^2\frac{m_n}{m_{\tilde{W}_3}} + 8(c_{b\tilde{d}}\lambda_{ud\tilde{b}}\sqrt{4\pi\alpha_s})^2\frac{m_n}{m_{\tilde{g}}} 3(c_{b\tilde{d}}\lambda_{ud\tilde{b}}\sqrt{4\pi\alpha})^2\frac{m_n}{m_{\tilde{W}}} + 3(c_{b\tilde{b}}\lambda_{ud\tilde{b}})^2 g^4 s_{\text{box}}\frac{m_n}{m_{\tilde{W}}}\right] \le 10^{-33}/\kappa_0 = 2.5\times 10^{-17}. \tag{31}$$

We will consider each case separately:

(i) Gluino exchange. Taking $\alpha_s = 1$ and $m_{\tilde{g}} = 500$ GeV we obtain

$$|c_{b\tilde{d}}\lambda_{ud\tilde{b}}| \lesssim 1.2 \times 10^{-8}. \tag{32}$$

(ii) A SUSY dark matter particle ($\tilde{W}_3$ or $\tilde{B}$) exchange [see Figs. 1(d) and 1(b)]. Taking $m_{\tilde{W}_3} = 500$ GeV we obtain

$$|c_{b\tilde{d}}\lambda_{ud\tilde{b}}| \lesssim 2.0 \times 10^{-7}. \tag{33}$$

(iii) Finally in the case of the box diagram taking $s_{\mathrm{box}} = 3.0 \times 10^{-6}$ and $m_{\tilde{W}} = 500$ GeV we obtain a weaker upper bound of the order

$$|c_{b\tilde{b}}\lambda_{ud\tilde{b}}| \lesssim 10^{-4}. \tag{34}$$

V. DISCUSSION

In a recent paper [5] a very interesting proposal was made to resolve the long-standing discrepancy on the determination of the neutron lifetime measured in experiments involving trapped neutrons in a "bottle" and neutrons decaying in flight ("beam" experiments). This model considers novel mechanisms for neutron decays involving new dark decay channels in the bottle case, where the decay products contain light dark matter particles, with mass in a slim range between the neutron and proton mass. The final state of this reaction might also involve visible particles such as photons. These scenarios sparked off a renewed activity on this issue and astrophysical as well as experimental constraints on the various decay modes have been discussed. Hence, in a recent analysis [17] decay channels involving a light dark matter particle and a visible photon were ruled out, while decays involving dark photons are subject to stringent constraints from astrophysical observations [18]. Furthermore, it has been suggested [19] (see also [20]) that neutron decay to dark matter is in conflict with neutron stars, but the argument does not involve free neutrons.

In the present work, we have explored two different aspects of this proposal, namely the implications on baryon number violation and the possible Majorana nature of the emitted dark matter light particle.

We firstly focused on the fact that this decay process is realized with the mediation of color triplets. In the context of the Standard Model and its obvious supersymmetric extensions, such particles generate other dangerous baryon and lepton number violating interactions, unless their coupling strengths to ordinary matter are unnaturally small. We have suggested that this problem can be remedied in the case of a class of SUSY GUTs derived in the framework of string theories where "fluxes" developed along the compact dimensions are capable of eliminating the superpotential terms associated with the undesired interactions.

Furthermore, we have considered the possibility that the neutral dark matter particle in the putative exotic neutron decay channel is of Majorana type. In this case we find, however, that the parameters employed in this model are in conflict with the neutron-antineutron oscillation limits. We have considered limits from such baryon number violating processes in the context of R-parity violating supersymmetry, both at tree as well as at the one-loop order. We find the most stringent limit on the parameter $|c_{b\tilde{d}}\lambda_{ud\tilde{b}}| \leq 1.2 \times 10^{-8}$ comes from gluino exchange. The weaker limit of $|c_{b\tilde{b}}\lambda_{ud\tilde{b}}| \lesssim 10^{-4}$ comes from the box diagram. The difference can be attributed to the fact that the tree diagrams involve both baryon and family flavor change of the participating s-quarks, while the loop diagram is diagonal in flavor.

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Note added.—We wish to emphasize that the masses of the supersymmetric particles such as squarks and gauginos used in the analysis are somewhat lower than the experimental bounds reported recently by the LHC collaboration (see for example [14]). However, the essential results of the present analysis do not change.

APPENDIX: THE BOX CONTRIBUTION

For the nonexpert reader we provide some details regarding the evaluation of the box diagram contribution.

The gluino exchange diagram requires nonminimal mixing which might not be present in simple supersymmetric models. Hence, we assume the case where $\tilde{b}_L, \tilde{b}_R$ have a nontrivial mixing term

$$m^2_{\tilde{b}_{L,R}} = m_b A_{\mathrm{eff}}, \tag{A1}$$

where $A_{\mathrm{eff}} = A - \mu \tan\beta$, A being a soft SUSY breaking parameter, μ the Higgs mixing (μ-term) and $\tan\beta$ the Higgs

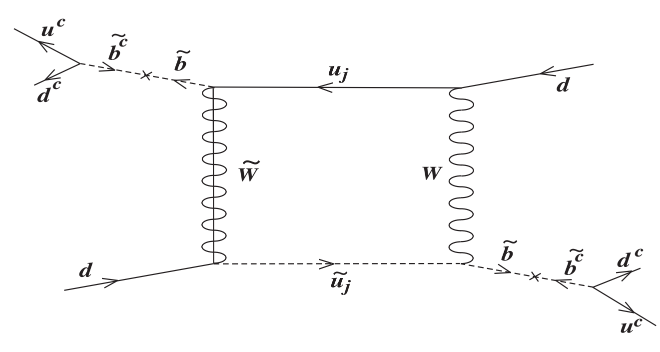

FIG. 3. $n - \bar{n}$ oscillations from box contributions.

vaccum expectation value ratio. Similar terms can exist for the other two families. As a result, the process receives contributions from one-loop box graphs involving Winos. This is depicted in Fig. 3. The possible R-parity violating terms contributing to the process are $\lambda''_{ud\tilde{b}} u^c d^c b^c$ and $\lambda''_{ud\tilde{s}} u^c d^c s^c$. Only $\lambda''_{ud\tilde{b}}$ is shown in the figure since, as explained above, $\lambda''_{ud\tilde{s}}$ is suppressed. Moreover, due to the larger b-quark mass m_b compared to m_s, factors such as m_b^2/m_W^2 enhance the effect.

The processes require the sequence of reactions: initially $d_R u_R + d_L \to d_L + \tilde{b}_R^*$ followed by $d_L + \tilde{b}_L^* \to \tilde{b}_L + \bar{d}_L$, from the W-boson and wino exchange box diagram. At the final stage we get $\bar{d}_L + \bar{d}_R \bar{u}_R$.

Calculation of the diagram gives the following relation for the decay rate [8],

$$\Gamma = -\frac{(\lambda''_{ud\tilde{b}})^2 g^4 m^4_{\tilde{b}_{LR}} m_{\tilde{w}}}{32\pi^2 (m_{\tilde{b}_L} m_{\tilde{b}_R})^4} |\psi(0)|^4 \sum_{j,k=1}^{3} \xi_{jk} \Omega(m^2_{\tilde{w}}, m^2_W, m^2_{u_j}, m^2_{\tilde{u}_k}), \tag{A2}$$

with $m^2_{\tilde{b}_{LR}}$ given by (A1) and ξ_{jk} being the following combination of CKM matrix parameters:

$$\xi_{jk} = V_{bj} V^\dagger_{jd} V_{bk} V^\dagger_{kd}. \tag{A3}$$

The computation of the loop integral in (A2) is parametrized by the function Ω which depends on the four masses circulating in the box and is given by

$$\Omega(m_1, m_2, m_3, m_4) = \sum_{j=1}^{4} \frac{m_j^4 \ln(m_j^2)}{\prod_{k\neq j}(m_j^2 - m_k^2)}. \tag{A4}$$

The current experimental lower bound on n-$\bar{n}$ oscillation period $\tau = \frac{1}{\Gamma}$ is $\tau \gtrsim 10^8$ s [16] (see Sec. IV).

In our notation $|\psi(0)|^2$ is the baryonic wave function matrix element for three quarks inside a nucleon estimated [5,15] to be $|\psi(0)|^2 = 0.014\ \mathrm{GeV}^3$. From Eq. (A2) we can recalculate the bounds on $\lambda''_{udb\tilde{b}}$ coupling using the latest LHC bounds on scalar masses involved in the box graph. However, knowing only the lower bounds on this large number of arbitrary mass parameters through this complicated formula is not very illuminating. Thus, before going to the most general case, in order to reduce the number of arbitrary mass parameters, and have a feeling of the contributions of the various components, we first examine the limit $m_{\tilde{u}} \to m_{\tilde{W}}$ and $m_{u,c} \ll m_{\tilde{u}}$. Then the various contributions of the integral become simpler. In particular, those involving only the CKM mixing of the first two generations are simplified as follows:

$$\Omega_{ij} = \frac{m^2_{\tilde{u}} - m^2_W - m^2_W \log(\frac{m^2_{\tilde{u}}}{m^2_W})}{(m^2_{\tilde{u}} - m^2_W)^2} \approx \mathcal{O}\left(\frac{1}{2m^2_W}\right), \qquad i,j = 1,2. \tag{A5}$$

The remaining contributions become

$$\begin{aligned}
\Omega_{i3} &= \frac{m_t^2 \log(m_t^2)}{(m_t^2 - m^2_{\tilde{u}})(m_t^2 - m^2_W)} + \frac{m^2_W \log(m^2_W)}{(m^2_W - m^2_t)(m^2_W - m^2_{\tilde{u}})} + \frac{m^2_{\tilde{u}} \log(m^2_{\tilde{u}})}{(m^2_t - m^2_{\tilde{u}})(m^2_W - m^2_{\tilde{u}})}\\
\Omega_{3j} &= \frac{m_t^4 \log(m_t^2)}{(m_t^2 - m^2_W)(m_t^2 - m^2_{\tilde{u}})^2} + \frac{m^4_{\tilde{u}}}{(m^2_t - m^2_{\tilde{u}})^2(m^2_{\tilde{u}} - m^2_W)}\left(1 + \frac{m^2_W \log(m^2_{\tilde{u}})}{m^2_W - m^2_{\tilde{u}}}\right)\\
&\quad + \frac{m^4_W \log(m^2_W)}{(m^2_W - m^2_t)(m^2_W - m^2_{\tilde{u}})^2} + \frac{m_t^2 m^2_{\tilde{u}}(m^2_W(2\log(m^2_{\tilde{u}}) + 1) - m^2_{\tilde{u}}(\log(m^2_{\tilde{u}}) + 1))}{(m^2_t - m^2_{\tilde{u}})^2 (m^2_W - m^2_{\tilde{u}})^2}\\
\Omega_{33} &= \frac{m^4_{\tilde{t}} \log(m^2_{\tilde{t}})}{(m^2_{\tilde{t}} - m^2_t)(m^2_{\tilde{t}} - m^2_{\tilde{u}})(m^2_{\tilde{t}} - m^2_W)} + \frac{m^4_W \log(m^2_W)}{(m^2_W - m^2_t)(m^2_W - m^2_{\tilde{t}})(m^2_W - m^2_{\tilde{u}})}\\
&\quad + \frac{m^4_{\tilde{u}} \log(m^2_{\tilde{u}})}{(m^2_{\tilde{u}} - m^2_t)(m^2_{\tilde{u}} - m^2_{\tilde{t}})(m^2_{\tilde{u}} - m^2_W)} + \frac{m^4_t \log(m^2_t)}{(m^2_t - m^2_{\tilde{t}})(m^2_t - m^2_W)(m^2_t - m^2_{\tilde{u}})}.
\end{aligned} \tag{A6}$$

We observe that in this simplified limiting case, where all scalar masses are taken equal, the contributions (A5) associated with the mixing parameters $\xi_{ij}, i, j = 1, 2$ (where here i, j are generation indices), have a very simple dependence on the boson mass m_W. Contributions involving the third family are given by (A6).

Notice that the CKM elements multiplying the above contributions are of the same order $\xi \equiv |\sum_{i,j=1}^{2} \xi_{ij}| \approx |\sum_{j=1}^{2} \xi_{3j}| \approx |\xi_{33}| \sim 0.75 \times 10^{-4}$. Focusing firstly on the contribution (A5) of the two lighter generations, we observe that the dependence on the unknown scalar SUSY masses is rather simple, and only ratios of these are involved. Then, a rough estimate from contributions coming only from (A5) gives

$$\frac{1}{\tau} = \Gamma \approx -\frac{(\lambda''_{ud\tilde{b}})^2 g^4 m^4_{\tilde{b}_{LR}} m_{\tilde{w}}}{32\pi^2 (m_{\tilde{b}_L} m_{\tilde{b}_R})^4} \frac{|\psi(0)|^4}{2m^2_W} \xi. \tag{A7}$$

We assume equal s-bottom masses $m_{\tilde{b}} = m_{\tilde{b}_R} \approx m_{\tilde{b}_R}$ and define the ratio

$$c_{b\tilde{b}} \approx m^2_{\tilde{b}_{LR}}/m^2_{\tilde{b}_R}. \tag{A8}$$

Then, we can turn the above expression into an upper bound for the product $\lambda''_{ud\tilde{b}} c_{b\tilde{b}}$

$$\lambda''_{ud\tilde{b}} c_{b\tilde{b}} \le \frac{8\pi m^2_{\tilde{b}} m_W}{g^2(\tau m_{\tilde{W}}\xi)^{1/2}} \frac{1}{|\psi(0)|^2}. \tag{A9}$$

From the lower bound $\tau_{n-\bar{n}} \gtrsim 10^8$ s in free neutron oscillation experiments, for the $\bar{n}$ annihilation in matter we can obtain a bound $\tau_m = \frac{1}{\Gamma_m} > 1.6 \times 10^{31}$ yr [16]. Assuming the scalar masses to be of the order $m_{\tilde{b}} \sim 1$ TeV and taking $m_{\tilde{W}} = 400$ GeV, we obtain

$$\lambda''_{ud\tilde{b}} c_{b\tilde{b}} \lesssim 0.5 \times 10^{-3}. \tag{A10}$$

The remaining contributions (A6) display a complicated dependence on the SUSY scalar masses, but for masses close to the experimental lower bounds, they can be of the same order. In such cases, depending on the signs of the CKM mixing parameters ξ_{jk} there might be cancellations which result in weaker bounds on the $\lambda''_{ud\tilde{b}}$ couplings.

To examine this general case, we use Eq. (A2) to recalculate the bounds on $\lambda''_{ud\tilde{b}}$ taking into account the latest experimental results for the SUSY mass parameters. We take $A_{\rm eff} = 400$ GeV which fixes the ratio (A8) to be $c_{b\tilde{b}} \sim 10^{-2}$.

In Fig. 4 we fix $m_{\tilde{b}_L} = m_{\tilde{b}_R} = 450$ GeV. The three curves correspond to top squark masses of 450, 625 and 750 GeV. As we can see, leaving aside accidental cancellations, the value of $\lambda''_{ud\tilde{b}}$ is constrained to be less than ∼0.15–0.3.

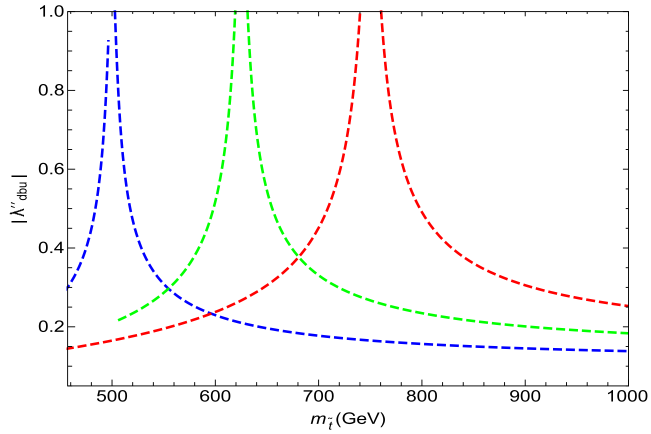

FIG. 4. Bounds on $\lambda''_{ud\tilde{b}}$ for degenerate up and bottom squark masses $m_{\tilde{u}} = m_{\tilde{c}} = m_{\tilde{b}_L} = m_{\tilde{b}_R} = 450$ GeV: $m_{\tilde{t}} = 500$ GeV (blue), $m_{\tilde{t}} = 625$ GeV (green) and $m_{\tilde{t}} = 750$ GeV (red).

We will now estimate $s_{\rm box}$ making use of Eq. (12) by writing

$$\frac{1}{m_{\tilde{w}}} s_{\rm box} = \frac{1}{3}\frac{1}{32\pi^2}\frac{m_{\tilde{w}}}{m^2_W}(0.75 \times 10^{-4}) \Rightarrow s_{\rm box} \approx 2.0 \text{ to } 3.0 \times 10^{-6},$$

assuming the range $m_{\tilde{w}} \approx 400$–500 GeV. We have, of course, removed the factors 3, (1/2), g^4 and the scalar masses from the expression of Eq. (A7) since they appear explicitly in Eq. (23).

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