Study of the isospin breaking decay $Y(2175) \to \phi f_0(980) \to \phi\eta\pi^0$ at BESIII

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Using the measured branching fraction of the decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-$ from the BESIII experiment, we estimate branching fraction of $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ decay, which proceeds via the $f_0(980)$-$a_0^0(980)$ mixing and the π^0-η mixing. The branching fraction is predicted to be about $O(10^{-6})$, which can be accessed with 10^{10} J/ψ events collected at BESIII. The decay is dominated by the contribution from $f_0(980)$-$a_0^0(980)$ mixing. We find that the interference between the amplitudes due to $f_0(980)$-$a_0^0(980)$ mixing and that due to π^0-η mixing is constructive. The branching fraction can be increased by about 10%, owing to the interference effect. We also study the $\eta\pi^0$ mass squared spectrum and find that a narrow peak due to the $f_0(980)$-$a_0^0(980)$ mixing in the $\eta\pi^0$ mass squared spectrum should be observed. The observation of this decay in experiment will be helpful to determine the $f_0(980)$-$a_0^0(980)$ mixing intensity and get information about the structures of the light scalar mesons.

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I. INTRODUCTION

The nature of the light scalar mesons $a_0^0(980)$ and $f_0(980)$ is still a hot topic in hadronic physics. Several models about the structure of the scalar mesons have been proposed, such as $q\bar{q}$ states, glueball, hybrid states, molecule states, tetra-quark states, and the superpositions of these contents [1–11]. Due to the absence of convincing evidence, a final consensus has not been reached so far. Therefore, more research in both theory and experiment is still needed.

The structure of $a_0^0(980)$ and $f_0(980)$ is closely related to the mixing of them, which was first suggested theoretically in Ref. [12]. Its mixing intensity has been studied extensively on its different aspects and possible manifestations in various processes [13–30]. Recently, the BESIII Collaboration has reported the first observation of $f_0(980) - a_0^0(980)$ mixing in the decays of $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ and $\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ [31]. In their work, the values of the mixing intensity ξ_{fa} for the $f_0(980) - a_0^0(980)$ transition was obtained:

$$\xi_{fa} = (0.99 \pm 0.35) \times 10^{-2} \quad \text{(solution-1)},$$
$$\xi_{fa} = (0.41 \pm 0.25) \times 10^{-2} \quad \text{(solution-2)}. \tag{1}$$

Here, ξ_{fa} is defined as

$$\xi_{fa} = \frac{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0)}{\mathcal{B}(J/\psi \to \phi f_0(980) \to \phi\pi^+\pi^-)}. \tag{2}$$

The theoretical calculation prefers the solution-1 result of BESIII [32]. Here, more works are needed to determine the final solution of ξ_{fa}.

The $Y(2175)$ resonance, which decays dominantly via a $\phi f_0(980)$ intermediate state, is a vector meson, its $J^{PC} = 1^{--}$ [33]. This resonance was first observed by the *BABAR* Collaboration [34] and then confirmed by the BESIII Collaboration [35] and Belle Collaboration [36]. At present, this state is listed by the Particle Data Group [37] as the excited ϕ meson, which is also referred as $\phi(2170)$ in the literature [38]. The recent result on $Y(2175)$ resonance in J/ψ decay from the BESIII Collaboration is obtained as [39]

$$\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-)$$
$$= (1.20 \pm 0.40) \times 10^{-4}. \tag{3}$$

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In this paper, we study the isospin breaking decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ and estimate its branching fraction by using recent measurements by BESIII [31,39]. We also study the distribution of $\eta\pi^0$ mass squared spectrum near the $K\bar{K}$ threshold.

II. TWO MECHANISMS OF THE DECAY

The isospin breaking decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ can proceed via the $f_0(980) - a_0^0(980)$ mixing and the π^0-η mixing. The amplitude can be written as

$$\begin{aligned}\mathcal{M}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0)\\ = \mathcal{M}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi a_0^0(980) \to \eta\phi\eta\pi^0)\\ + \mathcal{M}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^0\pi^0 \to \eta\phi\eta\pi^0).\end{aligned} \tag{4}$$

The corresponding graphs are shown in Figs. 1 and 2. For the contribution of $f_0(980)$-$a_0^0(980)$ mixing, the mixing intensity ξ_{fa} can be expressed in a way similar to that in Eq. (2) and here is defined as

$$\xi_{fa} = \frac{\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi a_0^0(980) \to \eta\phi\eta\pi^0)}{\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-)}. \tag{5}$$

Combining Eqs. (1), (3), and (5), one can obtain the branching fraction of $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi a_0^0(980) \to \eta\phi\eta\pi^0$.

With $\xi_{fa} = (0.99 \pm 0.35) \times 10^{-2}$, one can obtain

$$\begin{aligned}\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi a_0^0(980)\\ \to \eta\phi\eta\pi^0) = (1.19 \pm 0.58) \times 10^{-6},\end{aligned} \tag{6}$$

while with $\xi_{fa} = (0.41 \pm 0.25) \times 10^{-2}$, one can get

$$\begin{aligned}\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi a_0^0(980)\\ \to \eta\phi\eta\pi^0) = (0.49 \pm 0.34) \times 10^{-6}.\end{aligned} \tag{7}$$

In the BESIII analysis for the decays of $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ and $\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0$ [31], they only assumed contribution from $f_0(980) - a_0^0(980)$ or $a_0^0(980) - f_0(980)$ mixing, which causes the isospin breaking decays. In fact, the final states of $\phi\eta\pi^0$ could be also induced by $J/\psi \to \phi f_0(980) \to \phi\pi^0\pi^0 \to \phi\eta\pi^0$ via π^0-η mixing. If it is the case that the isospin breaking decay is due to both $f_0(980) - a_0^0(980)$ and π^0-η mixings, actually the BESIII measured values for the mixing intensity ξ_{fa} in Eq. (1) have already included both effects. Therefore, the results given in Eqs. (6) and (7) include both the contributions of $f_0(980)$-$a_0^0(980)$ mixing and π^0-η mixing physically.

As for the sole contribution of π^0-η mixing, the relative ratio of $\mathcal{B}(f_0(980) \to \pi^0\pi^0 \to \eta\pi^0)$ to $\mathcal{B}(f_0(980) \to \pi^0\pi^0)$ is

$$\frac{\mathcal{B}(f_0(980) \to \pi^0\pi^0 \to \eta\pi^0)}{\mathcal{B}(f_0(980) \to \pi^0\pi^0)} = 4\frac{f(m_{f_0}, m_\eta, m_{\pi^0})}{f(m_{f_0}, m_{\pi^0}, m_{\pi^0})}\left|\frac{\lambda_{\pi^0\eta}}{m_\eta^2 - m_{\pi^0}^2}\right|^2, \tag{8}$$

where m_{f_0}, m_η, and m_{π^0} are the masses of $f_0(980)$, η, and π^0, respectively. The factor 4 is from the sum of the contributions of Fig. 2 and its symmetric diagram with changing the π^0-η transition to another external π^0 line [40]. The function f is

$$f(x,y,z) = \sqrt{x^4 + y^4 + z^4 - 2x^2y^2 - 2x^2z^2 - 2y^2z^2}. \tag{9}$$

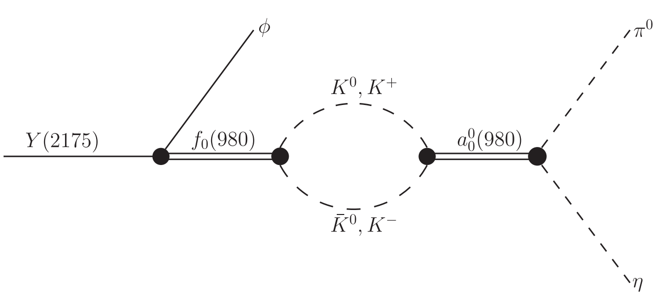

FIG. 1. Feynman diagram for the reaction $Y(2175) \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$.

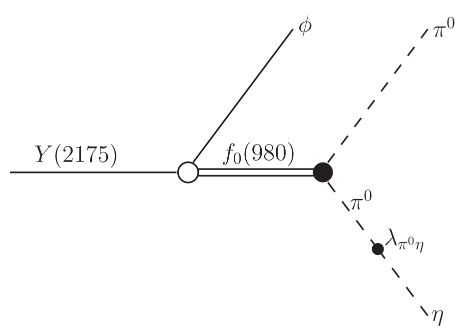

FIG. 2. Feynman diagram for the reaction $Y(2175) \to \phi f_0(980) \to \phi\pi^0\pi^0 \to \phi\eta\pi^0$.

TABLE I. The masses of the particles in final states.

$m_{\pi^+} = 139.6$ MeV [37]	$m_{\pi^0} = 135$ MeV [37]
$m_{K^+} = 493.7$ MeV [37]	$m_{K^0} = 497.6$ MeV [37]
$m_\eta = 547.9$ MeV [37]	$m_{\eta'} = (957.8 \pm 0.1)$ MeV [37]

$\lambda_{\pi^0\eta}$ is the π^0-η transition amplitude [13,41], which can be extracted from the ratio of $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0)$ and $\mathcal{B}(\eta' \to \pi^+\pi^-\eta)$ decays [42],

$$\left|\frac{\lambda_{\pi^0\eta}}{m_\eta^2 - m_{\pi^0}^2}\right|^2 = \frac{\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0)}{\mathcal{B}(\eta' \to \pi^+\pi^-\eta)} \frac{\phi_s(\eta' \to \pi^+\pi^-\eta)}{\phi_s(\eta' \to \pi^+\pi^-\pi^0)}, \quad (10)$$

where $\phi_s(\eta' \to \pi^+\pi^-\eta) = \int_{4m_{\pi^+}^2}^{(m_{\eta'}-m_\eta)^2} \frac{dq^2}{q^2} f(m_{\eta'}, \sqrt{q^2}, m_\eta) \times f(\sqrt{q^2}, m_{\pi^+}, m_{\pi^+})$, is the phase-space integral, while $\phi_s(\eta' \to \pi^+\pi^-\pi^0)$ is the relevant phase-space integral that changes m_η to m_{π^0} in $\phi_s(\eta' \to \pi^+\pi^-\eta)$. The relative ratio of $\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0)/\mathcal{B}(\eta' \to \pi^+\pi^-\eta)$ has been measured by the BESIII [43] and CLEO collaborations [44]. The recent value measured by the BESIII Collaboration is $(8.8 \pm 1.2) \times 10^{-3}$ [45,46]. Then, with Eq. (10) and the measured value of the relative branching ratio, we can obtain

$$\left|\frac{\lambda_{\pi^0\eta}}{m_\eta^2 - m_{\pi^0}^2}\right|^2 = (5.21 \pm 0.71) \times 10^{-4}, \quad (11)$$

where the particle masses listed in Tables I and II are used here. Next, employing the relation $\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 2\mathcal{B}(f_0(980) \to \pi^0\pi^0)$ and combining Eqs. (3), (8), and (11), one can obtain

$$\frac{\mathcal{B}(f_0(980) \to \pi^0\pi^0 \to \eta\pi^0)}{\mathcal{B}(f_0(980) \to \pi^0\pi^0)} = (1.43 \pm 0.19) \times 10^{-3}, \quad (12)$$

and

$$\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^0\pi^0 \to \eta\phi\eta\pi^0) = (0.86 \pm 0.31) \times 10^{-7}. \quad (13)$$

Obviously, this is much smaller than the results given in Eqs. (6) and (7), which implies that the contribution of $f_0(980)$-$a_0^0(980)$ mixing dominates over that of π^0-η mixing in decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$.

III. BRANCHING FRACTION

As mentioned in Eq. (4), both the $f_0(980)$-$a_0^0(980)$ mixing and the π^0-η mixing can contribute to the decay of $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$. The most characteristic feature of the first contribution is the narrow peak in the $\eta\pi^0$ mass spectrum, which is due to the property of the $f_0(980)$-$a_0^0(980)$ mixing amplitude [12,16,29]. As far as π^0-η mixing is concerned, however, the width in the $\eta\pi^0$ mass spectrum should be the natural width of the $f_0(980)$ state, which is broad. Fortunately, the contribution from the π^0-η mixing is much smaller than that from the $f_0(980)$-$a_0^0(980)$ mixing, so the narrow structure caused by the $f_0(980)$-$a_0^0(980)$ mixing is expected to be observed, while the broad width from the effect of π^0-η mixing is negligibly small. The corresponding decay amplitude contributed by $f_0(980)$-$a_0^0(980)$ mixing is [29,48]

$$\begin{aligned}&\mathcal{M}(Y(2175) \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0)\\ &= \mathcal{M}(Y(2175) \to \phi f_0(980))\\ &\cdot \frac{\Pi_{a_0 f_0}(q^2)}{D_{a_0}(q^2)D_{f_0}(q^2) - \Pi_{a_0 f_0}^2(q^2)} \cdot g_{a_0\eta\pi^0},\end{aligned} \quad (14)$$

where $q^2 = (p_\eta + p_{\pi^0})^2$ and $g_{a_0\eta\pi^0}$ is the coupling of $a_0^0(980)$ to $\eta\pi^0$. $\mathcal{M}(Y(2175) \to \phi f_0(980))$ is the invariant amplitude for the decay $Y(2175) \to \phi f_0(980)$, which can be used to calculate the branching fraction

$$\mathcal{B}(Y(2175) \to \phi f_0(980)) = |\mathcal{M}(Y(2175) \to \phi f_0(980))|^2 \cdot \frac{f(m_Y, m_\phi, m_{f_0})}{16\pi\Gamma_Y m_Y^3}, \quad (15)$$

where Γ_Y is the decay width of $Y(2175)$; m_Y, m_ϕ, and m_{f_0} are the masses of the resonances $Y(2175)$, ϕ, and $f_0(980)$, respectively. $\Pi_{a_0 f_0}(q^2)$ is the $f_0(980) - a_0^0(980)$ mixing amplitude and defined as

TABLE II. Properties of the resonances, here, f_0, a_0, and Y denote $f_0(980)$, $a_0^0(980)$, and $Y(2175)$, respectively.

$m_{f_0} = (0.99 \pm 0.02)$ GeV [37]	$\Gamma_{f_0} = 0.074$ GeV [47]
$m_{a_0} = (0.98 \pm 0.02)$ GeV [37]	$\Gamma_{a_0} = (0.092 \pm 0.008)$ GeV [37]
$m_Y = (2.188 \pm 0.010)$ GeV [37]	$\Gamma_Y = (0.083 \pm 0.012)$ GeV [37]
$m_\phi = 1019$ MeV [37]	$g_{a_0\eta\pi^0} = 2.43$ GeV [1,49]
$g_{a_0K^+K^-} = (2.76 \pm 0.46)$ GeV [50,51]	$g_{a_0K^0\bar{K}^0} = (2.76 \pm 0.46)$ GeV [50,51]
$g_{f_0\pi^+\pi^-} = 1.39$ GeV [1,49]	$g_{f_0\pi^0\pi^0} = 0.98$ GeV [1,49]
$g_{f_0K^+K^-} = 3.17$ GeV [29]	$g_{f_0K^0\bar{K}^0} = 3.17$ GeV [29]

$$\Pi_{a_0f_0}(q^2) = \frac{g_{a_0K^+K^-}g_{f_0K^+K^-}}{16\pi}\left[i(R_{K^+K^-}(q^2) - R_{K^0\bar{K}^0}(q^2)) - \frac{R_{K^+K^-}(q^2)}{\pi}\ln\frac{1+R_{K^+K^-}(q^2)}{1-R_{K^+K^-}(q^2)} + \frac{R_{K^0\bar{K}^0}(q^2)}{\pi}\ln\frac{1+R_{K^0\bar{K}^0}(q^2)}{1-R_{K^0\bar{K}^0}(q^2)}\right], \tag{16}$$

where for $q^2 > 4m_a^2$, $R_{aa}(q^2) = \sqrt{1-4m_a^2/q^2}$, while for $0 < q^2 \le 4m_a^2$, $R_{aa}(q^2) = i\sqrt{4m_a^2/q^2 - 1}$, here $a = K^\pm$, K^0. $D_r(q^2)$ in Eq. (14) is the denominator for the propagator of the resonance r,

$$D_r(q^2) = q^2 - m_r^2 - \sum_{ab}[\mathrm{Re}\Pi_r^{ab}(m_r^2) - \Pi_r^{ab}(q^2)]. \tag{17}$$

For $r = a_0^0(980)$, $ab = (\eta\pi^0, K^+K^-, K^0\bar{K}^0)$, and for $r = f_0(980)$, $ab = (\pi^+\pi^-, \pi^0\pi^0, K^+K^-, K^0\bar{K}^0)$. Π_r^{ab} stands for the diagonal matrix of the polarization operator of the resonance r corresponding to the one loop contribution from the two-particle intermediate states ab [29,48], for $q^2 \ge (m_a + m_b)^2$, we have

$$\Pi_r^{ab}(q^2) = \frac{g_{rab}^2}{16\pi}\left[\frac{m_{ab}^{(+)}m_{ab}^{(-)}}{\pi q^2}\ln\frac{m_b}{m_a} + \rho_{ab}(q^2)\left(i - \frac{1}{\pi}\ln\frac{\sqrt{q^2 - m_{ab}^{(-)2}} + \sqrt{q^2 - m_{ab}^{(+)2}}}{\sqrt{q^2 - m_{ab}^{(-)2}} - \sqrt{q^2 - m_{ab}^{(+)2}}}\right)\right]; \tag{18}$$

for $(m_a - m_b)^2 < q^2 < (m_a + m_b)^2$,

$$\Pi_r^{ab}(q^2) = \frac{g_{rab}^2}{16\pi}\left[\frac{m_{ab}^{(+)}m_{ab}^{(-)}}{\pi q^2}\ln\frac{m_b}{m_a} - \rho_{ab}(q^2)\left(1 - \frac{2}{\pi}\arctan\frac{\sqrt{m_{ab}^{(+)2} - q^2}}{\sqrt{q^2 - m_{ab}^{(-)2}}}\right)\right]; \tag{19}$$

for $q^2 \le (m_a + m_b)^2$,

$$\Pi_r^{ab}(q^2) = \frac{g_{rab}^2}{16\pi}\left[\frac{m_{ab}^{(+)}m_{ab}^{(-)}}{\pi q^2}\ln\frac{m_b}{m_a} + \rho_{ab}(q^2)\frac{1}{\pi}\ln\frac{\sqrt{m_{ab}^{(+)2} - q^2} + \sqrt{m_{ab}^{(-)2} - q^2}}{\sqrt{m_{ab}^{(+)2} - q^2} - \sqrt{m_{ab}^{(-)2} - q^2}}\right], \tag{20}$$

where g_{rab} is the coupling of resonance r to final states ab, $m_{ab}^{(\pm)} = |m_a \pm m_b|$, and $\rho_{ab}(q^2)$ is

$$\rho_{ab}(q^2) = \frac{\sqrt{|q^2 - m_{ab}^{(+)2}|}\sqrt{|q^2 - m_{ab}^{(-)2}|}}{q^2}. \tag{21}$$

As for the contribution of $\eta - \pi^0$ mixing, the transition amplitude is

$$\mathcal{M}(Y(2175) \to \phi f_0(980) \to \phi\pi^0\pi^0 \to \phi\eta\pi^0) = 2\mathcal{M}(Y(2175) \to \phi f_0(980)) \cdot \frac{g_{f_0\pi^0\pi^0} \cdot i}{D_{f_0}(q^2)} \cdot \frac{\lambda_{\pi^0\eta}}{m_\eta^2 - m_{\pi^0}^2}, \tag{22}$$

where the factor i is due to the phase of the $f_0(980)$ resonance contribution and the large and smooth background phase, which are approximately equal to 90° totally in the $f_0(980)$ region. Adding Eqs. (14) and (22) together, we then arrive at

$$\mathcal{M}(Y(2175) \to \phi f_0(980) \to \phi\eta\pi^0) = \mathcal{M}(Y(2175) \to \phi f_0(980)) \cdot \left[\frac{\Pi_{a_0f_0}(q^2) \cdot g_{a_0\eta\pi^0}}{D_{a_0}(q^2)D_{f_0}(q^2) - \Pi_{a_0f_0}^2(q^2)} + \frac{2g_{f_0\pi^0\pi^0} \cdot i}{D_{f_0}(q^2)} \cdot \frac{\lambda_{\pi^0\eta}}{m_\eta^2 - m_{\pi^0}^2}\right], \tag{23}$$

where $g_{a_0\eta\pi^0}$ and $g_{f_0\pi^0\pi^0}$ are the couplings of $a_0^0(980)$ to $\eta\pi^0$ and $f_0(980)$ to $\pi^0\pi^0$, respectively, which can be extracted from

$$\mathcal{B}(r \to ab) = \frac{g_{rab}^2}{16\pi m_r^3 \Gamma_r} f(m_r, m_a, m_b). \tag{24}$$

By combining Eqs. (10), (15), (23), and (24), we can obtain the distribution of the $\eta\pi^0$ mass squared spectrum for $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$, i.e.,

$$\begin{aligned}\mathcal{B}(J/\psi \to \eta Y(2175)) \cdot \frac{d\Gamma(Y(2175) \to \phi f_0(980) \to \phi\eta\pi^0)}{dq^2} \\ = \mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-) \cdot \varphi_S \\ \cdot |\delta_{f_0 a_0^0} + \delta_{\pi^0\eta}|^2,\end{aligned} \tag{25}$$

where φ_S is the phase-space factor of the involved decays

$$\varphi_S = \frac{\Gamma_Y}{\pi q^2} \cdot \frac{f(m_Y, m_\phi, \sqrt{q^2})}{f(m_Y, m_\phi, m_{f_0})} \cdot f\left(\sqrt{q^2}, m_\eta, m_{\pi^0}\right); \tag{26}$$

here, Γ_Y is the total decay width of $Y(2175)$. $\delta_{\pi^0\eta}$ and $\delta_{f_0 a_0^0}$ in Eq. (25) denote the contributions from the π^0-η mixing and the $f_0(980)$-$a_0^0(980)$ mixing, respectively, which are given in the following,

$$\begin{aligned}\delta_{\pi^0\eta} = -\frac{\sqrt{2}i}{D_{f_0}(q^2)} \sqrt{\frac{\mathcal{B}(\eta' \to \pi^+\pi^-\pi^0)}{\mathcal{B}(\eta' \to \pi^+\pi^-\eta)} \frac{\phi_s(\eta' \to \pi^+\pi^-\eta)}{\phi_s(\eta' \to \pi^+\pi^-\pi^0)}} \\ \times \sqrt{\frac{\Gamma_{f_0} m_{f_0}^3}{f(m_{f_0}, m_{\pi^0}, m_{\pi^0})}};\end{aligned} \tag{27}$$

here, the minus sign is associated with the $\lambda_{\pi^0\eta}$ vertex corresponding to the $\pi^0 \leftrightarrow \eta$ transition [13,52,53],

$$\begin{aligned}\delta_{f_0 a_0^0} = \sqrt{\frac{\mathcal{B}(a_0^0(980) \to \eta\pi^0)}{\mathcal{B}(f_0(980) \to \pi^+\pi^-)}} \cdot \sqrt{\frac{\Gamma_{a_0} m_{a_0}^3}{f(m_{a_0}, m_\eta, m_{\pi^0})}} \\ \cdot \frac{\Pi_{a_0 f_0}(q^2)}{D_{a_0}(q^2) D_{f_0}(q^2) - \Pi_{a_0 f_0}^2(q^2)},\end{aligned} \tag{28}$$

where Γ_{f_0} and Γ_{a_0} are the decay widths of $f_0(980)$ and $a_0^0(980)$, respectively. From Refs. [1] and [49], the branching fractions $\mathcal{B}(f_0(980) \to \pi^+\pi^-)$ and $\mathcal{B}(a_0^0(980) \to \eta\pi^0)$ are obtained as

$$\mathcal{B}(f_0(980) \to \pi^+\pi^-) = 0.50^{+0.07}_{-0.09}, \tag{29}$$

$$\mathcal{B}(a_0^0(980) \to \eta\pi^0) = 0.845 \pm 0.017. \tag{30}$$

Using the input parameters listed in Tables I and II, we obtain the result for the distribution curve of the $\eta\pi^0$ mass squared spectrum for $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ decay, which is shown in Fig. 3. In this figure, the narrow peak due to the $f_0(980)$-$a_0^0(980)$ mixing can be clearly observed.

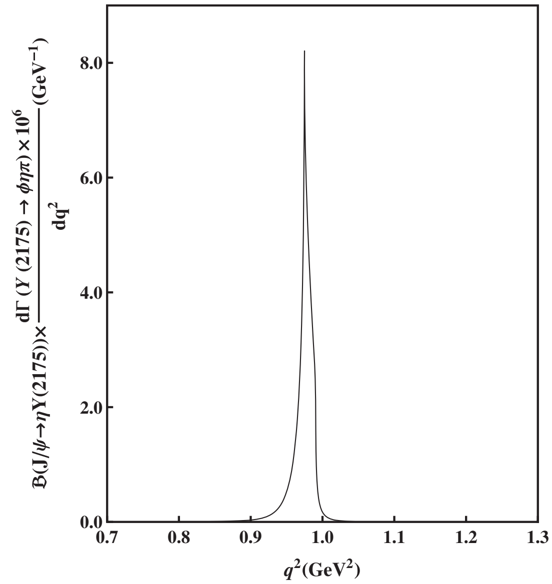

FIG. 3. The distribution of the $\eta\pi^0$ mass squared spectrum ($q^2 = (p_\eta + p_{\pi^0})^2$) for the decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$.

Furthermore, the branching fraction of the decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ is obtained by performing the integration in the effective region $(m_\eta + m_{\pi^0})^2 \leq q^2 \leq (m_Y - m_\phi)^2$, and the result is

$$\begin{aligned}\mathcal{B}(J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0) \\ = (1.61^{+0.84}_{-1.03}) \times 10^{-6},\end{aligned} \tag{31}$$

where we have considered the errors of the mass and width of $a_0^0(980)$ and $f_0(980)$, the errors from the branching fractions of the decays $f_0(980) \to \pi^+\pi^-$ and $a_0^0(980) \to \eta\pi^0$, as well as uncertainty from the branching fraction of the decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-$. The contribution from the $f_0(980)$-$a_0^0(980)$ mixing dominates the predicted branching fraction. In addition, the interference of the amplitudes from the $f_0(980)$-$a_0^0(980)$ mixing and π^0-η mixing is constructive, and the branching fraction is increased by about 10% owing to the interference effect.

IV. PROSPECTS FOR THE MEASUREMENT AT BESIII

The final states π^0, η, and ϕ in the cascade decay process $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ are reconstructed through the decays $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, and $\phi \to K^+K^-$. By employing the data reported by the Particle Data Group [37], we obtain

$$\begin{aligned}\mathcal{B}(\eta \to \gamma\gamma) \cdot \mathcal{B}(\phi \to K^+K^-) \cdot \mathcal{B}(\eta \to \gamma\gamma) \cdot \mathcal{B}(\pi^0 \to \gamma\gamma) \\ = (7.55 \pm 0.09) \times 10^{-2}.\end{aligned} \tag{32}$$

Because of the narrow peak near the $K\bar{K}$ thresholds in the $\eta\pi^0$ invariant mass spectrum, the event selection criteria for the $a_0^0(980)$ candidates have high efficiency. In addition, the final states contain six photons and two charged tracks, and the detection efficiency for $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ decay can be as large as 8% after the final selection [39,45,54,55]. The BESIII experiment will accumulate a huge data sample of 10×10^9 J/ψ decays by the end of 2019 [55–57]. Therefore, about 100 events for the decay of $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ are expected in the J/ψ decay sample at the BESIII. Therefore, the isospin breaking decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$ will be helpful to determine the final value of ξ_{fa} in addition to the process $J/\Psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$.

V. CONCLUSIONS

Basing on the branching fraction of the decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\pi^+\pi^-$ and the $f_0(980)$-$a_0^0(980)$ mixing intensity ξ_{fa} measured recently by BESIII, we study the isospin violation decay $J/\psi \to \eta Y(2175) \to \eta\phi f_0(980) \to \eta\phi\eta\pi^0$, which proceeds via the $f_0(980)$-$a_0^0(980)$ mixing and the π^0-η mixing. It is found that the decay can reach a branching fraction of the order of 10^{-6}, which can be accessed with 10^{10} J/ψ events collected at BESIII by the end of 2019. The contribution from the $f_0(980)$-$a_0^0(980)$ mixing dominates the decay. The interference between the amplitude caused by the $f_0(980)$-$a_0^0(980)$ mixing and the amplitude caused by the π^0-η mixing is constructive, and the branching fraction will be increased by about 10% because of the interference effect between the two mixings. In the distribution of the $\eta\pi^0$ mass square spectrum, we find that the narrow peak due to $f_0(980)$-$a_0^0(980)$ mixing should be expected, and the effect on the peak from π^0-η mixing is negligibly small. This decay will be complementary to the decay $J/\psi \to \phi f_0(980) \to \phi\eta\pi^0$, which will be helpful to determine the final solution of the $f_0(980)$-$a_0^0(980)$ mixing intensity and understand the nature of the light scalar mesons.

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