Prediction for 5^{++} mesons

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In this paper, we study the spectrum and decay behavior of the 5^{++} meson family which is still missing in experiment. By the modified Godfrey-Isgur model with a color screening effect, we obtain the mass spectrum of a_5, f_5, and f_5' mesons. And we predict their two-body strong decays by means of a phenomenology quark pair creation model. This study is crucial to establish $J^{PC} = 5^{++}$ meson family and it is also helpful to search for these states in the future.

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I. INTRODUCTION

In the past decades, the quark model has made great achievements. There are six quarks, up(u), down(d), strange(s), charm(c), bottom(b), and top(t), in the quark model. Except t quark, the other quarks can form mesons, baryons, and other hadrons. As an important part of hadron, the meson family is phenomenologically studied by many works [1–9]. When checking the experimental status of mesons [10], we notice an interesting phenomenon, the whole meson family with $J^{PC} = 5^{++}$ (with $q\bar{q}$ component, here q defines u, d, or s quark) is still missing, yet the other families for a and f meson (such as a_0, a_1, a_2, a_3, a_4, and a_6) have been reported by Particle Data Group(PDG) [10]. This phenomenon stimulates our interest in exploring where the mesons with $J^{PC} = 5^{++}$ are and their properties.

The mesons with $J^{PC} = 5^{++}$ have two families, which are isovector a_5 meson family, and isoscalar meson family [f_5 ($n\bar{n}$, here n defines u or d quark) and f_5' ($s\bar{s}$)]. Due to the present experimental progress on mesons, it is a suitable time to systematically carry out phenomenological study of missing 5^{++} states. This study is not only crucial to establish $J^{PC} = 5^{++}$ meson family and helpful to search for these states in the future, but also important for verifying quark model.

As 5^{++} mesons have a higher spin, the screening effect will be strong for the large angular momentum and larger average distance between quark pair. So we need introduce the screening effect into the quark model in this work when we deal with the spectrum. In this work, we calculate the mass spectra of the 5^{++} meson family by using the unquenched Godfrey-Isgur(GI) model [11,12], which contains the screening effect. According to the former studies [8,11–17], the GI model was tested by different systems, which shows that the GI model works well for describing hadron spectroscopy. In this work, we continue to apply this model to explore high spin states, especially estimate the mass spectrum of high spin mesons. Since the experimental information of high spin states is absent, we hope that these predicted states can be accessible at future experiment, which can provide a further test of the GI model to high spin states. In this paper, we fix the parameters in the model by fitting some well-established meson states, which are adopted when calculating the mass of 5^{++} meson states. Then, for further study the properties of 5^{++} mesons, we study their Okubo-Zweig-Iizuka (OZI)-allowed two-body strong decays taking input with the spatial wave functions obtaining in mass spectrum numerically calculation. Their partial and total decay widths are predicted by using a quark pair creation (QPC) model which was proposed by Micu [18] and extensively applied to studies of strong decay of other hadrons [1,3–7,15, 19–38]. We hope that our effort will be helpful to establish a_5, f_5, and f_5' meson families.

This paper is organized as follows. In Sec. II, the mass spectrum analysis of the meson family with $J^{PC} = 5^{++}$ will be performed. In Sec. III, we further study the two-body OZI-allowed strong decay behavior of these discussed states. The paper ends with a conclusion in Sec. IV.

II. THE MASS SPECTRUM ANALYSIS

In this work, the modified GI quark model is utilized to calculate the mass spectrum and wave functions of the meson family. In the following, this model will be illustrated in detail.

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A. The modified GI model

In 1985, Godfrey and Isgur raise the GI model for describing relativistic meson spectra with great success, exactly in low-lying mesons [13]. As for the excited states, the screening potential must be taken into account for coupled-channel effect.

The interaction between quark and antiquark is depicted by the Hamiltonian of potential model including kinetic energy pieces and effective potential piece,

$$\tilde{H} = \sqrt{m_1^2 + \mathbf{p}^2} + \sqrt{m_2^2 + \mathbf{p}^2} + \tilde{V}_{\text{eff}}(\mathbf{p}, \mathbf{r}), \tag{2.1}$$

where m_1 and m_2 denote the mass of quark and antiquark respectively, and effective potential $\tilde{V}_{\text{eff}}$ contains two ingredients, a short-range $\gamma^\mu \otimes \gamma_\mu$ one-gluon-exchange interaction and a $1 \otimes 1$ linear confinement interaction. The meaning of tilde will be explained later.

In the nonrelativistic limit, effective potential has a familiar format [13,39]

$$V_{\text{eff}}(r) = H^{\text{conf}} + H^{\text{hyp}} + H^{\text{so}}, \tag{2.2}$$

with

$$\begin{aligned} H^{\text{conf}} &= \left[-\frac{3}{4}(c + br) + \frac{\alpha_s(r)}{r}\right](\boldsymbol{F}_1 \cdot \boldsymbol{F}_2), \\ &= S(r) + G(r) \end{aligned} \tag{2.3}$$

$$\begin{aligned} H^{\text{hyp}} = -\frac{\alpha_s(r)}{m_1 m_2} \bigg[& \frac{8\pi}{3} \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \delta^3(\boldsymbol{r}) \\ & + \frac{1}{r^3}\left(\frac{3\boldsymbol{S}_1 \cdot \boldsymbol{r} \boldsymbol{S}_2 \cdot \boldsymbol{r}}{r^2} - \boldsymbol{S}_1 \cdot \boldsymbol{S}_2\right)\bigg](\boldsymbol{F}_1 \cdot \boldsymbol{F}_2), \end{aligned} \tag{2.4}$$

$$H^{\text{so}} = H^{\text{so(cm)}} + H^{\text{so(tp)}}, \tag{2.5}$$

where H^{conf} includes the spin-independent linear confinement piece $S(r)$ and Coulomb-like potential from one-gluon-exchange $G(r)$, H^{hyp} denotes the color-hyperfine interaction consists tensor and contact terms, and H^{SO} is the spin-orbit interaction with

$$H^{\text{so(cm)}} = \frac{-\alpha_s(r)}{r^3}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(\frac{\boldsymbol{S}_1}{m_1} + \frac{\boldsymbol{S}_2}{m_2}\right) \cdot \boldsymbol{L}(\boldsymbol{F}_1 \cdot \boldsymbol{F}_2), \tag{2.6}$$

colour magnetic term causing of one-gluon-exchange and

$$H^{\text{so(tp)}} = -\frac{1}{2r}\frac{\partial H^{\text{conf}}}{\partial r}\left(\frac{\boldsymbol{S}_1}{m_1^2} + \frac{\boldsymbol{S}_2}{m_2^2}\right) \cdot \boldsymbol{L}, \tag{2.7}$$

the Thomas precession term.

For above formulas, $\boldsymbol{S}_1/\boldsymbol{S}_2$ indicates the spin of quark/antiquark and $\boldsymbol{L}$ the orbital momentum between them. $\boldsymbol{F}$ is relevant to the Gell-Mann matrix, i.e., $\boldsymbol{F}_1 = \boldsymbol{\lambda}_1/2$ and $\boldsymbol{F}_1 = -\boldsymbol{\lambda}_2^*/2$, and for a meson, $\langle \boldsymbol{F}_1 \cdot \boldsymbol{F}_2 \rangle = -4/3$.

Now relativistic effects of distinguish influence must be considered especially in meson system, which is embedded in two ways. First, based on the nonlocal interactions and new $\mathbf{r}$ dependence, a smearing function is introduced for a meson $q\bar{q}$

$$\rho(\mathbf{r} - \mathbf{r}') = \frac{\sigma^3}{\pi^{3/2}} e^{-\sigma^2(\mathbf{r} - \mathbf{r}')^2}, \tag{2.8}$$

which is applied to $S(r)$ and $G(r)$ to obtain smeared potentials $\tilde{S}(r)$ and $\tilde{G}(r)$ by

$$\tilde{f}(r) = \int d^3 r' \rho(\mathbf{r} - \mathbf{r}') f(r'), \tag{2.9}$$

with

$$\sigma_{12}^2 = \sigma_0^2\left[\frac{1}{2} + \frac{1}{2}\left(\frac{4m_1 m_2}{(m_1 + m_2)^2}\right)^4\right] + s^2\left(\frac{2m_1 m_2}{m_1 + m_2}\right)^2, \tag{2.10}$$

where the values of σ_0 and s are defined later.

Second, owning to relativistic effects, a general potential should rely on the mass-of-center of interacting quarks. Momentum-dependent factors, which will be unity in the nonrelativistic limit, are applied as

$$\tilde{G}(r) \to \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2} \tilde{G}(r) \left(1 + \frac{p^2}{E_1 E_2}\right)^{1/2}, \tag{2.11}$$

and

$$\frac{\tilde{V}_i(r)}{m_1 m_2} \to \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2 + \epsilon_i} \frac{\tilde{V}_i(r)}{m_1 m_2} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2 + \epsilon_i}, \tag{2.12}$$

where $\tilde{V}_i(r)$ delegates the contact, tensor, vector spin-orbit, and scalar spin-orbit terms, and ϵ_i the relevant modification parameters.

Diagonalizing and solving the Hamiltonian in Eq. (2.1) by exploiting a simple harmonic oscillator (SHO) basis, we will obtain the mass spectrum and wave functions. As we know, a series of SHO wave function with different radial quantum number n can be regarded as a complete basis to expand the exact radial wave function of meson state. The base can produce all kind of states, even spurious ones(if the motion of the meson is taken into account [40]), However, this model is calculated in center-mass frame, so there are no spurious states in our numerical result.

In configuration and momentum space, SHO wave functions have explicit forms, respectively,

$$\Psi_{nLM_L}(\mathbf{r}) = R_{nL}(r,\beta)Y_{LM_L}(\Omega_r),$$
$$\Psi_{nLM_L}(\mathbf{p}) = R_{nL}(p,\beta)Y_{LM_L}(\Omega_p), \tag{2.13}$$

with

$$R_{nL}(r,\beta) = \beta^{3/2}\sqrt{\frac{2n!}{\Gamma(n+L+3/2)}}(\beta r)^L e^{\frac{-r^2\beta^2}{2}} \times L_n^{L+1/2}(\beta^2 r^2), \tag{2.14}$$

$$R_{nL}(p,\beta) = \frac{(-1)^n(-i)^L}{\beta^{3/2}} e^{-\frac{p^2}{2\beta^2}}\sqrt{\frac{2n!}{\Gamma(n+L+3/2)}}\left(\frac{p}{\beta}\right)^L \times L_n^{L+1/2}\left(\frac{p^2}{\beta^2}\right), \tag{2.15}$$

where $Y_{LM_L}(\Omega)$ is spherical harmonic function, and $L_{n-1}^{L+1/2}(x)$ is the associated Laguerre polynomial, and $\beta = 0.4$ GeV for our calculation.

The space-spin wave function $R_{nL}(r,\beta)\phi_{LSJM}$ with total angular quantum number J can be constructed by coupling $L \otimes S$

$$\phi_{LSJM} = \sum_{M_L M_S} C(LM_L SM_S; JM) Y_{LM_L}(\Omega_r)\chi_{SM_S}, \tag{2.16}$$

here $C(LM_L SM_S; JM)$ is Clebsch-Gordan coefficient. For the matrix element $\langle\alpha|\hat{V}(r,\hat{p})|\beta\rangle$ where $|\alpha\rangle$ and $|\beta\rangle$ are arbitrary SHO basis with quantum number $\{n, J, L, S\}$ and $\{n', J', L', S'\}$. The matrix element can be calculated conveniently by using SHO base as follows.

$$\langle\alpha|\hat{V}(r,\hat{p})|\beta\rangle = \langle\alpha|f(p)g(r)|\beta\rangle = \sum_n \langle\alpha|f(p)|n\rangle\langle n|g(r)|\beta\rangle. \tag{2.17}$$

After diagonalizing the Hamiltonian matrix, we can obtain the mass and wave function of meson which available to the following strong decay process.

5^{++} mesons have a higher spin, so the quarks and antiquarks will have large angular momentum and larger average distance which is greater than about 1 fm. In this circumstance, light quark antiquark pairs will be spontaneously created and the screening effect will be strong. So we introduce the screening effect into GI model in this work when we deal with the spectrum. In the previous work [11], the modified GI model was proposed, and the prediction results for the charm-strange mesons are consistent with the experimental data. For higher excitation states, the screen effect is considered to be very important by the authors of Ref. [11]. It could be introduced by the transformation $br + c \to \frac{b(1-e^{-\mu r})}{\mu} + c$, where μ is screened parameter whose particular value is need to be fixed by the comparisons between theory and experiment. Modified confinement potential also requires similar relativistic correction, which has been mentioned in the GI model. Then, we further write $V^{\rm scr}(r)$ as the way given in Eq. (2.18),

$$\tilde{V}^{\rm scr}(r) = \int d^3\mathbf{r}'\rho(\mathbf{r}-\mathbf{r}')\frac{b(1-e^{-\mu r'})}{\mu}. \tag{2.18}$$

By inserting the form of $\rho(\mathbf{r}-\mathbf{r}')$ in Eq. (2.9) into the above expression and finishing this integration, the concrete expression for $\tilde{V}^{\rm scr}(r)$ is given by

$$\tilde{V}^{\rm scr}(r) = \frac{b}{\mu r}\left[r + e^{\frac{\mu^2}{4\sigma^2}+\mu r}\frac{\mu+2r\sigma^2}{2\sigma^2}\left(\frac{1}{\sqrt{\pi}}\int_0^{\frac{\mu+2r\sigma^2}{2\sigma}} e^{-x^2}dx - \frac{1}{2}\right) - e^{\frac{\mu^2}{4\sigma^2}-\mu r}\frac{\mu-2r\sigma^2}{2\sigma^2}\left(\frac{1}{\sqrt{\pi}}\int_0^{\frac{\mu-2r\sigma^2}{2\sigma}} e^{-x^2}dx - \frac{1}{2}\right)\right]. \tag{2.19}$$

Notably, except for converting the confinement potential to the screened potential, the other processing contents and the Hamiltonian matrix elements contained in the original GI model are calculated. In our calculation, we need the spatial wave functions of the discussed meson family with $J^{PC} = 5^{++}$ which can be numerically obtained by the modified GI model.

B. Mass spectrum analysis

GI model can describe the mass of ground states of the mesons successfully, yet it does not describe the excited states well. Since unquenched effects are important for a heavy-light system, it is better to adopt the modified GI model (MGI) [11,12] which uses a screening potential with a new parameter μ. The parameter μ describes inverse of the size of screening. In our previous work [8], we calculate the kaon family spectra by using the MGI model. In this work we will use this MGI model to obtain the mass spectrum of meson with $J^{PC} = 5^{++}$. Beforehand, we need to adjust the parameters of MGI model by fitting with the experimental data. So we fix the following twelve parameters listed in Table I. In Table II, we select forty one experimental data of meson listed in PDG and optimize these meson masses to determine twelve parameters in Table I. This optimization has $\chi^2/n = 82$ which is smaller than 2638 for the GI model as shown in Table II.

Of course, besides the mass spectrum mesons with $J^{PC} = 5^{++}$ was calculated by the GI model, Ref. [42] also gives a spectrum for 5^{++} meson and we will compare them later. Finally, we can obtain the mass spectrum of these four 5^{++} states by the MGI mode list in Table III and compare our numerical result with GI model [13] and [42].

So we can conclude that

(1) The ground states of the 5^{++} states are still missing in experiments, and the predicted mass is 2.492 GeV for a_5/f_5, which are smaller than Ref. [13] and

TABLE I. Parameters and their values in this work and GI models.

Parameter	This work	GI [13]
m_u(GeV)	0.163	0.22
m_d(GeV)	0.163	0.22
m_s(GeV)	0.387	0.419
b(GeV^2)	0.221	0.18
c(GeV)	−0.240	−0.253
σ_0(GeV)	1.799	1.80
s(GeV)	1.497	1.55
μ(GeV)	0.0635	0
ϵ_c	−0.138	−0.168
ϵ_{sov}	0.157	−0.035
ϵ_{sos}	0.9726	0.055
ϵ_t	0.893	0.025

close to the result of Ref. [42]. $f_5'(1H)$ has the mass of 2.771 GeV which is smaller than Ref. [13] and Ref. [42].

(2) The first excited states of a_5/f_5 and f_5' have the mass of 2.719 GeV and 2.922 GeV, respectively. For the second excited states, $a_5/f_5(3\mathrm{H})$ and $f_5'(3\mathrm{H})$ have the mass of 2.914 GeV and 3.118 GeV, respectively, which are also smaller than Ref. [13].

The above conclusions are only from the point of mass spectra view and we will study their strong decays in the next section.

III. THE DECAY BEHAVIOR ANALYSIS

A. QPC model

The QPC model is used to obtain Okubo-Zweig-Iizuka (OZI) allowed hadronic strong decays. The QPC model is firstly proposed by Micu [18], which is further developed by Orsay group.[19,43–46]. This model was widely applied to the OZI-allowed two-body strong decay of hadrons in Refs. [1,3,5–7,20,21,24,26,28,30–35,38,47–51].

For a decay process $A \to B + C$, we can write

$$\langle BC|\mathcal{T}|A\rangle = \delta^3(\mathbf{P}_B + \mathbf{P}_C)\mathcal{M}^{M_{J_A}M_{J_B}M_{J_C}}, \tag{3.1}$$

where $\mathbf{P}_{B(C)}$ is a three-momentum of a meson $B(C)$ in the rest frame of a meson A. A superscript $M_{J_i}(i = A, B, C)$ denotes an orbital magnetic momentum. The transition operator $\mathcal{T}$ is introduced to describe a quark-antiquark pair creation from vacuum, which has the quantum number $J^{PC} = 0^{++}$, i.e., $\mathcal{T}$ can be expressed as

$$\mathcal{T} = -3\gamma\sum_m\langle 1m; 1-m|00\rangle\int d\mathbf{p}_3 d\mathbf{p}_4\delta^3(\mathbf{p}_3+\mathbf{p}_4) \times \mathcal{Y}_{1m}\left(\frac{\mathbf{p}_3-\mathbf{p}_4}{2}\right)\chi^{34}_{1,-m}\phi^{34}_0(\omega^{34}_0)_{ij}b^\dagger_{3i}(\mathbf{p}_3)d^\dagger_{4j}(\mathbf{p}_4). \tag{3.2}$$

TABLE II. The experimental data [41] fitted in our work. $\chi^2 = \frac{((\text{Th-Exp})/\text{Error})^2}{\text{D.O.F}}$, where This work, Exp, and Error represent the theoretical value, experimental results, and experimental error, respectively. We select some established $q\bar{q}$ meson states in PDG [41] for our fitting. The unit of the mass is GeV.

Component	State	This work	GI [13]	Exp	Error
$n\bar{n}$	1^1S_0	0.1397	0.1524	0.13957	0.0001
	2^1S_0	1.294	1.293	1.3	0.1
	3^1S_0	1.806	1.874	1.812	0.012
	1^1P_1	1.228	1.219	1.2295	0.0032
	2^1P_1	1.736	1.777	1.96	0.03
	3^1P_1	2.117	2.236	2.24	0.035
	1^1D_2	1.677	1.68	1.6722	0.00025
	2^1D_2	2.056	2.135	2.005	0.025
	3^1D_2	2.366	2.534	2.285	0.03
	1^1F_3	2.002	2.033	2.032	0.04
	2^1F_3	2.312	2.431	2.245	0.019
	1^3S_1	0.7744	0.7713	0.775 26	0.026
	2^3S_1	1.424	1.456	1.465	0.04
	3^3S_1	1.907	1.998	1.9	0.022
	4^3S_1	2.258	2.435	2.265	0.0005
	1^3P_0	1.191	1.087	1.474	0.016
	1^3P_1	1.235	1.238	1.23	0.015
	2^3P_1	1.756	1.818	1.647	0.032
	1^3P_2	1.317	1.307	1.3183	0.012
	2^3P_2	1.777	1.823	1.732	0.05
	1^3D_1	1.646	1.664	1.72	0.04
	2^3D_1	2.048	2.153	2.0	0.035
	3^3D_1	2.364	2.557	2.265	0.02
	2^3D_2	2.058	2.155	1.94	0.03
	3^3D_2	2.370	2.553	2.225	0.04
	1^3D_3	1.708	1.683	1.6888	0.0021
	2^3D_3	2.074	2.131	1.982	0.014
	1^3F_4	2.019	2.008	1.996	0.035
	2^3F_4	2.322	2.407	2.237	0.01
	1^3G_5	2.278	2.296	2.33	0.005
	1^3H_6	2.501	2.558	2.45	0.13
$n\bar{s}$ or $s\bar{n}$	1^1S_0	0.4953	0.4625	0.4976	0.0004
	2^1S_0	1.471	1.454	1.46	0.02
	1^3S_1	0.916	0.9028	0.8958	0.0008
	2^3S_1	1.567	1.579	1.414	0.015
	1^3P_0	1.325	1.234	1.425	0.05
	1^3P_2	1.450	1.428	1.4324	0.0013
	1^3D_1	1.765	1.776	1.717	0.027
	1^3D_3	1.826	1.794	1.776	0.007
	1^3F_4	2.126	2.108	2.045	0.009
	1^3G_5	2.378	2.388	2.382	0.024
	χ^2	82	2638		

This is completely constructed in the form of a visual representation to reflect the creation of a quark-antiquark pair from vacuum, where the quark and antiquark are denoted by indices 3 and 4, respectively.

A dimensionless parameter γ depicts the strength of the creation of $q\bar{q}$ from vacuum, where the concrete values of

TABLE III. The mass spectrum of 5^{++} states. The unit of the mass is GeV.

State	This work	GI [13]	Ebert [42]
$a_5(1H)$	2.492	2.610	2.359
$a_5(2H)$	2.719	2.941	…
$a_5(3H)$	2.922	3.255	…
$f_5(1H)$	2.492	2.610	2.359
$f_5(2H)$	2.719	2.941	…
$f_5(3H)$	2.922	3.255	…
$f'_5(1H)$	2.679	2.771	2.720
$f'_5(2H)$	2.914	3.097	…
$f'_5(3H)$	3.118	3.406	…

the parameter R which will be discussed in the later section. $\mathcal{Y}_{\ell m}(\mathbf{p}) = |\mathbf{p}|^{\ell} Y_{\ell m}(\mathbf{p})$ are the solid harmonics. χ, ϕ, and ω denote the spin, flavor, and color wave functions respectively, which can be treated separately. Subindices i and j denote the color of a $q\bar{q}$ pair.

By the Jacob-Wick formula [52], the decay amplitude is expressed as

$$\mathcal{M}^{JL}(\mathbf{P}) = \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{J_B} M_{J_C}} \langle L0; JM_{J_A} | J_A M_{J_A} \rangle \times \langle J_B M_{J_B}; J_C M_{J_C} | J_A M_{J_A} \rangle \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \quad (3.3)$$

and the general decay width reads

$$\Gamma = \frac{\pi}{4} \frac{|\mathbf{P}|}{m_A^2} \sum_{J,L} |\mathcal{M}^{JL}(\mathbf{P})|^2, \quad (3.4)$$

where m_A is the mass of an initial state A. In our calculation, we need the spatial wave functions of the discussed mesons. which can be numerically obtained by the modified GI model.

In the previous section, we obtain the mass spectrum and wave functions of the mesons. At the same time, we can use QPC model to study the strong decay of the $J^{PC} = 5^{++}$ meson families by the means of these wave functions.

As a phenomenological model of calculating strong decays of hadron, Quark pair creation (QPC) model was employed to estimate the decay behaviors of hadron. However, QPC model cannot very precisely describe experimental data. To some extent, $\rho \to \pi\pi$ cannot be reproduced well as shown in Table IV. In fact, it is a long-standing question not only for QPC model but also for other quark models like flux-tube model [53]. Just considering this point, usually we selected more typical channels to fix γ value in QPC model, where a global fit is adopted. And then, this fixed γ value is applied to calculate other decays. In this work we obtain $\gamma = 11.6$ by fitting the partial decay widths of 30 decay channels as shown in Table IV.

TABLE IV. The measured partial decay widths of 30 decay channels and the comparison with theoretical calculation (the third column). Here, the minimum of χ^2 is 636.

Decay channel	Exp (MeV) [5,8]	This work
$\rho \to \pi\pi$	151.2 ± 1.2	68.9
$b_1 \to \omega\pi$	142 ± 8	191
$\phi \to KK$	2.08 ± 0.02	1.53
$a_2 \to \eta\pi$	15.5 ± 0.7	2.65
$a_2 \to \rho\pi$	75.0 ± 4.5	66.2
$a_2 \to KK$	5.2 ± 0.2	4.53
$\pi_2 \to f_2(1270)\pi$	145.8 ± 5.1	48.6
$\pi_2 \to \rho\pi$	80.3 ± 2.8	220
$\pi_2 \to K^*K$	10.1 ± 3.4	26.7
$\rho_3 \to \pi\pi$	38 ± 2.4	61.8
$\rho_3 \to \omega\pi$	25.8 ± 1.6	44.3
$\rho_3 \to K\bar{K}$	2.5 ± 0.2	2.43
$f_2 \to \pi\pi$	156.8 ± 3.2	136
$f_2 \to K\bar{K}$	8.6 ± 0.8 s	8.31
$f_4(2050) \to \omega\omega$	54 ± 13	70.1
$f_4(2050) \to \pi\pi$	35.4 ± 3.8	79
$f_4(2050) \to KK$	1.4 ± 0.7	0.941
$f'_2(1525) \to K\bar{K}$	61 ± 5	53.7
$K^*(892) \to K\pi$	48.7 ± 0.8	18.6
$K^*(1410) \to K\pi$	15.3 ± 1.4	62.2
$K_0^*(1430) \to K\pi$	251 ± 74	291
$K_2^*(1430) \to K\pi$	54.4 ± 2.5	50.2
$K_2^*(1430) \to K^*\pi$	26.9 ± 1.2	19.9
$K_2^*(1430) \to K\rho$	9.5 ± 0.4	7.18
$K_2^*(1430) \to K\omega$	3.16 ± 0.15	2.13
$K_3^*(1780) \to K\rho$	74 ± 10	25.8
$K_3^*(1780) \to K^*\pi$	45 ± 7	28.3
$K_3^*(1780) \to K\pi$	31.7 ± 3.7	38.1
$K_4^*(2045) \to K\pi$	19.6 ± 3.8	22
$K_4^*(2045) \to K^*\phi$	2.8 ± 1.4	34.8
$\gamma = 11.6$		$\chi^2 = 636$

Next, we will analyze the strong decay behavior of these 5^{++} states.

B. The ground states

The ground state a_5 which is not observed in experiment is predicted in this work, with the mass of 2492 MeV ($a_5(2492)$), and the total width is 396 MeV. $\rho_3\pi$ is its dominant decay channel, the width is about 137 MeV, and the branch ratio is 0.36. $a_2\rho$, $\omega\rho$, $\rho\pi$, and $h_1\rho$, are its important decay channels which have the branch ratio about 0.08 each one, just as shown in Table V. The final states $b_1\omega$, $f_2\pi$, $f_4(2050)\pi$, $a_1\rho$, and $\rho(1450)\pi$, also have sizable decay widths, in which $b_1\omega$ and $f_2\pi$ almost have the same width about 20 MeV.

As the isospin partner of $a_5(1H)$, we predict $f_5(1H)$ will have the mass of 2.49 GeV and the width of 327 MeV, respectively. In the final decay channels of $f_5(1H)$, $\rho\rho$, and $b_1\rho$ will be the most important final states which have the widths of 73 MeV, and their branch ratios are about 0.22.

TABLE V. The partial decay widths of the ground states for 5^{++} family, the unit of widths is MeV.

a_5(1H)		f_5(1H)		f'_5(1H)	
Total	396	Total	327	Total	851
Channel	Value	Channel	Value	Channel	Value
$\rho_3\pi$	137	$b_1\rho$	72.9	$KK_3^*(1780)$	161
$a_2\rho$	54.2	$\rho\rho$	72.7	$K^*K_2^*(1430)$	144
$\omega\rho$	35.6	$a_2\pi$	58.4	K_1K^*	111
$\rho\pi$	31.5	$a_1\pi$	54.2	K^*K^*	85.6
$h_1\rho$	31.0	$h_1\omega$	30.3	$KK_2^*(1430)$	76.3
$b_1\omega$	23.6	$\pi_2\pi$	27.6	$KK^*(1410)$	73.0
$f_2\pi$	21.4	$KK_3^*(1780)$	5.63	KK^*	60.9
$f_4(2050)\pi$	19.0	$\rho(1450)\rho$	5.38	$KK_4^*(2045)$	49.4
$a_1\rho$	18.1			KK_1	48.9
$\rho(1450)\pi$	16.7			K'_1K^*	26.2
$KK_3^*(1780)$	5.63			$KK^*(1680)$	13.4
$\omega(1420)\rho$	2.05			$f_1(1425)\eta'$	1.31

$a_2\pi$ and $a_1\pi$ are the important decay channels too, with the widths of 58 MeV and 54 MeV, respectively. In addition, $h_1\omega$ and $\pi_2\pi$ also have visible widths of 30 MeV and 28 MeV which are presented in Table V. The widths of $KK_3(1780)$ and $\rho(1450)\rho$ are very small (see Table V), their branch ratios are about 0.017.

$f'_5(1H)$ is the $s\bar{s}$ partner of $f_5(1H)$, has the mass of 2.68 GeV and the width more than 850 MeV in our prediction. As shown in Table V, $f'_5(1H)$ mainly decays to two strange mesons for its $s\bar{s}$ component. $KK_3^*(1780)$ is the dominant decay mode with the width 161 MeV. $K^*K_2^*(1430)$, K_1K^* are also the important decay channels whose widths are over 100 MeV. Besides, K^*K^*, $KK_2^*(1430)$, $KK^*(1410)$, KK^* are its sizable final channels with the branch ratios about 0.08. KK_1, $KK_4^*(2045)$, $K^*K'_1$, $KK^*(1680)$, $KK_0^*(1430)$ are the visible decay channels of $f'_5(1H)$. Here, we do not consider the mix of the flavor between $f_5(1H)$ and $f'_5(1H)$.

C. The first excited states

In this section, we will analyze the strong decay behavior of the first excited states of 5^{++} family.

$a_5(2H)$ has the mass of 2719 MeV and narrow width of 159 MeV in our theory result. According to Table VI, $\rho_3\pi$ is the dominant decay channel of $a_5(2H)$ which is similar to $a_5(1H)$, the branch ratio is about 0.49.$\rho(1450)\pi$ and $a_0(1450)\rho$ are its important decay channels which have the branch ratios about 0.1, just as Table VI shown. The final decay modes $\pi_2\rho$, $\omega_3\rho$, a_1b_1, $\omega\rho$ and $\rho_3\omega$, also have sizable decay widths, with the width of 5–10 MeV. The other decay information is shown in Table VI.

$f_5(2H)$ as the isospin partner ($I = 0$) of $a_5(2H)$ has the mass of 2.72 GeV and width of 113 MeV, respectively. $f_5(2H)$ mainly decays to $\rho(1450)\rho$ which has the width of 44 MeV and the branch ratio 0.25. $\omega(1420)\omega$, $\rho\rho$ and $a_1\pi$ are the important final states which have the widths of 18 MeV, 9.7 MeV, and 8.2 MeV, respectively, and their branch ratios are 0.15, 0.11, and 0.10, respectively. $a_2\pi$, $\omega_3\omega a_1a_2$, and $\omega\omega$ also have visible widths from 2.3 MeV to 7 MeV which are presented in Table VI. The widths of other channels are very small (see Table VI) and their branch ratios are less than 0.02.

TABLE VI. The partial decay widths of the first excited states for 5^{++} family, the unit of widths is MeV.

a_5(2H)		f_5(2H)		f'_5(2H)	
Channel	Value	Channel	Value	Channel	Value
Total	159	Total	113	Total	637
$\rho_3\pi$	76.9	$\rho(1450)\rho$	43.5	$K^*K^*(1680)$	111
$a_0(1450)\rho$	17.9	$\omega(1420)\omega$	18.0	$KK_3^*(1780)$	96.0
$\rho(1450)\pi$	16.9	$\rho\rho$	9.66	$KK^*(1680)$	54.5
$\pi_2\rho$	9.81	$a_1\pi$	8.19	KK^*	51.6
$\omega_3\rho$	7.51	$\omega_3\omega$	7.01	K^*K^*	46.7
a_1b_1	8.97	$a_2\pi$	6.91	$K^*K^*(1410)$	43.3
$\rho_3\omega$	5.50	a_1a_2	5.88	$K_1K_2^*(1430)$	38.8
$\omega\rho$	4.76	a_2a_2	2.51	KK_1	37.6
$f_2\pi$	2.82	$\omega\omega$	2.34	$K^*K_3^*(1780)$	28.1
a_2b_1	1.72	$\pi_2\pi$	2.01	K_1K^*	27.8
f_2a_1	2.90	f_1f_2	1.75	$KK_2^*(1430)$	26.8
a_2f_2	2.08	f_2f_2	1.63	$K^*K_2^*(1430)$	22.7
a_2f_1	1.65	b_1b_1	1.63	$K'_1K^*(1410)$	16.7
		a_1a_1	1.45	$K_1K^*(1410)$	15.8
				$K^*(1410)K_2^*(1430)$	7.32
				$K_1K'_1$	5.67
				KK'_1	3.97
				$KK^*(1410)$	2.72

As the $s\bar{s}$ partner of $f_5(2H)$, $f'_5(2H)$ has the mass of 2.91 GeV and the width of 637 MeV in our prediction. According to Table VI, $f'_5(2H)$ also mainly decays into two kaon mesons. $K^*K^*(1680)$ and $KK_3^*(1780)$ are the most important decay modes with the widths of 111 MeV and 96 MeV, respectively. $KK^*(1680)$, KK^*, K^*K^*, and $K^*K^*(1410)$ are also the more important decay channels whose widths are in the range of 43–54 MeV. In addition, $K_1K_2^*(1430)$, KK_1, $K^*K_3^*(1780)$, K_1K^*, $KK_2^*(1430)$, and $K^*K_2^*(1430)$, are its sizable final channels whose branch ratios in the range of 0.04–0.07.

D. The second excited states

We also calculate the two body strong decays of the second excited states of 5^{++} family.

As the isovector meson of 5^{++} family, $a_5(3H)$ has the mass of 2.92 GeV, the total width of 103 MeV which is very narrow. $\rho_3\pi$ also is its dominant decay channel as shown in Table VII, the width is about 37.7 MeV, and the branch ratio is 0.4. $\rho(1450)\pi$ has a large ratio (0.13) in its decay final channels. $\rho\pi$, $\omega\rho$, a_1b_1, $\pi_2\rho$, $\omega_3\rho$, $\omega(1420)\rho$, and $\rho(1450)\omega$ also have sizable contribution in the total widths.

TABLE VII. The partial decay widths of the second excited states for 5^{++} family, the unit of widths is MeV.

a_5(3H)		f_5(3H)		f'_5(3H)	
Channel	Value	Channel	Value	Channel	Value
Total	103	Total	57	Total	522
$\rho_3\pi$	37.7	$\rho_3\rho$	5.8	$K_1K_2^*(1430)$	101
$\rho(1450)\pi$	12.8	$\rho\rho$	13.6	$K^*K^*(1410)$	94.8
a_1b_1	6.97	$\rho(1450)\rho$	10.1	$KK_3^*(1780)$	58.0
$\omega\rho$	6.74	a_1a_2	6.23	$KK^*(1680)$	52.3
$\rho\pi$	6.68	$a_0(1450)a_1$	5.1	$K^*K_3^*(1780)$	48.5
$\pi_2\rho$	6.58	$\omega(1420)\omega$	4.48	KK^*	41.5
$\omega_3\rho$	5.82	a_2a_2	3.82	$K^*K^*(1680)$	30.9
$\omega(1420)\rho$	4.70	$\pi_2\pi$	2.76	KK_1	20.4
$\omega\rho(1450)$	3.80	a_1a_1	2.76	$K_1K^*(1410)$	11.9
$a_2\rho$	2.82	b_1b_1	1.87	K^*K_1	11.3
f_2a_2	2.57	$a_1\pi$	0.674	$K'_1K^*(1410)$	10.7
a_2b_1	2.14			$K^*K_2^*(1430)$	8.55
f_2a_1	2.90			$K^*(1410)K^*(1410)$	8.03
b_1h_1	1.43			$KK^*(1410)$	7.72
				K_1K_1	7.37
				$K_1K'_1$	6.78
				$K_1K^*(1680)$	2.61

$f_5(3H)$ state is the second radial excited state of f_5, with the mass of 2.92 GeV and width of 57 MeV. $f_5(3H)$ mainly decays into $\rho\rho$ and $\rho(1450)\rho$, whose decay widths are 13.6 MeV and of 10.1 MeV, respectively, and each channel almost has the branch ratio 0.2. $a_1a_2, \rho_3\rho$, and $\omega(1420)\omega$ modes are the important decay channels too, with the widths about 5 MeV. In addition, a_2a_2, $a_0(1450)a_1$, and $\pi_2\pi$ also have visible widths which are presented in Table VII. The width of other modes are very small (see Table VII).

$f'_5(3H)$ has the $s\bar{s}$ component as the partner of $f_5(3H)$ which has the mass of 3.12 GeV. $f'_5(3H)$ has the total width of 522 MeV in our calculation. Just as shown in Table VII, $f'_5(3H)$ mainly decays to $K_1K_2^*(1430)$ and $KK^*(1410)$ with the widths of 101 MeV and 95 MeV. $KK_3^*(1780)$, $KK^*(1680)$, $K^*K_3^*(1780)$, and KK^* are its important decay channels with the widths of 58 MeV, 52 MeV, 49 MeV, and 42 MeV, respectively.

$K^*K^*(1680)$, KK_1, $K_1K^*(1410)$, K^*K_1, and $K'_1K^*(1410)$ are its sizable final channels with the branch ratios of 0.06, 0.04, 0.023, 0.022, and 0.02, respectively. Besides, $K^*K_2^*(1430)$, $K^*(1410)K^*(1410)$, $KK^*(1410)$, K_1K_1, and $K_1K'_1$) have the visible contribution to the total width too. The other modes have very small widths in the final states of $f'_5(3H)$.

IV. CONCLUSION

In this paper, we study the spectrum and two body strong decay of the family with $J^{PC} = 5^{++}$ which is still missing in experiment. By the modified Godfrey-Isgur model with a color screening effect, we analyze the mass spectrum of a_5 and f_5 mesons, in which we find that the ground states of the 5^{++} states, a_5, f_5, and f'_5 have the mass of 2.492 GeV, 2.492 GeV, and 2.68 GeV and the widths of 400 MeV, 330 MeV, and 850 MeV, respectively. The first excited states of a_5/f_5 and f'_5 have the mass of 2.719 GeV and 2.922 GeV and $a_5(3H)/f_5(3H)$ and $f'_5(3H)$ have the mass of 2.914 GeV and 3.118 GeV, respectively. The total widths are predicted to be 160 MeV(a_5(2H)), 110 MeV(f_5(2H)), and 640 MeV(f'_5(2H)) for the first excited states. For the second excited states of 5^{++}, a_5(3H), f_5(3H), and $f'_5(3H)$ have the widths of 100 MeV(a_5), 60 MeV(f_5), and 430 MeV(f'_5), respectively.

We also predict the detail of decay information of 5^{++} family using QPC model which can be helpful to search the mesons in the future experiments just as BESIII and COMPASS.

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