Exotic $bc\bar{q}\bar{q}$ four-quark states

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We carry out a systematic study of exotic $QQ'\bar{q}\bar{q}$ four-quark states containing distinguishable heavy flavors, b and c. Different generic constituent models are explored in an attempt to extract general conclusions. The results are robust, predicting the same sets of quantum numbers as the best candidates to lodge bound states independently of the model used, the isoscalar $J^P = 0^+$ and $J^P = 1^+$ states. The first state would be strong and electromagnetic-interaction stable, while the second would decay electromagnetically to $\bar{B}D\gamma$. Isovector states are found to be unbound, preventing the existence of charged partners. The interest on exotic heavy-light tetraquarks with nonidentical heavy flavors comes reinforced by the recent estimation of the production rate of the isoscalar $bc\bar{u}\bar{d}$ $J^P = 1^+$ state, 2 orders of magnitude larger than that of the $bb\bar{u}\bar{d}$ analogous state.

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I. INTRODUCTION

Among the flavor sectors where four-quark bound states may exist, there is one of particular interest, the so-called exotic heavy-light four-quark sector. The possible existence of stable $QQ\bar{q}\bar{q}$ states has been addressed using different approaches since the pioneering work of Ref. [1]. Exotic heavy-light four-quark states represent a very interesting exception in the landscape of exotic hadronic physics, because there is a broad theoretical consensus about its adequacy to lodge bound states for large M_Q/m_q ratios. In particular, there is a long-standing prediction, strengthened by several independent studies during the last years, about the existence of a deeply bound $bb\bar{u}\bar{d}$ isoscalar state with quantum numbers $J^P = 1^+$ [1–10]. In the charm sector, the decrease of the mass ratio M_Q/m_q might give rise just to a shallow bound state with the same quantum numbers [11,12].

In between $bb\bar{q}\bar{q}$ and $cc\bar{q}\bar{q}$, one finds the case with two distinguishable heavy quarks, $bc\bar{q}\bar{q}$, which has not received the same attention in the literature. The nonidentity of the heavy flavors enlarges the Hilbert space, and, thus, conclusions cannot be straightforwardly extrapolated from the case of identical heavy flavors. $QQ'\bar{q}\bar{q}$ states have been studied in Refs. [13,14] solving the four-body problem by expanding the wave function up to eight quanta in a harmonic oscillator basis. Two isoscalar $bc\bar{u}\bar{d}$ states close to threshold were identified as candidates to be bound, the $J^P = 0^+$ and 1^+. Note, however, that systematic expansions on the eigenstates of a harmonic oscillator is not very efficient to account for short-range correlations and could miss binding when it is induced by chromomagnetic effects [7,11]. Reference [3] has estimated the mass of the isoscalar $J^P = 0^+$ $bc\bar{u}\bar{d}$ state obtaining a central value 11 MeV below the $\bar{B}D$ threshold, although it is cautioned that the precision of the calculation is not sufficient to determine whether the tetraquark is actually above or below the corresponding two-meson threshold. Unfortunately, this reference has not analyzed the isoscalar $bc\bar{u}\bar{d}$ $J^P = 1^+$ state. QCD sum rules analysis concluded that the $bc\bar{q}\bar{q}$ $J^P = 0^+$ and 1^+ states are below the $D^*\bar{B}^*$ threshold [15]. The interest on exotic heavy-light tetraquarks with nonidentical heavy flavors comes reinforced by the recent estimation of the production rate of the isoscalar $bc\bar{u}\bar{d}$ $J^P = 1^+$ state at the LHCb, 2 orders of magnitude larger than that of the $bb\bar{u}\bar{d}$ analogous state [16].

In this work, we adopt generic constituent models to address four-quark systems containing distinguishable charm and bottom heavy flavors. We use two different methods to look for possible bound states, a variational approach with generalized Gaussians and the scattering of two mesons with different heavy flavor content. The

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manuscript is organized as follows. In Sec. II, we outline the relevant properties of the constituent models and the methods considered. In Sec. III, we present and discuss the results. Finally, the main conclusions are summarized in Sec. IV.

II. SOLVING THE $bc\bar{q}\bar{q}$ SYSTEM

For the sake of generality and to judge the independence of the results from the particular model considered, two different constituent models widely used in the tetraquark literature are implemented. The first one is the AL1 potential by Semay and Silvestre-Brac [14]. It contains a chromoelectric part made of a Coulomb-plus-linear interaction together with a chromomagnetic spin-spin term described by a regularized Breit-Fermi interaction with a smearing parameter that depends on the reduced mass of the interacting quarks. The second one is the constituent quark cluster (CQC) model of Ref. [17]. Besides chromoelectric and chromomagnetic terms analogous to the AL1 potential, it considers a chiral potential between light quarks. The main advantage of these models is that they reasonably describe the heavy meson spectra, and, thus, the thresholds relevant for each particular set of quantum numbers are correctly described within the same model.

Two different methods are used to tackle the possible existence of four-quark bound states. In the first one, we use a variational approach, where the wave function is expanded as a linear combination of all allowed vectors in color, spin, flavor, and radial subspaces. For the radial part, we make use of generalized Gaussians. The basis dimension quickly escalates with the number of allowed vectors, and therefore the numerical treatment becomes increasingly challenging although tractable. In the second approach, an expansion in terms of all contributing physical meson-meson states is considered. Within this scheme, the meson-meson interaction is obtained from the quark-quark potential, and then a two-body coupled-channel problem is solved. The equivalence of the two methods for the two-baryon system was theoretically derived in Ref. [18]. For the two-meson case, it was mathematically proven in Ref. [19] and numerically checked in Refs. [19,20].

To be a bit more specific, let us note that four-quark systems present a richer color structure than standard baryons or mesons. The color wave function for standard hadrons leads to a single vector, but dealing with four-quark hadrons, there are different vectors driving to a color singlet state out of colorless meson-meson (**11**) or colored two-body (**88**, $\bar{\mathbf{3}}\mathbf{3}$, or $\mathbf{6}\bar{\mathbf{6}}$) components. Note, however, that any colored two-body component can be expanded as an infinite sum of colorless singlet-singlet states [18]. This has been explicitly done for $QQ\bar{q}\bar{q}$ states in Ref. [19].

The lowest lying tetraquark configuration for systems with two heavy flavors presents a separate dynamics for the heavy quarks, in a color $\bar{\mathbf{3}}$ state, and for the light quarks, bound to a color **3** state, to construct a color singlet [4] (see the probabilities in Table II of Ref. [19] for the isoscalar axial vector $bb\bar{u}\bar{d}$ tetraquark). This argument has been recently revised in Ref. [7], showing in Fig. 8 how the probability of the $\mathbf{6}\bar{\mathbf{6}}$ component in a compact $QQ\bar{q}\bar{q}$ tetraquark tends to zero for $M_Q \to \infty$. Therefore, heavy-light compact bound states would be a dominant $\bar{\mathbf{3}}\mathbf{3}$ color state and not a single colorless meson-meson molecule, **11**. Such compact states with two-body colored components can be expanded as the mixture of several physical meson-meson channels [18] (see Table II of Ref. [19]), and, thus, they can be also studied as an involved coupled-channel problem of physical meson-meson states [20,21].

Let us summarize in the following subsections the main properties of the two methods used to look for bound states throughout this work.

A. Four-quark systems

The $bc\bar{q}\bar{q}$ four-quark problem has been solved following the variational method outlined in Ref. [22], expanding the radial wave function in terms of generalized Gaussians. The constituent model used is AL1. The variational wave function must include all possible flavor-spin-color channels contributing to a given configuration. Thus, for each channel s, the wave function will be the tensor product of color ($|C_n\rangle$), spin ($|S_m\rangle$), flavor ($|T_k\rangle$), and radial ($|R_r\rangle$) components,

$$|n\,m\,k\,r\rangle = |C_n\rangle \otimes |S_m\rangle \otimes |T_k\rangle \otimes |R_r\rangle. \qquad (1)$$

Once the color, spin, and flavor parts are integrated out, the coefficients of the radial wave function are obtained by solving the system of linear equations,

$$\sum_{s's}\sum_{i}\beta_r^{(i)}[\langle R_{r'}^{(j)}|H|R_r^{(i)}\rangle - E\langle R_{r'}^{(j)}|R_r^{(i)}\rangle\delta_{s,s'}] = 0 \quad \forall\, j, \qquad (2)$$

where the eigenvalues are obtained by a minimization procedure.

Let us discuss briefly the different terms outlined in the wave function of Eq. (1). The flavor part is uniquely determined by the isospin of the light antiquark pair: $|T_1\rangle$ for $T = 0$ and $|T_2\rangle$ for $T = 1$. The spin part of the wave function can be written as $[(\frac{1}{2}\frac{1}{2})_{S_{12}}(\frac{1}{2}\frac{1}{2})_{S_{34}}]_S \equiv |S_{12}S_{34}\rangle$, where the spin of the two quarks (antiquarks) is coupled to S_{12} (S_{34}). In Table I, we have summarized the vectors contributing to each total spin state, S.

TABLE I. Spin basis vectors, $|S_{12}S_{34}\rangle$, for the different total spin states, S.

$S = 0$	$S = 1$	$S = 2$
$\lvert S_1\rangle = \lvert 00\rangle$	$\lvert S_1\rangle = \lvert 10\rangle$	$\lvert S_1\rangle = \lvert 11\rangle$
$\lvert S_2\rangle = \lvert 11\rangle$	$\lvert S_2\rangle = \lvert 01\rangle$	
	$\lvert S_3\rangle = \lvert 11\rangle$	

The most general radial wave function with total orbital angular momentum $L = 0$ is constructed as a linear combination of generalized Gaussians depending on a set of variational parameters. The usual four-body H-like Jacobi coordinates are considered,

$$\vec{x} = \vec{r}_1 - \vec{r}_2$$
$$\vec{y} = \vec{r}_3 - \vec{r}_4$$
$$\vec{z} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} - \frac{m_3\vec{r}_3 + m_4\vec{r}_4}{m_3 + m_4}$$
$$\vec{R} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}, \tag{3}$$

where 1 and 2 stand for the quarks and 3 and 4 stand for the antiquarks. Thus, we define the function

$$g(\alpha_1,\alpha_2,\alpha_3) = \mathrm{Exp}(-a^i\vec{x}^2 - b^i\vec{y}^2 - c^i\vec{z}^2 - \alpha_1 d^i\vec{x}\cdot\vec{y} - \alpha_2 e^i\vec{x}\cdot\vec{z} - \alpha_3 f^i\vec{y}\cdot\vec{z}), \tag{4}$$

and the vectors

$$\overrightarrow{G^i} = \begin{pmatrix} g(+,+,+) \\ g(-,+,-) \\ g(-,-,+) \\ g(+,-,-) \end{pmatrix}, \tag{5}$$

and

$$\overrightarrow{\alpha_{SS}} = (+,+,+,+),$$
$$\overrightarrow{\alpha_{SA}} = (+,-,+,-),$$
$$\overrightarrow{\alpha_{AS}} = (+,+,-,-),$$
$$\overrightarrow{\alpha_{AA}} = (+,-,-,+), \tag{6}$$

where $S(A)$ stands for symmetric (antisymmetric) under the exchange of quarks $1 \leftrightarrow 2$ and antiquarks $3 \leftrightarrow 4$. Then, four different radial wave functions can be constructed depending on their permutation properties:

$$(SS) \Rightarrow |R_1\rangle = \sum_{i=1}^{n} \beta_1^{(i)}(\overrightarrow{\alpha_{SS}}\cdot\overrightarrow{G^i}),$$
$$(SA) \Rightarrow |R_2\rangle = \sum_{i=1}^{n} \beta_2^{(i)}(\overrightarrow{\alpha_{SA}}\cdot\overrightarrow{G^i}),$$
$$(AS) \Rightarrow |R_3\rangle = \sum_{i=1}^{n} \beta_3^{(i)}(\overrightarrow{\alpha_{AS}}\cdot\overrightarrow{G^i}),$$
$$(AA) \Rightarrow |R_4\rangle = \sum_{i=1}^{n} \beta_4^{(i)}(\overrightarrow{\alpha_{AA}}\cdot\overrightarrow{G^i}). \tag{7}$$

The scalar product $\overrightarrow{\alpha_{jk}}\cdot\overrightarrow{G^i}$ generates the appropriate combination of generalized Gaussians to have the specified symmetry jk (see Sec. 2.9 of Ref. [22] for a thorough discussion of the technical details), and n is the number of generalized Gaussians required to reach convergence. The terms mixing Jacobi coordinates, i.e., $\vec{x}\cdot\vec{y}$, $\vec{x}\cdot\vec{z}$, and $\vec{y}\cdot\vec{z}$, allow for nonzero internal orbital angular momenta, although the total orbital angular momentum is coupled to $L = 0$. This ensures the positive parity of the states studied.

Finally, regarding the color structure, there are three different ways to couple two quarks and two antiquarks into a colorless state:

$$[(q_1q_2)(\bar{q}_3\bar{q}_4)] \equiv \{|\bar{3}_{12}3_{34}\rangle, |6_{12}\bar{6}_{34}\rangle\} \equiv \{|\bar{3}3\rangle, |6\bar{6}\rangle\}, \tag{8}$$

$$[(q_1\bar{q}_3)(q_2\bar{q}_4)] \equiv \{|1_{13}1_{24}\rangle, |8_{13}8_{24}\rangle\} \equiv \{|11\rangle, |88\rangle\}, \tag{9}$$

$$[(q_1\bar{q}_4)(q_2\bar{q}_3)] \equiv \{|1_{14}1_{23}\rangle, |8_{14}8_{23}\rangle\} \equiv \{|1'1'\rangle, |8'8'\rangle\}. \tag{10}$$

Each coupling scheme represents a color orthonormal basis where the four-quark problem can be studied. Only two of these states have well-defined permutation properties: $|\bar{3}3\rangle$ is antisymmetric under the exchange of both quarks and antiquarks, and $|6\bar{6}\rangle$ is symmetric. Therefore, the basis (8) is the most suitable to deal with the Pauli principle. The other two, Eqs. (9) and (10), are hybrid bases containing singlet-singlet (physical) and octet-octet (hidden-color) vectors, that are required to extract meson-meson physical components from the final wave function. In the following, we denote $|C_1\rangle = |\bar{3}3\rangle$ and $|C_2\rangle = |6\bar{6}\rangle$.

The system $bc\bar{q}\bar{q}$ contains two identical light antiquarks; therefore, the Pauli principle has to be applied to this pair. A summary of all vectors allowed for the different spin-isospin channels is given in Table II. Further details on the formalism can be obtained from Refs. [22,23], and references therein.

TABLE II. $|n\,m\,k\,r\rangle$ basis vectors, with the notation of Eq. (1), contributing to each total spin and isospin state, (S, T).

(0,0)	(0,1)	(1,0)	(1,1)	(2,0)	(2,1)
$\|1111\rangle$	$\|1122\rangle$	$\|1112\rangle$	$\|1121\rangle$	$\|1112\rangle$	$\|1121\rangle$
$\|1113\rangle$	$\|1124\rangle$	$\|1114\rangle$	$\|1123\rangle$	$\|1114\rangle$	$\|1123\rangle$
$\|1212\rangle$	$\|1221\rangle$	$\|1211\rangle$	$\|1122\rangle$	$\|2111\rangle$	$\|2122\rangle$
$\|1214\rangle$	$\|1223\rangle$	$\|1213\rangle$	$\|1224\rangle$	$\|2113\rangle$	$\|2124\rangle$
$\|2112\rangle$	$\|2121\rangle$	$\|1312\rangle$	$\|1321\rangle$		
$\|2114\rangle$	$\|2123\rangle$	$\|1314\rangle$	$\|1323\rangle$		
$\|2211\rangle$	$\|2222\rangle$	$\|2111\rangle$	$\|2122\rangle$		
$\|2213\rangle$	$\|2224\rangle$	$\|2113\rangle$	$\|2124\rangle$		
		$\|2212\rangle$	$\|2221\rangle$		
		$\|2214\rangle$	$\|2223\rangle$		
		$\|2311\rangle$	$\|2322\rangle$		
		$\|2313\rangle$	$\|2324\rangle$		

TABLE III. Meson-meson channels contributing to each total spin and isospin state, (S, T).

(0,0),(0,1)	(1,0),(1,1)	(2,0),(2,1)
$\bar{B}D$	$\bar{B}^*D$	$\bar{B}^*D^*$
$\bar{B}^*D^*$	$\bar{B}D^*$	
	$\bar{B}^*D^*$	

B. Meson-meson systems

The $bc\bar{q}\bar{q}$ four-quark problem has also been addressed by solving the Lippmann-Schwinger equation for a two-meson coupled-channel problem. All allowed meson-meson components made of the lowest S-wave mesons, $\bar{B}$, D, $\bar{B}^*$, and D^*, have been considered; see Table III. The number of coupled channels in the meson-meson approach increases with the number of allowed vectors in the four-quark formalism, as seen in Table II.

Thus, we consider a system of two mesons interacting through a potential V that has been obtained from the CQC model of Ref. [17]. If we denote the different meson-meson systems as channel A_i, the Lippmann-Schwinger equation for the meson-meson scattering becomes

$$\begin{aligned} t_{\alpha\beta;JT}^{L_\alpha S_\alpha, L_\beta S_\beta}(p_\alpha, p_\beta; E) &= V_{\alpha\beta;JT}^{L_\alpha S_\alpha, L_\beta S_\beta}(p_\alpha, p_\beta) \\ &+ \sum_{\gamma=A_1,A_2,\ldots} \sum_{L_\gamma=0,2} \int_0^\infty p_\gamma^2 dp_\gamma V_{\alpha\gamma;JT}^{L_\alpha S_\alpha, L_\gamma S_\gamma}(p_\alpha, p_\gamma) \\ &\times G_\gamma(E; p_\gamma) t_{\gamma\beta;JT}^{L_\gamma S_\gamma, L_\beta S_\beta}(p_\gamma, p_\beta; E), \alpha,\beta = A_1, A_2, \ldots, \end{aligned} \tag{11}$$

where t is the two-body scattering amplitude; J and T are the total angular momentum and isospin of the system; and $L_\alpha S_\alpha$, $L_\gamma S_\gamma$, and $L_\beta S_\beta$ are the initial, intermediate, and final orbital angular momentum and spin, respectively. p_α (p_β) stands for the initial (final) relative momentum of the two-body system that enters in the Fourier transform of the potential in configuration space, and E is the total energy of the two-body system. p_γ is the relative momentum of the intermediate two-body system γ. We refer the reader to Ref. [24] for a thorough discussion of the technical details about the solution of the momentum-space Lippmann-Schwinger equation.

The propagators $G_\gamma(E; p_\gamma)$ are given by

$$G_\gamma(E; p_\gamma) = \frac{2\mu_\gamma}{k_\gamma^2 - p_\gamma^2 + i\epsilon}, \tag{12}$$

with

$$E = \frac{k_\gamma^2}{2\mu_\gamma}, \tag{13}$$

where μ_γ is the reduced mass of the two-body system γ. For bound-state problems, $E < 0$ so that the singularity of the propagator is never touched and we can forget the $i\epsilon$ in the denominator. If we make the change of variables

$$p_\gamma = b\frac{1 + x_\gamma}{1 - x_\gamma}, \tag{14}$$

where b is a scale parameter, and the same for p_α and p_β, we can write Eq. (11) as

$$\begin{aligned} t_{\alpha\beta;JT}^{L_\alpha S_\alpha, L_\beta S_\beta}(x_\alpha, x_\beta; E) = V_{\alpha\beta;JT}^{L_\alpha S_\alpha, L_\beta S_\beta}(x_\alpha, x_\beta) + \sum_{\gamma=A_1,A_2,\ldots} \sum_{L_\gamma=0,2} \int_{-1}^1 b^2 \left(\frac{1+x_\gamma}{1-x_\gamma}\right)^2 \frac{2b}{(1-x_\gamma)^2} dx_\gamma \\ \times V_{\alpha\gamma;JT}^{L_\alpha S_\alpha, L_\gamma S_\gamma}(x_\alpha, x_\gamma) G_\gamma(E; p_\gamma) t_{\gamma\beta;JT}^{L_\gamma S_\gamma, L_\beta S_\beta}(x_\gamma, x_\beta; E). \end{aligned} \tag{15}$$

We solve this equation by replacing the integral from -1 to 1 by a Gauss-Legendre quadrature, which results in the set of linear equations,

$$\sum_{\gamma=A_1,A_2,\ldots} \sum_{L_\gamma=0,2} \sum_{m=1}^N M_{\alpha\gamma;JT}^{nL_\alpha S_\alpha, mL_\gamma S_\gamma}(E) t_{\gamma\beta;JT}^{L_\gamma S_\gamma, L_\beta S_\beta}(x_m, x_k; E) = V_{\alpha\beta;JT}^{L_\alpha S_\alpha, L_\beta S_\beta}(x_n, x_k), \tag{16}$$

with

$$M_{\alpha\gamma;JT}^{nL_\alpha S_\alpha, mL_\gamma S_\gamma}(E) = \delta_{nm}\delta_{L_\alpha L_\gamma}\delta_{S_\alpha S_\gamma} - w_m b^2 \left(\frac{1+x_m}{1-x_m}\right)^2 \frac{2b}{(1-x_m)^2} V_{\alpha\gamma;JT}^{L_\alpha S_\alpha, L_\gamma S_\gamma}(x_n, x_m) G_\gamma(E; p_{\gamma m}), \tag{17}$$

and where w_m and x_m are the weights and abscissas of the Gauss-Legendre quadrature while $p_{\gamma m}$ is obtained by putting $x_\gamma = x_m$ in Eq. (14). If a bound state exists at an energy E_B, the determinant of the matrix $M_{\alpha\gamma;JT}^{nL_\alpha S_\alpha, mL_\gamma S_\gamma}(E_B)$ vanishes, i.e., $|M_{\alpha\beta;JT}(E_B)| = 0$.

TABLE IV. S-wave thresholds for each total spin, S, state.[1] Energies are in MeV.

$S=0$	$S=1$	$S=2$
$\bar{B}D$ (7155)		
	$\bar{B}^*D$ (7212)	
	$\bar{B}D^*$ (7309)	
$\bar{B}^*D^*$ (7366)	$\bar{B}^*D^*$ (7366)	$\bar{B}^*D^*$ (7366)

C. Thresholds

One of the most important aspects for stability studies on tetraquark spectroscopy, often overlooked in the literature, is the determination of the two-meson breakup thresholds using the same interacting model, hypothesis, and approximations considered for the four-quark or two-meson study. Due to the presence of heavy quarks of different flavors, no antisymmetry restrictions apply to the final two-meson states; therefore, all possible isospin values, $T=0$ and 1, are allowed for each spin state. The structure and energy of the different S-wave thresholds contributing to each spin state are given in Table IV.

III. RESULTS

The results obtained following the procedure outlined in Sec. II A are given in Table V. In all cases, the results have been converged till the energy difference between $n-1$ and n generalized Gaussians is less than 1 MeV. Using six generalized Gaussians, the energy of all states is fully converged. The results obtained with the meson-meson formalism of Sec. II B are shown in Fig. 1, where we have plotted the Fredholm determinant as a function of the energy, $E=0$ being the mass of the lowest threshold allowed for the corresponding channel.

The overall conclusion that can be drawn from Table V and Fig. 1 is the existence of isoscalar bound states with $J^P=0^+$ and $J^P=1^+$, independently of the constituent model considered. Note that, although both constituent models correctly reproduce the meson masses entering the thresholds, which guarantees the similarity of the meson wave functions, there are particularities that coherently explain the small differences between them. The CQC model of Ref. [17] contains boson exchanges between the light quarks and thus a weaker one-gluon exchange chromomagnetic interaction. The slightly smaller bindings derived from the CQC model do also give rise to the unbound nature of the isoscalar $J^P=2^+$ channel, barely bound with the AL1 model. In both approaches, isovector states are found to be unbound precluding the existence of charged counterparts. The $J^P=0^+$ state would be strong and electromagnetically stable, while the $J^P=1^+$ would decay electromagnetically to $\bar{B}D\gamma$. It is worth noting that the stability of the isoscalar $J^P=0^+$ $bc\bar{u}\bar{d}$ state has recently been suggested in Ref. [3] with a central value for its mass 11 MeV below the $\bar{B}D$ threshold, although, as mentioned in the Introduction, it is cautioned that the precision of the calculation is not sufficient to determine whether the $bc\bar{u}\bar{d}$ tetraquark is actually above or below the corresponding two-meson threshold. Anyhow, it could manifest itself as a narrow resonance just at threshold. Unfortunately, this reference has not analyzed the $J^P=1^+$ state. The production rate of the isoscalar $bc\bar{u}\bar{d}$ $J^P=1^+$ state at the LHCb has been recently estimated in Ref. [16]. They have obtained a cross section II orders of magnitude larger than that of the production of the $bb\bar{u}\bar{d}$ analogous state. Thus, exotic heavy-light tetraquarks with nonidentical heavy flavors have an excellent discovery potential at the LHCb.

TABLE V. Four-quark energy, E_{4q}, probability of the different color channels, P_{C_1} and P_{C_2}, energy of the lowest threshold, E_{Th}, and binding energy, $B=E_{4q}-E_{Th}$, of the different $bc\bar{q}\,\bar{q}$ spin-isospin states, (S,T), obtained using the AL1 model. Energies are in MeV.

(S,T)	E_{4q}	P_{C_1}	P_{C_2}	E_{Th}	B
(0,0)	7132	0.49	0.51	7155	−23
(0,1)	7194	$\frac{1}{3}$	$\frac{2}{3}$	7155	+39
(1,0)	7189	0.61	0.39	7212	−23
(1,1)	7245	$\frac{1}{3}$	$\frac{2}{3}$	7212	+33
(2,0)	7363	0.26	0.74	7366	−3
(2,1)	7383	$\frac{1}{3}$	$\frac{2}{3}$	7366	+17

At first glance, the $bc\bar{q}\bar{q}$ system may look deceptively similar to the $cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ ones. However, the existence of two distinguishable heavy quarks is a major difference. This makes it possible that a large number of basis vectors contributes to a particular set of quantum numbers, some of which are forbidden in a system with identical heavy flavors. As we discuss below, they are relevant to understanding the dynamics of the $bc\bar{q}\bar{q}$ bound states.

Let us analyze how the dynamics of thresholds, see Fig. 2, a consequence of the larger Hilbert space, helps in understanding the results obtained for the $bc\bar{q}\bar{q}$ system [25]. For this purpose, and without loss of generality, we restrict ourselves to the isoscalar axial vector $J^P=1^+$ bound state, existing in both the sector with identical and the sector with nonidentical heavy flavors. We consider the AL1 model, where the $bb\bar{u}\bar{d}$ state is bound by about 150 MeV [7] while the $cc\bar{u}\bar{d}$ state is bound by about 3 MeV [11]. The chromomagnetic interaction, suppressed by M_Q, is stronger in the charm sector than in the bottom one, which generates larger matrix elements between color-spin

[1]For the sake of simplicity, only the energies corresponding to the AL1 constituent model are shown, those of the CQC being rather similar [17].

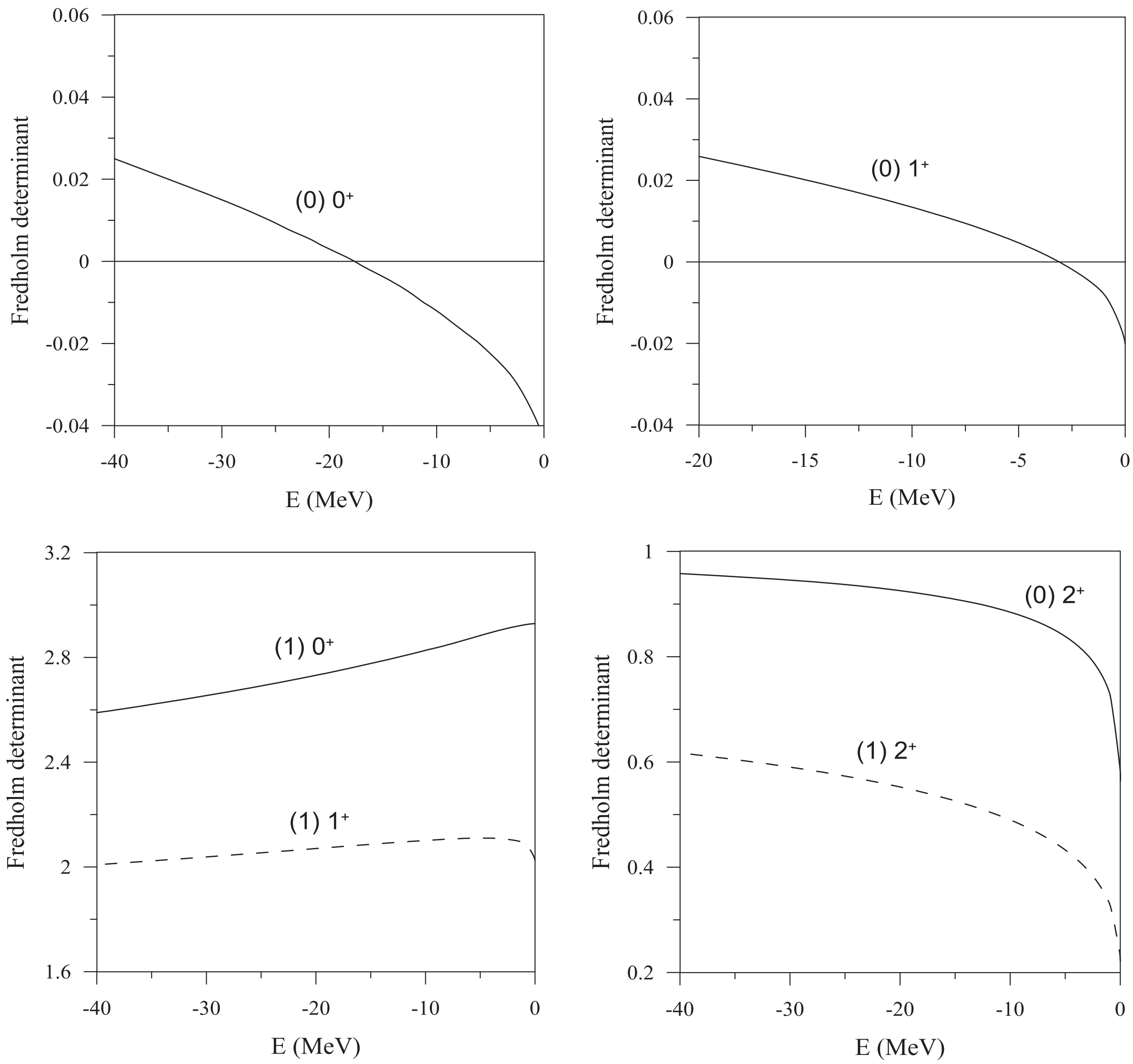

FIG. 1. Fredholm determinant of the different $(T)J^P$ $bc\bar{q}\,\bar{q}$ channels.

vectors of the pseudoscalar-vector and vector-vector two-meson thresholds. However, the mass difference between the allowed thresholds increases from 57 MeV in the bottom sector to 154 in the charm one, which weakens the coupling between different color-spin vectors of the pseudoscalar-vector and vector-vector two-meson thresholds. Thus, as the uncoupled pseudoscalar-vector two-meson systems, DD^* or $\bar{B}\bar{B}^*$, do not show bound states [20,22] and the coupling to the vector-vector two-meson channels, D^*D^* or $\bar{B}^*\bar{B}^*$, is weaker in the charm that in the bottom sector, one unavoidable arrives to a reduction of the binding energy from 150 to 3 MeV when moving from $bb\bar{q}\bar{q}$ to $cc\bar{q}\bar{q}$ systems. If we now consider the isoscalar $bc\bar{u}\bar{d}$ $J^P = 1^+$ state, the mass difference between the $\bar{B}^*D$ and $\bar{B}^*D^*$ thresholds is the same as in the charm case, but the chromomagnetic interaction involving the bottom quark is weakened by a factor $m_b/m_c \sim 3$. Then, one would expect to get a smaller binding energy than in the charm sector. However, the results shown in Table V exhibit a different trend, with a larger binding energy. The nice feature of the $bc\bar{q}\bar{q}$ state is that it contains distinguishable

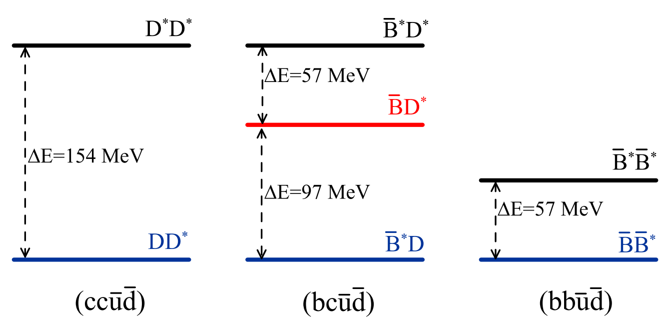

FIG. 2. Two-meson thresholds for the isoscalar $J^P = 1^+$ $cc\bar{u}\bar{d}$, $bc\bar{u}\bar{d}$, and $bb\bar{u}\bar{d}$ states.

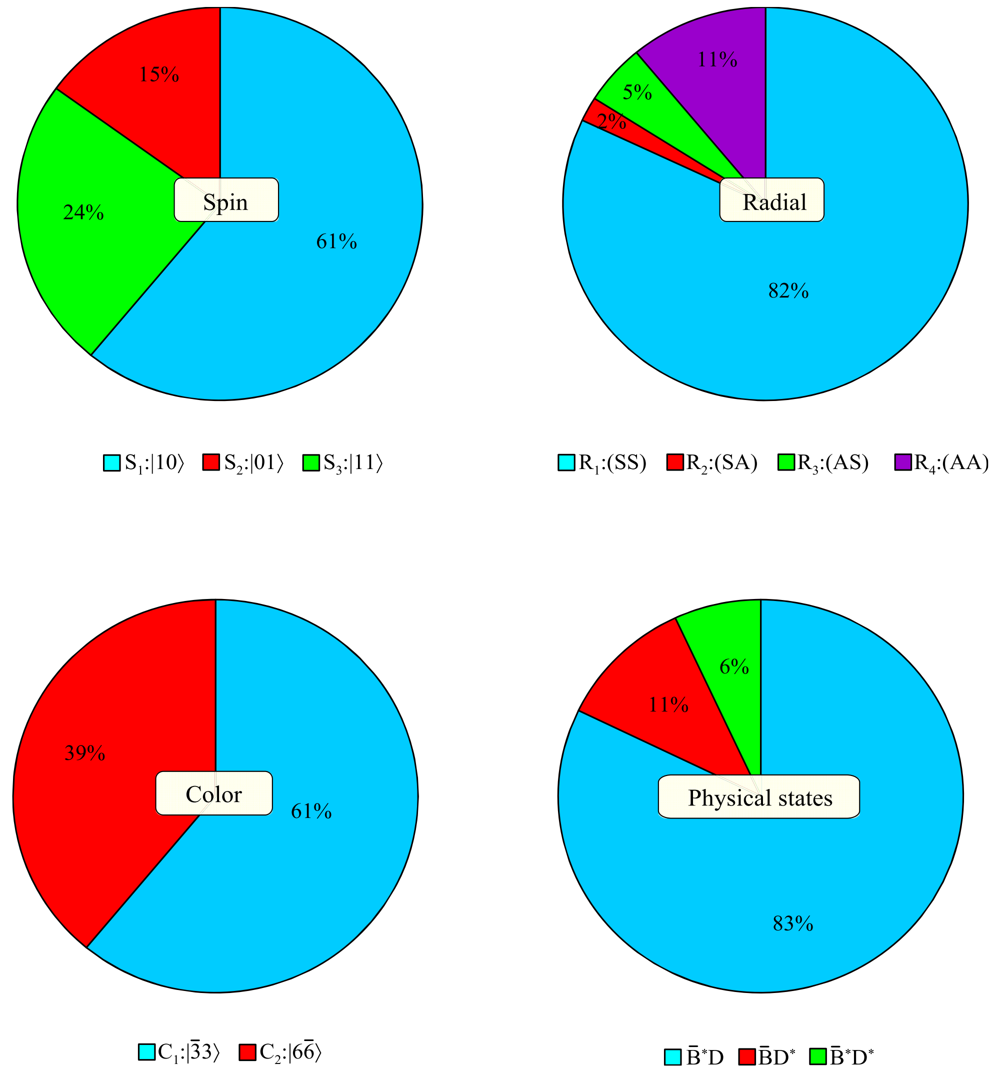

FIG. 3. Detailed structure of the isoscalar $bc\bar{u}\,\bar{d}$ $J^P = 1^+$ wave function. The first three panels show the probability of the different spin, radial, and color vectors. The last panel shows the decomposition of the wave function in terms of the singlet-singlet color vectors of bases (9) and (10).

heavy quarks, and, thus, a new threshold (a larger Hilbert space in the language of four-quark states) appears in the $J^P = 1^+$ state, the $\bar{B}D^*$, in between $\bar{B}^*D$ and $\bar{B}^*D^*$. Although the $\bar{B}^*D$ and $\bar{B}D^*$ systems cannot couple directly, nevertheless, they are coupled through the higher $\bar{B}^*D^*$ state, i.e., $\bar{B}^*D \leftrightarrow \bar{B}^*D^* \leftrightarrow \bar{B}D^*$. Because the mass difference between $\bar{B}^*D$ and $\bar{B}D^*$ is smaller than between DD^* and D^*D^*, the mixing is reinforced as compared to the charm case, driving to a binding energy larger than in the charm sector. The dynamics of thresholds to enhance or diminish coupled-channel effects has been illustrated at length in the literature [25–28], although to the best of our knowledge, this is the first example where the presence of an additional intermediate threshold induced by the nonidentity of the heavy quarks helps in increasing the binding.

Thus, the connection between the two proposed methodologies is amazing, and it can be analytically derived through the formalism developed in Ref. [19]. It allows one to extract the probabilities of meson-meson physical channels out of a four-quark wave function expressed as a linear combination of color-spin-flavor-radial vectors. We show in Fig. 3 a summary of the color, spin, radial, and meson-meson component probabilities for the isoscalar $J^P = 1^+$ $bc\bar{u}\bar{d}$ bound state. It is worth noting the 11% probability of the $\bar{B}D^*$ component, induced by the indirect coupling to the lowest $\bar{B}^*D$ state through the highest $\bar{B}^*D^*$ component. As has been recently discussed [7], these results present sound evidence about the importance of including a complete basis, i.e., not discarding any set of components *a priori*. In addition, the importance of a complete radial wave function considering terms mixing

Jacobi coordinates, thus able to accommodate the antisymmetric terms reported in Fig. 3, becomes apparent. Unless it is done that way, one is in front of approximations driving to unchecked results [7]. In particular, removing the $6\bar{6}$ color components or antisymmetric terms in the wave function leads to unbound states for all quantum numbers.

IV. OUTLOOK

Heavy-light four-quark states containing a pair of identical b or c heavy quarks have been widely discussed in the literature for the last 40 years. However, this has not been the case when heavy quarks of different flavor are considered. Thus, in this work, the possible existence of $bc\bar{q}\bar{q}$ bound states has been addressed.

Independently of the constituent model used, isoscalar states are found to be attractive, while isovector states are repulsive, which precludes the existence of exotic charged heavy-light four-quark states with distinguishable heavy flavors. The isoscalar $J^P = 1^+$ state, holding a bound state for the case of identical bottom quarks, is found to be bound in the $bc\bar{u}\bar{d}$ case. Besides, the isoscalar $J^P = 0^+$ state, forbidden in S waves for identical heavy flavors, is also found to be bound. These two states are bound independently of the constituent model used. While the $J^P = 0^+$ state would be strong and electromagnetic-interaction would be stable, the $J^P = 1^+$ would decay electromagnetically to $\bar{B}D\gamma$. Recent estimations of the production rate of double heavy tetraquarks at the LHCb conclude the enhancement of the production of nonidentical heavy flavors bc compared to the identical bottom case by 2 orders of magnitude. In particular, with the LHCb integrated luminosity of 50 fb^{-1}, to be reached in Runs 1–4, well over 10^9 $bc\bar{u}\bar{d}$ events will be produced [16].

In spite of the supposed similarity with the case of identical heavy flavors, the dynamics is richer, and the interplay among different thresholds drives to unexpected results, as it is the large binding of the isoscalar axial vector state and the existence of a strong and electromagnetic-interaction stable isoscalar scalar state. It is hoped that the relevance of the present predictions for the understanding of basic properties of low energy QCD and the current capability of existing experiments, like the LHCb, to detect these exotic structures would encourage experimentalists to investigate heavy-light four-quark systems also for the case of nonidentical heavy flavors.

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Note added.—While this paper was under review, other independent calculations made in different frameworks arrived to similar conclusions. Among them, it is important to emphasize that the lattice QCD results of Ref. [29] find evidence for the existence of a strong-interaction-stable $(T)J^P = (0)1^+$ $ud\bar{c}\bar{b}$ four-quark state with a mass in the range of 15 to 61 MeV below the $\bar{D}B^*$ threshold.

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