Metric for a rotating object in an infrared corrected nonlocal gravity model

Utkarsh Kumar,[*] Sukanta Panda,[†] and Avani Patel[‡]
Indian Institute of Science Education and Research, Bhopal, Bhauri, Bhopal 462066, Madhya Pradesh, India

(Received 22 August 2018; published 28 December 2018)

Here, we derive the metric for the spacetime around a rotating object for the gravity action having a nonlocal correction of $R\Box^{-2}R$ to the Einstein-Hilbert action. Starting with the generic stationary, axisymmetric metric, we solve the equations of motion in a linearized gravity limit for the modified action, including the energy-momentum tensor of the rotating mass. We also derive the rotating metric from the static metric using the Demiański-Janis-Newman algorithm. Finally, we obtain the constraint on the value of M by calculating the frame-dragging effect in our theory and comparing it to that of general relativity and Gravity Probe B data, where M is the mass scale of the theory.

DOI: 10.1103/PhysRevD.98.124040

I. INTRODUCTION

The framework of an effective field theory of quantum gravity predicts nonlocal correction terms in the gravity action. The effects of these nonlocal terms correspond to both the infrared (IR) and the ultraviolet (UV) behavior of gravity. The cosmological effect of nonlocal terms added to the Einstein-Hilbert (EH) action in the late-time era are poorly studied. It is well known that the cosmic microwave background and supernova data suggest that our Universe has gone through two accelerating expansion phases in the early Universe and in the recent era, respectively, known as inflation and late-time acceleration. Therefore, explanations of inflation and late-time acceleration are the primary motivations for the study of any nonlocal correction in the cosmological context.

The correction term required to explain the accelerating expansion of our Universe in recent times has to dominate at large distance. Nonlocal correction term in the EH action was first proposed by Wetterich [1]. However, this model does not produce the desired cosmological evolution. As the first attempt in this direction, in 2008, Deser and Woodard introduced a class of models considering a general nonlocal term of $Rf(\Box^{-1}R)$. The functional form of $f(\Box^{-1}R)$ is obtained by fitting it with the supernova data. The structure formations of various functional forms of $f(\Box^{-1}R)$ have since been studied by several authors [2–18].

In this work, we consider the "RR model" developed in Ref. [9], which contains a nonlocal term like $R\Box^{-2}R$. This nonlocal term mimics the cosmological constant at late times. Such a nonlocal term also appears in the effective action of pure gravity theory [19–21].

In 1918, Lense and Thirring derived a metric which describes the spacetime structure outside the rotating object in the weak gravity regime which is known as the Lense-Thirring metric [22,23]. One can obtain the Lense-Thirring metric by taking a weak field and slow rotation limit to the Kerr metric. The progress of the work presented here is similar to that of Ref. [24], wherein the rotating metric (Lense-Thirring metric) is derived for the nonsingular infinite derivative gravity. In this work, we derive the Lense-Thirring metric for the RR model of nonlocal gravity taking two different approaches. We start with writing a field equation for the model in the linearized gravity limit which assumes the perturbative field $h_{\alpha\beta}$ on top of the background Minkowski metric $\eta_{\alpha\beta}$. This field equation is solved for different components of the general rotating metric. Thus the achieved metric is characterized by two scalar potentials (denoted by Φ and Ψ) and one vector potential. In the general relativistic limit, the two scalar potentials are the same, which is nothing but the Newtonian potential. We also derive the same metric using Demiański-Janis-Newman (DJN) algorithm [25,26].

The Kerr metric in general relativity (GR) is singular at $r = 0$ and $\theta = \pi/2$, where (r, θ, ϕ) are the Boyer-Lindquist coordinates. This is called ring singularity since $r = 0$ and $\theta = \pi/2$ correspond to the equation of a ring $x^2 + y^2 = a^2$ and $z = 0$ in Cartesian coordinates [27]. We find that the rotating metric in nonlocal gravity model considered here also bears the ring singularity at $r = 0$ and $\theta = \pi/2$.

Since GR has been proven to be correct in all experiments to date, any modification to GR needs to be checked against the experimental data. We calculate the geodetic precession and the Lense-Thirring precession in an orbit around Earth considering the rotating metric obtained here

[*]utkarshk@iiserb.ac.in
[†]sukanta@iiserb.ac.in
[‡]avani@iiserb.ac.in

for the RR model. We constrain the value of mass scale M present as a coefficient of the $R\Box^{-2}R$ term in the action by comparing the calculated values of geodetic and Lense-Thirring precession to the Gravity Probe B satellite data [28].

This paper is organized as follows: In Sec. II, we review the RR model of the nonlocal gravity and write the field equations for the model. In Sec. II A, we derive the field equations in the linearized gravity limit. The Lense-Thirring metric or rotating metric for the model is derived in Sec. III. The analysis of the ring singularity for the metric obtained in Sec. III is done in Sec. IV. We calculate the geodetic precession and the Lense-Thirring precession for the abovementioned metric and compare it to the experimental values of both precessional motions observed by the Gravity Probe B satellite in Sec. V. Finally, we conclude our work in Sec. VI.

II. THE MODEL AND BASIC EQUATIONS

Let us begin with the RR model of the nonlocal gravity specified by the following action:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R + \frac{M^2}{3}R\frac{1}{\Box^2}R\right] + \mathcal{L}_m. \quad (1)$$

Here, M is the mass scale associated with the nonlocal correction to the EH action. In the limit $M \to 0$, the above action (1) reduces to the EH action. The extensive study of the action under consideration has been done in Refs. [6,7,29].

The equation of motion for the field $g_{\alpha\beta}$ [30–33] corresponding to action (1) is

$$\begin{aligned}2\kappa^2 T_{\alpha\beta} = {}& G_{\alpha\beta} + \frac{2}{3}M^2 G_{\alpha\beta}\frac{1}{\Box^2}R + \frac{2M^2}{3}g_{\alpha\beta}R\frac{1}{\Box^2}R \\ & - \frac{2}{3}M^2(\nabla_\alpha\nabla_\beta + g_{\alpha\beta}\Box)\frac{1}{\Box^2}R + \frac{2}{3}M^2\nabla_{(\alpha}R^{-1}\nabla_{\beta)}R^{-2} \\ & - \frac{2M^2}{3}g_{\alpha\beta}(\nabla_{(\gamma}R^{-1}\nabla^{\gamma)}R^{-2} + 2R^{-2}),\end{aligned} \quad (2)$$

where $G_{\alpha\beta}$ is the Einstein tensor and $T_{\alpha\beta}$ is the energy-momentum tensor of the matter. Another equivalent form of the field equation can be obtained by redefining the operation of the inverse of the d'Alembertian operator on R as the scalar field and converting Eq. (1) into an equivalent scalar-tensor action. This approach gives rise to the following equation of motion:

$$\begin{aligned}\kappa^2 T_{\alpha\beta} = {}& G_{\alpha\beta} - \frac{M^2}{3}\Big\{2(G_{\alpha\beta} - \nabla_\alpha\nabla_\beta + g_{\alpha\beta}\Box)S + g_{\alpha\beta}\nabla^\gamma U\nabla_\gamma S \\ & - \nabla_{(\alpha}U\nabla_{\beta)}S - \frac{1}{2}g_{\alpha\beta}U^2\Big\},\end{aligned} \quad (3)$$

where $U = -\frac{1}{\Box}R$ and $S = -\frac{1}{\Box}U$.

A. Linearized limit

We consider the linearized gravity limit of the field equation written in Eq. (2). In the linearized gravity limit, we take $g_{\alpha\beta}$ as a perturbed metric around the Minkowski background $\eta_{\alpha\beta}$ by a small amount,

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}, \qquad |h| \ll 1. \quad (4)$$

Using Eq. (4), one can find the expressions for the Riemann tensor, the Ricci tensor, and the Ricci scalar as follows:

$$R_{\gamma\alpha\delta\beta} = \frac{1}{2}(\partial_\delta\partial_\alpha h_{\gamma\beta} + \partial_\beta\partial_\gamma h_{\alpha\delta} - \partial_\beta\partial_\alpha h_{\gamma\delta} - \partial_\delta\partial_\gamma h_{\beta\alpha}), \quad (5)$$

$$R_{\alpha\beta} = \frac{1}{2}(\partial^\gamma\partial_\alpha h_{\gamma\beta} + \partial_\beta\partial_\gamma h^\gamma_\alpha - \partial_\beta\partial_\alpha h - \Box h_{\alpha\beta}), \quad (6)$$

$$R = \partial_\alpha\partial_\beta h^{\alpha\beta} - \Box h. \quad (7)$$

Then the field equation (2) becomes

$$\begin{aligned}2\kappa^2 T_{\alpha\beta} = -\Big[&\Box h_{\alpha\beta} - \partial_\gamma\partial_{(\alpha}h^\gamma_{\beta)} \\ & + \left(1 - \frac{2M^2}{3}\Box^{-1}\right)(\partial_\alpha\partial_\beta h + \eta_{\alpha\beta}\partial_\gamma\partial_\delta h^{\gamma\delta}) \\ & - \left(-1 + \frac{2M^2}{3}\Box^{-1}\right)\eta_{\alpha\beta}\Box h \\ & + \frac{2M^2}{3}\Box^{-2}\nabla_\alpha\nabla_\beta\partial_\gamma\partial_\delta h^{\gamma\delta}\Big].\end{aligned} \quad (8)$$

III. METRIC FOR ROTATING OBJECT

In this section, we calculate the spacetime metric for the exterior region of the rotating object for the nonlocal gravity model considered in Eq. (1). Consider the generic rotating metric as

$$ds^2 = -(1 + 2\Phi)dt^2 + 2\vec{h}.dxdt + (1 - 2\Psi)dx^2. \quad (9)$$

In the case of general relativity, $\Phi = \Psi$ is the Newtonian potential. Note that the components of $h_{\mu\nu}$ are

$$h_{00} = -2\Phi, \quad (10)$$

$$h_{ij} = -2\Psi\eta_{ij}, \quad (11)$$

$$\vec{h} = h_{0x}\hat{x} + h_{0y}\hat{y} + h_{0z}\hat{z}. \quad (12)$$

The components of the stress-energy tensor for the rotating object having energy density $\rho = m\delta^3(\vec{r})$ with mass m and angular velocity v_i are given by

$$T_{00} = \rho, \qquad T_{0i} = -\rho v_i. \quad (13)$$

Notice that the rotation of the object explains the presence of the angular momentum terms T_{0i} in the stress-energy tensor. Taking the trace of linearized field equation (8) and substituting the metric given in Eq. (9) and the stress-energy tensor components given in Eq. (13), we obtain

$$\rho = -2(1 - M^2\Box^{-1})(\Box h - \partial_\alpha\partial_\beta h^{\alpha\beta}). \qquad (14)$$

Using Eqs. (8) and (14), we obtain the equations of motion for h_{00}, h_{ij}, and h_{0i} as

$$\rho = 4(1 - M^2\Box^{-1})(\nabla^2\Phi - 2\nabla^2\Psi), \qquad (15)$$

$$\rho = -\frac{4}{3}M^2\Box^{-1}(\nabla^2\Phi - 2\nabla^2\Psi) - 4\nabla^2\Psi, \qquad (16)$$

$$\kappa\rho v_i = -2\nabla^2 h_{0i}. \qquad (17)$$

One can see that there are no off-diagonal terms h_{0i} in the Eqs. (15) and (16). It is also apparent from Eq. (17) that the equation for the off-diagonal terms is unaffected by the nonlocal gravity correction and has the same form as in GR. Solving Eqs. (15)–(17), we get

$$\Phi(r) = \frac{m}{24\pi M_p^2 r}(e^{-Mr} - 4) = \frac{Gm}{r}\left(\frac{e^{-Mr} - 4}{3}\right), \qquad (18)$$

$$\Psi(r) = \frac{m}{24\pi M_p^2 r}(-e^{-Mr} - 2) = \frac{Gm}{r}\left(\frac{-e^{-Mr} - 2}{3}\right), \qquad (19)$$

$$h_{0x} = -\frac{mv_x}{2\pi M_p^2 r} = -\frac{4Gmv_x}{r}, \qquad (20)$$

$$h_{0y} = -\frac{mv_y}{2\pi M_p^2 r} = -\frac{4Gmv_y}{r}, \qquad (21)$$

$$h_{0z} = -\frac{mv_z}{2\pi M_p^2 r} = -\frac{4Gmv_z}{r}. \qquad (22)$$

We consider the case in which the source is moving in such direction so that its angular momentum points in the z direction. Therefore, we can write the velocities as follows:

$$v_x = -y\omega, \qquad v_y = x\omega, \qquad v_z = 0. \qquad (23)$$

Using Eqs. (18) and (19), we rewrite Eqs. (20)–(22) as

$$h_{0x} = -2y\omega(\Phi(r) + \Psi(r)),$$
$$h_{0y} = 2x\omega(\Phi(r) + \Psi(r)), \qquad h_{0z} = 0. \qquad (24)$$

The resulting metric is given by

$$ds^2 = -(1 + 2\Phi)dt^2 + 4(\Phi + \Psi)(x\omega dtdy - y\omega dtdx) + (1 - 2\Psi)d\mathrm{x}^2. \qquad (25)$$

Furthermore, we can convert the above metric from Cartesian coordinates to Boyer-Lindquist coordinates (t, r, θ, ϕ) via the transformations

$$x = \sqrt{r^2 + a^2}\sin\theta\cos\phi, \qquad y = \sqrt{r^2 + a^2}\sin\theta\sin\phi,$$
$$z = r\cos\theta, \qquad (26)$$

and as a result we get

$$ds^2 = -(1 + 2\Phi)dt^2 + 4\frac{J\sin^2\theta}{m}(\Phi + \Psi)d\phi dt + (1 - 2\Psi)(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2), \qquad (27)$$

where J is the angular momentum, defined as $v = \frac{r\times J}{mr^2}$. This is a case of a very slowly rotating object, and therefore we take $r^2 + a^2 \sim r^2$ in the transformation equations. If we take the $r \to \infty$ limit, then the metric in Eq. (27) reduces to the rotating metric in GR, which shows that the nonlocal gravity correction attenuates at very large distances.

A. Rotating metric from DJN algorithm

The Demiański-Janis-Newman algorithm provides a scheme to transform a static metric into a rotating metric using complex coordinate transformations [25,26,34,35]. In this section, we apply this algorithm on the weak gravity static metric for the RR model written in Eq. (1) to reproduce the metric in Eq. (27). First, we write the static metric for the RR model

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)d\mathrm{x}^2, \qquad (28)$$

where Φ and Ψ are given by Eqs. (18) and (19), respectively. Transforming the above metric into spherical polar coordinates, one can rewrite metric (28) in the following form:

$$ds^2 = -f_t dt^2 + f_r dr^2 + f_\Omega(d\theta^2 + \sin^2\theta d\phi^2), \qquad (29)$$

where $f_t = 1 + 2\Phi$, $f_r = 1–2\Psi$, and $f_\Omega = r^2 f_r$.

Performing the null coordinate transformation [25,26,34,35] $t = u + (\frac{1-2\Psi}{1+2\Phi})^{1/2}r$, Eq. (29) yields

$$ds^2 = -(1 + 2\Phi)du^2 - 2\sqrt{(1 + 2\Phi)(1 - 2\Psi)}dudr + f_\Omega d\Omega^2, \qquad (30)$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The next step is to complexify the coordinates in metric (30) as follows:

$$r \to r' = r + ai\cos\theta, \qquad u \to u' = u - ai\cos\theta. \qquad (31)$$

In Eq. (31), we introduced rotation parameter a defined by $a \equiv \frac{J}{m}$. Using the above transformations and the ansatz $id\theta = \sin\theta d\phi$, we obtain the differential transformations of r and u as

$$dr = dr' - a\sin^2\theta d\phi, \tag{32}$$

$$du = du' + a\sin^2\theta d\phi. \tag{33}$$

In the DJN approach, we have to be careful while choosing transformations for r, r^2, and $\frac{1}{r}$ that the functions f_i remain real, and their angle dependence should be of $\cos\theta$. Ensuring this, the transformations we find are

$$r \to r', \tag{34}$$

$$\frac{1}{r} \to \frac{\mathrm{Re}(r')}{|r'|^2}, \tag{35}$$

$$r^2 \leftrightarrow |r'|^2. \tag{36}$$

Therefore, our functions become

$$f_t(r) \to \tilde{f}_t(r,\theta) = 1 + \frac{mr}{24\pi M_p^2\Sigma}(e^{-mr} - 4), \tag{37}$$

$$f_r(r) \to \tilde{f}_r(r,\theta) = 1 + \frac{mr}{24\pi M_p^2\Sigma}(e^{-mr} + 2), \tag{38}$$

$$r^2 \to \Sigma \equiv r^2 + a^2\cos^2\theta. \tag{39}$$

Then we write down the null rotating metric

$$ds^2 = -\tilde{f}_t(du + \alpha dr + \omega\sin\theta d\phi)^2 + 2\beta dr d\phi + \Sigma\tilde{f}_r(d\theta^2 + \sigma^2\sin^2\theta d\phi^2), \tag{40}$$

where

$$\omega = a\sin\theta - \sqrt{\frac{\tilde{f}_r}{\tilde{f}_t}}a\sin\theta, \tag{41}$$

$$\sigma^2 = 1 + \frac{a^2\sin^2\theta}{r^2 + a^2}, \tag{42}$$

$$\alpha = \sqrt{\frac{\tilde{f}_r}{\tilde{f}_t}}, \tag{43}$$

$$\beta = -\tilde{f}_r a\sin^2\theta. \tag{44}$$

The last step is to convert the null metric into Boyer-Lindquist form. To do that, we have to make sure that

$$g(r) = \frac{\sqrt{(\tilde{f}_t\tilde{f}_r)^{-1}}\tilde{f}_\Omega - F'G'}{\Delta}, \tag{45}$$

$$h(r) = \frac{F'}{H(\theta)\Delta} \tag{46}$$

are functions of r only, where $\Delta = (\tilde{f}_\Omega/\tilde{f}_r)\sigma^2$. It is true provided that $\Phi \ll 1$, such that $f_r^{-1} = f_t$. These transformations are valid only when we consider the very small perturbation around the Minkowski background. After some trivial algebra, we obtain

$$\begin{aligned} ds^2 = &-(1 + 2\tilde{\Phi})dt^2 + 4a(\tilde{\Phi} + \tilde{\Psi})\sin^2\theta d\phi dt \\ &+ \frac{\Sigma(1 - 2\tilde{\Psi})}{r^2 + a^2}dr^2 \\ &+ \Sigma(1 - 2\tilde{\Psi})\left(d\theta^2 + \sin^2\theta\left(\frac{r^2 + a^2}{\Sigma}\right)d\phi^2\right), \end{aligned} \tag{47}$$

where

$$\tilde{\Phi} = \frac{mr}{24\pi M_p^2\Sigma}(e^{-Mr} - 4), \tag{48}$$

$$\tilde{\Psi} = \frac{mr}{24\pi M_p^2\Sigma}(-e^{-Mr} - 2). \tag{49}$$

IV. RING SINGULARITY

In this section, we investigate the ring singularity in the metric (27). We follow the same procedure as in Ref. [36]. Let us consider a rotating ring having mass m and radius a. The ring lies in the X-Y plane with $z = 0$, and the angular velocity of the ring points in the direction of the Z axis. The (00) component of the energy-momentum tensor of the source is given by

$$T_{00} = m\delta(z)\frac{\delta(x^2 + y^2 - a^2)}{\pi}. \tag{50}$$

The above distribution for the energy-momentum tensor is similar to that of the distributional form of the energy-momentum tensor of the Kerr metric [37]. We also have the following nonvanishing components of the stress-energy tensor:

$$T_{0i} = T_{00}v_i, \tag{51}$$

where v_i is the same angular velocity as defined earlier.

Now we rewrite the general linearized metric given in Eq. (9),

$$ds^2 = -(1 + 2\Phi)dt^2 + 2\vec{h}.dxdt + (1 - 2\Psi)dx^2. \tag{52}$$

The differential equations for each component of the metric are given by

$$\frac{(\Box+M^2)}{(3\Box+4M^2)}\nabla^2\Phi(\vec{r})=4Gm\delta(z)\delta(x^2+y^2-a^2),$$
$$\frac{(\Box+M^2)}{(3\Box+2M^2)}\nabla^2\Psi(\vec{r})=4Gm\delta(z)\delta(x^2+y^2-a^2),$$
$$\nabla^2 h_{0x}(\vec{r})=-8Gm\omega y\delta(z)\delta(x^2+y^2-a^2),$$
$$\nabla^2 h_{0y}(\vec{r})=8Gm\omega x\delta(z)\delta(x^2+y^2-a^2). \qquad (53)$$

In the next section, we solve Eq. (53) and examine the presence of ring singularity. Before going to the next section, let us look at the Kerr metric given by [38]

$$ds^2=-\left(1-\frac{2mr}{\Sigma}\right)dt^2-\frac{4mar\sin^2\theta}{\Sigma}dtd\phi+\frac{\Sigma}{\Delta}dr^2$$
$$+\Sigma d\theta^2+\sin^2\theta\left(r^2+a^2+\frac{2ma^2r\sin^2\theta}{\Sigma}\right)d\phi^2, \qquad (54)$$

where $\Sigma\equiv r^2+a^2\cos^2\theta$ and $\Delta\equiv r^2-2mr+a^2$, with a being the rotation parameter. It is easy to observe that the Kerr metric in Eq. (54) becomes singular (see Ref. [27]) when Σ becomes zero. The Kretschmann scalar blows up as $\Sigma=0$ at $r=0$ and $\theta=\pi/2$, which in Cartesian coordinates means that [27]

$$x^2+y^2=a^2, \qquad z=0, \qquad (55)$$

which is nothing but the equation of a ring of radius a. Thus GR admits the ring singularity in the Kerr spacetime.

A. Computation of metric components h_{00}, h_{0i}, and h_{ij}

Solving the differential equations written in Eq. (53) and obtaining expressions for the metric components is easier in momentum space. Therefore, we first find the Fourier transform for $\delta(z)\delta(x^2+y^2-a^2)$,

$$\mathcal{F}[\delta(z)\delta(x^2+y^2-a^2)]$$
$$=\int dxdydz\delta(z)\delta(x^2+y^2-a^2)e^{ik_xx}e^{ik_yy}e^{ik_zz}. \qquad (56)$$

The above integral can be computed in cylindrical coordinates:

$$x=\rho\cos\varphi, \qquad y=\rho\sin\varphi, \qquad z=z. \qquad (57)$$

We have

$$\mathcal{F}[\delta(z)\delta(x^2+y^2-a^2)]$$
$$=\int_{-\infty}^{\infty}dz\delta(z)e^{ik_zz}\int_0^{\infty}d\rho\rho\delta(\rho^2-a^2)\int_0^{2\pi}d\varphi e^{ik_x\rho\cos\varphi}e^{ik_y\rho\sin\varphi}$$
$$=\pi J_0\left(a\sqrt{k_x^2+k_y^2}\right), \qquad (58)$$

where J_0 is a Bessel function.

Using Eq. (58) in the differential equation for Φ and taking the inverse Fourier transform, we get the following expression for the potential Φ:

$$\Phi(\vec{r})=-4\pi Gm\int\frac{d^3k}{(2\pi)^3}\frac{(3k^2+4M^2)}{k^2(k^2+M^2)}$$
$$\times J_0\left(a\sqrt{k_x^2+k_y^2}\right)e^{ik_xx}e^{ik_yy}e^{ik_zz}, \qquad (59)$$

where d^3k is the differential volume in momentum space. To study the ring singularity in our model of interest, we restrict ourselves to the ring plane (i.e., the $X-Y$ plane, $z=0$) and transform Eq. (59) in cylindrical coordinates via the following transformations:

$$k_x=\zeta\cos\phi, \qquad k_y=\zeta\sin\phi, \qquad k_z=0. \qquad (60)$$

Then we have the final expression for $\Phi(\rho)$ being

$$\Phi(\rho)=-Gm\int_0^{\infty}d\zeta J_0(a\zeta)J_0(\zeta\rho)\frac{(3\zeta^2+4M^2)}{(\zeta^2+M^2)}, \qquad (61)$$

which in the limit $M\to 0$ gives the metric potential in the case of GR:

$$\Phi_{GR}(\rho)=-3Gm\int_0^{\infty}d\zeta J_0(a\zeta)J_0(\zeta\rho). \qquad (62)$$

A similar expression can be found for Ψ,

$$\Psi(\rho)=-Gm\int_0^{\infty}d\zeta J_0(a\zeta)J_0(\zeta\rho)\frac{(3\zeta^2+2M^2)}{(\zeta^2+M^2)}. \qquad (63)$$

To compute the h_{0i}, first we find the Fourier transform of $x\delta(z)\delta(x^2+y^2-a^2)$,

$$\mathcal{F}[x\delta(z)\delta(x^2+y^2-a^2)]$$
$$=\int_{-\infty}^{\infty}dz\delta(z)e^{ik_zz}\int_0^{\infty}d\rho\rho^2\delta(\rho^2-a^2)$$
$$\times\int_0^{2\pi}d\varphi e^{ik_x\rho\cos\varphi}e^{ik_y\rho\sin\varphi}\cos\varphi$$
$$=\pi a\frac{k_x}{\sqrt{k_x^2+k_y^2}}J_1\left(a\sqrt{k_x^2+k_y^2}\right), \qquad (64)$$

and similarly we also obtain

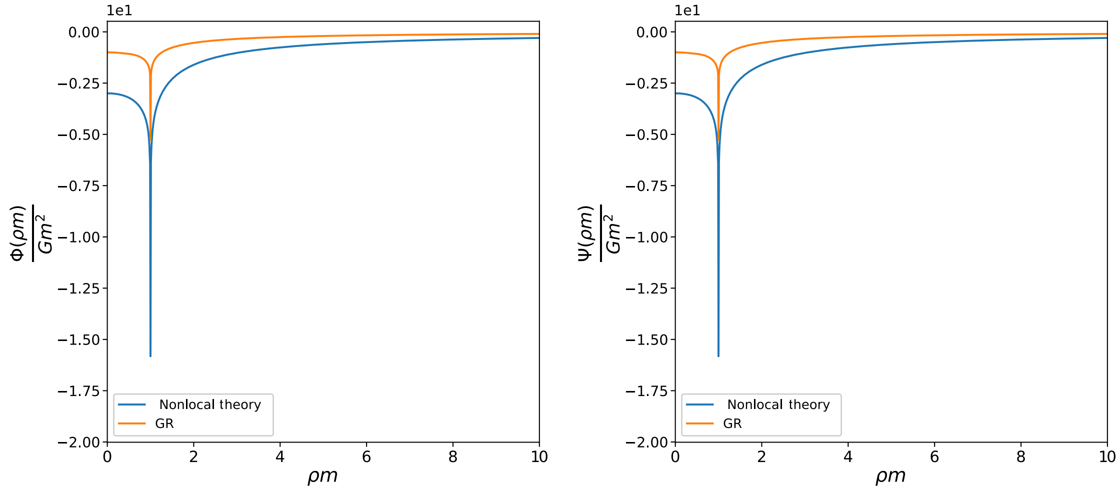

FIG. 1. Plots for $\Phi(\rho m)/Gm^2$ and $\Psi(\rho m)/Gm^2$. We have taken $ma = 1.0$.

$$\mathcal{F}[y\delta(z)\delta(x^2+y^2-a^2)] = \pi a \frac{k_y}{\sqrt{k_x^2+k_y^2}} J_1\left(a\sqrt{k_x^2+k_y^2}\right). \tag{65}$$

Then h_{0i} are given by the following expressions:

$$h_{0x}(\vec{r}) = 16Gm\omega a \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \frac{k_x}{\sqrt{k_x^2+k_y^2}} \times J_1\left(a\sqrt{k_x^2+k_y^2}\right) e^{ik_x x} e^{ik_y y} e^{ik_z z}, \tag{66}$$

$$h_{0y}(\vec{r}) = -16Gm\omega a \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \frac{k_y}{\sqrt{k_x^2+k_y^2}} \times J_1\left(a\sqrt{k_x^2+k_y^2}\right) e^{ik_x x} e^{ik_y y} e^{ik_z z}. \tag{67}$$

By using cylindrical coordinates and setting $z = 0$, we can obtain similar expressions for the cross terms,

$$h_{0x}(x,y) = 2Gm\omega a \frac{y}{\rho} \int_0^\infty d\zeta J_1(a\zeta) J_1(\zeta\rho), \tag{68}$$

$$h_{0y}(x,y) = -2Gm\omega a \frac{x}{\rho} \int_0^\infty d\zeta J_1(a\zeta) J_1(\zeta\rho), \tag{69}$$

where we remember that $\rho = \sqrt{x^2+y^2}$ is the radial cylindrical coordinate in the plane $z = 0$. Note that since $\theta = \pi/2$, we have

$$\frac{x}{\rho} = \cos\phi, \qquad \frac{y}{\rho} = \sin\phi. \tag{70}$$

Thus all of the radial dependence and the singularity structure are taken into account by the following integral:

$$H(\rho) \equiv \int_0^\infty d\zeta J_1(a\zeta) J_1(\zeta\rho), \tag{71}$$

which has the same form as $H_{GR}(\rho)$ [36]:

$$H_{GR}(\rho) \equiv \int_0^\infty d\zeta J_1(a\zeta) J_1(\zeta\rho). \tag{72}$$

Computation of the integrals involved in expressions of gravitational potentials Φ and Ψ in Eqs. (61) and (63) and of $H(\rho)$ in Eq. (71) is not possible analytically. Here, we solve them numerically and show the results in Figs. 1 and 2. We divide and multiply Eqs. (62), (63), and (71) by m and reparametrize ζ, a, and ρ as $\zeta' = \zeta/m$, $a' = am$, $\rho' = \rho m$ in order to make all of the quantities dimensionless. In our numerical calculation, we consider $M/m = 0.001$. It is evident from the figures that the gravitational potentials Φ and Ψ and the off-diagonal components of the metric h_{0i} show the singular characteristic at $m\rho = 1$.

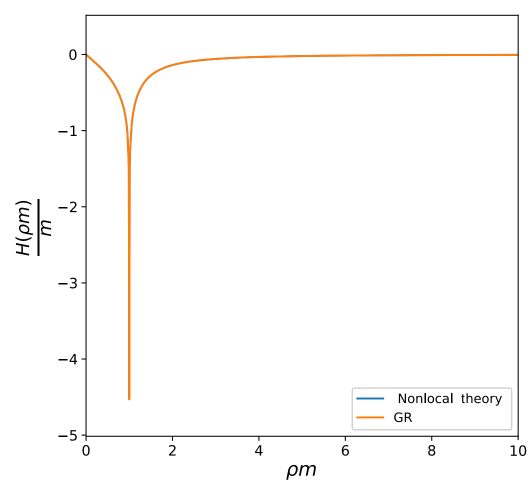

FIG. 2. $H(\rho m)/m$ vs ρm. We have taken $ma = 1.0$. $H(\rho m)/m$ for the RR model and general relativity overlap with each other.

V. FRAME-DRAGGING EFFECT

The rotating metric is valid in the weak field limit for any rotating astrophysical object which possesses the axial symmetry. Thus the metric derived in Eq. (47) can be applied to the spacetime around Earth, under the assumption that the underlying theory of gravity has a nonlocal correction like that given in Eq. (1) to the GR action. Any object which revolves around Earth will experience two types of precessional motion of general relativistic origin. One is due to the geodetic drift, and another is due to the frame-dragging drift. The Gravity Probe B satellite has measured these two precessions, and they were compared to the predicted value from general relativity.

The geodetic precession is caused by the curvature of the spacetime due to the rotating object's mass, and the Lense-Thirring precession or the frame-dragging effect is the result of drifting of the frame due to the rotation of the object. As we can see in Eq. (9), the metric is composed of a scalar field Φ and a vector field $\vec{h}$. These scalar fields and the vector field depend on the shape of the body and mass and velocity distribution of the body. These fields can be expanded in terms of multipole moments. For our calculation, we consider only the monopole moment. The formulas for instantaneous geodetic precession and instantaneous Lense-Thirring precession in Cartesian coordinates are given by [39]

$$\Omega_G = \frac{3}{2}\nabla\Phi \times \vec{V}, \qquad \Omega_{LT} = \frac{1}{2}\nabla \times \vec{h}, \tag{73}$$

where $\vec{V}$ is the four velocity of the orbiting gyroscope and h_{0i} are the off-diagonal terms of the rotating metric. The instantaneous geodetic precession and Lense-Thirring precession for nonlocal gravity theory can be expressed as

$$\Omega_{G(NL)} = \frac{1}{3}e^{-Mr}(-1 + 4e^{Mr} - Mr)\Omega_{G(GR)}, \tag{74}$$

$$\Omega_{LT(NL)} = \Omega_{LT(GR)}, \tag{75}$$

where general relativistic expressions of geodetic precession and Lense-Thirring precession are given by

$$\Omega_{G(GR)} = \frac{3Gm}{2r^3}(\vec{r} \times \vec{V}), \qquad \Omega_{LT(GR)} = \frac{2G}{r^3}\vec{J}. \tag{76}$$

The unchanging Lense-Thirring precession $\Omega_{LT(NL)}$ can be also explained by the fact that the off-diagonal terms [see Eqs. (20)–(22)] of the metric which are due to rotation of the object are not affected by the nonlocal correction of the RR model.

The Gravity Probe B satellite containing four gyroscopes was launched in 2004. These gyroscopes measured geodetic and frame-dragging precessions in orbit with an altitude $r = 650$ km from the surface of Earth. The result of the measurements done by Gravity Probe B for the geodetic drift rate was $\Omega_G = 6601.8 \pm 18.3$ milliarc sec/yr and for the frame-dragging drift rate was $\Omega_{LT} = 37.2 \pm 7.2$ milliarc sec/yr [28]. The GR predicted geodetic drift rate is $\Omega_{G(GR)} = 6606.1$ milliarc sec/yr and frame-dragging drift rate is $\Omega_{LT(GR)} = 39.2$ milliarc sec/yr.

One can constrain the value of M by checking for what values of M, $\Omega_{G(NL)}$, and $\Omega_{LT(NL)}$ match with $\Omega_{G(GR)}$ and $\Omega_{LT(GR)}$ having the difference well within the error bars of the Gravity Probe B results. Since M comes multiplied by r in Eq. (74), we find a constraint on Mr which comes out to be $Mr \leq 0.117$. Considering the value of the radial distance of the Gravity Probe B satellite from the center of Earth, which is $r = 7021$ km, and which we can obtain by converting the above constraint into the constraint on M, which is $M \leq 3.299 \times 10^{-15}$ eV. This is the reverse of the condition obtained in Ref. [24], which can be justified in a way that IR corrections to GR are taken here, while in Ref. [24] UV corrections were considered.

VI. CONCLUSIONS

In this work, we derived the metric for the exterior spacetime of the rotating body starting from the general rotating metric in the modified gravity theory having nonlocal gravity corrections to the Einstein-Hilbert action. The rotating metric which we found in Eq. (27) reduces to GR form in the large r limit. We also found that the off-diagonal terms of the metric are unchanged from the rotating metric in GR.

In the last section, we calculated the instantaneous geodetic precession and instantaneous Lense-Thirring precession of a satellite orbiting Earth for the model considered in Eq. (1) using the rotating metric derived in Eq. (27). We found that the instantaneous geodetic precession $\Omega_{G(NL)}$ for the model (1) differs from that of GR $\Omega_{G(GR)}$ by a multiplicative factor, while the instantaneous Lense-Thirring precession is the same as in GR. We compared the values of geodetic precession and Lense-Thirring precession to the Gravity Probe B satellite's data and put the constraint on the value of the scale M, which comes out to be $M \leq 3.299 \times 10^{-15}$ eV.

The rotating metric obtained in this paper can be utilized further in the studies of the metric for the Kerr black hole and tests of nonlocal gravity theory using gravitational wave astronomy. In particular, one can derive the metric for the Kerr black hole as the Kerr metric with small perturbation and can calculate the deviations in the frequencies of the gravitational waves emitted by the test particle orbiting a super massive black hole, which consequently can be useful in modeling extreme mass ratio inspirals for the modified gravity with nonlocal gravity corrections.

ACKNOWLEDGMENTS

This work was partially supported by DST Grant No. SERB/PHY/2017041.

[1] C. Wetterich, Effective nonlocal Euclidean gravity, Gen. Relativ. Gravit. **30**, 159 (1998).
[2] R. P. Woodard, Nonlocal models of cosmic acceleration, Found. Phys. **44**, 213 (2014).
[3] A. O. Barvinsky, Nonlocal action for long distance modifications of gravity theory, Phys. Lett. B **572**, 109 (2003).
[4] A. O. Barvinsky, Dark energy and dark matter from nonlocal ghost-free gravity theory, Phys. Lett. B **710**, 12 (2012).
[5] A. O. Barvinsky, Serendipitous discoveries in nonlocal gravity theory, Phys. Rev. D **85**, 104018 (2012).
[6] Y. Dirian and E. Mitsou, Stability analysis and future singularity of the $m^2 R\Box^{-2} R$ model of non-local gravity, J. Cosmol. Astropart. Phys. 10 (2014) 065.
[7] H. Nersisyan, Y. Akrami, L. Amendola, T. S. Koivisto, and J. Rubio, Dynamical analysis of $R\frac{1}{\Box^2}R$ cosmology: Impact of initial conditions and constraints from supernovae, Phys. Rev. D **94**, 043531 (2016).
[8] L. Amendola, N. Burzilla, and H. Nersisyan, Quantum gravity inspired nonlocal gravity model, Phys. Rev. D **96**, 084031 (2017).
[9] M. Maggiore and M. Mancarella, Nonlocal gravity and dark energy, Phys. Rev. D **90**, 023005 (2014).
[10] S. Foffa, M. Maggiore, and E. Mitsou, Cosmological dynamics and dark energy from nonlocal infrared modifications of gravity, Int. J. Mod. Phys. A **29**, 1450116 (2014).
[11] A. Kehagias and M. Maggiore, Spherically symmetric static solutions in a nonlocal infrared modification of general relativity, J. High Energy Phys. 08 (2014) 029.
[12] N. C. Tsamis and R. P. Woodard, Improved cosmological model, Phys. Rev. D **94**, 043508 (2016).
[13] N. C. Tsamis and R. P. Woodard, Stochastic quantum gravitational inflation, Nucl. Phys. **B724**, 295 (2005).
[14] N. C. Tsamis and R. P. Woodard, A phenomenological model for the early Universe, Phys. Rev. D **80**, 083512 (2009).
[15] S. Deser and R. P. Woodard, Observational viability and stability of nonlocal cosmology, J. Cosmol. Astropart. Phys. 11 (2013) 036.
[16] G. Narain and T. Li, Ultraviolet complete dark energy model, Phys. Rev. D **97**, 083523 (2018).
[17] G. Narain and T. Li, Universe **4**, 82 (2018).
[18] U. Kumar and S. Panda, Non-local cosmological models, arXiv:1806.09616.
[19] A. Codello and R. K. Jain, Leading gravitational corrections and a unified Universe, Int. J. Mod. Phys. D **25**, 1644023 (2016).
[20] A. Codello and R. K. Jain, Covariant effective field theory of gravity, Proc. Sci., DSU2015 (**2016**) 008.
[21] A. Codello and R. K. Jain, A unified Universe, Eur. Phys. J. C **78**, 357 (2018).
[22] J. Lense and H. Thirring, Phys. Z. **19**, 156 (1918).
[23] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, New York, 1973).
[24] A. S. Cornell, G. Harmsen, G. Lambiase, and A. Mazumdar, Rotating metric in nonsingular infinite derivative theories of gravity, Phys. Rev. D **97**, 104006 (2018).
[25] E. T. Newman and A. I. Janis, Note on the Kerr spinning particle metric, J. Math. Phys. (N.Y.) **6**, 915 (1965).
[26] M. Demiaski, New Kerr-like space-time, Phys. Lett. **42A**, 157 (1972).
[27] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).
[28] C. W. F. Everitt *et al.*, Gravity Probe B: Final Results of a Space Experiment to Test General Relativity, Phys. Rev. Lett. **106**, 221101 (2011).
[29] G. Calcagni and G. Nardelli, Non-local gravity and the diffusion equation, Phys. Rev. D **82**, 123518 (2010).
[30] J. Edholm, A. S. Koshelev, and A. Mazumdar, Behavior of the Newtonian potential for ghost-free gravity and singularity-free gravity, Phys. Rev. D **94**, 104033 (2016).
[31] A. Conroy, T. Koivisto, A. Mazumdar, and A. Teimouri, Generalized quadratic curvature, non-local infrared modifications of gravity and Newtonian potentials, Classical Quantum Gravity **32**, 015024 (2015).
[32] T. Biswas, A. Conroy, A. S. Koshelev, and A. Mazumdar, Generalized ghost-free quadratic curvature gravity, Classical Quantum Gravity **31**, 015022 (2014); Erratum, Classical Quantum Gravity **31**, 159501(E) (2014).
[33] T. Biswas, T. Koivisto, and A. Mazumdar, Nonlocal theories of gravity: The flat space propagator, arXiv:1302.0532.
[34] H. Erbin, Deciphering and generalizing Demiański-Janis-Newman algorithm, Gen. Relativ. Gravit. **48**, 56 (2016).
[35] H. Erbin, Janis-Newman algorithm: Simplifications and gauge field transformation, Gen. Relativ. Gravit. **47**, 19 (2015).
[36] L. Buoninfante, A. S. Cornell, G. Harmsen, A. S. Koshelev, G. Lambiase, J. Marto, and A. Mazumdar, Non-singular rotating metric in ghost-free infinite derivative gravity, Phys. Rev. D **98**, 084041 (2018).
[37] H. Balasin and H. Nachbagauer, Distributional energy momentum tensor of the Kerr-Newman space-time family, Classical Quantum Gravity **11**, 1453 (1994).
[38] M. Visser, The Kerr spacetime: A brief introduction, arXiv: 0706.0622.
[39] R. J. Adler and A. S. Silbergleit, A general treatment of orbiting gyroscope precession, Int. J. Theor. Phys. **39**, 1291 (2000).