Flavor violation in chromo- and electromagnetic dipole moments induced by Z' gauge bosons and a brief revisit of the Standard Model

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The electromagnetic dipole moments of the tau lepton and the chromoelectromagnetic dipole moments of the top quark are estimated via flavor-changing neutral currents, mediated by a new neutral massive gauge boson. We predict them in the context of models beyond the Standard Model with extended current sectors, in which simple analytic expressions for the dipole moments are presented. For the different Z' gauge boson considered, the best prediction for the magnetic dipole moment of the tau lepton, $|a_\tau|$, is of the order of 10^{-8}, while the highest value for the electric one, $|d_\tau|$, corresponds to 10^{-24} e cm; our main result for the chromomagnetic dipole moment of the top quark, $|\hat{\mu}_t|$, is 10^{-6}, and the value for the chromoelectric one, $|d_t|$, can be as high as 10^{-22} e cm. We compare our results, revisiting the corresponding Standard Model predictions, in which the chromomagnetic dipole moment of the top quark is carefully evaluated, finding explicit imaginary contributions.

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I. INTRODUCTION

In the Standard Model (SM), flavor-changing transitions promoted by neutral gauge bosons can be found in the quark sector; however, these are strongly suppressed by the Glashow-Iliopoulos-Maiani mechanism and because they are induced at the one-loop level [1]. On the other hand, in the leptonic sector Lagrangian, the SM contains an exact flavor symmetry, which implies that transitions between charged leptons mediated by neutral gauge bosons are forbidden to any perturbative order. Although in the SM the flavor violation phenomenon is suppressed, it is known that the impact of flavor-changing neutral currents (FCNCs) could be increased by new physics effects due, for example, to both extended Yukawa [2] or new current sectors [3–5]. The study of flavor violation has gained much interest due to the discovery of neutrino oscillations [6]. However, this phenomenon occurs exclusively between neutral fermions (neutrinos), and therefore transitions between charged leptons would play a complementary role by offering clear signals of flavor violation, enriching such a phenomenon. According to this, the proposal of this work is to study the effects of new physics on the electromagnetic and chromoelectromagnetic properties of charged fermions due to the presence of FCNCs mediated by a new neutral massive gauge boson identified as Z'. The existence of this boson has been proposed in numerous extended models, the simplest those being ones that involve an extra $U'(1)$ gauge symmetry group [7]. The simplest model that predicts the existence of the Z' boson is founded on the $SU_L(2) \times U_Y(1) \times U'(1)$ extended electroweak gauge group [8–11].

At present, the experimental collaborations ATLAS and CMS, at the LHC, have devoted many studies to the search for new elementary particles, such as new neutral massive gauge bosons [12,13] or new scalar bosons [14]. As far as the search for new neutral massive gauge bosons is concerned, the experimental results indicate that the existence of Z' bosons is not excluded for masses slightly above 3 TeV. Specifically, the ATLAS Collaboration establishes lower limits on the Z' masses ranging from 2.74 up to 3.36 TeV at 95% C.L. [12,15]. In contrast, the CMS Collaboration reports that the existence of Z' gauge bosons would be excluded for masses below the range between 2.57 and 2.9 TeV at 95% C.L. [13,15].

The flavor violation (FV) issue has allowed us to relate the hypothetical Z' particle with several processes such as single top production [3,4], the $D^0 - \bar{D}^0$ mixing system [4], the $b_q^0 - \bar{b}_q^0$ mixing system [16], lepton flavor–violating decays [5,11,17,18], etc. In this way, by using the most general renormalizable Lagrangian that includes FV mediated by a new neutral massive gauge boson, we will estimate the impact of FCNC on the electromagnetic dipole moments of the tau lepton and the chromoelectromagnetic dipole moments of the top quark, resorting to different grand unification models (GUT) with extended current sectors [3,19].

The static magnetic properties of charged leptons in the context of the Standard Model have developed the predictive power of this theory [20]. However, little is known about the static electric properties of charged leptons, referring alike to the electron, the muon, and the tau lepton. The experimental measurement of the magnetic dipole moment (MDM) of the electron (a_e) has been the main argument to establish the SM as a rather successful theory. In contrast, although the MDM of the muon (a_μ) has been studied exhaustively, a discrepancy persists between the experimental measurement [21] and the SM theoretical prediction [22], which turns out to be around three standard deviations [23]. Therefore, new measurements will be carried out in order to increase the experimental precision and look for possible systematic errors [24]. At the same time, theoretical efforts are realized in order to try to reduce the uncertainty in the theoretical prediction coming from hadronic light-by-light contributions [23,25]. If such a discrepancy were reduced, it would imply that possible new physics effects would be very restricted. On the other hand, there is practically no information regarding the static electromagnetic properties of the tau lepton, mainly due to its short lifetime [20]. For the tau magnetic dipole moment there are only experimental bounds, that restrict it with enormous uncertainty, $-0.052 < a_\tau < 0.013$ at 95% C.L. [15]. In this sense, we have revisited the so-called SM electroweak contribution for the tau lepton MDM. Similarly, given that for the electric dipole moment (EDM) of charged leptons there are only experimental bounds on their real value, we turn our attention to the EDM of the tau lepton as a source of study of possible new physics effects, related to FV, and given its nature, it would also be related to CP violation. Since the SM does not predict appreciable effects of CP violation in the leptonic sector [26], the study of the tau EDM is an ideal testing ground for the search of new physics effects. The experimental measurement attempts of the tau EDM have resulted in the following constraints [15,26]: $-2.2\times10^{-17}\,e\,\mathrm{cm} < \mathrm{Re}(d_\tau) < 4.5\times10^{-17}\,e\,\mathrm{cm}$ and $-2.5\times10^{-17}\,e\mathrm{cm} < \mathrm{Im}(d_\tau) < 8.0\times10^{-19}\,e\mathrm{cm}$. Studies on the EDM have been carried out in Refs. [27–29].

Moreover, given the great mass of the top quark, 173 GeV [15], which is of the order of the Fermi scale, it is thought that this particle could be related to new physics effects present at the TeV energy scale. Thereby, it is interesting to study the physical properties of this particle, our proposal being the characterization of possible flavor-violating effects due to the presence of FCNCs, which would be impacting the chromoelectromagnetic properties of the top quark. Because in the SM the chromomagnetic dipole moment (CMDM) of the top quark appears at the one-loop level and its chromoelectric dipole moment (CEDM) arises at three-loop level, the impact of new physics effects becomes relevant. In addition, appreciable new physics effects on the top CEDM are of great importance as they would directly impact the CP-violation phenomenon, which would be indicative of new sources of CP violation and, in our case, of FV. Currently, the spin correlations of top-antitop pairs and the polarization of the top quark have been measured in pp collisions at $\sqrt{s} = 8$ TeV [30]. These results were obtained by the CMS Collaboration at CERN, where constraints on extended models are imposed, finding new exclusion limits at 95% of C.L. for the CMDM and CEDM of the top quark, namely, $-0.053 < \mathrm{Re}(\hat{\mu}_t) < 0.026$ and $-0.068 < \mathrm{Im}(\hat{d}_t) < 0.067$ [30], respectively. The top-quark CMDM and CEDM have been calculated in the SM [31] as well as in other extensions such as the two-Higgs doublet model [32], the minimal supersymmetric Standard Model [33,34], 3-3-1 models [35], technicolor models [36], models with vector-like multiplets [37], effective operators [38], and the two-Higgs doublet model with four fermion generations [39]. However, the SM CMDM contribution of the top quark coming from the three-gluon vertex is in fact divergent when the gluon is on shell, but in Ref. [35], the authors claim that it is finite. Indeed, Refs. [40] and [41] are in agreement with the ill behavior when the gluon is on shell. In view of such an issue, we were forced to revisit in depth the complete one-loop SM calculations for the CMDM of the top quark, finding novelties that will be commented on below.

The rest of this paper is organized as follows. In Sec. II, the basis of FCNCs induced by a new neutral massive gauge boson of spin 1 is presented, and it is explained how bounds over $Z'f_if_j$ (for $f_if_j = \tau\mu, \tau e, tc, tu$) couplings are determined. In Sec. III, we exhibit the theoretical results for the electromagnetic and chromoelectromagnetic dipole moments induced by FCNCs. Also, we present the numerical analysis for the MDM (CMDM) and the EDM (CEDM) of the tau lepton (top quark), respectively; in addition, we present a brief revisit of the CMDM of the top quark in the SM. Finally, Sec. IV gives the conclusions.

II. THEORETICAL FRAMEWORK

Since it is required to estimate the strength of the $Z'f_if_j$ couplings (where $f_{i,j}$ represents any SM charged fermion) in order to determine its impact on the MDM, EDM, CMDM, and CEDM, it is necessary establish the Lagrangian that comprises FCNCs mediated by the Z' gauge boson. The most general renormalizable Lagrangian that includes FV mediated by a new neutral massive gauge boson, coming from any extended model or GUT [42–44], is

$$\mathcal{L}_{NC} = \sum_{i,j}[\bar{f}_i\gamma^\alpha(\Omega_{Lf_if_j}P_L + \Omega_{Rf_if_j}P_R)f_j + \bar{f}_j\gamma^\alpha(\Omega^*_{Lf_jf_i}P_L + \Omega^*_{Rf_jf_i}P_R)f_i]Z'_\alpha, \qquad (2.1)$$

TABLE I. Chiral-diagonal couplings of the extended models.

	Z_S	Z_{LR}	Z_χ	Z_ψ	Z_η
$Q_L^{l_i}$	−0.2684	0.2548	$\frac{3}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{1}{2\sqrt{15}}$
$Q_R^{l_i}$	0.2316	−0.3339	$\frac{-3}{2\sqrt{10}}$	$\frac{-1}{\sqrt{24}}$	$\frac{-1}{2\sqrt{15}}$
$Q_L^{u_i}$	0.3456	−0.08493	$\frac{-1}{2\sqrt{10}}$	$\frac{1}{\sqrt{24}}$	$\frac{-2}{2\sqrt{15}}$
$Q_R^{u_i}$	−0.1544	0.5038	$\frac{1}{2\sqrt{10}}$	$\frac{-1}{\sqrt{24}}$	$\frac{2}{2\sqrt{15}}$

where f_i is any fermion of the SM, $P_{L,R} = \frac{1}{2}(1 \pm \gamma_5)$ are the chiral projectors, and Z'_α is a new neutral massive gauge boson predicted by several extensions of the SM [42–45]. The $\Omega_{Lf_if_j}$, $\Omega_{Rl_il_j}$ parameters represent the strength of the $Z'f_if_j$ coupling, where f_i is any charged fermion of the SM. From now on, we will assume that $\Omega_{Lf_if_j} = \Omega_{Lf_jf_i}$ and $\Omega_{Rf_if_j} = \Omega_{Rf_jf_i}$. The Lagrangian in Eq. (2.1) includes both flavor-conserving and flavor-violating couplings mediated by a Z' gauge boson. In this work, the following Z' bosons are considered: the Z_S of the sequential Z model, the Z_{LR} of the left-right symmetric model, the Z_χ boson that arises from the breaking of $SO(10) \to SU(5) \times U(1)$, the Z_ψ that emerges as a result of $E_6 \to SO(10) \times U(1)$, and the Z_η appearing in many superstring-inspired models [9]. Concerning to the flavor-conserving couplings, $Q_{L,R}^{f_i}$ [3,8,9], the values of which are shown in Table I, for different extended models are related to the Ω couplings as $\Omega_{Lf_if_i} = -g_2 Q_L^{f_i}$ and $\Omega_{Rf_if_i} = -g_2 Q_R^{f_i}$, where g_2 is the gauge coupling of the Z' boson. For the extended models we are interested in, the gauge couplings of Z''s are

$$g_2 = \sqrt{\frac{5}{3}} \sin\theta_W g_1 \lambda_g, \tag{2.2}$$

where $g_1 = g/\cos\theta_W$, λ_g depends on the symmetry-breaking pattern being of $\mathcal{O}(1)$ [46], and g is the weak coupling constant. In the sequential Z model, the gauge coupling $g_2 = g_1$.

A. Bounding the $Z'f_if_j$ couplings

The subject of this work is to study the impact of flavor-violating couplings mediated by a Z' gauge boson on the MDM and the EDM of the tau lepton and the CMDM and the CEDM of the top quark. To do this task, we will use bounds on the lepton flavor–violating couplings $Z'\tau\mu$ and $Z'\tau e$, which have been previously computed by using the experimental constraints for the lepton flavor–violating $\tau \to \mu\mu^+\mu^-$ and $\tau \to \mu e^+e^-$ decays [5]. Finally, we will use the results of a previous work in which the strength of the $Z'tc, Z'tu$ couplings is estimated by means of the $D^0 - \bar{D}^0$ mixing system [4].

1. Three-body $\tau \to \mu\mu^+\mu^-$, ee^+e^- decays

The contribution of the flavor-violating $Z'l_il_j$ vertex to the $\tau \to l_il_i^+l_i^-$ decay is depicted in Fig. 1, in which l_il_j represents $\tau\mu$ or τe and $l_il_i^+l_i^-$ symbolizes $\mu\mu^+\mu^-$ or ee^+e^-. The three-body decay of the tau lepton comes from the tree-level Feynman diagram, the associated branching ratio of which was computed in a previous work [5],

$$\mathrm{Br}(\tau \to l_il_i^+l_i^-) = \frac{g_2^2}{384\pi^3} h_1(m_{Z'})(|Q_L^e \Omega_{Ll_il_j}|^2 + |Q_R^e \Omega_{Rl_il_j}|^2) \frac{m_\tau}{\Gamma_\tau}, \tag{2.3}$$

where

$$h_1(m_{Z'}) = \int_0^1 dx \frac{2x-1}{(x-1+m_{Z'}^2/m_\tau^2)^2} (2(7-4x)x-5), \tag{2.4}$$

and Γ_τ is the total decay width of the tau lepton. The branching ratio in Eq. (2.3) must be less than the corresponding experimental bounds to the processes $\tau \to \mu\mu^+\mu^-$ and $\tau \to ee^+e^-$, as applicable. It is considered that $\mathrm{Br}_{\mathrm{Exp}}(\tau \to \mu\mu^+\mu^-) < 2.1 \times 10^{-8}$ [15] and $\mathrm{Br}_{\mathrm{Exp}}(\tau \to ee^+e^-) < 2.7 \times 10^{-8}$ [15], which allow us to get constraints on the flavor-violating parameters: $|\Omega_{L\tau\mu}|^2$, $|\Omega_{R\tau\mu}|^2$, $|\Omega_{L\tau e}|^2$, $|\Omega_{R\tau e}|^2$.

2. $D^0 - \bar{D}^0$ mixing system

For FCNCs mediated by a new neutral massive gauge boson, in a previous work [4], the mass difference, ΔM_D, coming from the $D_0 - \overline{D_0}$ mixing system, was estimated. Explicitly, ΔM_D can be written as

$$\Delta M_D = \frac{1}{12} \frac{\Omega_{uc}^2}{m_{Z'}^2} f_D^2 M_D B_D \left[1 + \frac{x}{8\pi^2}(32f(x) - 5g(x))\right], \tag{2.5}$$

where B_D is the bag model parameter and f_D symbolizes the D_0-meson constant decay. Here, we are taking $B_D \sim 1$, $f_D = 222.6$ MeV [47], and $M_D = 1.8646$ GeV [15]. By assuming that ΔM_D does not exceed the experimental uncertainty, we are able to constraint the Ω_{uc} parameter [4]

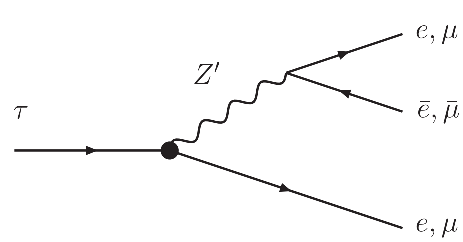

FIG. 1. Feynman diagrams corresponding to the $\tau \to \mu\mu^+\mu^-$ and $\tau \to ee^+e^-$ decays.

$$|\Omega_{uc}| < \frac{3.6\times 10^{-7} m_{Z'}\mathrm{GeV}^{-1}}{\sqrt{1+\frac{x}{8\pi^2}(32f(x)-5g(x))}}. \tag{2.6}$$

From this bound, we can estimate the Ω_{tc} and Ω_{tu} parameters by considering that $|\Omega_{uc}| \approx |\Omega_{tc}\Omega_{tu}|$ and $\Omega_{tc} = 10\Omega_{tu}$; the details of the calculation and the justification for such assumptions can be found in Ref. [4]. Therefore, the coupling parameters are given as

$$|\Omega_{tc}|^2 < \frac{3.6\times 10^{-6} m_{Z'}\mathrm{GeV}^{-1}}{\sqrt{1+\frac{x}{8\pi^2}(32f(x)-5g(x))}},$$
$$|\Omega_{tu}|^2 < \frac{3.6\times 10^{-8} m_{Z'}\mathrm{GeV}^{-1}}{\sqrt{1+\frac{x}{8\pi^2}(32f(x)-5g(x))}}. \tag{2.7}$$

It is pertinent to comment that another possibility for bounding flavor-violating couplings is that coming from experimental limits on the electric dipole moment of the neutron [48].

III. RESULTS AND DISCUSSION

In this section, we exhibit the analytical results for the MDM, EDM, CMDM, and CEDM induced by FCNCs mediated by the Z' gauge boson. Subsequently, the corresponding numerical results will be presented.

A. Static electromagnetic dipole moments

The effective electromagnetic dipole moment Lagrangian for charged leptons, $f = l$, is

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2}\bar{f}\sigma^{\mu\nu}(F_M + iF_E\gamma^5)fF_{\mu\nu}, \tag{3.1}$$

where F_M is the magnetic and F_E is the electric form factor, $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field strength. The associated vertex is

$$\Gamma^\mu = \sigma^{\mu\nu}q_\nu(F_M + iF_E\gamma^5). \tag{3.2}$$

On the other hand, the invariant amplitude is

$$\mathcal{M} = \mathcal{M}^\mu\epsilon_\mu(\vec{q}), \tag{3.3}$$

where $\mathcal{M}^\mu = \bar{u}(p')\Gamma^\mu u(p)$.

The static properties arise when the photon is on shell, $q^2 = 0$, and hence the static anomalous magnetic, a_f, and electric, d_f, dipole moments [49] are

$$F_M \equiv \frac{eQ_f}{2m_f}a_f, \qquad F_E \equiv Q_f d_f. \tag{3.4}$$

It is usual to express them as a single complex dipole form factor,

$$F_C = F_M + iF_E = |F_C|e^{i\phi_f}, \tag{3.5}$$

with

$$|F_C| = \sqrt{F_M^2 + F_E^2}, \qquad \tan\phi_f = \frac{F_E}{F_M}, \tag{3.6}$$

where ϕ_f is the phase that parametrizes the relative size of the EDM and its MDM.

To compare the results derived in this section we have also calculated the corresponding SM contributions at one-loop level to the tau MDM. Our approximate analytical expressions, which excellently agree with the complete calculations, are

$$a_{l_i}(\gamma) = \frac{\alpha}{2\pi}, \tag{3.7}$$

$$a_{l_i}(W) \simeq \frac{5G_F m_{l_i}^2}{12\sqrt{2}\pi^2}, \tag{3.8}$$

$$a_{l_i}(Z) \simeq \frac{G_F m_{l_i}^2}{6\sqrt{2}\pi^2}\frac{(1-4s_W^2)^2-5}{4}, \tag{3.9}$$

$$a_{l_i}(H) \simeq -\frac{G_F m_{l_i}^2}{24\sqrt{2}\pi^2}\frac{m_{l_i}^2}{m_H^2}\left(7 + 6\log\frac{m_{l_i}^2}{m_H^2}\right). \tag{3.10}$$

These are valid for any charged lepton and can be compared with those given for the muon in Sec. 4.2.1 of Ref. [50]. Notice that in our expression for the Higgs contribution we also conserve the first term, which is not relevant for the electron and muon cases but is important for the tau lepton. The numerical values are given in Table II, in which the electroweak contribution means $a_{l_i}(\mathrm{EW}) = a_{l_i}(W) + a_{l_i}(Z) + a_{l_i}(H)$.

B. One-loop Z' contribution to the static electromagnetic and chromoelectromagnetic dipole moments

In analogy to the SM $\bar{f}fZ$ coupling, for the $\bar{f}_i f_j Z'$ coupling, we rewrite this as

TABLE II. Anomalous magnetic dipole moment of the tau lepton at one loop in the SM with $m_H = 125.18$ GeV [15].

Contribution	a_τ
γ	1.16×10^{-3}
W	1.10×10^{-6}
Z	-5.48×10^{-7}
H	9.76×10^{-10}
EW	5.52×10^{-7}

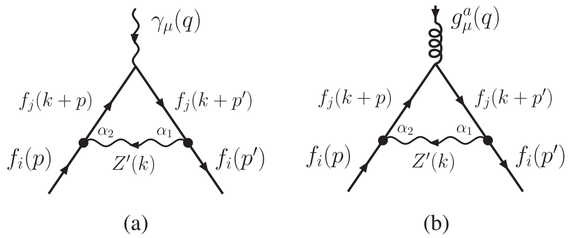

FIG. 2. (a) Electromagnetic ($f = l$) and (b) chromoelectromagnetic ($f = q$) dipole moments at one-loop level mediated by a Z' gauge boson with FV.

$$\Omega_{Lf_if_j}P_L + \Omega_{Rf_if_j}P_R = g^{f_if_j}_{VZ'} - g^{f_if_j}_{AZ'}\gamma^5. \tag{3.11}$$

$$g^{f_if_j}_{VZ'} \equiv \frac{1}{2}(\Omega_{Lf_if_j} + \Omega_{Rf_if_j}), \quad g^{f_if_j}_{AZ'} \equiv \frac{1}{2}(\Omega_{Lf_if_j} - \Omega_{Rf_if_j})\gamma^5. \tag{3.12}$$

The general one-loop quantum fluctuation that generates the static electromagnetic dipole moments, depicted in Fig. 2, is

$$\mathcal{M}^\mu_{f_if_j} = eQ_{f_j}\int \frac{d^4k}{(2\pi^4)}\frac{\bar{u}(p')\gamma^{\alpha_1}(g^{f_if_j}_{VZ'} - g^{f_if_j}_{AZ'}\gamma^5)(\not{k} + \not{p}' + m_{f_j})\gamma^\mu(\not{k} + \not{p} + m_{f_j})}{(k^2 - m^2_{Z'})[(k+p')^2 - m^2_{f_j}][(k+p)^2 - m^2_{f_j}]}$$
$$\times \gamma^{\alpha_2}\left(g^{f_if_j*}_{VZ'} - g^{f_if_j*}_{AZ'}\gamma^5\right)u(p)\left(-g_{\alpha_1\alpha_2} + \frac{k_{\alpha_1}k_{\alpha_2}}{m^2_{Z'}}\right). \tag{3.13}$$

For the chromoelectromagnetic case, the factor eQ_{f_j} must be replaced by g_sT^a. From this loop integral, the complete analytical results for the static electromagnetic dipole moments can be obtained, given in terms of the form factors $F_{M,E}(q^2 = 0)$; nevertheless, we present more suitable approximate expressions that have been cross-checked, matching excellently.

The MDM form factor is

$$F_{Mf_if_j} \simeq \frac{eQ_{f_j}}{48\pi^2 m^4_{Z'}}\Big\{|g^{f_if_j}_{VZ'}|^2[m_{f_i}(3m^2_{f_j} - 4m^2_{Z'}) + 6m_{f_j}m^2_{Z'}]$$
$$+ |g^{f_if_j}_{AZ'}|^2[m_{f_i}(3m^2_{f_j} - 4m^2_{Z'}) - 6m_{f_j}m^2_{Z'}]\Big\}, \tag{3.14}$$

where

$$|g^{f_if_j}_{VZ'}|^2 = \frac{1}{4}[(\mathrm{Re}\Omega_{Lf_if_j} + \mathrm{Re}\Omega_{Rf_if_j})^2 + (\mathrm{Im}\Omega_{Lf_if_j} + \mathrm{Im}\Omega_{Rf_if_j})^2],$$
$$|g^{f_if_j}_{AZ'}|^2 = \frac{1}{4}[(\mathrm{Re}\Omega_{Lf_if_j} - \mathrm{Re}\Omega_{Rf_if_j})^2 + (\mathrm{Im}\Omega_{Lf_if_j} - \mathrm{Im}\Omega_{Rf_if_j})^2]. \tag{3.15}$$

Correspondingly, the EDM form factor is

$$F_{Ef_if_j} \simeq \frac{ieQ_{f_j}m_{f_j}}{8\pi^2 m^2_{Z'}}(g^{f_if_j}_{VZ'}g^{f_if_j*}_{AZ'} - g^{f_if_j}_{AZ'}g^{f_if_j*}_{VZ'}), \tag{3.16}$$

where

$$g^{f_if_j}_{VZ'}g^{f_if_j*}_{AZ'} - g^{f_if_j}_{AZ'}g^{f_if_j*}_{VZ'} = i(\mathrm{Re}\Omega_{Lf_if_j}\mathrm{Im}\Omega_{Rf_if_j} - \mathrm{Re}\Omega_{Rf_if_j}\mathrm{Im}\Omega_{Lf_if_j}). \tag{3.17}$$

1. CP property

The electromagnetic dipole moments can be distinguished in two scenarios due to the CP property:

(i) The CP conservation (CP-c) case, which only allows a_{fi} (d_{f_i} is forbidden), can happen when

$$\mathrm{Re}\Omega_L \neq 0, \quad \mathrm{Im}\Omega_L \neq 0, \quad \mathrm{Re}\Omega_R = 0, \quad \mathrm{Im}\Omega_R = 0.$$

(ii) The CP violation (CP-v) case, which gives rise to both a_{fi} and d_{f_i}, can occur when

$$\mathrm{Re}\Omega_L \neq 0, \quad \mathrm{Im}\Omega_L = 0, \quad \mathrm{Re}\Omega_R = 0, \quad \mathrm{Im}\Omega_R \neq 0.$$

C. Predictions on the tau electromagnetic dipole moments

In this section, we carry out the phenomenological analysis on the tau MDM and EDM by considering the different Z' gauge bosons, Z'_S, Z'_{LR}, Z'_χ, Z'_ψ, and Z'_η, the coupling parameters, $\Omega_{L,R}$, of which were computed in Ref. [5].

The tau MDM is conformed by

$$a_\tau = a_{\tau e} + a_{\tau\mu} + a_{\tau\tau}, \tag{3.18}$$

where $a_{l_il_j}$ are given in Eq. (3.4) in terms of $F_{Mf_if_j}$, the explicit expressions of which were given in Eq. (3.14).

Otherwise, the tau EDM contributions are

$$d_\tau = d_{\tau e} + d_{\tau\mu} + d_{\tau\tau}, \tag{3.19}$$

where $d_{l_il_j}$ are given in Eq. (3.4). The explicit expressions for the $F_{Ef_if_j}$ form factors are given in Eq. (3.16). Below, we are going to analyze the EDM in $e$cm units, as is common in the literature.

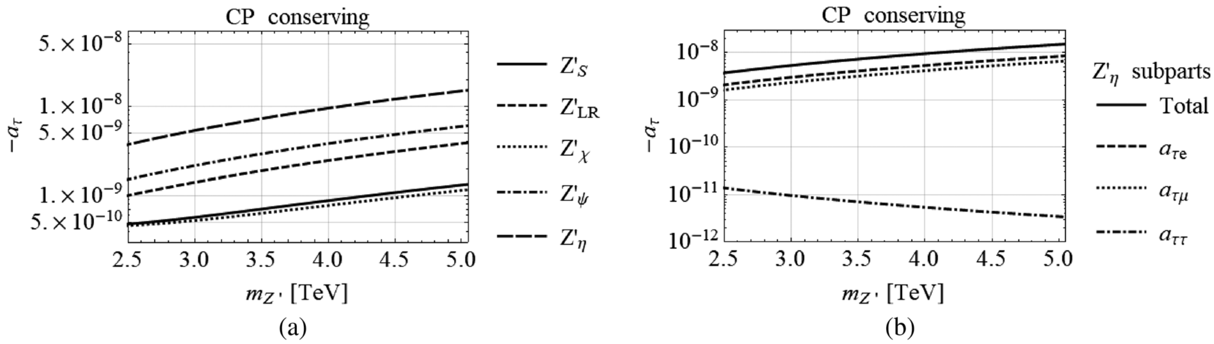

FIG. 3. *CP* conservation: tau static anomalous magnetic dipole moment. (a) Contributions of the Z' gauge bosons. (b) Main contribution due to Z'_η and its subparts.

1. CP conservation: a_τ

For the CP-c analysis, we follow the scenario $\mathrm{Re}\Omega_L \neq 0$, $\mathrm{Im}\Omega_L \neq 0$, $\mathrm{Re}\Omega_R = 0$, $\mathrm{Im}\Omega_R = 0$. Here, a_τ is provided by Eq. (3.18); the $a_{\tau e}$ and $a_{\tau\mu}$ quantities receive contributions from the coupling parameters, $\Omega_{L,R\tau e}$ and $\Omega_{L,R\tau\mu}$, which can be derived from Eq. (2.3) (for more details, see Ref. [5]), and $a_{\tau\tau}$ depends on the $\Omega_{L,R\tau\tau}$ parameter [5]. Regarding the Z' boson mass, we are going to explore the mass interval, $m_{Z'} = [2.5, 5]$ TeV, which respects the current experimental bounds on the Z' boson mass [15]. The a_τ results in the CP-c scenario as a function of the Z' gauge boson mass, for the interval $m_{Z'} = [2.5, 5]$ TeV, are illustrated in Fig. 3. In Fig. 3(a), the contributions from the various Z' gauge bosons are shown; the highest signal is provided by the Z'_η boson, which goes from 10^{-9} to 10^{-8}, barely 1 order of magnitude below the SM electroweak (EW) contribution $a_\tau(\mathrm{EW}) = 5.52\times10^{-7}$ with opposite sign, while the lowest one corresponds to the Z'_χ boson, which ranges between 10^{-10} and 10^{-9}. In Fig. 3(b), the main contribution belonging to Z'_η is detailed, in which the $a_{\tau e}$ and $a_{\tau\mu}$ components essentially represent the signal, while $a_{\tau\tau}$ is 3 orders of magnitude below. To contextualize our results, we cite some predictions of a_τ in some extended models. The estimations for a_τ coming from two-Higgs doublet models [51], the minimal supersymmetric Standard Model [52], and the unparticle model [53] are of the order of 10^{-6}, whereas for leptoquark models, a_τ can be as high as 10^{-8} [54], which coincides with the strongest prediction of the simplest little Higgs model [55].

2. CP violation: a_τ and d_τ

For the CP-v analysis, we follow the scenario $\mathrm{Re}\Omega_L \neq 0$, $\mathrm{Im}\Omega_L = 0$, $\mathrm{Re}\Omega_R = 0$, $\mathrm{Im}\Omega_R \neq 0$. Figure 4 presents the results of the tau MDM and EDM in the CP-v case. The MDM (a_τ) is displayed in Figs. 4(a) and 4(b): in (a), the contributions from the different Z' gauge bosons essentially reproduce the same signals as in the CP-c case but are slightly enhanced, and also the Z'_η prediction is the leading signal, being of the order of 10^{-8}, and the Z'_χ signal is the minor one reaching 10^{-9}; in (b), the components of the main signal (Z'_η) are displayed.

On the other hand, in 4(c) and 4(d), the EDM of the tau lepton is displayed. In (c) the strongest prediction corresponds to the Z'_η gauge boson, while the lower is offered by Z'_ψ (Z'_S) in the interval $m_{Z'} = [2.5, 3.9)$ TeV ($m_{Z'} = [3.9, 5)$ TeV), respectively; in (d) the subparts of the main prediction are shown, where $d_{\tau\mu}$ represents the main contribution.

In the Fig. 4(e), the ϕ_τ phase [see Eq. (3.6)] is depicted and represents the relative size of the EDM respect to the MDM. From this plot, we can appreciate that the Z'_χ signal provides the values closest to 1, while the smallest one corresponds to Z'_ψ.

D. Chromoelectromagnetic dipole moments

The effective Lagrangian that comprises chromoelectromagnetic dipole moments for quarks, $f = q$, is

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{2} T^a \bar{f} \sigma^{\mu\nu} (\mu + i d \gamma^5) f G^a_{\mu\nu}, \tag{3.20}$$

where T^a is the color generator, μ is the chromomagnetic and d the chromoelectric form factor, and $G^a_{\mu\nu}$ is the gluon strength field. The CMDM μ_f and the CEDM d_f [15,30,56] can be defined dimensionless as $\hat{\mu}_f$ and $\hat{d}_f$:

$$\mu \equiv \frac{g_s}{m_f} \hat{\mu}_f, \qquad d \equiv \frac{g_s}{m_f} \hat{d}_f. \tag{3.21}$$

In analogy to the electromagnetic dipoles given in (3.4), then, $\mu \equiv F_M$ and $d \equiv F_E$.

In general, the chromoelectromagnetic dipoles are complex quantities. The current available experimental bounds from PDG [15,30] to the top-quark dipoles are $-0.053 < \mathrm{Re}\,\hat{\mu}_t < 0.026$ and $-0.068 < \mathrm{Im}\,\hat{d}_t < 0.067$, obtained in the context of an off-shell top-gluon vertex with a timelike scenario $q^2 > 0$ in hadronic $t\bar{t}$ production, in which absorptive imaginary parts for both dipoles are expected. On the other hand, in contrast to the fermion electromagnetic dipole moments defined with the on-shell photon,

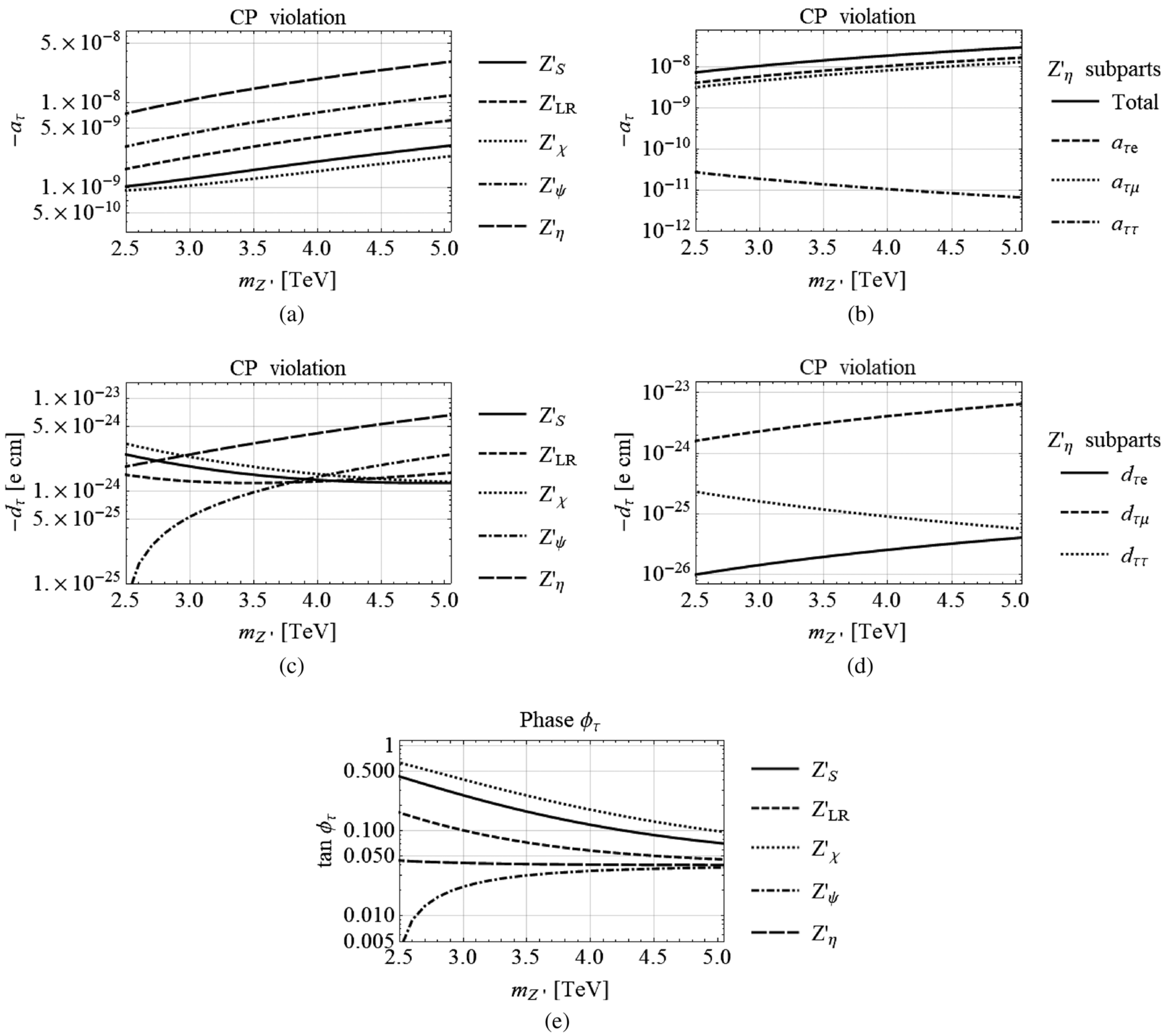

FIG. 4. *CP* violation: tau static electromagnetic dipole moments. (a) a_τ, contributions of the Z' gauge bosons. (b) a_τ, main contribution due to Z'_η and its subparts; (c) d_τ, contributions of the Z' gauge bosons. (d) d_τ, main contribution due to Z'_η and its subparts. (e) Phase between EDM and MDM.

$q^2 = 0$, in perturbative QCD, the chromoelectric dipoles cannot be defined on shell because this does not make sense; they are not quantities physically sensitive to that case, and instead, they must be measured off shell at large gluon momentum transfer $q^2 \neq 0$ [40].

To properly compare our obtained results in this section with the SM predictions, we have to revisit the chromomagnetic dipole moment of the top quark in the SM at the one-loop level, for which we have chosen to evaluate at $q^2 = \pm m_Z^2$. We must keep in mind that the weak-mixing angle, $\sin^2\theta_W(m_Z) = 0.23122$, and alpha strong, $\alpha_s(m_Z) = 0.1181$, are experimentally known at the scale of the Z mass [15]. Reference [40] only calculated the $q^2 = -m_Z^2$ case, and the authors allowed a small mass of the virtual gluons; nevertheless, we cannot reproduce their Eq. (9). On the other hand, we agree with these authors in the observation that the three-gluon vertex diagram considered in Ref. [35] was not properly calculated; such a diagram is in fact divergent when $q^2 = 0$. In advance, our derived results given in Table III show that the contributions at $q^2 = \pm m_Z^2$ coming from the virtual particles γ, Z, H, and g barely change, while the W contribution changes sign for its real part; besides, the three-gluon vertex contribution, at which we refer as $3g$, cures its ill behavior when it is off shell. Furthermore, we have found that the contributions from W and $3g$ provide imaginary parts, and as far as we know, this characteristic has not been carefully reported in the literature. Notice that the on-shell gluon scenario, $q^2 = 0$, for γ, Z, H, and g, the diagrams of which have in common the same quark as virtual and off shell, serves as an

TABLE III. Anomalous chromomagnetic dipole moment of the top quark at one-loop level in the SM as function of the gluon momentum transfer $q^2 = -m_Z^2$, 0, m_Z^2. The total value for $q^2 = 0$ does not take into account the triple gluon contribution because it diverges.

	q^2		
$\hat{\mu}_t$	$-m_Z^2$	0	m_Z^2
γ	2.47×10^{-4}	2.58×10^{-4}	2.71×10^{-4}
Z	-1.79×10^{-3}	-1.85×10^{-3}	-1.91×10^{-3}
W	$-3.42 \times 10^{-5} - 9.43 \times 10^{-4} i$	$-2.64 \times 10^{-6} - 1.23 \times 10^{-3} i$	$1.44 \times 10^{-4} - 1.19 \times 10^{-3} i$
H	1.89×10^{-3}	1.95×10^{-3}	2.02×10^{-3}
g	-1.50×10^{-3}	-1.57×10^{-3}	-1.64×10^{-3}
$3g$	-2.13×10^{-2}	indeterminate	$-1.22 \times 10^{-2} - 2.56 \times 10^{-2} i$
Total	$-2.24 \times 10^{-2} - 9.43 \times 10^{-4} i$	$-1.20 \times 10^{-3} - 1.23 \times 10^{-3} i$	$-1.34 \times 10^{-2} - 2.68 \times 10^{-2} i$

approximate or rough average with respect to the $q^2 = \pm m_Z^2$ evaluations. These results will soon be presented in depth elsewhere, where in addition we will show that in our calculations it is unnecessary to consider a small mass of the virtual gluons [57].

E. Predictions on the chromoelectromagnetic dipole moments of the top quark induced by FCNCs

To calculate the chromoelectromagnetic dipoles of the top quark, we are going to consider the gluon off shell with a 4-momentum transfer $q^2 = \pm m_Z^2$; nevertheless, despite being aware that the chromodipoles must be computed with $q^2 \neq 0$, for comparison purposes, we also are going to evaluate the on-shell scenario ($q^2 = 0$). In advance, as it will be shown below, the $\mathrm{Re}\hat{\mu}_t(Z')$ and $\mathrm{Re}\hat{d}_t(Z')$ result is essentially invariant to any of the three cases $q^2 = 0, \pm m_Z^2$, while only the timelike scenario, $q^2 = m_Z^2$, gives rise to $\mathrm{Im}\hat{\mu}_t(Z')$ and $\mathrm{Im}\hat{d}_t(Z')$.

The chromoelectromagnetic one-loop diagram is analogous to the photon case, as already commented on in Sec. III B, except that for the gluon in the loop integral [see Eq. (3.13)] eQ_{f_j} must be replaced by $g_s T^a$.

The top-quark CMDM is conformed by the contributions

$$\hat{\mu}_t = \hat{\mu}_{tu} + \hat{\mu}_{tc} + \hat{\mu}_{tt}, \tag{3.22}$$

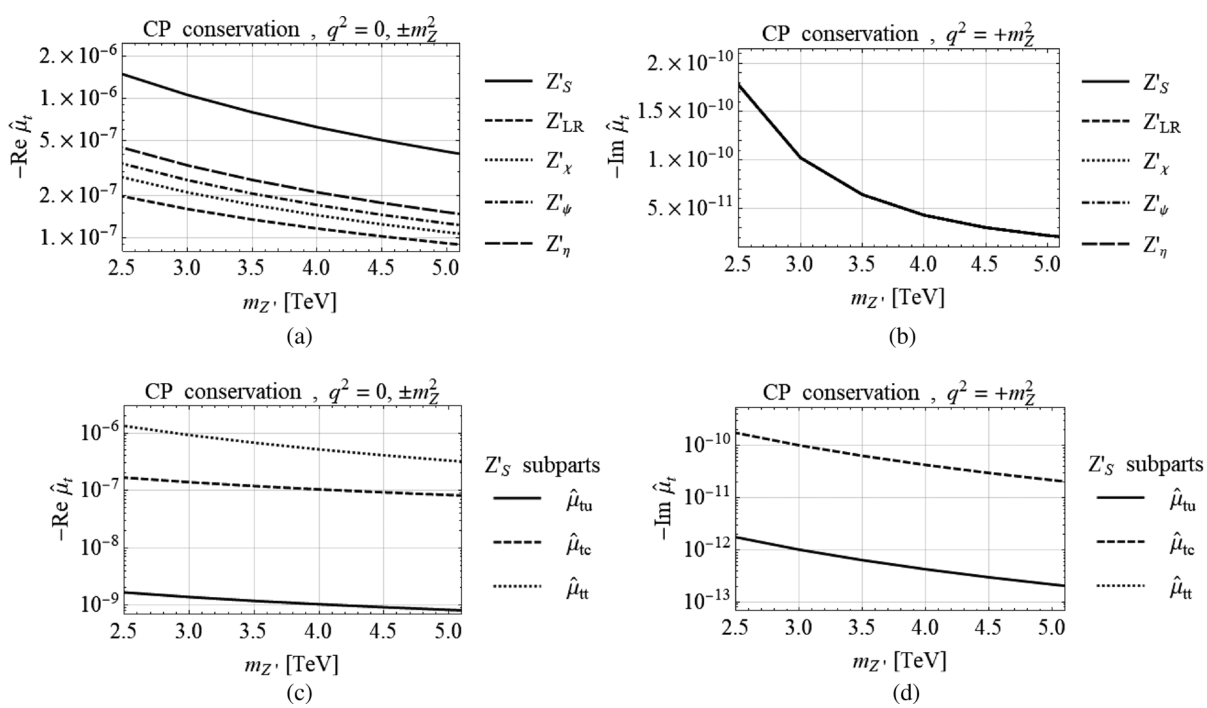

FIG. 5. *CP* conservation: top magnetic dipole moment. (a) Contributions of the Z' gauge bosons from different models to the $\mathrm{Re}\hat{\mu}_t$ generated by $q^2 = 0, \pm m_Z^2$ and (b) $\mathrm{Im}\hat{\mu}_t$ generated by $q^2 = m_Z^2$, where all the different Z' bosons share essentially the same imaginary value. (c) Main contribution due to Z'_S to $\mathrm{Re}\hat{\mu}_t$ and (d) $\mathrm{Im}\hat{\mu}_t$, which arise only from the nondiagonal subparts $\hat{\mu}_{tu,tc}$.

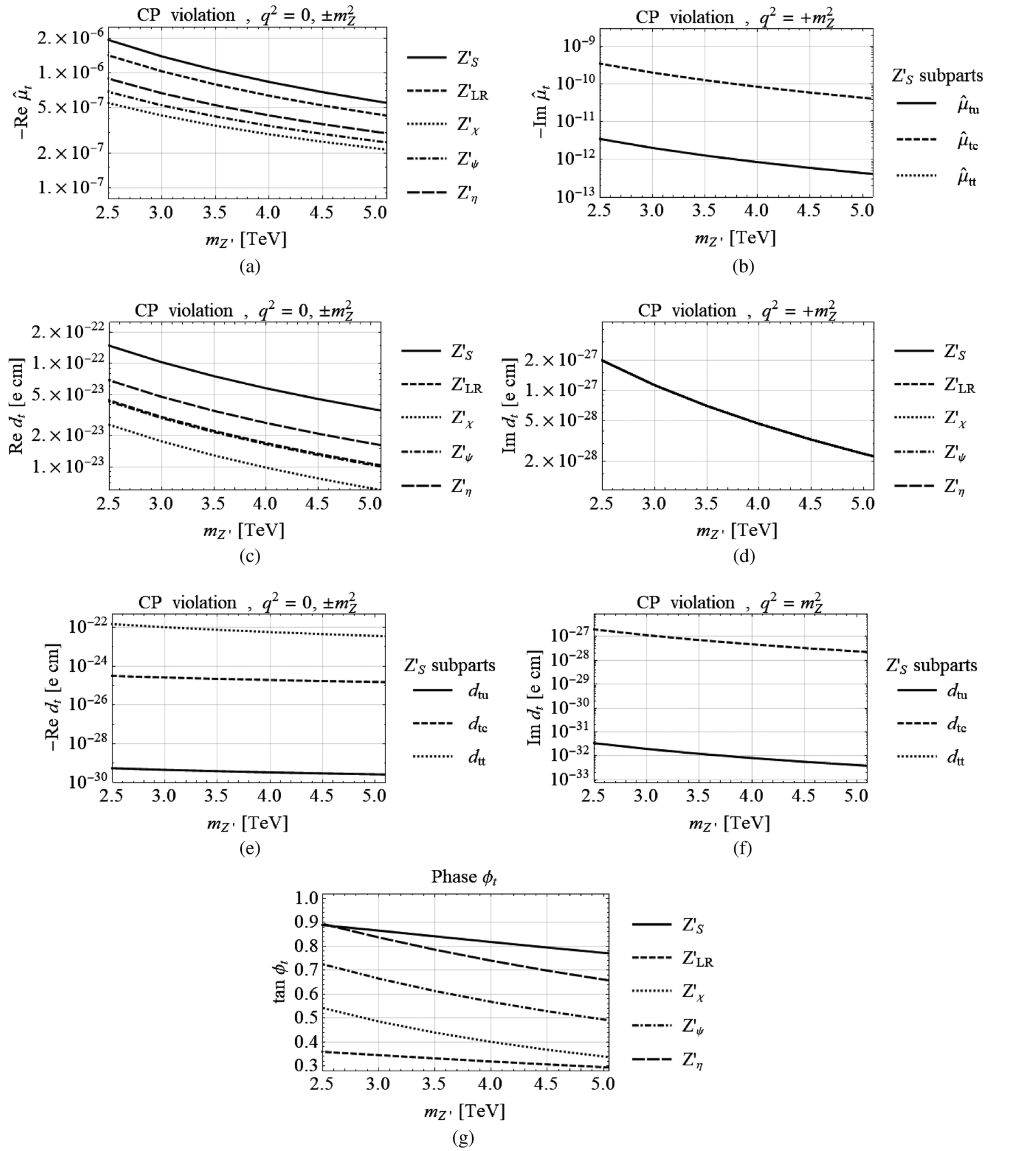

FIG. 6. *CP* violation: top electromagnetic dipole moments. (a) Contributions of the Z' gauge bosons from different models to $\mathrm{Re}\hat{\mu}_t$ and (b) the $\mathrm{Im}\hat{\mu}_t$. (c) The $\mathrm{Re}\hat{d}_t$ coming from the different Z' and (d) the $\mathrm{Im}\hat{d}_t$. (e),(f) The respective real and imaginary parts generated by the subparts of the main contributor Z'_S. (g) The phase.

and similarly for the top CEDM,

$$\hat{d}_t = \hat{d}_{tc} + \hat{d}_{tc} + \hat{d}_{tt}, \tag{3.23}$$

where the components are defined in (3.21). Below, we are going to present the CEDM in units of $e$cm.

As already commented on above, the $\mathrm{Re}\hat{\mu}_t(Z')$ and $\mathrm{Re}\hat{d}_t(Z')$ parts are essentially invariant to the $q^2 = 0$, $\pm m_Z^2$ scenarios, and the differences are away from the significant numbers; hence, the same form factors F_M and F_E derived for the on-shell case in Eqs. (3.14) and (3.16), respectively, allow us now to compute $\mathrm{Re}\hat{\mu}_t(Z') = F_M$ and

$\mathrm{Re}\hat{d}_t(Z') = F_E$. These form factors were already used to evaluate the tau static dipoles, where $m_\tau \ll m_{Z'}$, but they are still appropriate to evaluate the top-quark dipoles because $m_t \ll m_{Z'}$; we have crossed-checked this by comparing with the unapproximated form factors, and they match excellently. On the other side, the imaginary parts of the chromoelectromagnetic top-quark dipoles, which arise when $q^2 = m_Z^2$, are computed with the exact form factors.

1. CP conservation: $\hat{\mu}$

For the analysis of the CP-c, we follow the scenario $\mathrm{Re}\Omega_L \neq 0$, $\mathrm{Im}\Omega_L \neq 0$, $\mathrm{Re}\Omega_R = 0$, $\mathrm{Im}\Omega_R = 0$. Since the coupling parameters $\Omega_{L,R\,tc}$, $\Omega_{L,R\,tu}$ in $\hat{\mu}_t$ were estimated in Ref. [4], we follow that procedure updated to the current permitted values for the Z' mass, where Eqs. (2.7) are employed. In Figs. 5(a)–5(d), the results for the $\hat{\mu}_t$ in the CP-c case are shown as a function of the Z' boson mass, $m_{Z'} = [2.5, 5]$ TeV: in (a), the contributions to $\mathrm{Re}\hat{\mu}_t$ from the different Z' gauge bosons are presented, and the leading contribution is due to the Z'_S gauge boson, which decreases from 10^{-6} to 10^{-7} in the interval, while Z'_{LR} is responsible for the smallest values, which go from 10^{-7} to 10^{-8}; in (b), the $\mathrm{Im}\hat{\mu}_t$ is shown, and all the different Z' bosons share the same imaginary value; in (c), the subparts of the main contributor, Z'_S, with its $\mathrm{Re}\hat{\mu}_t$ are displayed, with $\hat{\mu}_{tt}$ being the highest one, while $\hat{\mu}_{tc}$ is 3 orders of magnitude below; in (d), the subparts of Z'_S that contribute to $\mathrm{Im}\hat{\mu}_t$ are exhibited, which are generated only by the nondiagonals $\hat{\mu}_{tu}$ and $\hat{\mu}_{tc}$. Now, we can compare with the closest SM value, which corresponds to $\hat{\mu}_t(W) = -3.419 \times 10^{-5} - 9.434 \times 10^{-4} i$, when $q^2 = -m_Z^2$, where the real part of the Z'_S starts 1 order of magnitude below, while the imaginary part is 6 orders lower.

2. CP violation: $\hat{\mu}$ and $\hat{d}$

The CP-v analysis is carried out according to the scenario $\mathrm{Re}\Omega_L \neq 0$, $\mathrm{Im}\Omega_L = 0$, $\mathrm{Re}\Omega_R = 0$, $\mathrm{Im}\Omega_R \neq 0$. The $\hat{\mu}_t$ results are presented in Figs. 6(a) and 6(b). In Fig. 6(a), the contributions from the different Z' gauge bosons can be appreciated, and the Z'_S provides again the highest signal to $\mathrm{Re}\hat{\mu}_t$ but a little higher than in the CP-c case, being 10^{-6} in $m_{Z'} = [2.5, 3.5)$ and 10^{-7} in $m_{Z'} = [3.5, 5]$ TeV. Here, Z'_χ produces the lowest value, while in the CP-c scenario was due to the Z'_{LR}. In Fig. 6(b), the imaginary part remains in the order of 10^{-10}. The corresponding subparts due to the main contributor, Z'_S, behave in a way similarly as in the CP-c case; we do not show them. Once again, these values are just below the SM subpart coming from the W gauge boson diagram.

Now, we turn our attention to the CEDM, which does not exist in the SM at the one-loop level. The results for $\hat{d}_t$ are shown in Figs. 6(c)–6(f) in units of $e$cm. Figure 6(c) displays the contributions to $\mathrm{Re}\hat{d}_t$ from the different Z' gauge bosons, and again the same results are provided by the scenarios $q^2 = 0, \pm m_Z^2$, the differences are away from the significant numbers, and also the Z'_S is responsible for the highest signal, being 10^{-22} e cm in $m_{Z'} = [2.5, 3.2)$ and 10^{-23} e cm in $m_{Z'} = [3.2, 5]$ TeV. In contrast, the Z'_χ boson offers the lowest signal that is 1 order of magnitude below the Z'_S one. Figure 6(d) exhibits the corresponding imaginary part. In Fig. 6(e), we can see that the diagonal $\mathrm{Re}\hat{\mu}_{tt}$ is the responsible for the highest value. In Fig. 6(f), the nondiagonal subparts generate the imaginary part. Finally, the ϕ_t phase is presented in Fig. 6(g), in which the Z'_S boson yields the most intense CP-violation behavior, whereas the lesser one is due to the Z'_{LR} boson.

IV. CONCLUSIONS

The new physics effects due to the possible presence of FCNCs mediated by a new neutral massive gauge boson, identified as Z', have been studied on the MDM (EDM) of the tau lepton and the CMDM (CEDM) of the top quark. The theoretical framework corresponds to the most general renormalizable Lagrangian that includes flavor violation mediated by a gauge boson type Z', which can be induced in grand unification models. By using constraints, calculated in a previous work, of the lepton flavor–violating couplings $Z'\tau\mu$ and $Z'\tau e$, coming from experimental bounds for the lepton flavor–violating $\tau \to \mu\mu^+\mu^-$ and $\tau \to \mu e^+ e^-$ decays, the MDM (a_τ) and the EDM (d_τ) of the tau lepton were estimated. Specifically, for the CP-conservation case, in which only a_τ is induced, we found that $|a_\tau| \sim 10^{-8}$ at best for the Z'_η boson, which is of the same order of magnitude as the respective predictions in the leptoquark models and the simplest little Higgs model; the remanning Z' bosons offer values for $|a_\tau|$ between 10^{-10} and 10^{-9}. Besides, for the CP-violation case, also $|a_\tau|$ can be as high as 10^{-8} for the Z'_η boson, while the other Z' boson contributions can reach 10^{-9}; in relation to the EDM (d_τ), the highest prediction for the $|d_\tau|$ corresponds to the Z'_η, with $|d_\tau|$ being of the order of 10^{-24} e cm, whereas the SM prediction is less than 10^{-34} e cm.

In addition, by considering the results of a previous work in which the strength of the $Z'tc$ and $Z'tu$ couplings were estimated through the $D^0 - \bar{D}^0$ mixing system, the FCNC predictions for the CMDM ($\hat{\mu}_t$) and the CEDM ($\hat{d}_t$) of the top quark were calculated. We have revisited the SM predictions in order to be able to compare the results of the chromodipoles induced by FCNCs, for which we have considered the off-shell gluon 4-momentum transfer $q^2 = \pm m_Z^2$, where imaginary contributions are generated. For the CP-conservation and CP-violation scenarios, the main signal is offered by the Z'_S boson, being of the order of $-\mathrm{Re}\hat{\mu}_t \sim 10^{-6}$–$10^{-7}$ and $-\mathrm{Im}\hat{\mu}_t \sim 10^{-10}$–$10^{-11}$, where the real part value starts barely 1 order of magnitude below the SM prediction due to the W boson. The CEDM, $\hat{d}_t$, is estimated to be in the interval $-\mathrm{Re}\hat{d}_t \sim 10^{-23}$–$10^{-22}$ e cm and $-\mathrm{Im}\hat{d}_t \leq 10^{-27}$ e cm, where signals provided by the Z'_S boson correspond to the best situation. All our predictions agree with the current experimental limits.

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