S_3 flavored left-right symmetric model of quarks

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We construct a model based on the electroweak gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ augmented by an S_3 symmetry. We assign nontrivial S_3 transformation properties to the quarks, and consequently we need two scalar bidoublets. Despite the extra bidoublet, we have only six Yukawa couplings thanks to the discrete symmetry. Diagonalization of the quark mass matrices shows that at the leading order only the first two generations mix, resulting in a block diagonal CKM matrix, and the first generation quarks are massless. Inclusion of subleading terms produces an acceptable CKM matrix up to corrections of $\mathcal{O}(\lambda^4)$. As for the first generation quark masses, we obtain a satisfactory value for m_u/m_d. The masses themselves, though in the same ballpark, turn out to be somewhat smaller than the accepted range.

I. INTRODUCTION

One very compelling extension of the standard model (SM) is the left-right symmetric (LRS) model [1,2] based on the electroweak gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Unlike the SM, the left-chiral and right-chiral fermions are treated similarly in these models.

In the minimal LRS model, the left-chiral and right-chiral quarks are assigned the following representations under the gauge group:

$$Q_{iL}: \left(2, 1, \frac{1}{3}\right), \qquad Q_{iR}: \left(1, 2, \frac{1}{3}\right), \tag{1}$$

where the index i runs from 1 to 3 to accommodate three generations. At a high scale where the $SU(2)_R$ is spontaneously broken by the vacuum expectation values (VEVs) of $(1,2,1)$ or $(1,3,2)$ scalar multiplets of the gauge group, the quarks do not acquire any mass because they do not have any Yukawa couplings with these scalars. Allowing the Yukawa couplings for the quarks requires the presence of a scalar bidoublet:

$$\Phi: (2, 2, 0). \tag{2}$$

It has two neutral components and, therefore, two possible VEVs. These VEVs break the standard model symmetry group and also provide masses to the quarks. The mass matrix has many free parameters. There are nine Yukawa couplings involving Φ that relate three generations of left-chiral quarks with three generations of right-chiral quarks. Besides, since the representation of Φ under the gauge group is real, there are nine more couplings where Φ is replaced by its complex conjugate,

$$\tilde{\Phi} = \tau_2 \Phi^* \tau_2, \tag{3}$$

suitably sandwiched by the antisymmetric Pauli matrix so that its transformation property is exactly the same as that of Φ. Because of this large number of parameters, the quark mass matrices do not have much of a predictive power.

In this article, we impose an S_3 symmetry between the generations and show that the number of Yukawa couplings is drastically reduced to the extent that predictions are possible. Such a symmetry has been explored extensively in the context of the SM gauge group [3–39]. However, to our knowledge, this discrete symmetry in the context of the left-right symmetric gauge group [40,41] has not been explored very much. We will show that the enhanced gauge symmetry, along with the discrete symmetry, leads to relations between the quark masses and mixings.

II. THE MODEL WITH A HORIZONTAL S_3 SYMMETRY

We extend the minimal LRS model with an extra S_3 symmetry that acts between different generations of fermions. This S_3 symmetry has three different irreducible representations, 1, 1', and 2, where the numbers signify the dimension of the representation matrices. The group has

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one order-two and one order-three generator. In the 2 representation, we take them to be

$$g_2 = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}, \qquad g_3 = \begin{bmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}. \qquad (4)$$

For this choice of basis, we assign the following representations to the fermions,

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} : 2, \qquad Q_3 : 1, \qquad (5)$$

following the same rule for left and right chiral quarks. In order to obtain an acceptable mass pattern, we now need scalars to be in the 2 representation of S_3. This means that we need to add an extra bidoublet over and above what was shown in Eq. (2) [42]. Calling the two scalar multiplets Φ_1 and Φ_2, we assign them the representation

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} : 2 \qquad (6)$$

under the S_3 symmetry. Keeping in mind the fact that for any term in the Lagrangian where there is a Φ, there is another term containing $\tilde{\Phi}$, we can write down the most general Yukawa couplings involving quarks as

$$\begin{aligned}
-\mathcal{L}_Y = &A(\bar{Q}_{1L}\Phi_1 + \bar{Q}_{2L}\Phi_2)Q_{3R} + C\bar{Q}_{3L}(\Phi_1 Q_{1R} + \Phi_2 Q_{2R}) \\
&+ B[(\bar{Q}_{1L}\Phi_2 + \bar{Q}_{2L}\Phi_1)Q_{1R} + (\bar{Q}_{1L}\Phi_1 - \bar{Q}_{2L}\Phi_2)Q_{2R}] \\
&+ \tilde{A}(\bar{Q}_{1L}\tilde{\Phi}_1 + \bar{Q}_{2L}\tilde{\Phi}_2)Q_{3R} + \tilde{C}\bar{Q}_{3L}(\tilde{\Phi}_1 Q_{1R} + \tilde{\Phi}_2 Q_{2R}) \\
&+ \tilde{B}[(\bar{Q}_{1L}\tilde{\Phi}_2 + \bar{Q}_{2L}\tilde{\Phi}_1)Q_{1R} + (\bar{Q}_{1L}\tilde{\Phi}_1 - \bar{Q}_{2L}\tilde{\Phi}_2)Q_{2R}] \\
&+ \text{H.c..} \qquad (7)
\end{aligned}$$

After symmetry breaking, both Φ_1 and Φ_2 develop VEVs:

$$\langle \Phi_a \rangle = \begin{pmatrix} \kappa_a & 0 \\ 0 & \kappa'_a \end{pmatrix}, \qquad a = 1, 2. \qquad (8)$$

The resulting mass matrices for the quarks are of the form

$$\mathcal{M}_u = F\kappa_1 + G\kappa_2 + \tilde{F}\kappa'_1 + \tilde{G}\kappa'_2, \qquad (9a)$$

$$\mathcal{M}_d = \tilde{F}\kappa_1 + \tilde{G}\kappa_2 + F\kappa'_1 + G\kappa'_2, \qquad (9b)$$

where

$$F = \begin{pmatrix} 0 & B & A \\ B & 0 & 0 \\ C & 0 & 0 \end{pmatrix}, \qquad G = \begin{pmatrix} B & 0 & 0 \\ 0 & -B & A \\ 0 & C & 0 \end{pmatrix}, \qquad (10)$$

and $\tilde{F}$ and $\tilde{G}$ are matrices which have exactly the same form, except that they involve the Yukawa couplings with

tilde marks. We will assume that all Yukawa couplings are real, and so are the VEVs. Our task is now to perform the diagonalization of the mass matrices given in Eq. (9) and show that, under some reasonable assumptions, the diagonalization can be performed and the quark mixing matrix can be obtained in a form that is consistent with the present data.

Of course, the matrices shown in Eq. (9) cannot be diagonalized in general with the help of unitary transformations. One needs bi-unitary transformations, which induces different transformations on the left-chiral and right-chiral quarks. For the sake of the CKM matrix, we need only the mixing of the left-chiral fermions. The relevant mixing matrices can be obtained by considering the diagonalization of $\mathcal{M}_q \mathcal{M}_q^\dagger$, where the index q takes two values, u and d, to distinguish the up-sector quarks from the down-sector quarks. Let us write

$$U_q \mathcal{M}_q \mathcal{M}_q^\dagger U_q^\dagger = D_q^2, \qquad (11)$$

where D_q^2 is a diagonal matrix whose diagonal elements are the mass-squared values of the quarks of type q (i.e., u or d). Then the CKM matrix will be given by

$$V_{\text{CKM}} = U_u U_d^\dagger. \qquad (12)$$

We, therefore, need to find the diagonalizing matrices U_u and U_d. For this, we need to proceed in steps, making some assumptions which we now describe.

III. LARGE AND SMALL TERMS

In order to perform the diagonalization, we will first make some assumptions about the relative magnitudes of different parameters. The first thing we assume is that the primed VEVs are much smaller compared to the unprimed ones:

$$\kappa'_1, \kappa'_2 \ll \kappa_1, \kappa_2. \qquad (13)$$

The opposite assumption $\kappa'_1, \kappa'_2 \gg \kappa_1, \kappa_2$ will serve as well and amounts to fixing a convention. Such an assumption can naturally suppress the mixing between the gauge bosons in the left and right sectors. The terms in Eq. (9) proportional to the unprimed VEVs will, therefore, be considered dominant, and the other terms, proportional to the primed VEVs, will be considered as perturbations. In this section, we consider diagonalization of the quark mass matrices in the limit $\kappa'_1 = \kappa'_2 = 0$, i.e., in the zeroth order of smallness.

There are only two VEVs at this level of approximation, κ_1 and κ_2. Since the other VEVs have been assumed to be negligible, we can write

$$\kappa_1^2 + \kappa_2^2 = v^2, \qquad (14)$$

where $v = 174$ GeV is the breaking scale of the SM. We also define

$$\tan \beta = \frac{\kappa_2}{\kappa_1}. \tag{15}$$

Henceforth, instead of using κ_1 and κ_2 directly, we will use the parameters v and β.

Thus, in this zeroth-order approximation, the mass matrices of the quarks are of the form

$$\mathcal{M}_q^{(0)} = v \begin{pmatrix} B_q \sin \beta & B_q \cos \beta & A_q \cos \beta \\ B_q \cos \beta & -B_q \sin \beta & A_q \sin \beta \\ C_q \cos \beta & C_q \sin \beta & 0 \end{pmatrix}, \tag{16}$$

where, for the ease of notation, we have renamed the Yukawa couplings by a subscript q according to the mass matrix to which they contribute:

$$A_u = A, \qquad B_u = B, \qquad C_u = C, \tag{17a}$$

$$A_d = \tilde{A}, \qquad B_d = \tilde{B}, \qquad C_d = \tilde{C}. \tag{17b}$$

The kind of mass matrix shown in Eq. (16) was obtained in our earlier work [39] in the context of the $SU(2)_L \times U(1)_Y$ model. In order to perform a diagonalization of the mass matrices at this level of approximation, we note that

$$|\det(\mathcal{M}_q^{(0)})| = v^3 A_q B_q C_q \sin 3\beta. \tag{18}$$

Since the first generation quark masses are very small, we assume that they are zero at this level and arise entirely from smaller corrections to the mass matrices. Then the determinant must vanish at this level. Without arbitrarily making some of the Yukawa couplings vanish, this can be achieved, in both up and down sectors, if we have

$$\sin 3\beta = 0. \tag{19}$$

This value can be nontrivially obtained by setting

$$\beta = \pi/3. \tag{20}$$

We assume that this is indeed the value of β that comes out of the minimization of the Higgs potential at this level of approximation, i.e., on assuming $\kappa_1' = \kappa_2' = 0$. Then, taking

$$U_q^{(0)} = \begin{pmatrix} -\frac{\sqrt{3}}{2} \sin \theta_q & \frac{1}{2} \sin \theta_q & \cos \theta_q \\ \frac{\sqrt{3}}{2} \cos \theta_q & -\frac{1}{2} \cos \theta_q & \sin \theta_q \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \end{pmatrix} \tag{21}$$

with

$$\tan \theta_q = \frac{C_q}{B_q}, \tag{22}$$

one finds

$$U_q^{(0)} \mathcal{M}_q^{(0)} \mathcal{M}_q^{(0)\dagger} U_q^{(0)\dagger} = v^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & B_q^2 + C_q^2 & 0 \\ 0 & 0 & A_q^2 + B_q^2 \end{pmatrix}. \tag{23}$$

At this stage, then, the CKM matrix is given by

$$V_{\text{CKM}}^{(0)} = U_u^{(0)} U_d^{(0)\dagger} = \begin{pmatrix} \cos(\theta_u - \theta_d) & -\sin(\theta_u - \theta_d) & 0 \\ \sin(\theta_u - \theta_d) & \cos(\theta_u - \theta_d) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{24}$$

This shows that, at the zeroth order, we have only the Cabibbo angle that mixes the first two generations of quarks, whereas the third generation is unmixed. This state of affairs is certainly consistent with the fact that the Cabibbo angle in the largest angle in the CKM matrix, and all others are much smaller. In Sec. IV, we will see how the small angles can arise from the small corrections that we have left out so far.

Before that, we want to summarize the information that we have already obtained about the masses and consequently about the Yukawa couplings. From Eq. (23), we see that, at the zeroth level of approximation,

$$m_t^2 = (A_u^2 + B_u^2)v^2, \qquad m_c^2 = (B_u^2 + C_u^2)v^2, \tag{25a}$$

$$m_b^2 = (A_d^2 + B_d^2)v^2, \qquad m_s^2 = (B_d^2 + C_d^2)v^2. \tag{25b}$$

Although these masses will receive some corrections which will be introduced later, such modifications are expected to be small, and therefore we can use Eq. (25) as a very good approximation to the actual masses. Knowledge of the hierarchy of quark masses then tells us that

$$A_u^2 \gg B_u^2, C_u^2, \qquad A_d^2 \gg B_d^2, C_d^2, \tag{26}$$

so that the third generation is much heavier than the second, and further,

$$A_u^2 \gg A_d^2, \tag{27}$$

to ensure that the top mass is much bigger than the bottom mass. Using Eq. (26) and the definition of Eq. (22), we can write the Yukawa couplings in the form

$$A_q \approx \frac{m_{3q}}{v}, \quad B_q \approx \frac{m_{2q}}{v} \cos \theta_q, \quad C_q \approx \frac{m_{2q}}{v} \sin \theta_q, \tag{28}$$

where m_{3q} and m_{2q} denote the masses of the third- and second-generation quarks in the sector marked by q, i.e.,

$$m_{3u} = m_t, \qquad m_{2u} = m_c, \qquad m_{3d} = m_b, \qquad m_{2d} = m_s. \tag{29}$$

At this point, perhaps it is worth reemphasizing the main conclusion of this section. Here, we have considered an approximate reality where $m_u = m_d = 0$ and $V_{i3} = V_{3i} = 0$ ($i = 1, 2$) as well. The vanishing of V_{3i} ($i = 1, 2$) will follow automatically from the vanishing of V_{i3} ($i = 1, 2$) due to the unitarity of the CKM matrix. We are, thus, left with four zeros, viz., $m_u = 0$, $m_d = 0$, and $V_{i3} = 0$ ($i = 1, 2$), which are disconnected in the SM; i.e., they are four different *accidents* in the framework of the SM. But, in our model, one needs only one *accident*, given by Eq. (19), to achieve all these zeros, i.e., the four zeros are connected. Therefore, concerning the small parameters in the quark Yukawa sector, our construction provides a sense of aesthetic connection that is absent in the SM. Moreover, this approximate reality with $\kappa'_a = 0$ forbids W_L-W_R mixing. In the next section, we will see that turning on small values of κ'_a leads to small CKM elements as well as the first generation quark masses. Therefore, in our scenario, these small masses and mixings in the quark sector owe their origin to the same parameters which govern the smallness of the W_L-W_R mixing.

IV. INCLUDING THE SMALLER TERMS

We now try to see the effects of nonzero values of κ'_1 and κ'_2. The extra contributions that appear in the mass matrices will be denoted by $\mathcal{M}'$, i.e.,

$$\mathcal{M}_q = \mathcal{M}_q^{(0)} + \mathcal{M}'_q. \tag{30}$$

These contributions will come from two sources. First, there are terms proportional to κ'_1 and κ'_2 in Eq. (9). Second, the minimization of the Higgs potential will now not give Eq. (20), but rather

$$\sin 3\beta = 3\delta, \tag{31}$$

with some small value of δ.

All correction terms in the mass matrices will have one factor of some Yukawa coupling. Motivated by the hierarchy among the Yukawa couplings noted in Sec. III, we will keep only the terms proportional to A_u as the dominant corrections to $\mathcal{M}_q^{(0)}$ defined in Eq. (16), with the understanding that the contribution from other terms are proportional to much smaller Yukawa couplings, and are negligible at the level of accuracy that we seek for. Keeping these in mind, we can write the dominant corrections are as follows:

$$\mathcal{M}'_u \approx v A_u \begin{pmatrix} 0 & 0 & \frac{\sqrt{3}}{2}\delta \\ 0 & 0 & -\frac{1}{2}\delta \\ 0 & 0 & 0 \end{pmatrix}, \qquad \mathcal{M}'_d \approx A_u \begin{pmatrix} 0 & 0 & \kappa'_1 \\ 0 & 0 & \kappa'_2 \\ 0 & 0 & 0 \end{pmatrix} \tag{32}$$

In order to set up a uniform notation for both up and down sectors, let us introduce some shorthands through the relations

$$(\mathcal{M}'_u)_{13} = m_t \epsilon_u \cos\chi_u, \qquad (\mathcal{M}'_u)_{23} = m_t \epsilon_u \sin\chi_u, \tag{33a}$$

$$(\mathcal{M}'_d)_{13} = m_t \epsilon_d \cos\chi_d, \qquad (\mathcal{M}'_d)_{23} = m_t \epsilon_d \sin\chi_d, \tag{33b}$$

so that

$$\epsilon_u = \delta, \qquad\qquad \chi_u = -\pi/6, \tag{34a}$$

$$\epsilon_d = \sqrt{\kappa'^2_1 + \kappa'^2_2}/v, \qquad \tan\chi_d = \kappa'_2/\kappa'_1. \tag{34b}$$

We now need to examine the mass matrices including these corrections, and the diagonalization procedure.

The first thing that we notice is that, after the inclusion of $\mathcal{M}'$, the determinant of the mass matrix is no more zero and is given by

$$|\det\mathcal{M}_q| = v^2 B_q C_q m_t \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right). \tag{35}$$

This quantity should be equal to the product of the three mass eigenvalues. Therefore, the mass of the first generation quark will be given by

$$m_{1q} = \frac{m_t m_{2q}}{m_{3q}} \sin\theta_q \cos\theta_q \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right)$$
$$= \epsilon'_q m_{2q} \sin\theta_q \cos\theta_q, \tag{36}$$

where we have substituted B_q and C_q using Eq. (28) and defined

$$\epsilon'_q = \frac{m_t}{m_{3q}} \epsilon_q \sin\left(\frac{\pi}{3} - \chi_q\right). \tag{37}$$

In a less cluttered but lengthier way, we can break up Eq. (36) as

$$m_u = \epsilon'_u m_c \sin\theta_u \cos\theta_u, \tag{38a}$$

$$m_d = \epsilon'_d m_s \sin\theta_d \cos\theta_d. \tag{38b}$$

We now look at the diagonalization of the matrices $\mathcal{M}_q \mathcal{M}_q^\dagger$. Referring back to Eq. (11) and its zeroth level analog, Eq. (23), we propose to incorporate the correction to the diagonalizing matrix by writing

$$U_q = X_q U_q^{(0)}, \qquad (39)$$

where X_q is supposed to inflict small corrections on $U_q^{(0)}$. We now parametrize X_q by writing

$$X_q = \begin{pmatrix} 1 & 0 & \alpha_q \\ 0 & 1 & \gamma_q \\ -\alpha_q & -\gamma_q & 1 \end{pmatrix}, \qquad (40)$$

ignoring higher-order terms in α_q and γ_q. We have checked that including a rotation in the 12 sector as well contributes only at a subleading order. Therefore, for our purposes, Eq. (40) constitutes a reasonable approximation for the correction to $U_q^{(0)}$.

If we now evaluate the left side of Eq. (11), using Eq. (39) for U_q and $\mathcal{M}_q$ as the sum of the expression of Eq. (16) with $\beta = \pi/3$ and the corrections from Eq. (32), we should obtain a diagonal matrix, to the accuracy employed in defining the small parameters. From this condition, one should be able to determine the relevant parameters of X_q.

First, we check the diagonal elements. The lower two diagonal elements will pick up small corrections to the formulas of the second and third generation quarks given in Eq. (25), and are unimportant for our purpose. The first diagonal element should give the mass squared of the first generation quark. Evaluation of Eq. (11) gives

$$m_{1q}^2 = (m_{2q}^2 \cos^2\theta_q + m_{3q}^2)\alpha_q^2 - 2m_t m_{3q}\epsilon_q \sin\left(\frac{\pi}{3} - \chi_d\right)\alpha_q$$
$$+ m_t^2 \epsilon_q^2 \sin^2\theta_q \sin^2\left(\frac{\pi}{3} - \chi_d\right). \qquad (41)$$

But the mass value has already been found in Eq. (36) from the consideration of the determinant. Putting in the value from there and neglecting terms which provide corrections of order m_{2q}^2/m_{3q}^2, we can determine α_q as:

$$\alpha_q = \frac{m_t}{m_{3q}}\epsilon_q \sin\theta_q \sin\left(\frac{\pi}{3} - \chi_q\right) \equiv \epsilon_q' \sin\theta_q. \qquad (42)$$

Quite nicely, the 13 element of the left side of Eq. (11) also vanishes at the leading order under the same condition, confirming the consistency of the approximation. Further, the vanishing of the 23 element at the leading order gives the expression for γ_q as

$$\gamma_q = -\frac{m_t}{m_{3q}}\epsilon_q \cos\theta_q \sin\left(\frac{\pi}{3} - \chi_q\right) \equiv -\epsilon_q' \cos\theta_q. \qquad (43)$$

V. THE CKM MATRIX

Using Eq. (12) in conjunction with Eq. (39), we can now write the CKM matrix as

$$V_{\text{CKM}} = X_u U_u^{(0)} U_d^{(0)\dagger} X_d^\dagger = X_u V_{\text{CKM}}^{(0)} X_d^\dagger \approx \begin{pmatrix} \cos\theta_C & -\sin\theta_C & -(\epsilon_d' - \epsilon_u')\sin\theta_u \\ \sin\theta_C & \cos\theta_C & (\epsilon_d' - \epsilon_u')\cos\theta_u \\ (\epsilon_d' - \epsilon_u')\sin\theta_d & -(\epsilon_d' - \epsilon_u')\cos\theta_d & 1 \end{pmatrix}, \qquad (44)$$

where $V_{\text{CKM}}^{(0)}$ has been defined already in Eq. (24) and the Cabibbo angle, θ_C, is defined as

$$\theta_C = \theta_u - \theta_d. \qquad (45)$$

In writing Eq. (44), we have also used the definition of X_q given in Eq. (40) along with the solutions of Eqs. (42) and (43).

In the Wolfenstein parametrization [45] of the CKM matrix, the off-diagonal 12 and 21 elements are of $\mathcal{O}(\lambda)$, where λ is a small parameter that is roughly equal to the

Cabibbo angle. The 23 and 32 elements are $\mathcal{O}(\lambda^2)$, whereas the 13 and 31 elements are $\mathcal{O}(\lambda^3)$. Since we have already produced the Cabibbo mixing of $\mathcal{O}(\lambda)$ at the zeroth order, the perturbations ϵ_q' should be at least of $\mathcal{O}(\lambda^2)$. Taking $\epsilon_q' \sim \mathcal{O}(\lambda^2)$ and $\sin\theta_q \sim \mathcal{O}(\lambda)$, we can see that Eq. (44) reproduces the correct orders of magnitudes for the different CKM elements. For easy comparison, we summarize below the current experimental values for the magnitudes of the elements of the CKM matrix [46,47]:

$$|V_{\text{CKM}}^{\text{exp}}| = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 0.00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}. \qquad (46)$$

While comparing with the experimental values, we should keep in mind that the inherent uncertainty of $\mathcal{O}(\lambda^4)$ in Eq. (44) is much larger than the experimental uncertainties in Eq. (46). Therefore, for the ij-th element of the CKM matrix, we take

$$V_{ij} = V_{ij}^{\text{cen}} \pm \lambda^4, \qquad (47)$$

where the central values are taken from Eq. (46). We assume that $\sin \theta_C \equiv -\lambda \approx -0.225$ has been measured quite accurately and use Eq. (45) to express θ_u in terms of θ_d. Our goal is to see, using Eq. (47), whether there exists a common region in the $\sin \theta_d$ vs $|\epsilon'_d - \epsilon'_u|$ plane, which is allowed by $|V_{ub}|$, $|V_{cb}|$, $|V_{ts}|$ and $|V_{td}|$ simultaneously. We display our result in Fig. 1 where we see that there is indeed some common solution region. Note that there are two different allowed regions from $|V_{ub}|$, which correspond to different signs for $\sin \theta_u$.

With $|\epsilon'_d - \epsilon'_u|$ and $\sin \theta_d$ nearly fixed from Fig. 1, now we have only one parameter, namely ϵ'_u (or equivalently ϵ'_d) to play around. Thus, using Eq. (38), we still need to reproduce *two* light quark masses, m_u and m_d, with only *one* parameter remaining at our disposal. As a matter of fact, the cyan region on the left in Fig. 1, is disfavored because it gives too small values for the down-quark mass. Keeping this in mind, we choose the following values

$$\epsilon'_d = 0.072, \qquad \epsilon'_u = 0.028, \qquad \sin \theta_d = 0.26, \quad (48)$$

which correspond to a benchmark point somewhere in the cyan region on the right in Fig. 1. Using these values we find

$$|V_{ub}| \approx 0.002, \qquad |V_{cb}| \approx 0.044,$$
$$|V_{td}| \approx 0.011, \qquad |V_{ts}| \approx 0.042. \qquad (49a)$$

We see that these values of the CKM elements are acceptable within an error bar of $\mathcal{O}(\lambda^4)$. As commented earlier, we also obtain the light quark masses from this exercise. Taking $m_s = 110$ MeV and $m_c = 1.2$ GeV, and the values of the parameters in Eq. (48), we obtain using Eq. (49)

$$m_u \approx 1.2 \text{ MeV}, \qquad m_d \approx 2.0 \text{ MeV}. \qquad (49b)$$

The values of m_u as well as the ratio m_u/m_d are within tolerable ranges, but the absolute value of m_d comes out to
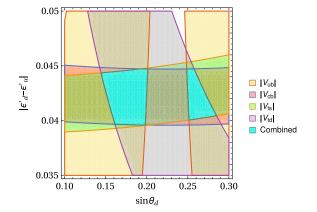

FIG. 1. Shaded areas in yellow, orange, green and gray represent allowed regions from $|V_{ub}|$, $|V_{cb}|$, $|V_{ts}|$ and $|V_{td}|$ respectively. The area shaded in cyan represents the common solution region.

be a bit too low. Having only one free remaining parameter prevents us from obtaining a good fit for both the up and down quark masses. We have checked that, as long as the down quark mass is concerned, the values given in Eq. (49) reflects the best case scenario.

VI. SUMMARY

We have considered a model where the left-right symmetric gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B\text{-}L}$ is augmented by an S_3 symmetry. The discrete symmetry drastically reduces the number of Yukawa couplings in the model. In fact, there are only six Yukawa couplings. Because of the small number of parameters, we can relate many aspects of quark masses and mixings satisfactorily in our model. We have demonstrated that the smallness of the first generation quark masses is related to the smallness of the 13 and 23 elements of the CKM matrix as well as to the smallness of the W_L-W_R mixing. We have also shown that, under some reasonable assumptions about the relative magnitudes of the VEVs, the CKM matrix can be reproduced within an accuracy of $\mathcal{O}(\lambda^4)$. The only sore point seems to be the light quark masses that we obtain from the model, which, although being in the same ballpark, turn out to be a bit smaller than expected. Yet the model deserves careful attention since we believe that the successes of the model with the CKM matrix outweigh the dissatisfaction with the light quark masses.

[1] R. N. Mohapatra and J. C. Pati, A natural left-right symmetry, Phys. Rev. D **11,** 2558 (1975).

[2] G. Senjanović and R. N. Mohapatra, Exact left-right symmetry and spontaneous violation of parity, Phys. Rev. D **12,** 1502 (1975).

[3] S. Pakvasa and H. Sugawara, Discrete symmetry and cabibbo angle, Phys. Lett. **73B,** 61 (1978).

[4] N. G. Deshpande, M. Gupta, and P. B. Pal, Flavor changing processes and CP violation in S(3) × Z(3) model, Phys. Rev. D **45,** 953 (1992).

[5] Y. Koide, Universal seesaw mass matrix model with an S(3) symmetry, Phys. Rev. D **60,** 077301 (1999).

[6] J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui, The flavor symmetry, Prog. Theor. Phys. **109,** 795 (2003); Erratum, Prog. Theor. Phys. **114,** 287(E) (2005).

[7] P. F. Harrison and W. G. Scott, Permutation symmetry, tribimaximal neutrino mixing and the S_3 group characters, Phys. Lett. B **557,** 76 (2003).

[8] S.-L. Chen, M. Frigerio, and E. Ma, Large neutrino mixing and normal mass hierarchy: A discrete understanding, Phys. Rev. D **70,** 073008 (2004); Erratum, Phys. Rev. D **70,** 079905(E) (2004).

[9] Y. Koide, S(3) symmetry and neutrino masses and mixings, Eur. Phys. J. C **50,** 809 (2007).

[10] C.-Y. Chen and L. Wolfenstein, Consequences of approximate S(3) symmetry of the neutrino mass matrix, Phys. Rev. D **77,** 093009 (2008).

[11] R. Jora, J. Schechter, and M. N. Shahid, Perturbed S(3) neutrinos, Phys. Rev. D **80,** 093007 (2009); Erratum, Phys. Rev. D **82,** 079902(E) (2010).

[12] T. Kaneko and H. Sugawara, Broken S_3 symmetry in flavor physics, Phys. Lett. B **697,** 329 (2011).

[13] Z.-z. Xing, D. Yang, and S. Zhou, Broken S_3 flavor symmetry of leptons and quarks: Mass Spectra and flavor mixing patterns, Phys. Lett. B **690,** 304 (2010).

[14] T. Teshima and Y. Okumura, Quark/lepton mass and mixing in S_3 invariant model and CP-violation of neutrino, Phys. Rev. D **84,** 016003 (2011).

[15] S. Dev, S. Gupta, and R. R. Gautam, Broken S_3 symmetry in the neutrino mass matrix, Phys. Lett. B **702,** 28 (2011).

[16] F. Gonzalez Canales, A. Mondragon, and M. Mondragon, The S_3 flavour symmetry: Neutrino masses and mixings, Fortschr. Phys. **61,** 546 (2013).

[17] F. G. Canales, A. Mondragón, M. Mondragón, U. J. S. Salazar, and L. Velasco-Sevilla, Quark sector of S3 models: Classification and comparison with experimental data, Phys. Rev. D **88,** 096004 (2013).

[18] A. G. Dias, A. C. B. Machado, and C. C. Nishi, An S_3 model for lepton mass matrices with nearly minimal texture, Phys. Rev. D **86,** 093005 (2012).

[19] D. Meloni, S_3 as a flavour symmetry for quarks and leptons after the Daya Bay result on θ_{13}, J. High Energy Phys. 05 (2012) 124.

[20] S. Dev, R. R. Gautam, and L. Singh, Broken S_3 symmetry in the neutrino mass matrix and non-zero θ_{13}, Phys. Lett. B **708,** 284 (2012).

[21] S. Zhou, Relatively large theta13 and nearly maximal theta23 from the approximate S3 symmetry of lepton mass matrices, Phys. Lett. B **704,** 291 (2011).

[22] E. Ma and B. Melic, Updated S_3 model of quarks, Phys. Lett. B **725,** 402 (2013).

[23] H. B. Benaoum, Broken S_3 neutrinos, Phys. Rev. D **87,** 073010 (2013).

[24] E. Ma and R. Srivastava, Dirac or inverse seesaw neutrino masses with $B - L$ gauge symmetry and S_3 flavor symmetry, Phys. Lett. B **741,** 217 (2015).

[25] D. Cogollo and J. P. Silva, Two Higgs doublet models with an S_3 symmetry, Phys. Rev. D **93,** 095024 (2016).

[26] S. Pramanick and A. Raychaudhuri, Neutrino mass model with S_3 symmetry and seesaw interplay, Phys. Rev. D **94,** 115028 (2016).

[27] I. de Medeiros Varzielas, R. W. Rasmussen, and J. Talbert, Bottom-up discrete symmetries for cabibbo mixing, Int. J. Mod. Phys. A **32,** 1750047 (2017).

[28] J. Kubo, H. Okada, and F. Sakamaki, Higgs potential in minimal S(3) invariant extension of the standard model, Phys. Rev. D **70,** 036007 (2004).

[29] T. Teshima, Flavor mass and mixing and S(3) symmetry: An S(3) invariant model reasonable to all, Phys. Rev. D **73,** 045019 (2006).

[30] Y. Koide, Permutation symmetry S(3) and VEV structure of flavor-triplet Higgs scalars, Phys. Rev. D **73,** 057901 (2006).

[31] G. Bhattacharyya, P. Leser, and H. Pas, Exotic Higgs boson decay modes as a harbinger of S_3 flavor symmetry, Phys. Rev. D **83,** 011701 (2011).

[32] G. Bhattacharyya, P. Leser, and H. Pas, Novel signatures of the Higgs sector from S3 flavor symmetry, Phys. Rev. D **86,** 036009 (2012).

[33] T. Teshima, Higgs potential in S_3 invariant model for quark/lepton mass and mixing, Phys. Rev. D **85,** 105013 (2012).

[34] E. Barradas-Guevara, O. Félix-Beltrán, and E. Rodríguez-Jáuregui, Trilinear self-couplings in an S(3) flavored Higgs model, Phys. Rev. D **90,** 095001 (2014).

[35] A. E. Cárcamo Hernández, I. de Medeiros Varzielas, and E. Schumacher, Fermion and scalar phenomenology of a two-Higgs-doublet model with S_3, Phys. Rev. D **93,** 016003 (2016).

[36] D. Emmanuel-Costa, O. M. Ogreid, P. Osland, and M. N. Rebelo, Spontaneous symmetry breaking in the S_3-symmetric scalar sector, J. High Energy Phys. 02 (2016) 154; Erratum, J. High Energy Phys. 08 (2016) 169(E).

[37] D. Das and U. K. Dey, Analysis of an extended scalar sector with S_3 symmetry, Phys. Rev. D **89,** 095025 (2014); Erratum, Phys. Rev. D **91,** 039905(E) (2015).

[38] D. Das, U. K. Dey, and P. B. Pal, S_3 symmetry and the quark mixing matrix, Phys. Lett. B **753,** 315 (2016).

[39] D. Das, U. K. Dey, and P. B. Pal, Quark mixing in an S_3 symmetric model with two Higgs doublets, Phys. Rev. D **96,** 031701 (2017).

[40] E. A. Garcés, J. Carlos Gómez-Izquierdo, and F. Gonzalez-Canales, Flavored non-minimal left-right symmetric model fermion masses and mixings, Eur. Phys. J. C **78,** 812 (2018).

[41] J. C. Gómez-Izquierdo, Non-minimal flavored $S_3 \otimes Z_2$ left–right symmetric model, Eur. Phys. J. C **77,** 551 (2017).

[42] LRS models with two scalar bidoublets have also been considered in Refs. [43,44].

[43] A. Damanik, M. Satriawan, Muslim, and P. Anggraita, Left-right symmetry model with two bidoublets and one doublet Higgs field for electroweak interaction, J. Theor. Comput. Stud. **6,** 0404 (2007).

[44] Y.-L. Wu and Y.-F. Zhou, Two Higgs bi-doublet left-right model with spontaneous P and CP violation, Sci. China G **51,** 1808 (2008).

[45] L. Wolfenstein, Parametrization of the Kobayashi-Maskawa Matrix, Phys. Rev. Lett. **51,** 1945 (1983).

[46] S. Descotes-Genon and P. Koppenburg, The CKM parameters, Annu. Rev. Nucl. Part. Sci. **67,** 97 (2017).

[47] Particle Data Group, CKM quark-mixing matrix, http://pdg .lbl.gov/2018/reviews/rpp2018-rev-ckm-matrix.pdf.