Form factors for semileptonic B_c decays into η, η' and glueballs

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We calculated the form factors of B_c transitions into η, η' meson and pseudoscalar glueballs, where the B_c meson is a bound state of two different heavy flavors and is treated as a nonrelativistic state, while the mesons η, η' and glueballs are treated as light-cone objects since their masses are small enough compared to the transition momentum scale. The mechanism of two gluon scattering into $\eta\prime$ dominated the form factors of B_c decays into $\eta\prime$. We considered the η-η'-glueball mixing effects, and then obtained their influences on the form factors. The form factors of B_c transition into η, η', and the pseudoscalar glueball in the maximum momentum recoil point were obtained as follows: $f^{\eta}_{0,+}(q^2=0) = 1.38^{+0.00}_{-0.02} \times 10^{-3}$, $f^{\eta'}_{0,+}(q^2=0) = 0.89^{+0.11}_{-0.10} \times 10^{-2}$, and $f^{G}_{0,+}(q^2=0) = 0.44^{+0.13}_{-0.05} \times 10^{-2}$. Also phenomenological discussions for semileptonic $B_c \to \eta^{(\prime)} + \ell + \bar{\nu}_\ell$, $B_c \to G(0^{-+}) + \ell + \bar{\nu}_\ell$, and $D_s \to \eta + \ell + \bar{\nu}_\ell$ decays are given.

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I. INTRODUCTION

Hadron-hadron colliders currently provide the unique platform to investigate the production and decay properties of the B_c meson as the bound state of two different heavy flavors. In pace with the running of the CERN Large Hadron Collider (LHC) with the luminosity of about $\mathcal{L} \sim 10^{33}$ cm^{-2} s^{-1}, one can expect around 10^9 B_c events per year [1]. When a tremendous number of B_c events are reconstructed, one can systematically and precisely test the golden decay channels of the B_c meson or hunt for its rare decays [2].

The B_c^- meson has two different heavy flavors, and its decay modes can be classified into three categories: (i) the anticharm quark decays with $\bar{c} \to \bar{d}, \bar{s}$; (ii) the bottom quark decays with $b \to u, c$; and (iii) the weak annihilation where both the bottom and anticharm decays. These three categories of decay modes contribute to the total decay width of the B_c^- meson and are around 70%, 20%, and 10%, respectively [3]. There are currently a lot of theoretical and experimental works on the singly heavy quark decays of the B_c meson, some of which can be found in Refs. [2,4–10]. And the studies of the rare weak annihilation decays of the B_c meson are few, some of which can be found in Refs. [11–16].

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In this paper, we will investigate the decay properties of the B_c meson into the light pseudoscalar mesons η, η' and glueball. The light pseudoscalar mesons with quark contents are organized into two representations: singlet and octet according to flavor $SU(3)$ symmetry. Because of isospin symmetry, the form factors of the B_c meson into the isospin triplet π^0 is trivial, where the contributions from the quark contents $u\bar{u}$ and $d\bar{d}$ in π^0 will be canceled out. Thus the form factors of the B_c meson into the light meson π^0 will only depend on a small isospin symmetry breaking effect, which is very similar to the case where the cross sections of $e^+e^- \to J/\psi\eta(\eta')$ are around pb while there is no signal for $e^+e^- \to J/\psi\pi^0$ [17,18].

In the flavor $SU(3)$ symmetry, the light mesons with quark contents form the flavor octet and the flavor singlet. The masses of these light mesons become identical and trivial in the limit of zero quark mass. For different masses for the light u, d, and s quarks and the flavor symmetry breaking, the light mesons in the flavor octet and the flavor singlet will gain their masses. On the other hand, the axial $U(1)$ anomaly will lead to a large mass difference between the η and η', which cannot be ignored [19–23]. Besides the flavor singlet and octet contents, even then the gluonium states will be mixed with each other with the identical J^{PC} and form the physical $\eta\prime$ states [19–24]. The η meson is viewed as the mixing state between flavor singlet and octet contents, and the gluonium content is usually suppressed. However, the conventional singlet-octet basis is not enough to explain the content of η'. For example, the gluonium contribution reached a few percents in $B \to \eta'$ decays. Thus the η' is viewed as the mixing state among $q\bar{q}$, $s\bar{s}$, and gg.

The B_c meson is treated as a nonrelativistic bound state, where the heavy quark relative velocity is small in the rest frame of the meson. The nonrelativistic QCD (NRQCD) effective theory is employed to deal with the decays of the B_c meson. Considering the mass of the light meson, P is less than the B_c meson, i.e., $m_P^2 \ll m_{B_c}^2$, and a large momentum is transferred in the B_c transitions into the light meson P. The light meson P can be treated as a light cone object in the rest frame of the B_c meson. In the maximum momentum recoil point with $q^2 = 0$, the form factors of the B_c transitions into the light meson P can be factored as the hadron long-distance matrix elements and the corresponding perturbative short distance coefficients.

We will discuss the properties of the form factors of the B_c transitions into the light pseudoscalar mesons η, η', and glueball. We will employ the form factor formulas into the related semileptonic decays, naming $B_c \to \eta^{(\prime)} + \ell + \bar{\nu}_\ell$ and $B_c \to G(0^{-+}) + \ell + \bar{\nu}_\ell$.

The paper is organized as followings: In Sec. II, we will introduce the NRQCD approach, the η-η'-glueball mixing effect, the light cone distribution amplitudes, and the scattering mechanism of two gluons into light mesons. In Sec. III, we will calculate the form factors of the B_c meson into $\eta\prime$ and glueball. Especially, we will determine the quark-antiquark pair and gluonium contributions to the form factors and discuss their properties. In Sec. IV, we will study the semileptonic decays of the B_c meson into $\eta\prime$ and glueball. And we will tentatively analyze the processes $D_s \to \eta + \ell + \bar{\nu}_\ell$. We summarize and conclude in the end.

II. FACTORIZATION FORMULAS

A. NRQCD effective theory

The heavy quark relative velocity is a small quantity inside the heavy quarkonium, and then the heavy quark pair is nonrelativistic in the rest frame of heavy quarkonium. The quark relative velocity squared is estimated as $v^2 \approx 0.3$ for J/ψ and $v^2 \approx 0.1$ for Υ [25]. The B_c meson is usually treated as a nonrelativitic state, and the quark reduced velocity squared is estimated in the region $0.1 < v^2 < 0.3$. The calculations of the productions and decays of the heavy quarkonium and the B_c meson with a large momentum transmitted usually refer to the NRQCD effective theory established by Bodwin, Braaten, and Lepage [25].

In the NRQCD effective theory, the Lagrangian is written as [25]

$$\begin{aligned}\mathcal{L}_{\rm NRQCD} &= \psi^\dagger\left(iD_t + \frac{\mathbf{D}^2}{2m}\right)\psi + \frac{c_F}{2m}\psi^\dagger\boldsymbol{\sigma}\cdot g_s\mathbf{B}\psi \\ &+ \psi^\dagger\frac{\mathbf{D}^4}{8m^3}\psi + \frac{c_D}{8m^2}\psi^\dagger(\mathbf{D}\cdot g_s\mathbf{E} - g_s\mathbf{E}\cdot\mathbf{D})\psi \\ &+ \frac{ic_S}{8m^2}\psi^\dagger\boldsymbol{\sigma}\cdot(\mathbf{D}\times g_s\mathbf{E} - g_s\mathbf{E}\times\mathbf{D})\psi \\ &+ (\psi\to i\sigma^2\chi^*, A_\mu \to -A_\mu^T) + \mathcal{L}_{\rm light}, \end{aligned} \tag{1}$$

where ψ and χ represent the two-component Pauli spinor field that annihilates a heavy quark and creates a heavy antiquark with quark mass m, respectively. $\boldsymbol{\sigma}$ is the Pauli matrix. The electric and magnetic color components of the gluon field strength tensor $G^{\mu\nu}$ are denoted as $E^i = G^{0i}$ and $B^i = \frac{1}{2}\epsilon^{ijk}G^{jk}$, respectively. The space and time components of the gauge-covariant derivative D^μ are denoted as $\mathbf{D}$ and D_t, respectively. $\mathcal{L}_{\rm light}$ denotes the Lagrangian for the light quarks and gluons. The short-distance coefficients c_D, c_F, and c_S can be perturbatively calculated according to the matching procedure between QCD and NRQCD calculations.

Within the framework of NRQCD, the heavy quarkonium inclusive annihilation decay width is factorized as [25]

$$\Gamma(H) = \sum_n \frac{2{\rm Im}f_n(\mu_\Lambda)}{m^{d_n-4}}\langle H|\mathcal{O}_n(\mu_\Lambda)|H\rangle, \tag{2}$$

where $\langle H|\mathcal{O}_n(\mu_\Lambda)|H\rangle$ are NRQCD decay long-distance matrix elements (LDMEs), which involve nonperturbative information and obey the power counting rules, which are ordered by the relative velocity between the heavy quark and the antiquark inside the heavy quarkonium H.

The leading order NRQCD decay operators for the decay of S-wave heavy quarkonium are

$$\mathcal{O}(^1S_0^{[1]}) = \psi^\dagger\chi\chi^\dagger\psi, \tag{3}$$

$$\mathcal{O}(^3S_0^{[1]}) = \psi^\dagger\boldsymbol{\sigma}\chi\cdot\chi^\dagger\boldsymbol{\sigma}\psi. \tag{4}$$

These operators are also valid for the B_c family with two different heavy flavors.

For a certain process, the matching coefficients multiplying decay LDMEs are determined through perturbative matching between QCD and NRQCD at the amplitude level. The covariant projection method is another equivalent but more convenient approach to extract the short-distance coefficients of the NRQCD LDMEs. The corresponding projection operators are defined as

$$\begin{aligned}\Pi_{S=0,1}(k) &= -i\sum_{\lambda_1,\lambda_2}u_1(p_1,\lambda_1)\bar{v}_2(p_2,\lambda_2)\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2|SS_z\rangle \\ &\otimes\left\{\frac{\mathbf{1}_c}{\sqrt{N_c}},\sqrt{2}\mathbf{T}^a\right\} \\ &= \frac{i}{4\sqrt{2E_1E_2}\omega}(\alpha\not{p}_H - \not{k} + m_1)\frac{\not{p}_H + E_1 + E_2}{E_1 + E_2} \\ &\times\Gamma_S(\beta\not{p}_H + \not{k} - m_2)\otimes\left\{\frac{\mathbf{1}_c}{\sqrt{N_c}},\sqrt{2}\mathbf{T}^a\right\},\end{aligned} \tag{5}$$

where $\omega = \sqrt{E_1+m_1}\sqrt{E_2+m_2}$ with $E_1 = \sqrt{m_1^2 - k^2} = \sqrt{m_1^2 + \mathbf{k}^2}$ and $E_2 = \sqrt{m_2^2 - k^2} = \sqrt{m_2^2 + \mathbf{k}^2}$. The parameters α and β satisfy the relations as $\alpha = E_1/(E_1+E_2)$, and

$\beta = 1 - \alpha$. We have the spin $S = 0$ and $\Gamma_{S=0} = \gamma^5$ for the spin-singlet combination. For the spin-triplet combination, we have the spin $S = 1$ and $\Gamma_{S=1} = \not{\epsilon}_H = \varepsilon_\mu(p_H)\gamma^\mu$. $\{\frac{\mathbf{1}_c}{\sqrt{N_c}}, \sqrt{2}\mathbf{T}^a\}$ denote the color-singlet and color-octet projection in the $SU(3)$ color space. For the decays of B_c^-, p_H is the B_c^- meson momentum; $p_1 = \alpha p_H - k$ is the bottom quark momentum with the mass $m_1 = m_b$; $p_2 = \beta p_H + k$ is the anticharm quark momentum with the mass $m_2 = m_c$; k is half of the relative momentum between the anticharm and bottom quarks with $k^2 = -\mathbf{k}^2$.

The heavy quarkonium state is not limited to heavy quark pairs in a color singlet configuration according to NRQCD. The heavy quark pairs in a color singlet configuration is only the leading order of the Fock state of the quarkonium. Other Fock states sometimes play an important role in the inclusive production of heavy quarkonium. In the form factors of the B_c meson into the light mesons, the dominant contribution is from the color singlet configuration.

B. η-η'-Glueball mixing schemes

The η-η'-glueball mixing effects are discussed in lots of literature. The popular mixing schemes which are widely employed in this literature are the quark-flavor bases [23,26–30] and the flavor singlet-octet bases [31–37]. In the quark-flavor scheme, the basic quark components are $\eta_q = q\bar{q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$, while the basic flavor components become $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ for the flavor singlet-octet scheme. In addition, the gluonium state $\eta_g = gg$ is introduced when it has the identical quantum numbers as the two light quark states. For a η-η'-glueball mixing with identical spin parity $J^{PC} = 0^{-+}$, one has

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \\ |G\rangle \end{pmatrix} = U(\phi, \phi_G) \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |\eta_g\rangle \end{pmatrix}, \qquad (6)$$

with the matrix[1]

$$U(\phi, \phi_G) = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi\cos\phi_G & \cos\phi\cos\phi_G & \sin\phi_G \\ -\sin\phi\sin\phi_G & -\cos\phi\sin\phi_G & \cos\phi_G \end{pmatrix}. \qquad (7)$$

The QCD states η_i with $i = q, s, g$ form the physical mass eigenstates η, η', and glueball. One candidate for the physical $J^{PC} = 0^{-+}$ glueball state is the $\eta(1405)$, where the corresponding analysis was performed in Ref. [33]. Here we assume that the physical η state does not mix with glueball, under which two mixing angles ϕ and ϕ_G are sufficient.

Another equivalent mixing approach for the flavor singlet-octet scheme can easily be obtained by the replacements of $\eta_q \to \eta_8$, $\eta_s \to \eta_1$, $\phi \to \theta$, and $\phi_G \to \phi'_G$. The small angle $\phi_G = \phi'_G$ is adopted for simplification in the literature [29,30,33]. When $\sin\phi_G \to 0$, the mixing only occurs between the two quark states.

Considering that the flavor singlet and octet states can be decomposed into the quark flavor states, one has

$$\begin{pmatrix} |\eta_8\rangle \\ |\eta_1\rangle \\ |\eta_g\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \\ |\eta_g\rangle \end{pmatrix}, \qquad (8)$$

where

$$\cos\theta_i = \sqrt{\frac{1}{3}}, \qquad \sin\theta_i = \sqrt{\frac{2}{3}}. \qquad (9)$$

The parameter $\theta_i = \arctan\sqrt{2} \simeq 54.74^0$ is always named as the ideal mixing angle. From the observations, vector or tensor meson mixing angles where the axial vector anomaly plays no role are always close to this ideal angle.

The relations between two mixing schemes can easily be obtained as

$$\cos\theta = \frac{\sqrt{2}\sin\phi + \cos\phi}{\sqrt{3}}, \qquad (10)$$

$$\sin\theta = \frac{\sin\phi - \sqrt{2}\cos\phi}{\sqrt{3}}. \qquad (11)$$

Equivalently, one can get the mixing angle for the flavor singlet-octet scheme as $\theta = \phi - \arctan\sqrt{2}$.

According to the quantum field theory definition, the decay constants of the mesons are defined as

$$\langle 0|\bar{q}'\gamma^\mu\gamma_5 q'|\eta^{(\prime)}(p)\rangle = i f^{q'}_{\eta^{(\prime)}} p^\mu \qquad (q' = q, s), \qquad (12)$$

where the decay constants of the mesons are also related to the decay constants of the quark components as

$$f^q_\eta = f_q\cos\phi, \qquad f^s_\eta = -f_s\sin\phi, \qquad (13)$$

$$f^q_{\eta'} = f_q\sin\phi\cos\phi_G, \qquad f^s_{\eta'} = f_s\cos\phi\cos\phi_G, \qquad (14)$$

where the relations will turn to the traditional form in Ref. [27] when $\phi_G \to 0$.

If one defines the meson decay constants through the flavor $SU(3)$ octet and singlet axial vector current as

[1] Here we do not consider the mixing between the η and glueball, which is consistent with the experimental constraints of the production and decays of η [23,26–37]. If considering the mixing between the η and glueball, one has to introduce an additional mixing angle, and the mixing matrix will have not zero element.

$$\langle 0|J_5^{\mu,i}|\eta^{(\prime)}(p)\rangle = if^i_{\eta^{(\prime)}}p^\mu \quad (i=8,1), \tag{15}$$

the relations of the decay constants of the mesons become

$$f_\eta^8 = f_8\cos\theta_8, \qquad f_\eta^1 = -f_1\sin\theta_1, \tag{16}$$

$$f_{\eta'}^8 = f_8\sin\theta_8, \qquad f_{\eta'}^1 = f_1\cos\theta_1. \tag{17}$$

The flavor singlet and octet decay constants can be obtained by the quark flavor decay constants [27]

$$f_8 = \sqrt{\frac{f_q^2+2f_s^2}{3}}, \qquad \theta_8 = \phi - \arctan(\sqrt{2}f_s/f_q), \tag{18}$$

$$f_1 = \sqrt{\frac{2f_q^2+f_s^2}{3}}, \qquad \theta_1 = \phi - \arctan(\sqrt{2}f_q/f_s). \tag{19}$$

From the above equations, one would have $\theta_8=\theta_1=\theta=\phi-\arctan\sqrt{2}$ in the strict flavor $SU(3)$ symmetry where $f_q=f_s$.

There are several experimental methods to extract the values of the mixing angle and the decay constants, which have been discussed in the literature [26–29,31,38–41].

In Ref. [27], the decays of $\eta'\to\rho\gamma$ and $\rho\to\eta\gamma$ are investigated, where the ratio of their decay widths is given as

$$\frac{\Gamma(\eta'\to\rho\gamma)}{\Gamma(\rho\to\eta\gamma)} = 3\tan^2\phi\cos^2\phi_G\left(\frac{m_{\eta'}\left(1-\frac{m_\rho^2}{m_{\eta'}^2}\right)}{m_\rho\left(1-\frac{m_\eta^2}{m_\rho^2}\right)}\right)^3. \tag{20}$$

In the $\eta'\to\rho\gamma$ decays, the contributions from η_s and η_g components are suppressed. Inputting the latest PDG results: $\mathrm{Br}(\eta'\to\rho\gamma)=(28.9\pm0.5)\%$, $\mathrm{Br}(\rho\to\eta\gamma)=(3.00\pm0.21)\times10^{-4}$, $\Gamma_\rho=149.1\pm0.8$ MeV, and $\Gamma_{\eta'}=0.196\pm0.009$ MeV [42], the mixing angles are extracted as $\tan\phi\cos\phi_G=0.827^{+0.39}_{-0.34}$.

In Ref. [28], a global analysis of radiative $V\to P\gamma$ and $P\to V\gamma$ decays was performed to determine the gluon content of the $\eta\prime$ mesons. Allowing for gluonium in the η', the mixing angles were found to be $\phi=41.4°\pm1.3°$ and $\sin^2\phi_G=0.04\pm0.09$, naming $\tan\phi\cos\phi_G=0.864^{+0.059}_{-0.078}$.

In Ref. [38], the KLOE Collaboration measured the mixing angles by looking for the radiative decays $\phi\to\eta\prime\gamma$. Ignoring the gluonium contribution, the mixing angle is fitted into $\phi=41.4°\pm0.3°_{\mathrm{stat}}\pm0.7°_{\mathrm{syst}}\pm0.6°_{\mathrm{th}}$. Allowing for gluonium in the η', they yielded $\phi=39.7°\pm0.7°$ and $\sin^2\phi_G=0.14\pm0.04$, naming $\tan\phi\cos\phi_G=0.770^{+0.037}_{-0.036}$.

The LHCb Collaboration recently has fitted the mixing angles as $\phi=(43.5^{+1.4}_{-2.8})°$ and $\phi_G=(0\pm24.6)°$ by a study of B or B_s^0 meson decays into $J/\psi\eta$ and $J/\psi\eta'$ at proton-proton collisions [41]. Compared to the small angle ϕ_G in Refs. [28,41], a larger mixing angle ϕ_G is obtained in Refs. [29,38,43].

The quark flavor decay constants can be obtained by their two-photon decays. One has [27]

$$f_q = \frac{5\alpha}{12\sqrt{2}\pi^{3/2}}\left[\sqrt{\Gamma(\eta\to\gamma\gamma)/m_\eta^3}\cos\phi + \sqrt{\Gamma(\eta'\to\gamma\gamma)/m_{\eta'}^3}\frac{\sin\phi}{\cos\phi_G}\right]^{-1}, \tag{21}$$

$$f_s = \frac{\alpha}{12\pi^{3/2}}\left[-\sqrt{\Gamma(\eta\to\gamma\gamma)/m_\eta^3}\sin\phi + \sqrt{\Gamma(\eta'\to\gamma\gamma)/m_{\eta'}^3}\frac{\cos\phi}{\cos\phi_G}\right]^{-1}. \tag{22}$$

The decay constant f_s is not well determined in this way and has a large error. To extract the values of f_s, one can use [27]

$$\frac{f_s}{f_q} = \frac{\sqrt{2}(m_\eta^2\cos^2\phi + m_{\eta'}^2\sin^2\phi - m_\pi^2)}{(m_{\eta'}^2-m_\eta^2)\sin2\phi}. \tag{23}$$

We input the latest PDG results: $\mathrm{Br}(\eta\to\gamma\gamma)=(38.41\pm0.20)\%$, $\mathrm{Br}(\eta'\to\gamma\gamma)=(2.22\pm0.08)\%$, $\Gamma_\eta=1.31\pm0.05$ keV, and $\Gamma_{\eta'}=0.196\pm0.009$ MeV [42]. Imposing the mixing angles $\phi=41.4°\pm1.3°$ and $\sin^2\phi_G=0.04\pm0.09$ in Ref. [28], the decay constants are extracted as $f_q=(1.05\pm0.02)f_\pi$ and $f_s=(1.34\pm0.03)f_\pi$ with $f_\pi=130.4$ MeV. When imposing the mixing angles $\phi=39.7°\pm0.7°$ and $\sin^2\phi_G=0.14\pm0.04$ in Ref. [38], the decay constants become $f_q=(1.03\pm0.02)f_\pi$ and $f_s=(1.28\pm0.02)f_\pi$. We will input these values in the following calculations.

C. Light cone distribution amplitudes

The light cone distribution amplitudes (LCDA) of η_q and η_s components in η have the form [20]

$$\Phi_\eta^{(q,s)}(x,\mu) = 6x\bar{x}\left[1+\sum_{n=2,4,\ldots}a_n(\mu)C_n^{3/2}(x-\bar{x})\right], \tag{24}$$

where x and $\bar{x}=1-x$ are the momentum fractions of the light quark and antiquark inside $\eta_{q,s}$, respectively. $C_n^{3/2}$ and the following $C_n^{5/2}$ are the Gegenbauer polynomials. $a_n(\mu)$ is obtained by the scale evolution at leading-order logarithmic accuracy

$$a_n(\mu) = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\frac{\gamma_n}{\beta_0}} a_n(\mu_0), \tag{25}$$

where $\beta_0=11C_A/3-2n_f/3$ with flavor number n_f and the anomalous dimension γ_n reads as

$$\gamma_n = 4C_F\left(\psi(n+2)+\gamma_E-\frac{3}{4}-\frac{1}{2(n+1)(n+2)}\right), \quad (26)$$

with the digamma function $\psi(n)$.

The evolution of the LCDA of the quark contents will mix with the gluonium state for η'. In Ref. [44], the evolution equation for the LCDA of the mixing $q\bar{q}$ and gg states has been calculated. The corresponding light cone distribution amplitudes are [45–48]

$$\Phi^{(q,s)}(x,\mu) = 6x\bar{x}\left\{1+\sum_{n=2,4,\ldots}[a_n^{(q,s)}(\mu)+\rho_n^g a_n^{(g)}(\mu)] \times C_n^{3/2}(x-\bar{x})\right\}, \quad (27)$$

$$\Phi^{(g)}(x,\mu) = x\bar{x}\sum_{n=2,4,\ldots}[\rho_n^{q,s}a_n^{(q,s)}(\mu)+a_n^{(g)}(\mu)]C_{n-1}^{5/2}(x-\bar{x}), \quad (28)$$

where $a_n^{(q,s;g)}(\mu)$ can be obtained by the scale evolution at leading-order logarithmic accuracy,

$$a_n^{(q,s)}(\mu) = a_n^{(q,s)}(\mu_0)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{-\gamma_+^n}, \quad (29)$$

$$a_n^{(g)}(\mu) = a_n^{(g)}(\mu_0)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{-\gamma_-^n}, \quad (30)$$

where the parameters $\gamma_\pm^n$ and ρ_n are

$$\gamma_\pm^n = \frac{1}{2}\left[\gamma_{qq}^n+\gamma_{gg}^n\pm\sqrt{(\gamma_{qq}^n-\gamma_{gg}^n)^2+4\gamma_{gq}^n\gamma_{qg}^n}\right], \quad (31)$$

with

$$\gamma_{qq}^n = -\frac{\gamma^n}{\beta_0}, \quad (32)$$

$$\gamma_{gq}^n = \frac{n_f}{\beta_0}\frac{2}{(n+1)(n+2)}, \quad (33)$$

$$\gamma_{qg}^n = \frac{C_F}{\beta_0}\frac{n(n+3)}{(n+1)(n+2)}, \quad (34)$$

$$\gamma_{gg}^n = \frac{4C_A}{\beta_0}\left[\frac{2}{(n+1)(n+2)}-\sum_{j=2}^{n+1}\frac{1}{j}-\frac{1}{12}-\frac{n_f}{6C_A}\right], \quad (35)$$

and

$$\rho_n^g = -\frac{1}{6}\frac{Q_n}{1-P_n}, \qquad \rho_n^q = 6\frac{P_n}{Q_n}, \quad (36)$$

$$P_n = \frac{\gamma_+^n-\gamma_{qq}^n}{\gamma_+^n-\gamma_-^n}, \qquad Q_n = \frac{\gamma_{qg}^n}{\gamma_+^n-\gamma_-^n}. \quad (37)$$

D. The scattering mechanism of two gluons into light mesons

For the B_c meson decays into $\eta^{(\prime)}$, the mechanism of two gluons scattering into $\eta^{(\prime)}$ will play an important role. The two gluons scattering mechanism is blind to quark charges and light quark flavors, so the amplitude is identical to $q\bar{q}$ and $s\bar{s}$ except for the mixing factor and the decay constant.

The amplitudes of two gluons scattering into quarks and gluonium contents in lowest-order perturbation theory can be obtained by calculating the corresponding Feynman diagrams that are plotted in Fig. 1.

The amplitude of two gluons scattering to each quark content is

$$\mathcal{M}^{(q,s)} = -iF_{g^*g^*}^{(q,s)}(q_1^2,q_2^2)\delta_{ab}\epsilon^{\mu\nu\rho\sigma}\varepsilon_{1\mu}^a\varepsilon_{2\nu}^b q_{1\rho}q_{2\sigma}, \quad (38)$$

where the momenta of two initial virtual gluons are denoted as q_1 and q_2, respectively. The polarization vectors of two initial gluons are denoted as $\varepsilon_1(q_1)$ and $\varepsilon_2(q_2)$, respectively. The form factor $F_{g^*g^*}^{(q,s)}$ of two gluons transitions to the quark-antiquark content can be written as [45,46]

$$F_{g^*g^*}^{(q,s)}(q_1^2,q_2^2) = \frac{2\pi\alpha_s(\mu^2)}{N_c}\int_0^1 dx\Phi^{(q,s)}(x,\mu) \times\left[\frac{1}{xq_1^2+\bar{x}q_2^2-x\bar{x}m_P^2+i\epsilon}+(x\leftrightarrow\bar{x})\right], \quad (39)$$

where m_P is the meson mass.

The amplitude of two gluons scattering into the gluon content can be written as

$$\mathcal{M}^{(g)} = -iF_{g^*g^*}^{(g)}\delta_{ab}\epsilon^{\mu\nu\rho\sigma}\varepsilon_{1\mu}^a\varepsilon_{2\nu}^b q_{1\rho}q_{2\sigma}, \quad (40)$$

where the form factors $F_{g^*g^*}^{(g)}$ of two gluons scattering to the gluonium content can be written as [45,46]

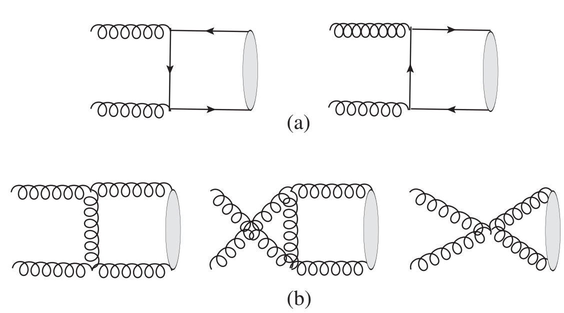

FIG. 1. Feynman diagrams for two gluons scattering into η_q, η_s, and η_g.

$$F^{(g)}_{g^*g^*}(q_1^2,q_2^2)=\frac{2\pi\alpha_s(\mu^2)}{Q^2}\int_0^1 dx\Phi^{(g)}(x,\mu)\times\left[\frac{xq_1^2+\bar{x}q_2^2-(1+x\bar{x})m_P^2}{\bar{x}q_1^2+xq_2^2-x\bar{x}m_P^2+i\epsilon}-(x\leftrightarrow\bar{x})\right], \tag{41}$$

where the typical scale Q^2 is introduced to preserve the dimensionless for the transition form factors. The choice of Q^2 has some freedom, and Q^2 is adopted to $|q_i^2|$ or $|q_1^2+q_2^2|$ in Refs. [45,46]. In this paper, Q^2 is adopted as m_P^2, and the LCDA of gluonium content is adopted as the form in Eq. (28).

For η, we assume it does not mix with glueball, which is consistent with the experimental constraints [23,26–37]. The amplitude of two gluons to η is written as

$$\mathcal{M}^{(q,s)}_\eta=C_\eta\mathcal{M}^{(q,s)}, \tag{42}$$

and the corresponding form factor of two gluons scattering to η is

$$F^{(q,s)}_{\eta g^*g^*}(q_1^2,q_2^2)=C_\eta F^{(q,s)}_{g^*g^*}(q_1^2,q_2^2). \tag{43}$$

According to the mixing scheme in Eq. (6), we have $C_\eta=\sqrt{2}f_q\cos\phi-f_s\sin\phi$, which can be viewed as the effective decay constant of η. In this case, the light meson mass in the formulas becomes $m_P=m_\eta$.

For η', the mixing between the quark and gluon contents should be considered. The amplitude of two gluons to η' is written as

$$\mathcal{M}^{(q,s;g)}_{\eta'}=\mathcal{M}^{(q,s)}_{\eta'}+\mathcal{M}^{(g)}_{\eta'}=C^{(q,s)}_{\eta'}\mathcal{M}^{(q,s)}+C^{(g)}_{\eta'}\mathcal{M}^{(g)}, \tag{44}$$

and the corresponding form factors of two gluons to η' are

$$F^{(q,s)}_{\eta'g^*g^*}(q_1^2,q_2^2)=C^{(q,s)}_{\eta'}F^{(q,s)}_{g^*g^*}(q_1^2,q_2^2), \tag{45}$$

$$F^{(g)}_{\eta'g^*g^*}(q_1^2,q_2^2)=C^{(g)}_{\eta'}F^{(g)}_{g^*g^*}(q_1^2,q_2^2). \tag{46}$$

In this case, the light meson mass in the formulas becomes $m_P=m_{\eta'}$. According to the mixing scheme in Eq. (6), we have $C^{(q,s)}_{\eta'}=\sqrt{2}f_q\sin\phi\cos\phi_G+f_s\cos\phi\cos\phi_G$, which can be viewed as the effective decay constant of η'. Following the two gluon scattering mechanism proposed in Refs. [45,46], we do not need to introduce the decay constant of η_g and we parametrize $C^{(g)}_{\eta'}=\sin\phi_G C^{(q,s)}_{\eta'}$.

For glueball, the mixing between the gluon and quark contents should also be considered. The amplitude of two gluons to glueball is written as

$$\mathcal{M}^{(q,s;g)}_G=\mathcal{M}^{(q,s)}_G+\mathcal{M}^{(g)}_G=C^{(q,s)}_G\mathcal{M}^{(q,s)}+C^{(g)}_G\mathcal{M}^{(g)}, \tag{47}$$

and the corresponding form factors of two gluons to glueball are

$$F^{(q,s)}_{Gg^*g^*}(q_1^2,q_2^2)=C^{(q,s)}_G F^{(q,s)}_{g^*g^*}(q_1^2,q_2^2), \tag{48}$$

$$F^{(g)}_{Gg^*g^*}(q_1^2,q_2^2)=C^{(g)}_G F^{(g)}_{g^*g^*}(q_1^2,q_2^2). \tag{49}$$

In this case, the light meson mass in the formulas becomes $m_P=m_G$ where the candidate of 0^{-+} glueball is $\eta(1405)$ with $m_{\eta(1405)}=1408.8\pm1.8$ MeV [33]. According to the mixing scheme in Eq. (6) and the two gluon scattering mechanism in Refs. [45,46], we have $C^{(q,s)}_G=-\sqrt{2}f_q\cos\phi\sin\phi_G-f_s\sin\phi\sin\phi_G$ and $C^{(g)}_G=\cos\phi_G C^{(q,s)}_G$.

III. FORM FACTORS OF B_c INTO η, η' AND GLUEBALL

The form factors of B_c into a light pseudoscalar meson P, i.e., f_+ and f_0, are defined in common [49,50],

$$\langle P(p)|\bar{c}\gamma^\mu b|B_c(p_{B_c})\rangle=f^P_+(q^2)\left(k'^\mu-\frac{m^2_{B_c}-m_P^2}{q^2}q^\mu\right)+f^P_0(q^2)\frac{m^2_{B_c}-m_P^2}{q^2}q^\mu, \tag{50}$$

where the momentum transfer is defined as $q=p_{B_c}-p$ with the B_c meson momentum p_{B_c} and the light meson momentum p, and the momentum k' is defined as $k'=p_{B_c}+p$. For the decays of B_c into $\eta^{(\prime)}$, the form factors $f^{\eta'}_+(q^2)$ and $f^{\eta'}_0(q^2)$ can be defined by the exchange of $P\to\eta^{(\prime)}$. For the decays of B_c into glueball, the form factors $f^G_+(q^2)$ and $f^G_0(q^2)$ can be defined by the exchange of $P\to G$.

It is convenient to write the unintegrated form factors as

$$\langle P(xp,(1-x)p)|\bar{c}\gamma^\mu b|B_c(p_{B_c})\rangle=f^P_+(q^2,x)\left(k'^\mu-\frac{m^2_{B_c}-m_P^2}{q^2}q^\mu\right)+f^P_0(q^2,x)\frac{m^2_{B_c}-m_P^2}{q^2}q^\mu. \tag{51}$$

After performing the integration, we have $f^P_{+,0}(q^2)=\int_0^1 f^P_{+,0}(q^2,x)dx$.

Typical Feynman diagrams for the form factors of B_c into a light pseudoscalar meson P, i.e., $\eta^{(\prime)}$ and glueball, are plotted in Fig. 2. Other Feynman diagrams can be obtained by changing the gluon vertex to the anticharm quark line.

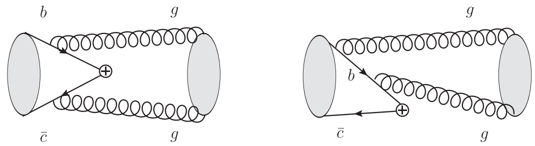

FIG. 2. Typical Feynman diagrams for the form factors of B_c into $\eta\prime$ and glueball.

Considering the hard scattering mechanism of two gluons transitioning into a light pseudoscalar meson P, i.e., $\eta^{(\prime)}$ and glueball, the leading order Feynman diagrams for B_c into a light pseudoscalar meson P by the charged vector current can be written as

$$\begin{aligned}\mathcal{M}^\mu &= \langle P(p)|\bar{c}\gamma^\mu b|B_c(p_{B_c})\rangle \\ &= \langle 0|\chi_c^\dagger\psi_b|B_c\rangle \mathrm{Tr}[\mathcal{A}^\mu(0)\Pi_{S=0}(k=0)], \end{aligned} \tag{52}$$

where ψ_b and χ_c represent the Pauli spinor field that annihilates a bottom quark and creates a charm antiquark, respectively.

$$\begin{aligned}\mathcal{A}^\mu(q) = &\frac{4\pi\alpha_s C_A C_F}{(m_{B_c}N_c)^{1/2}}\sum_{i=q,s;g}\int\frac{d^4\ell}{(2\pi)^4}\varepsilon^{\alpha\beta\rho\sigma}\ell_\rho(p_P-\ell)_\sigma \\ &\times F^{(i)}_{Pg^*g^*}(\ell^2,(p_P-\ell)^2)\frac{1}{\ell^2(p_P-\ell)^2} \\ &\times\left[\frac{\gamma^\beta(m_c+\not{p}_P-\not{\ell}-\not{p}_2)\gamma^\alpha(m_c+\not{p}_P-\not{p}_2)\gamma^\mu}{((p_2-p_P)^2-m_c^2)((p_2+\ell-p_P)^2-m_c^2)}\right. \\ &+\frac{\gamma^\mu(m_b+\not{p}_1-\not{p}_P)\gamma^\beta(m_b+\not{p}_1-\not{\ell})\gamma^\alpha}{((p_1-p_P)^2-m_b^2)((p_1-\ell)^2-m_b^2)} \\ &\left.+\frac{\gamma^\beta(m_c+\not{p}_P-\not{p}_2-\not{\ell})\gamma^\mu(m_b+\not{p}_1-\not{\ell})\gamma^\alpha}{((p_2+\ell-p_P)^2-m_c^2)((p_1-\ell)^2-m_b^2)}\right], \end{aligned} \tag{53}$$

where p_1 is the bottom quark momentum, p_2 is the anticharm quark momentum, and p_P is the momentum of the light pseudoscalar meson P, i.e., η, η', and glueball.

The *Mathematica* software is employed with the help of the packages FeynCalc [51], FeynArts [52], and LoopTools [53] in the calculation of the form factors. In order to obtain the values of form factors at the maximum recoil point, we will adopt the parameter values as follows: $m_{B_c} = 6.276$ GeV, $m_\eta = 547.85$ MeV, and $m_{\eta'} = 957.78$ MeV [42]. The heavy quark masses are adopted as $m_c = (1.5\pm 0.1)$ GeV and $m_b = (4.8\pm 0.1)$ GeV [8,9]. The Gegenbauer momenta are adopted as $a_2(1\ \mathrm{GeV}) = a_2^{q,s}(1\ \mathrm{GeV}) = 0.44\pm 0.22$ [36] and $a_2^g(1\ \mathrm{GeV}) = 0.1$ [18]. Their values at other scales can be obtained by the scale evolution equations in Eqs. (25), (29), and (30). The running strong coupling constant is adopted around the η' mass, and one has $\alpha_s(1\ \mathrm{GeV}) = 0.42$. If one chooses the scale at the charm quark mass with $m_c = (1.5\pm 0.1)$ GeV, one has $\alpha_s(m_c) = 0.32$–0.34, and in this case the values of the form factors will be reduced by (30–40)%.

TABLE I. Form factors of the B_c into η in the maximum recoil point with $q^2 = 0$. Here and in the following tables, the uncertainty is from the choice of the bottom and charm quark masses. Note that $f_+(0) = f_0(0)$.

Contributions	$10^{-3} f_0^\eta(q^2=0)$
$q\bar{q}$ with LO Gegenbauer	$1.23^{+0.04}_{-0.05}$
$q\bar{q}$ with NLO Gegenbauer	$1.38^{+0.00}_{-0.02}$

TABLE II. Form factors of the B_c into η' and 0^{-+} glueball in the maximum recoil point with $q^2 = 0$. Here and in the following tables, the uncertainty is from the choice of the bottom and charm quark masses. Note that $f_+(0) = f_0(0)$.

Contributions	$10^{-2} f_0^{\eta'}(q^2=0)$	$10^{-2} f_0^{G}(q^2=0)$
$q\bar{q}$ with NLO Gegenbauer	$0.97^{+0.10}_{-0.09}$	$0.04^{+0.04}_{-0.02}$
gg	$-0.08^{+0.00}_{-0.01}$	$-0.48^{+0.09}_{-0.03}$
Total	$0.89^{+0.11}_{-0.10}$	$-0.44^{+0.13}_{-0.05}$

In the maximum momentum recoil point, the form factors appearing in expression (50) can be perturbatively calculated reliably. We give the form factors in the maximum momentum recoil point in Tables I and II, where we input the mixing angles $\phi = 39.7°$ and $\sin^2\phi_G = 0.14$ [38]. When inputting other values of the mixing angles such as $\phi = 39.7° \pm 0.7°$ and $\sin^2\phi_G = 0.14\pm 0.04$ in Ref. [38] and $\phi = 41.4° \pm 1.3°$ and $\sin^2\phi_G = 0.04\pm 0.09$ in Ref. [28], one can easily get the corresponding values of the form factors by the definition of the parameters $C_\eta^{(q,s)}$, $C_{\eta'}^{(q,s)}$, $C_{\eta'}^{(q,s)}$, $C_G^{(q,s)}$, and $C_G^{(g)}$.

The unintegrated form factors dependent on the meson momentum fraction are sensitive to the shapes of the Gegenbauer series of the light meson. We give the unintegrated form factors dependent on the meson momentum fraction in Figs. 3–5. For the quark content contributions, the momentum fraction dependent shapes of form factors with leading order (LO) Gegenbauer momentum have only one peak, while that of form factors with next-to-leading (NLO) Gegenbauer momentum will have two peaks. From Fig. 4, one sees that the gluonium content will contribute the form factors of B_c into η'. From Fig. 5, the quark contents will contribute the form factors of B_c into glueball.

At the minimum momentum recoil point, the perturbative calculations for the form factors become invalid. In these regions, one has to refer to lattice QCD simulations or some certain models. In order to extrapolate the form factors to the minimum momentum recoil region, the pole model is generally adopted in the literature [54,55]. Thus the q^2 distribution of the form factors can be parametrized as

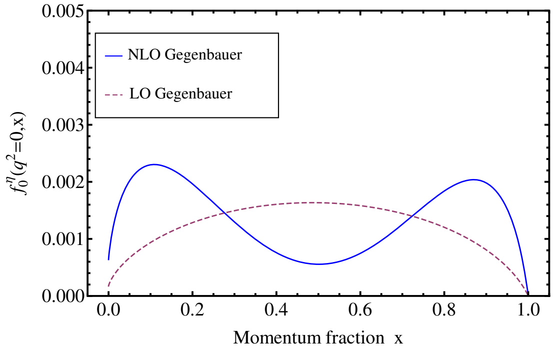

FIG. 3. The unintegrated form factor of B_c into η dependent on the meson momentum fraction. Here we input the mixing angle $\phi = 39.7°$. Note that $f_+^\eta(q^2 = 0, x) \equiv f_0^\eta(q^2 = 0, x)$.

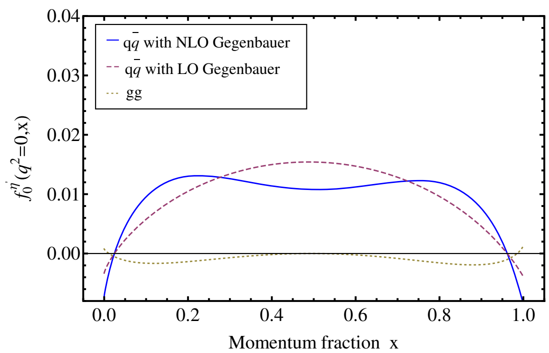

FIG. 4. The unintegrated form factor of B_c into η' dependent on the meson momentum fraction. Here and in the following, we input the mixing angles $\phi = 39.7°$ and $\sin^2\phi_G = 0.14$ [38]. Note that $f_+^{\eta'}(q^2 = 0, x) \equiv f_0^{\eta'}(q^2 = 0, x)$.

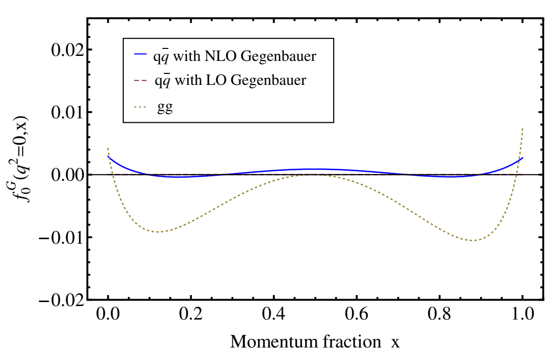

FIG. 5. The unintegrated form factor of B_c into 0^{-+} glueball dependent on the meson momentum fraction. Note that $f_+^G(q^2 = 0, x) \equiv f_0^G(q^2 = 0, x)$.

$$f_{0,+}^P(q^2) = \frac{f_{0,+}^P(0)}{(1 - \frac{q^2}{m_{B_c}^2})(1 - a\frac{q^2}{m_{B_c}^2} + b\frac{q^4}{m_{B_c}^4})}, \quad (54)$$

where a and b are model independent parameters and can be fitted when the data are available. Here we let $a = b = 0$

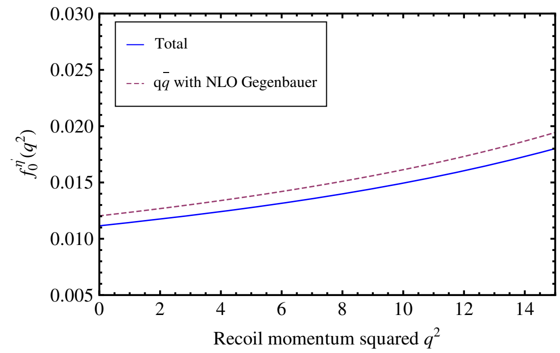

FIG. 6. The form factor of B_c into η dependent on the recoil momentum squared.

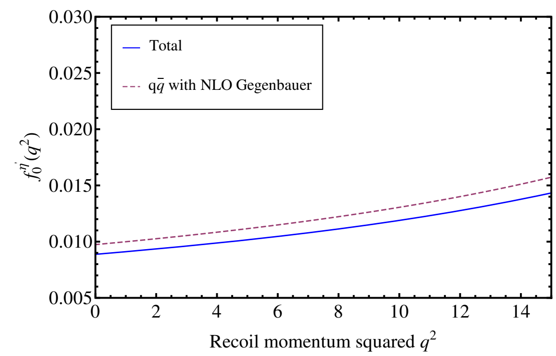

FIG. 7. The form factor of B_c into η' dependent on the recoil momentum squared.

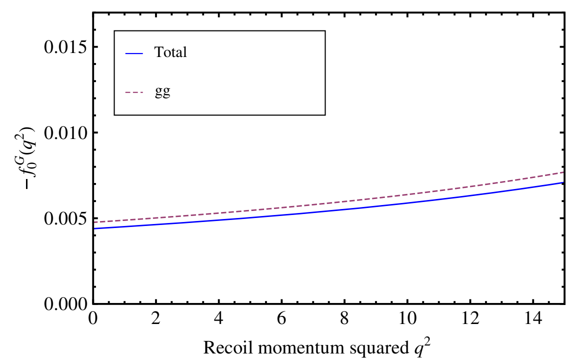

FIG. 8. The form factor of B_c into 0^{-+} glueball dependent on the recoil momentum squared. Considering that the value of the form factors is negative, we denoted "$-f_0^G(q^2)$" in the longitudinal coordinates.

for simplification. We plotted the form factors dependent on the momentum transfer squared in Figs. 6–8.

IV. SEMILEPTONIC DECAYS OF B_c INTO η, η' AND GLUEBALL

In this section, we will employ the form factors into the semileptonic decays of B_c into $\eta^{(\prime)}$ and glueball. We retain the lepton masses, and the semileptonic partial decay width of B_c into η can be written as

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \lambda(m_{B_c}^2, m_\eta^2, q^2)^{1/2} |V_{cb}|^2}{384\pi^3 m_{B_c}^3} \left(\frac{q^2 - m_\ell^2}{q^2}\right)^2 \frac{1}{q^2} \times [(m_\ell^2 + 2q^2)\lambda(m_{B_c}^2, m_\eta^2, q^2)(f_+^\eta(q^2))^2 + 3m_\ell^2 (m_{B_c}^2 - m_\eta^2)^2 (f_0^\eta(q^2))^2], \tag{55}$$

where $\lambda(m_{B_c}^2, m_\eta^2, q^2) = (m_{B_c}^2 + m_\eta^2 - q^2)^2 - 4m_{B_c}^2 m_\eta^2$. And the similar formulas can be obtained for the semileptonic decay width of B_c into η' and glueball with the replacement of $\eta \to \eta'(G)$.

The semileptonic decay widths and the branching ratios can be obtained after integrating the momentum recoil squared q^2. In Table III, we give the results for the $B_c \to \eta + \ell + \bar{\nu}_\ell$ with $\ell = e, \mu, \tau$. The masses of the leptons are $m_e = 0.50$ MeV, $m_\mu = 105.6$ MeV, and $m_e = 1777$ MeV [42]. The form factors with both LO and NLO Gegenbauer series are considered in the semileptonic decays. From the table, their decay widths are around 10^{-19} GeV, while the branching ratios are around 10^{-7}. For $\ell = e, \mu$, their decay widths are nearly identical since their masses can be discarded in the B_c meson decays to η. Besides, the LO and NLO Gegenbauer series have less influence in the semileptonic decay width of B_c into η. In Table IV, we give the results for the $B_c \to \eta' + \ell + \bar{\nu}_\ell$ with $\ell = e, \mu, \tau$. The form factors from the quark content with NLO Gegenbauer series and from the gluonium contribution are considered in the semileptonic decays. From the table, their decay widths are around 10^{-17} GeV, while the branching ratios are around 10^{-5}. We give the results for the $B_c \to G(0^{-+}) + \ell + \bar{\nu}_\ell$ in Table V, where the decay widths are around $(10^{-18}$–$10^{-17})$ GeV, while the branching ratios are around $(10^{-6}, 10^{-5})$.

TABLE III. The semileptonic decay widths and the branching ratios of B_c into η. Here and in the following $q\bar{q}$ denote all the quark contents, and the lifetime $\tau_{B_c} = 0.50$ ps.

$B_c \to \eta + \ell + \bar{\nu}_\ell$		$\Gamma_\eta(\times 10^{-19}$ GeV)	$\text{Br}_\eta(\times 10^{-7})$
$\ell = e, \mu$	$q\bar{q}$ (LO)	$2.44^{+0.15}_{-0.18}$	$1.85^{+0.12}_{-0.13}$
	$q\bar{q}$ (NLO)	$3.07^{+0.00}_{-0.07}$	$2.33^{+0.00}_{-0.05}$
$\ell = \tau$	$q\bar{q}$ (LO)	$2.37^{+0.15}_{-0.17}$	$1.80^{+0.11}_{-0.13}$
	$q\bar{q}$ (NLO)	$2.99^{+0.00}_{-0.08}$	$2.27^{+0.00}_{-0.05}$

TABLE IV. The semileptonic decay widths and the branching ratios of B_c into η'.

$B_c \to \eta' + \ell + \bar{\nu}_\ell$		$\Gamma_{\eta'}(\times 10^{-17}$ GeV)	$\text{Br}_{\eta'}(\times 10^{-5})$
$\ell = e, \mu$	$q\bar{q}$ (NLO)	$1.24^{+0.28}_{-0.21}$	$0.94^{+0.21}_{-0.18}$
	Total	$1.02^{+0.24}_{-0.16}$	$0.78^{+0.18}_{-0.12}$
$\ell = \tau$	$q\bar{q}$ (NLO)	$1.04^{+0.23}_{-0.18}$	$0.79^{+0.23}_{-0.13}$
	Total	$0.86^{+0.19}_{-0.14}$	$0.65^{+0.14}_{-0.11}$

TABLE V. The semileptonic decay widths and the branching ratios of B_c into 0^{-+} glueball.

$B_c \to G(0^{-+}) + \ell + \bar{\nu}_\ell$		$\Gamma_G(\times 10^{-18}$ GeV)	$\text{Br}_G(\times 10^{-6})$
$\ell = e, \mu$	gg	$2.21^{+0.15}_{-0.78}$	$1.68^{+0.14}_{-0.60}$
	Total	$2.03^{+0.00}_{-0.69}$	$1.54^{+0.00}_{-0.52}$
$\ell = \tau$	gg	$1.59^{+0.14}_{-0.57}$	$1.21^{+0.10}_{-0.43}$
	Total	$1.46^{+0.00}_{-0.49}$	$1.11^{+0.00}_{-0.38}$

In Ref. [11], the semileptonic branching ratios of B_c into $\eta\prime$ have already been predicted in perturbative QCD, where the $\text{Br}(B_c \to \eta + \ell + \bar{\nu}_\ell) = 3.98 \times 10^{-6}$ and $\text{Br}(B_c \to \eta' + \ell + \bar{\nu}_\ell) = 5.24 \times 10^{-5}$ with $\ell = e, \mu$ and $m_u = 2$ MeV, $m_d = 4$ MeV, and $m_s = 80$ MeV. Compared with these predictions in perturbative QCD, our results are smaller due to the choice of the decay constant of $\eta\prime$, and the two gluon scattering mechanism is employed. Currently, there is no report on semileptonic decays of B_c into $\eta\prime$. However, the hunting for the signals of B_c into $\eta\prime$ is accessible in future LHCb experiments when considering the large cross section of B_c meson.

In the end it is very interesting to find out whether the formulas above can guide the studies of the processes $D_s \to \eta + \ell + \bar{\nu}_\ell$. The BESIII Collaboration has measured these channels and given $\text{Br}(D_s \to \eta + \ell + \bar{\nu}_\ell) = (2.42 \pm 0.46 \pm 0.11)\%$ and $\text{Br}(D_s \to \eta' + \ell + \bar{\nu}_\ell) = (1.06 \pm 0.54 \pm 0.07)\%$ [56]. For $D_s \to \eta^{(\prime)} + \ell + \bar{\nu}_\ell$, the $c \to s$ transition with another spectator strange quark will be present in the $D_s \to \eta\prime$ form factors. Considering the transferred momentum is small in $D_s \to \eta' + \ell + \bar{\nu}_\ell$, the perturbative calculation may be invalid, so we only consider the channel $D_s \to \eta + \ell + \bar{\nu}_\ell$. As the tentative analysis, it is interesting to find out how large the mechanism of two gluon transitions contributes to processes $D_s \to \eta + \ell + \bar{\nu}_\ell$. Employing the above formulas, and taking the replacement of $b \to c$, $c \to s$, and $B_c \to D_s$, we may tentatively give the order of magnitude of their decay widths since the D_s meson is not really a nonrelativistic bound state. We found that the mechanism of two gluon transitions gives $\text{Br}(D_s \to \eta + \ell + \bar{\nu}_\ell) \sim 10^{-4}$ and only contributes to 0.5% in the channel $D_s \to \eta + \ell + \bar{\nu}_\ell$. The $c \to s$ transition thus dominates the form factor of $D_s \to \eta$. To extrapolate the form factors of $D_s \to \eta$ to the minimum momentum recoil region, the pole model is still useful [55,57]. Combining the experimental data, the $c \to s$ transition leads to the $f_{0,+}^{D_s \eta}(q^2 = 0) = 0.50 \pm 0.05$.

V. CONCLUSION

We investigated the form factors of B_c into the $\eta\prime$ and pseudoscalar glueball and employed the form factors into their semileptonic decays. Unlike the decay of D_s into $\eta^{(\prime)}$ where the $c \to s$ transition is dominant, the two gluon scattering mechanism dominated the contribution for the form factors of B_c into $\eta\prime$. We considered the η-η'-glueball mixing effects and studied their influences in the form factors. At the maximum momentum recoil point, the form factors of B_c into the light pseudoscalar mesons are factored as the LDMEs along with the corresponding short-distance perturbatively calculable coefficients. The results of form factors in the maximum momentum recoil point were obtained. Using the pole model, the form factors of B_c into the $\eta^{(\prime)}$ and pseudoscalar glueball are extrapolated into the minimum momentum recoil region.

The corresponding semileptonic decay widths and the branching ratios were calculated. The results are as follows: the branching ratio of $B_c \to \eta + \ell + \bar{\nu}_\ell$ is around 10^{-7}; the branching ratio of $B_c \to \eta' + \ell + \bar{\nu}_\ell$ is around 10^{-5}; and the branching ratio of $B_c \to G(0^{-+}) + \ell + \bar{\nu}_\ell$ is around $(10^{-6}, 10^{-5})$. Future LHCb experiments shall test these predictions, which is helpful to understand the η-η'-glueball mixing and the decay properties of B_c meson.

ACKNOWLEDGMENTS

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[1] C. H. Chang and X. G. Wu, Eur. Phys. J. C **38**, 267 (2004).
[2] C. F. Qiao, P. Sun, D. Yang, and R. L. Zhu, Phys. Rev. D **89**, 034008 (2014).
[3] S. Kar, P. C. Dash, M. Priyadarsini, S. Naimuddin, and N. Barik, Phys. Rev. D **88**, 094014 (2013).
[4] C. H. Chang and Y. Q. Chen, Phys. Rev. D **49**, 3399 (1994).
[5] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Rev. D **68**, 094020 (2003).
[6] W. Wang, Y. L. Shen, and C. D. Lu, Phys. Rev. D **79**, 054012 (2009).
[7] S. Dubnicka, A. Z. Dubnickova, A. Issadykov, M. A. Ivanov, and A. Liptaj, Phys. Rev. D **96**, 076017 (2017).
[8] R. Zhu, Nucl. Phys. **B931**, 359 (2018).
[9] R. Zhu, Y. Ma, X. L. Han, and Z. J. Xiao, Phys. Rev. D **95**, 094012 (2017).
[10] X. G. He, W. Wang, and R. L. Zhu, J. Phys. G **44**, 014003 (2017).
[11] G. Li, W. G. Ma, Y. Jiang, R. Y. Zhang, L. Han, and L. Xue-Qian, Phys. Rev. D **73**, 094010 (2006).
[12] Z. J. Xiao and X. Liu, Phys. Rev. D **84**, 074033 (2011).
[13] X. Liu, R. H. Li, Z. T. Zou, and Z. J. Xiao, Phys. Rev. D **96**, 013005 (2017).
[14] W. Wang and R. L. Zhu, Eur. Phys. J. C **75**, 360 (2015).
[15] L. B. Chen and C. F. Qiao, Phys. Lett. B **748**, 443 (2015).
[16] R. Zhu, X. L. Han, Y. Ma, and Z. J. Xiao, Eur. Phys. J. C **78**, 740 (2018).
[17] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. D **94**, 032009 (2016).
[18] C. F. Qiao and R. L. Zhu, Phys. Rev. D **89**, 074006 (2014).
[19] C. Rosenzweig, A. Salomone, and J. Schechter, Phys. Rev. D **24**, 2545 (1981).
[20] T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).
[21] G. M. Shore, Lect. Notes Phys. **737**, 235 (2008).
[22] K. Kawarabayashi and N. Ohta, Nucl. Phys. **B175**, 477 (1980).
[23] V. Mathieu and V. Vento, Phys. Rev. D **81**, 034004 (2010).
[24] R. Zhu, J. High Energy Phys. 09 (2015) 166.
[25] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[26] P. Ball, J. M. Frere, and M. Tytgat, Phys. Lett. B **365**, 367 (1996).
[27] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998).
[28] R. Escribano and J. Nadal, J. High Energy Phys. 05 (2007) 006.
[29] R. Escribano, Eur. Phys. J. C **65**, 467 (2010).
[30] H. W. Ke, X. H. Yuan, and X. Q. Li, Int. J. Mod. Phys. A **26**, 4731 (2011).
[31] A. Bramon, R. Escribano, and M. D. Scadron, Phys. Lett. B **403**, 339 (1997).
[32] R. Escribano and J. M. Frere, J. High Energy Phys. 06 (2005) 029.
[33] H. Y. Cheng, H. n. Li, and K. F. Liu, Phys. Rev. D **79**, 014024 (2009).
[34] G. Li, Q. Zhao, and C. H. Chang, J. Phys. G **35**, 055002 (2008).
[35] X. Liu, H. n. Li, and Z. J. Xiao, Phys. Rev. D **86**, 011501 (2012).
[36] Z. J. Xiao, Y. Li, D. T. Lin, Y. Y. Fan, and A. J. Ma, Phys. Rev. D **90**, 114028 (2014).
[37] L. A. Harland-Lang, V. A. Khoze, M. G. Ryskin, and A. G. Shuvaev, Phys. Lett. B **770**, 88 (2017).
[38] F. Ambrosino *et al.* (KLOE Collaboration), Phys. Lett. B **648**, 267 (2007).
[39] C. Michael, K. Ottnad, and C. Urbach (ETM Collaboration), Phys. Rev. Lett. **111**, 181602 (2013).
[40] M. C. Chang *et al.*, Phys. Rev. D **85**, 091102 (2012).
[41] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 01 (2015) 024.

[42] M. Tanabashi *et al.* (PDG Collaboration), Phys. Rev. D **98**, 030001 (2018).
[43] E. Kou and A. I. Sanda, Phys. Lett. B **525**, 240 (2002).
[44] T. Ohrndorf, Nucl. Phys. **B186**, 153 (1981).
[45] T. Muta and M. Z. Yang, Phys. Rev. D **61**, 054007 (2000).
[46] A. Ali and A. Y. Parkhomenko, Phys. Rev. D **65**, 074020 (2002).
[47] P. Kroll and K. Passek-Kumericki, Phys. Rev. D **67**, 054017 (2003).
[48] P. Kroll and K. Passek-Kumericki, J. Phys. G **40**, 075005 (2013).
[49] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985).
[50] C. F. Qiao and R. L. Zhu, Phys. Rev. D **87**, 014009 (2013).
[51] R. Mertig, M. Bohm, and A. Denner, Comput. Phys. Commun. **64**, 345 (1991).
[52] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. **118**, 153 (1999).
[53] T. Hahn, Nucl. Phys. B, Proc. Suppl. **89**, 231 (2000).
[54] R. C. Verma, J. Phys. G **39**, 025005 (2012).
[55] G. Duplancic and B. Melic, J. High Energy Phys. 11 (2015) 138.
[56] M. Ablikim *et al.*, Phys. Rev. D **97**, 012006 (2018).
[57] H. Y. Cheng and X. W. Kang, Eur. Phys. J. C **77**, 587 (2017); **77**, 863(E) (2017).