Heavy-quark symmetry partners of the $P_c(4450)$ pentaquark

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The spectrum of heavy-hadron molecules is constrained by heavy-quark symmetry in its different manifestations. Heavy-quark spin symmetry for instance connects the properties of the ground and excited states of heavy hadrons, while heavy-antiquark-diquark symmetry connects the properties of heavy antimesons ($\bar{D}$, $\bar{D}^*$) and doubly heavy baryons (Ξ_{cc}, Ξ_{cc}^*). A prediction of these symmetries is that if the $P_c(4450)$ is indeed a $\bar{D}^*\Sigma_c$ bound state, then there should be a series of $\bar{D}^*\Sigma_c^*$, $\Xi_{cc}\Sigma_c$, $\Xi_{cc}^*\Sigma_c$, $\Xi_{cc}\Sigma_c^*$ and $\Xi_{cc}^*\Sigma_c^*$ partners. The concrete application of heavy-quark spin symmetry indicates that, if the $P_c(4450)$ is a $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecule, the existence of a $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ partner with similar binding energy—which we call $P_c(4515)$, given its expected mass—is likely. Conversely, the application of heavy-antiquark-diquark symmetry indicates that the 0^+ $\Xi_{cc}\Sigma_c$, 1^+ $\Xi_{cc}\Sigma_c^*$, 2^+ $\Xi_{cc}^*\Sigma_c$ and 3^+ $\Xi_{cc}^*\Sigma_c^*$ molecules are likely to bind too, with binding energies in the 20–30 MeV range.

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Exotic hadrons—hadrons that are neither a quark-antiquark nor a three-quark state—are an interesting window into low-energy QCD dynamics. From a theoretical perspective the simplest types of exotic hadrons are hadronic molecules, which are bound states of two or more hadrons. They were theorized decades ago [1,2] on the analogy of how the nuclear forces among nucleons generate the deuteron and other nuclei. The discovery of the $X(3872)$ [3] provided the first solid candidate for a hadronic molecule and suggested that the early speculations about their existence [4,5] were on the right track. The proximity of the $X(3872)$ to the open-charm threshold $D^{*0}\bar{D}^0$ provides circumstantial evidence that the $X(3872)$ is molecular [6–8], while the isospin-breaking decays into $J/\psi 2\pi$ and $J/\psi 3\pi$ [9] represent a stronger case for its molecular nature [10–12]. The most stringent test of the $X(3872)$ nature will eventually be provided by its decays into $D^0\bar{D}^0\gamma$ and $D^0\bar{D}^0\pi^0$ [7,13,14], but as for now the detailed experimental information required about them is not available. Other molecular candidates include the $Z_b(10610)$ and $Z_b(10650)$ [15,16], the $Z_c(3900)$ [17,18] and $Z_c(4020)$ [19,20]; see Ref. [21] for a recent review. Recently a narrow pentaquarklike resonance, the $P_c(4450)$, was discovered by the LHCb [22], which has been assumed to be a $\bar{D}^*\Sigma_c$ [23–27], a $\bar{D}^*\Sigma_c^*$ [28,29], or a $\chi_{c1}p$ molecule [30]. Besides the molecular hypothesis, there are other competing explanations for the $P_c(4450)$: a genuine pentaquark [31–36], a threshold effect [37,38] (see [39] for a detailed discussion), baryocharmonium [40] or other more exotic possibilities [41,42].

Hadronic molecules have a high degree of symmetry. If the hadrons conforming a molecule contain light quarks, chiral and SU(3)-flavor symmetries strongly constrain the interactions and the spectra of these molecules. Conversely if the hadrons contain heavy quarks, then heavy-quark symmetry in its different manifestations [43–47] will influence the way these molecules organize in multiplets. The light and heavy symmetries of hadronic molecules can indeed be used to understand their known spectrum and to predict the existence of new states [48–57]. The present manuscript deals with how these symmetries apply for the particular case of a molecular $P_c(4450)$.

We begin by considering the $P_c(4450)$ (the P_c^* from now on) from the point of view of heavy-quark spin symmetry (HQSS). HQSS states that the dynamics of a heavy hadron is independent of the spin of the heavy quark inside it. In the molecular picture the P_c^* is commonly pictured as a $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecule [23–27], or less commonly as a $J^P = \frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecule [28]. Here for concreteness we work under the first of these assumptions, namely that the

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P_c^* is a $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ bound state. In either case we have to define heavy-hadron superfields that group the heavy-hadron fields into units that are well behaved with respect to heavy-quark rotations. The nonrelativistic superfield for the heavy pseudoscalar and vector mesons D and D^* is [58]

$$H_c = \frac{1}{\sqrt{2}}[D + \vec{D}^* \cdot \vec{\sigma}], \tag{1}$$

where H_c is a 2×2 matrix and $\vec{\sigma}$ refers to the Pauli matrices. For the heavy-baryon field we define the superfield as [59]

$$\vec{S}_c = \frac{1}{\sqrt{3}}\vec{\sigma}\Sigma_c + \vec{\Sigma}_c^*, \tag{2}$$

which is a 2×3 matrix, basically the tensor product between the spin-$1/2$ heavy and spin-1 light degrees of freedom. In this representation the spin-$3/2$ heavy-baryon field is subjected to the condition $\vec{\sigma} \cdot \vec{\Sigma}_c^* = 0$, which ensures that the $\vec{\Sigma}_c^*$ is a spin-$3/2$ field. From the heavy-meson and baryon superfield the most general contact-range Lagrangian that we can construct without derivatives is

$$\mathcal{L} = C_a \vec{S}_c^\dagger \cdot \vec{S}_c \mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] + C_b \sum_{i=1}^{3} \vec{S}_c^\dagger \cdot (J_i \vec{S}_c) \mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c], \tag{3}$$

where J_i with $i = 1, 2, 3$ refers to the spin-1 angular momentum matrices, which we recall here,

$$J_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad J_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},$$
$$J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{4}$$

and C_a and C_b are coupling constants. This Lagrangian leads to the contact-range potential of Table I, which in turn can be considered as the leading-order potential of an effective field theory for the $\bar{H}_c S_c$ family of molecules (in line with the analogous effective field theories for $H_c\bar{H}_c$ [52] and $S_c\bar{S}_c$ molecules [57]).

If the $P_c(4450)$ is a $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecule, its potential is

$$V\left(\bar{D}^*\Sigma_c, J^P = \frac{3}{2}^-\right) = C_a + \frac{2}{3}C_b. \tag{5}$$

Curiously, the potential for a prospective $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecule is similar,

TABLE I. The lowest-order contact-range potential for the $\bar{H}_c S_c$ system, which contains two unknown couplings C_a and C_b. We show the potential for each particle and spin channel (the "molecule" and "J^P" columns). The potential is suspected to bind for the $J^P = \frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ channel, forming the $P_c(4450)$ pentaquark, which strongly suggests that the $J^P = \frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ channel binds too.

Molecule	J^P	V	B (MeV)
$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	C_a	?
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	C_a	?
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$C_a - \frac{4}{3}C_b$	?
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$C_a + \frac{2}{3}C_b$	12 ± 3
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$C_a - \frac{5}{3}C_b$	?
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$C_a - \frac{2}{3}C_b$	?
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$C_a + C_b$	12

$$V\left(\bar{D}^*\Sigma_c^*, J^P = \frac{5}{2}^-\right) = C_a + C_b, \tag{6}$$

which strongly suggests that this molecule should also bind (probably with a binding energy similar to that of the $\frac{3}{2}^-$ state). This conclusion is subject to a series of uncertainties, from which the most obvious one is that the potential is not exactly the same. We do not know how much of the binding is due to the individual couplings C_a and C_b. It could indeed happen that one of these states binds, but the other does not, which could be the case if $|C_b|$ is disproportionately bigger than $|C_a|$. The existence of the $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ partner is very likely if the two couplings are of similar size, i.e., $|C_a| \sim |C_b|$, or alternatively if $|C_a| > |C_b|$. In this regard we note that the phenomenological model of Ref. [60] (which predicts $\bar{D}\Sigma_c$ and $\bar{D}^*\Sigma_c$ bound states at 4261 and 4412 MeV respectively, in the latter case independently of the total spin of the $\bar{D}^*\Sigma_c$ system) indeed suggests that $|C_a| > |C_b|$. Besides the issue with C_a and C_b, we have that HQSS is not exact but expected to have a level of uncertainty of the order of $\Lambda_{\mathrm{QCD}}/m_Q$, with $\Lambda_{\mathrm{QCD}} \sim$ 200–300 MeV and m_Q being the mass of the heavy quark. For the charm sector this uncertainty is of the order of 15%, which is how much the potential in the $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecule is expected to differ from its HQSS expectation. The existence of subleading-order effects, in particular one-pion exchange, induces an additional source of uncertainty. This is usually dealt with by including a floating cutoff in the calculations and varying it within a reasonable window, as we show later for the triply heavy molecules. Another effect from one-pion exchange is the curious coupled-channel dynamics between the $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ and $\bar{D}\Lambda_c(2595)$ channels ($\bar{D}\Lambda_{c1}$ from now on). This involves the exchange of a pion near the mass shell, resulting in a long-range $1/r^2$ type of interaction that renders binding easier [27]. For the $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c - \bar{D}\Lambda_{c1}$ system this effect is modest, but still

noticeable: the short-range attraction (i.e., the $C_a + \frac{2}{3}C_b$ coupling combination) required to bind the coupled $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c - \bar{D}\Lambda_{c1}$ system is about 70%–90% of that required to bind the uncoupled $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ system, depending on whether we include pions or not [27].

For the doubly heavy baryons we define the superfield [61],

$$\vec{T}_{cc} = \frac{1}{\sqrt{3}}\vec{\sigma}\Xi_{cc} + \vec{\Xi}^*_{cc}, \tag{7}$$

which is formally analogous to the $\vec{S}_c$ superfield. But the interpretation of the $\vec{T}_{cc}$ superfield is different from the $\vec{S}_c$ superfield: for $\vec{T}_{cc}$ the light spin is $1/2$ while the heavy spin is 1. The application of heavy-antiquark-diquark symmetry (HADS) [45] can actually be encapsulated in the following two substitutions,

$$\mathrm{Tr}[\bar{H}_c^\dagger \bar{H}_c] \to \vec{T}^\dagger_{cc}\cdot\vec{T}_{cc}, \tag{8}$$

$$\mathrm{Tr}[\bar{H}_c^\dagger \sigma_i \bar{H}_c] \to \vec{T}^\dagger_{cc}\cdot(\sigma_i\vec{T}_{cc}), \tag{9}$$

which are derived from the formalism of Ref. [61]. From these substitutions we arrive at the Lagrangian that describes the contact-range interaction between a heavy baryon and a doubly heavy baryon,

$$\mathcal{L} = C_a \vec{S}_c^\dagger\cdot\vec{S}_c\vec{T}^\dagger_{cc}\cdot\vec{T}_{cc} + C_b\sum_{i=1}^{3}\vec{S}_c^\dagger\cdot(J_i\vec{S}_c)\vec{T}^\dagger_{cc}\cdot(\sigma_i\vec{T}_{cc}), \tag{10}$$

where the corresponding contact-range potential can be consulted in Table II. The following configurations are worth considering:

$$V(\Xi_{cc}\Sigma_c, J^P = 0^+) = C_a + \frac{2}{3}C_b, \tag{11}$$

$$V(\Xi_{cc}\Sigma_c^*, J^P = 1^+) = C_a + \frac{5}{9}C_b, \tag{12}$$

$$V(\Xi_{cc}^*\Sigma_c, J^P = 2^+) = C_a + \frac{2}{3}C_b, \tag{13}$$

$$V(\Xi_{cc}^*\Sigma_c^*, J^P = 3^+) = C_a + C_b, \tag{14}$$

because they imply a potential that is either identical to that of a molecular P_c^* or very similar. From this it is sensible to expect that in a first approximation these four molecules will bind. More concrete predictions are possible from solving a nonrelativistic bound state equation with the contact-range potentials. If we work in momentum space, we can solve the integral equation

TABLE II. The lowest-order contact-range potential for the $T_{cc}S_c$ system, which we derive from the $\bar{H}_cS_c$ potential and HADS. The potential depends on two unknown couplings C_a and C_b, with different linear combinations depending on the particle and spin channel (the molecule and J^P columns). The combination $C_a + \frac{2}{3}C_b$ can be determined from the hypothesis that the P_c^* is a $\bar{H}_cS_c$ molecule. From this we can compute the binding energy of two $T_{cc}S_c$ molecules and estimate the binding energy of another two. The binding energies are expressed in MeV. For the 0^+ and 2^+ molecules we show the results for the cutoff $\Lambda = 0.5(1.0)$ GeV, where the errors come from the uncertainty of HADS and the P_c^* mass. For the 1^+ and 3^+ molecules we simply show the cutoff variation: the uncertainty in the binding of these states is difficult to estimate because the potential is in fact not identical to that of the P_c^*.

Molecule	J^P	V	B (MeV)
$\Xi_{cc}\Sigma_c$	0^+	$C_a + \frac{2}{3}C_b$	$19^{+15}_{-13}(29^{+32}_{-23})$
$\Xi_{cc}\Sigma_c$	1^+	$C_a - \frac{2}{9}C_b$	?
$\Xi_{cc}\Sigma_c^*$	1^+	$C_a + \frac{5}{9}C_b$	20–30
$\Xi_{cc}\Sigma_c^*$	2^+	$C_a - \frac{1}{3}C_b$	?
$\Xi_{cc}^*\Sigma_c$	1^+	$C_a - \frac{10}{9}C_b$	?
$\Xi_{cc}^*\Sigma_c$	2^+	$C_a + \frac{2}{3}C_b$	$19^{+15}_{-12}(30^{+33}_{-24})$
$\Xi_{cc}^*\Sigma_c^*$	0^+	$C_a - \frac{5}{3}C_b$	?
$\Xi_{cc}^*\Sigma_c^*$	1^+	$C_a - \frac{11}{9}C_b$	?
$\Xi_{cc}^*\Sigma_c^*$	2^+	$C_a - \frac{1}{3}C_b$	?
$\Xi_{cc}^*\Sigma_c^*$	3^+	$C_a + C_b$	20–30

$$\phi(k) + \int\frac{d^3p}{(2\pi)^3}\langle k|V|p\rangle\frac{\phi(p)}{B + \frac{p^2}{2\mu}} = 0, \tag{15}$$

where ϕ is the vertex function, B the binding energy, and μ the reduced mass. To solve this equation we have to regularize the contact-range potential,

$$\langle p|V_\Lambda|p'\rangle = C_{P_c^*}(\Lambda) f\left(\frac{p}{\Lambda}\right) f\left(\frac{p'}{\Lambda}\right), \tag{16}$$

with Λ being a cutoff, $f(x)$ a regulator function and $C_{P_c^*} = C_a + \frac{2}{3}C_b$ the coupling of the contact-range potential for the P_c^* and the 0^+ $\Xi_{cc}\Sigma_c$ and 2^+ $\Xi_{cc}^*\Sigma_c$ molecules; see Tables I and II. A typical choice of the cutoff is $\Lambda =$ 0.5–1.0 GeV, while for the regulator we choose $f(x) = e^{-x^2}$. For the masses we use $m(D^*) = 2009$ MeV, $m(\Sigma_c) = 2454$ MeV, $m(\Sigma_c^*) = 2518$ MeV (i.e., the isospin average of their PDG values [62]), $m(\Xi_{cc}) = 3621$ MeV and $m(\Xi_{cc}^*) = 3727$ MeV, where the Ξ_{cc}^* mass has been deduced from the HADS relation $m(\Xi_{cc}^*) - m(\Xi_{cc}) = \frac{3}{4}(m(D^*) - m(D))$ [45].

We can use the existence of the P_c^* as a renormalization condition, that is, for a given cutoff Λ and regulator function we fix the coupling $C_{P_c^*}(\Lambda)$ from the condition

of reproducing the P_c^* pole. With the coupling determined in this way, we can make predictions for the 0^+ and 2^+ triply heavy molecules,

$$B(0^+) \simeq B(2^+) \simeq 19\text{–}29 \text{ MeV}, \tag{17}$$

which are more bound than the original P_c^* state simply because the reduced mass is bigger for the heavy-baryon / doubly heavy-baryon system. The above range represents the cutoff variation, which is expected to give the uncertainty from not taking into account subleading-order interactions such as pion exchanges though we comment on this later. For the 1^+ and 3^+ molecules the binding energy should be similar, but it is difficult to be more concrete as the potentials are not exactly the same. We mention in passing that phenomenological predictions of 0^+ $\Xi_{cc}\Sigma_c$ and 1^+ $\Xi_{cc}\Sigma_c^*$ molecules also exist in the one boson exchange model [63], though they are more vague owing to the nonuniqueness of physically acceptable form factor and cutoff choices.

Of course there are several sources of uncertainty that have to be taken into account. The most conspicuous one is the binding energy of the P_c^*, i.e., $B = 12 \pm 3$ MeV. A second source of uncertainty is HADS itself, which is expected to be only accurate at the $\Lambda_{\rm QCD}/(m_Q v)$ level [45] with v being the velocity of the heavy diquark pair.[1] This is easily included by assuming that the relative error of the 0^+ and 2^+ potential is of the same size as the HADS uncertainty, which we estimate to be about 30%. Combining these two effects, we arrive at

$$B(0^+) \simeq B(2^+) \simeq 19^{+15}_{-13}(29^{+33}_{-24}) \text{ MeV}, \tag{18}$$

for $\Lambda = 0.5(1.0)$ GeV. For a more comprehensive list we refer to Table II, where it should be noticed that for the 1^+ and 3^+ states this detailed error analysis is not possible because the potential does not exactly match that of the P_c^*. The errors are dominated by the HADS uncertainty, with the binding uncertainty playing a secondary role. There are additional sources of uncertainly which are not so easily modeled. One of these sources is the contribution of the $\bar{D}\Lambda_{c1}$ channel to the binding of the P_c^*. As already explained, this contribution basically reduces the strength of the contact-range interaction required to bind the P_c^*. However the error induced by this effect is expected to be noticeably smaller than the HADS uncertainty. One-pion exchange will be another important factor, and so will coupled-channel effects ($\bar{D}^*\Sigma_c - \bar{D}^*\Sigma_c^*$). Yet these uncertainty sources fall into the category of subleading-order contributions and are in principle expected to be covered by the cutoff variation. There is a caveat though: when connecting the double and triply heavy molecules, calculations do not converge in the $\Lambda \to \infty$ limit (the binding energy eventually develops a quadratic divergence). This is analogous to what happens when connecting different heavy-flavor sectors [66]. As a consequence the calculations cannot be interpreted as the results of a genuine effective field theory, but instead have a distinct phenomenological taste. In practical terms this means that there are systematic errors that have not been accounted for properly. Luckily the size of these unaccounted errors seems to be moderate: despite the impossibility of removing the cutoff, the predictions are relatively independent of the choice of regulator (provided we use a cutoff of the order of the typical hadronic scale). This can be illustrated with the use of a delta-shell regulator in coordinate space

$$V(r; R_c) = C_{P_c^*}(R_c)\frac{\delta(r - R_c)}{4\pi R_c^2}, \tag{19}$$

with $R_c = 0.5$–1.0 fm (i.e., the typical size of hadrons). With this regulator the predictions are

$$B(0^+) \simeq B(2^+) \simeq 20^{+18}_{-14}(31^{+37}_{-25}) \text{ MeV}, \tag{20}$$

for $R_c = 1.0(0.5)$ fm, which is fairly consistent with the previous results with the Gaussian regulator in momentum space; see Eq. (18).

Be it as it may, the biggest source of systematic uncertainty is the fact that we do not know the nature of the P_c^* for sure. Here we have simply followed the hypothesis that it is a $\frac{3}{2}^-$ $\bar{D}^*\Sigma_c$ molecule [23–27] with a binding energy of about 12 MeV and a radius of $1/\sqrt{2\mu B} \sim 1.2$ fm. Other authors consider the P_c^* to be a $\frac{5}{2}^-$ $\bar{D}^*\Sigma_c^*$ molecule [28] with a binding energy of about 77 MeV, implying a radius of 0.5 fm. In this second scenario, the P_c^* is considerably more compact to the extent that it is somewhere in the limit between a molecule and a multiquark state. We do not consider this scenario in detail here, but merely mention that the qualitative predictions do not change: we still expect a $\frac{3}{2}^-$ pentaquark partner and the 0^+, 1^+, 2^+ and 3^+ $T_{cc}S_c$ molecules. The quantitative predictions are fairly different though. The $\frac{3}{2}^-$ pentaquark is bound by about 70–80 MeV and is located in the 4380 MeV region, where the second LHCb pentaquark is. Their identification is problematic, in part owing to the large decay width of the $P_c(4380)$ (mostly into $J/\psi p$), $\Gamma = 205 \pm 90$ MeV, which is definitely large for a composed state, but not that excessive if we take into account that the size of the $P_c(4380)$ in this scenario is similar to the size of the J/ψ, namely 0.47 fm [67]. Actually the strongest argument against the $P_c(4380)$ being the HQSS partner of the $P_c(4450)$ comes from the statistical analysis of the

[1]Reference [61] estimates $m_Q v \sim 0.8$ GeV for the case of the charm quark ($Q = c$), while Ref. [64] argues that the c quark is too light for HADS to be applicable. From lattice QCD [65] it seems apparent that the $J = \frac{1}{2}$ Ξ_{cc} and $J = \frac{3}{2}$ Ξ_{cc}^* mass splitting is close to the HADS prediction. The eventual discovery of the Ξ_{cc}^* doubly charmed baryon and its properties will probably settle the question of how accurate HADS is.

LHCb data, from which it is unlikely that the two pentaquarklike resonances have the same parity [68]. For the 3^+ $\Xi_{cc}^*\Sigma_c^*$ molecule, the binding energy is $B(3^+) \simeq 90$–110 MeV, with similar binding energies expected for its 0^+, 1^+ and 2^+ partners. Finally there is a series of works that do not consider the P_c^* to be a molecule: the predictions of this manuscript are not likely to hold if these scenarios are confirmed. But this depends on whether it is the dynamics of the P_c^* or heavy-quark symmetry itself which plays the greatest role in the eventual existence of these partners.

To summarize, we have explored what consequences can be derived from HQSS and HADS and the hypothesis that the P_c^* is a molecular state. From HQSS it is plausible to expect that the P_c^* has a $\frac{5}{2}^-$ partner, with a similar binding energy to that of the P_c^*, which we may call the $P_c(4515)$. From HADS we expect the existence of up to four triply heavy baryon-baryon molecules, with quantum numbers 0^+, 1^+, 2^+ and 3^+ and binding energies in the 20–30 MeV range. These predictions are subjected to a series of uncertainties, which include the approximate nature of HQSS and HADS, the existence of a long-range $1/r^2$ potential in the P_c^* molecular candidate, subleading contributions to the potential such as pion exchanges, and the fact that in a few instances the form of the potential is not identical to that of the P_c^*. The most important systematic uncertainty is the nature of the P_c^* itself, which requires further experiments. While the eventual experimental observation of the theorized HQSS partner of the P_c^* seems possible, the detection of its triply charmed partners is more tricky as it requires triple charm production. This suggests that the lattice might be a more expedient way to determine the existence of $\Xi_{cc}\Sigma_c$, $\Xi_{cc}\Sigma_c^*$, $\Xi_{cc}^*\Sigma_c$ and $\Xi_{cc}^*\Sigma_c^*$ molecules.

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