

P -wave contributions to $B \rightarrow \psi\pi\pi$ decays in the perturbative QCD approach

Zhou Rui*

*College of Sciences, North China University of Science and Technology,
Tangshan, Hebei 063210, People's Republic of China*Ya Li[†]*Department of Physics, College of Science, Nanjing Agricultural University,
Nanjing, Jiangsu 210095, People's Republic of China*Hsiang-nan Li[‡]*Institute of Physics, Academia Sinica, Taipei, Taiwan 115, Republic of China*

(Received 21 September 2018; published 7 December 2018)

We present the differential branching fractions for the $B \rightarrow \psi\pi\pi$ decays with the charmonia $\psi = J/\psi, \psi(2S)$ in the invariant mass of the P -wave pion pairs in the perturbative QCD approach. The two-pion distribution amplitudes (DAs) corresponding to both longitudinal and transverse polarizations are constructed to capture important final state interactions in the processes. The timelike form factors, normalizing the two-pion DAs, contain contributions from the ρ resonance and radial excitations fitted to the *BABAR* e^+e^- annihilation data. Given the hadronic parameters for the two-pion DAs associated with the longitudinal polarization which were determined in our previous study, and tuning those associated with the transverse polarization, we accommodate well the observed branching ratios and polarization fractions of the $B \rightarrow J/\psi\pi\pi$ decays. Our predictions for the $B \rightarrow \psi(2S)\pi\pi$ modes from the same set of parameters can be tested in future LHCb and BelleII experiments. We also investigate the sources of theoretical uncertainties in our calculation.

DOI: [10.1103/PhysRevD.98.113003](https://doi.org/10.1103/PhysRevD.98.113003)**I. INTRODUCTION**

The $B_{(s)}$ meson decay chains with charmonia and pion pairs in final states, providing rich opportunities to search for intermediate resonances, have caught both experimental and theoretical attention. The neutral and charged $B \rightarrow J/\psi\pi\pi$ modes, first observed by the *BABAR* Collaboration [1,2], may involve the $J/\psi\pi$ and $\pi\pi$ intermediate channels. No obvious exotic structures were found through the former, and a series of resonant and nonresonant components with different $\pi\pi$ invariant masses has been extracted though the latter in the LHCb experiment [3]. Recent LHCb data [3,4] have indicated that the $B \rightarrow J/\psi\pi^+\pi^-$ decay spectrum is well described by six resonances in the $\pi^+\pi^-$ channel, $f_0(500)$, $\rho(770)$, $\rho(1450)$, $\rho(1700)$, $\omega(782)$,

$f_2(1270)$, with $\rho(770)$ being the dominant component, and that there is no evidence for $f_0(980)$ production. The corresponding $B_s \rightarrow J/\psi\pi^+\pi^-$ decay can be described by an interfering sum of five resonances, $f_0(980)$, $f_0(1500)$, $f_0(1790)$, $f_2(1270)$, and $f'_2(1525)$ [5,6], among which the S -wave $f_0(980)$ is the largest component [5–7], and the D -wave ones amount only up to a few percents. Because the $s\bar{s}$ pair produced in this mode is an isoscalar ($I = 0$), it must form a zero isospin meson, and P -wave resonances, such as the isovector $\rho(770)$, are forbidden. The resonance structures in the $B_{(s)}$ meson decays into $\psi(2S)$ have not been analyzed in detail due to a limited number of events [8].

On the theoretical side, order-of-magnitude estimations for the rates of the above modes have been performed in the chiral unitary approach [9], where a $B_{(s)}$ meson decay amplitude is followed by hadronization of a quark-antiquark pair into two mesons and their further rescattering. Given the input from a well-measured intermediate channel, the others can be derived via their relations to the input one under the above rescattering picture, and were found to compare reasonably well with present data. The authors in Ref. [10] calculated the $B \rightarrow J/\psi\pi\pi$ branching ratios in the generalized factorization and improved QCD factorization approaches, where the ρ intermediate

*jindui1127@126.com

†liyakelly@163.com

‡Corresponding author.
hnli@phys.sinica.edu.tw

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resonance was described by a Breit-Wigner (BW) propagator. More recently, final state interactions in the $B_{(s)} \rightarrow J/\psi\pi\pi$ decays were extracted from data in a framework based on dispersion theory [11]. These works mainly focus on the $\rho(770)$ contribution to the P -wave dipion system, with the two radial excitations $\rho(1450)$ and $\rho(1700)$ and the ρ - ω interference being neglected. As stressed in Ref. [12], the contributions from the two excited ρ states to the time-like pion form factor are indispensable, if one intends to accommodate the measured space-like pion form factor from the time-like one through analytic continuation. Several collaborations [13,14] have also successfully fitted the $e^+e^- \rightarrow \pi^+\pi^-$ cross section in the vicinity of the $\rho(770)$ resonance, with a small but clearly visible ω -meson admixture.

It has been argued [15] that the dominant kinematic region for three-body B meson decays is restricted to the edges of a Dalitz plot, where two of the three final state mesons form a collimated pair in the rest frame of the B meson. In this region, the proof of the corresponding factorization theorem is basically similar to that for the two-body cases [16–18]. Hence, the perturbative QCD (PQCD) approach [19,20] is applicable to three-body B meson decays, albeit the underlying k_T factorization has not been proven rigorously [21,22]. With the introduction of two-hadron distribution amplitudes (DAs) [23–26] to absorb the final state interaction involved in the meson pair, the factorization formalism can be greatly simplified. The factorization theorem holds for B meson decays containing charmonia in the heavy quark limit under the power counting specified in [27]. As a result, a typical amplitude for the $B \rightarrow \psi\pi\pi$ decays, $\psi = J/\psi, \psi(2S)$, is written as [15]

$$\mathcal{A} = \Phi_B \otimes H \otimes \Phi_{\pi\pi} \otimes \Phi_\psi, \quad (1)$$

in which Φ_B and Φ_ψ are the B meson and charmonium DAs, respectively. The two-pion DA $\Phi_{\pi\pi}$ collects the nonperturbative dynamics in the $\pi\pi$ hadronization process. The hard kernel H , similar to that in two-body decays, can be evaluated in perturbation theory. The symbol \otimes denotes the convolution in parton momenta of all the perturbative and nonperturbative objects.

In this paper we will analyze the decays $B \rightarrow \psi(\pi\pi)_P$ with the P -wave dipion system. We do not consider the corresponding decays of a B_s meson, in which the isovector resonant contributions are forbidden as explained before. The decays $B_{(s)} \rightarrow J/\psi(\pi\pi)_S$ and $B_{(s)} \rightarrow J/\psi(K\pi)_S$ as well as the $\psi(2S)$ counterparts, with the S -wave $\pi\pi$ and $K\pi$ pairs, have been studied under the quasi-two-body approximation in the PQCD approach [28–30]. The charmless B meson decays into P -wave pion pairs in the longitudinal polarization were investigated in Refs. [31–33]. The three possible polarizations of the spin-1 ψ meson generate the longitudinal (0), parallel (\parallel), and perpendicular (\perp) amplitudes, such that the two-pion DAs corresponding to

both the longitudinal and transverse polarizations are necessary nonperturbative inputs in our analysis. We will include the two-pion P -wave DAs corresponding to the transverse polarization into the PQCD formalism for the $B \rightarrow \psi(\pi\pi)_P$ decays. It will be explained that the total momentum (angular momentum) of the pion pair mimics the longitudinal (transverse) polarization of the P -wave dipion system.

The decomposition of the longitudinal two-pion DAs up to the twist-3 accuracy has been presented in Ref. [31], but that of the transverse DAs is not yet available. Following the derivation in Refs. [34,35], the two-pion DAs can be parametrized in terms of the Gegenbauer polynomials that depend on parton momentum fractions, and the Legendre polynomials that depend on meson momentum fractions. Moreover, the two-pion DAs are normalized to the timelike form factors, which contain both resonant and nonresonant contributions to the dipion system. To be specific, we adopt the vector-dominance-model parametrization for these form factors, which has been used to fit the pion form factor measured via the e^+e^- annihilation process [14]. Apart from the dominant $\rho(770)$ component, the two radial excitations $\rho(1450)$ and $\rho(1700)$ as well as the ρ - ω interference effect were also taken into account. Besides, the $B \rightarrow J/\psi\pi\pi$ modes are relevant to the determination of the CP violation phases in the B system, which is, however, not the theme of the present work. For recent progresses on this subject, refer to [36–40].

The paper is organized as follows. In Sec. II we define the involved kinematic variables and construct the two-pion DAs for the longitudinal and transverse polarizations. The numerical results are presented and discussed in Sec. III. The last section contains the conclusion. The factorization formulas for the considered decay amplitudes are collected in the Appendix.

II. FRAMEWORK

We begin with the parametrization of the kinematic variables involved in the decay $B(P_B) \rightarrow \psi(P_\psi)(\pi\pi)(P)$. The momenta in the light-cone coordinates are chosen as

$$\begin{aligned} P_B &= \frac{M}{\sqrt{2}}(1, 1, \mathbf{0}_T), & P_\psi &= \frac{M}{\sqrt{2}}(r^2, 1 - \eta, \mathbf{0}_T), \\ P &= \frac{M}{\sqrt{2}}(1 - r^2, \eta, \mathbf{0}_T), \end{aligned} \quad (2)$$

in the B meson rest frame, with the mass ratio $r = m/M$, $m(M)$ being the charmonium (B meson) mass, and the variable $\eta = \omega^2/(M^2 - m^2)$, $\omega^2 = P^2$ being the invariant mass squared of the pion pair. The momenta p_1 and p_2 of the two pions, obeying $p_1 + p_2 = P$, are defined as

$$\begin{aligned} p_1 &= (\zeta P^+, (1 - \zeta)\eta P^+, \sqrt{\zeta(1 - \zeta)}\omega, 0), \\ p_2 &= ((1 - \zeta)P^+, \zeta\eta P^+, -\sqrt{\zeta(1 - \zeta)}\omega, 0), \end{aligned} \quad (3)$$

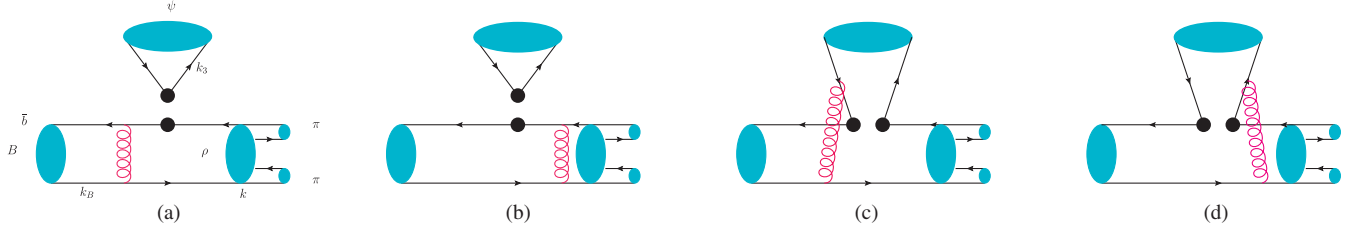


FIG. 1. Leading-order Feynman diagrams for the quasi-two-body decays $B \rightarrow \psi\rho(\rightarrow \pi\pi)$, where ρ represents a P -wave $\pi\pi$ intermediate state, with (a) and (b) the factorizable amplitudes, and (c) and (d) the nonfactorizable amplitudes.

with the pion momentum fraction ζ . We focus on the kinematic configuration, where p_1 and p_2 are almost collimated to each other with small amount of relative transverse momenta. The valence quark momenta labeled by k_B , k_3 , and k in Fig. 1(a) are parametrized as

$$\begin{aligned} k_B &= \left(0, \frac{M}{\sqrt{2}}x_B, \mathbf{k}_{BT}\right), \\ k_3 &= \left(\frac{M}{\sqrt{2}}r^2x_3, \frac{M}{\sqrt{2}}(1-\eta)x_3, \mathbf{k}_{3T}\right), \\ k &= \left(\frac{M}{\sqrt{2}}z(1-r^2), 0, \mathbf{k}_T\right), \end{aligned} \quad (4)$$

in which x_B , x_3 , z denote the longitudinal momentum fractions, and k_{iT} represent the transverse momenta.

The hadronic matrix element for the B meson is written as [41]

$$\Phi_B(x, b) = \frac{i}{\sqrt{2N_c}} [(\not{v}_B + M)\gamma_5\phi_B(x, b)], \quad (5)$$

with the impact parameter b conjugate to the transverse momentum \mathbf{k}_{BT} , and the number of colors N_c . The B meson DA $\phi_B(x, b)$ is the same as in Refs. [41,42],

$$\phi_B(x, b) = N x^2(1-x)^2 \exp\left(-\frac{x^2 M^2}{2\omega_b^2} - \frac{\omega_b^2 b^2}{2}\right), \quad (6)$$

where the shape parameter $\omega_b = 0.40 \pm 0.04$ GeV has been fixed in the study of the B meson transition form factors [43,44], and the coefficient N is determined by the normalization $\int_0^1 dx \phi_B(x, b=0) = 1$.

The hadronic matrix elements for the longitudinally and transversely polarized vector charmonia are decomposed into

$$\begin{aligned} \Phi_\psi^L &= \frac{1}{\sqrt{2N_c}} [m\phi_{3L}\psi^L(x_3, b_3) + \phi_{3L}\not{v}_3\psi^T(x_3, b_3)], \\ \Phi_\psi^T &= \frac{1}{\sqrt{2N_c}} [m\phi_{3T}\psi^V(x_3, b_3) + \phi_{3T}\not{v}_3\psi^T(x_3, b_3)], \end{aligned} \quad (7)$$

respectively, with the longitudinal and transverse polarization vectors

$$\epsilon_{3L} = \frac{1}{\sqrt{2(1-\eta)r}} (-r^2, 1-\eta, \mathbf{0}_T), \quad \epsilon_{3T} = (0, 0, \mathbf{1}_T). \quad (8)$$

The explicit expressions of ψ^i are referred to our previous works [45,46].

The two-pion DAs can be related to the pion DAs through a perturbative evaluation of the matrix elements [34,35],

$$\langle \pi(p_1)\pi(p_2) | \bar{q}'(y^-)\Gamma q(0) | 0 \rangle, \quad (9)$$

as a timelike dipion production process, where Γ denotes the possible spin projectors I , γ_5 , γ_μ , $\gamma_\mu\gamma_5$, $\sigma_{\mu\nu}$, and $\sigma_{\mu\nu}\gamma_5$. The complete set of pion meson DAs up to twist 3 is given by

$$\begin{aligned} \Phi_{P_1}(p_1, x_1) &= \frac{i}{\sqrt{2N_c}} \gamma_5 \left[\not{p}_1 \phi_{P_1}^A(x_1) + m_0 \phi_{P_1}^P(x_1) + m_0 \left(\frac{\not{p}_1 \not{v}_B}{p_1 \cdot v_B} - 1 \right) \phi_{P_1}^T(x_1) \right], \\ \Phi_{P_2}(p_2, x_2) &= \frac{i}{\sqrt{2N_c}} \gamma_5 \left[\not{p}_2 \phi_{P_2}^A(x_2) + m_0 \phi_{P_2}^P(x_2) + m_0 \left(\frac{\not{p}_2 \not{v}_B}{p_2 \cdot v_B} - 1 \right) \phi_{P_2}^T(x_2) \right], \end{aligned} \quad (10)$$

with the chiral scale m_0 . The above decompositions, in which the B meson four-velocity $v_B = (1, 0, 0, 0)$ is invariant under the frame rotation, hold for the pion momenta p_1

and p_2 in arbitrary directions. It is easy to see that the third structure in Eq. (10) approaches to the conventional one in [42],

$$\frac{\not{p}_1 \not{p}_B}{p_1 \cdot v_B} \rightarrow \not{p}_+ \not{p}_-, \quad (11)$$

as p_1 is aligned with the plus direction $n_+ = (1, 0, 0_T)$, where the dimensionless vector $n_- = (0, 1, 0_T)$ is along the direction of the displacement between the quarks q and q' in Eq. (9).

The key to construct the transverse polarization vector $\epsilon_{T\mu}$ for the di-pion system in terms of the kinematic variables in Eq. (10) is to relate it to the orbital angular momentum

$$\epsilon_{T\mu} \propto \epsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho v_B^\sigma, \quad (12)$$

with the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ under the convention $\epsilon_{0123} = -1$. The transverse polarization vector is then normalized into

$$\epsilon_{T\mu} = \frac{\epsilon_{\mu\nu\rho\sigma} p_1^\nu P^\rho n_-^\sigma}{\sqrt{\zeta(1-\zeta)\omega P \cdot n_-}}. \quad (13)$$

To arrive at the above expression, we have added p_1^ρ to p_2^ρ in Eq. (12) to get the total momentum P^ρ of the pion pair without changing the result, and replaced v_B by n_- , because P is dominated by the plus component.

Employing the pion DAs in Eq. (10), adopting the definition in Eq. (13), and following the prescription in [34,35], we obtain the nonlocal matrix elements in Eq. (9) for various spin projectors Γ up to twist 3:

$$\begin{aligned} \langle \pi\pi | \bar{q}'(y^-) \gamma_\mu q(0) | 0 \rangle &= (2\zeta - 1) P_\mu \int_0^1 dz e^{izP \cdot y} \phi^0(z, \omega) \\ &\quad - 2\sqrt{\zeta(1-\zeta)} \omega \frac{\epsilon_{\mu\nu\rho\sigma} \epsilon_T^\nu P^\rho n_-^\sigma}{P \cdot n_-} \int_0^1 dz e^{izP \cdot y} \phi^v(z, \omega), \end{aligned} \quad (14)$$

$$\langle \pi\pi | \bar{q}'(y^-) I q(0) | 0 \rangle = \omega \int_0^1 dz e^{izP \cdot y} \phi^s(z, \omega), \quad (15)$$

$$\langle \pi\pi | \bar{q}'(y^-) \sigma_{\mu\nu} \gamma_5 q(0) | 0 \rangle = -\sqrt{\zeta(1-\zeta)} \epsilon_{T\nu} P_\mu \int_0^1 dz e^{izP \cdot y} \phi^T(z, \omega), \quad (16)$$

$$\langle \pi\pi | \bar{q}'(y^-) \gamma_\mu \gamma_5 q(0) | 0 \rangle = i\sqrt{\zeta(1-\zeta)} \omega \epsilon_{T\mu} \int_0^1 dz e^{izP \cdot y} \phi^a(z, \omega), \quad (17)$$

$$\langle \pi\pi | \bar{q}'(y^-) \sigma_{\mu\nu} q(0) | 0 \rangle = -i \frac{P_{1\mu} P_{2\nu} - P_{1\nu} P_{2\mu}}{\omega} \int_0^1 dz e^{izP \cdot y} \phi^l(z, \omega), \quad (18)$$

$$\langle \pi\pi | \bar{q}'(y^-) \gamma_5 q(0) | 0 \rangle = 0, \quad (19)$$

with the two-pion DAs $\phi^{0,T}$ and $\phi^{s,t,v,a}$ being of twist 2 and twist 3, respectively.

Some detailed derivation of Eqs. (14)–(19) are outlined here. For Eq. (14), we have applied the parametrizations for the longitudinal and transverse components of $p_1 - p_2$,

$$\begin{aligned} (p_1 - p_2)_\mu &\approx (2\zeta - 1) P_\mu, \\ (p_1 - p_2)^\sigma &= -2\sqrt{\zeta(1-\zeta)} \omega \frac{\epsilon^{\sigma\nu\rho\sigma} \epsilon_{T\nu} P_\rho n_{-\sigma}}{P \cdot n_-}, \end{aligned} \quad (20)$$

where the ζ -dependent factors will be absorbed into the corresponding two-pion DAs below. The matrix

element in Eq. (16) for the choice $\mu, \nu = +, y$ is proportional to

$$\epsilon^{+y\rho\sigma} p_{1\rho} p_{2\sigma} = \epsilon^{\gamma y\rho\sigma} p_{1\rho} P_\sigma n_{-\gamma} = -\sqrt{\zeta(1-\zeta)} \omega P^+ \epsilon_T^y, \quad (21)$$

in which Eq. (13) has been inserted. It is pointed out that the structure $(p_{1\mu} p_{2\nu} - p_{1\nu} p_{2\mu})$ in Eq. (18) corresponds to $\not{p}_L \not{P}$ for the twist-3 DAs in the longitudinally polarized pseudoscalar-vector meson pair [34,35].

We summarize the hadronic matrix elements $\Phi_{\pi\pi}^L$ ($\Phi_{\pi\pi}^T$) for the pion pair associated with the longitudinal (transverse) polarization from Eqs. (14)–(18) as

$$\begin{aligned} \Phi_{\pi\pi}^L &= \frac{1}{\sqrt{2N_c}} \left[\not{P} \phi^0(z, \zeta, \omega) + \omega \phi^s(z, \zeta, \omega) + \frac{\not{p}_1 \not{p}_2 - \not{p}_2 \not{p}_1}{\omega(2\zeta - 1)} \phi^l(z, \zeta, \omega) \right], \\ \Phi_{\pi\pi}^T &= \frac{1}{\sqrt{2N_c}} \left[\gamma_5 \not{p}_T \not{P} \phi^T(z, \zeta, \omega) + \omega \gamma_5 \not{p}_T \phi^a(z, \zeta, \omega) + i\omega \frac{\epsilon^{\mu\nu\rho\sigma} \gamma_\mu \epsilon_{T\nu} P_\rho n_{-\sigma}}{P \cdot n_-} \phi^v(z, \zeta, \omega) \right], \end{aligned} \quad (22)$$

where the projectors $\gamma_5 \not{P}$, $\gamma_5 \not{T}$, and $\epsilon^{\mu\nu\rho\sigma} \gamma_\mu \epsilon_{T\nu} P_\rho n_{-\sigma}$ come from Eq. (16), Eq. (17), and the second line of Eq. (14), respectively. Our result for the longitudinal piece $\Phi_{\pi\pi}^L$ has the same form as in [31], while the transverse one $\Phi_{\pi\pi}^T$ is new. The two-pion DAs for various twists are expanded in terms of the Gegenbauer polynomials, such as $C_2^{3/2}(1-2z)$:

$$\begin{aligned}\phi^0(z, \zeta, \omega) &= \frac{3F^\parallel(\omega^2)}{\sqrt{2N_c}} z(1-z)[1 + a_2^0 C_2^{3/2}(1-2z)](2\zeta - 1), \\ \phi^s(z, \zeta, \omega) &= \frac{3F^\perp(\omega^2)}{2\sqrt{2N_c}} (1-2z)[1 + a_2^s(1-10z+10z^2)](2\zeta - 1), \\ \phi^t(z, \zeta, \omega) &= \frac{3F^\perp(\omega^2)}{2\sqrt{2N_c}} (1-2z)^2[1 + a_2^t C_2^{3/2}(1-2z)](2\zeta - 1), \\ \phi^T(z, \zeta, \omega) &= \frac{3F^\perp(\omega^2)}{\sqrt{2N_c}} z(1-z)[1 + a_2^T C_2^{3/2}(1-2z)]\sqrt{\zeta(1-\zeta)}, \\ \phi^a(z, \zeta, \omega) &= \frac{3F^\parallel(\omega^2)}{4\sqrt{2N_c}} (1-2z)[1 + a_2^a(10z^2 - 10z + 1)]\sqrt{\zeta(1-\zeta)}, \\ \phi^v(z, \zeta, \omega) &= \frac{F^\parallel(\omega^2)}{2\sqrt{2N_c}} \left\{ \frac{3}{4} [1 + (1-2z)^2] + a_2^v [3(2z-1)^2 - 1] \right\} \sqrt{\zeta(1-\zeta)},\end{aligned}\quad (23)$$

in which we have introduced one Gegenbauer moment a_2 for each DA. The decomposition of the above DAs is similar to that of the ρ meson DAs, but with the vector (tensor) decay constant f_ρ (f_ρ^T) being replaced by the timelike pion form factors F^\parallel (F^\perp).

For the form factor $F^\parallel(\omega^2)$, we adopt the parametrization in Ref. [14],

$$F^\parallel(\omega^2) = \left[BW_\rho^{GS}(\omega^2, m_\rho, \Gamma_\rho) \frac{1 + c_\omega BW_\omega^{KS}(\omega^2, m_\omega, \Gamma_\omega)}{1 + c_\omega} + \sum_i c_i BW_i^{GS}(\omega^2, m_i, \Gamma_i) \right] \left(1 + \sum_i c_i \right)^{-1}, \quad (24)$$

with $i = \rho'(1450)$ and $\rho''(1700)$. The values of the masses m_i , the widths Γ_i , the complex coefficients c_i , and the BW functions of various resonances are referred to [14]. For the form factor $F^\perp(\omega^2)$, we employ the approximate relation $F^\perp(\omega^2)/F^\parallel(\omega^2) \approx f_\rho^T/f_\rho$ for the $\rho(770)$ resonance [31]. Because the tensor decay constants f^T for $\rho(1450)$ and

$\rho(1700)$ are not known yet, we treat the corresponding modules $|c_i|$ in F^\perp as free parameters, but keep their phases the same as in [14]. The global fit to the existing data for the $B \rightarrow J/\psi\pi\pi$ branching ratios and polarization fractions [4] determines the central values of the dimensionless parameters appearing in the two-pion DAs,

$$a_2^0 = 0.2, \quad a_2^s = 0.7, \quad a_2^t = -0.4, \quad a_2^T = 0.5, \quad a_2^a = 0.4, \quad a_2^v = -0.5, \quad |c_{\rho'}| = 0.316, \quad |c_{\rho''}| = 0.272. \quad (25)$$

The differential branching fraction for the $B \rightarrow \psi\pi\pi$ decays into P -wave pion pairs is expressed as

$$\frac{dB}{d\omega} = \frac{\tau\omega |\vec{p}_1| |\vec{p}_3|}{32\pi^3 M^3} \sum_{i=0, \parallel, \perp} |\mathcal{A}_i|^2, \quad (26)$$

where the pion and charmonium three-momenta in the $\pi\pi$ center-of-mass frame are given by

$$|\vec{p}_1| = \frac{\sqrt{\lambda(\omega^2, m_\pi^2, m_\pi^2)}}{2\omega}, \quad |\vec{p}_3| = \frac{\sqrt{\lambda(M^2, m^2, \omega^2)}}{2\omega}, \quad (27)$$

respectively, with the pion mass m_π and the Källén function $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$. The terms

\mathcal{A}_0 , \mathcal{A}_\parallel , and \mathcal{A}_\perp represent the longitudinal, parallel, and perpendicular polarization amplitudes in the transversity basis, respectively. The polarization fractions f_λ with $\lambda = 0, \parallel, \perp$ are then defined by

$$f_\lambda = \frac{|\mathcal{A}_\lambda|^2}{|\mathcal{A}_0|^2 + |\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}. \quad (28)$$

III. NUMERICAL RESULTS

To proceed with the numerical analysis, we first collect all the input quantities below. The meson masses and the heavy quark masses take the central values (in units of GeV) [47]

TABLE I. PQCD results for the branching ratios and the polarization fractions of the P -wave resonance channels in the $B^0 \rightarrow J/\psi\pi^+\pi^-$ decay. The theoretical errors are attributed to the variation of the Gegenbauer moments a_2^0 and a_2^T , and the hard scales t , respectively. The data are taken from [3,4,39], where the first uncertainty is statistical and the second is systematic. The uncertainties from [39] are statistical only.

R	$\mathcal{B}(B^0 \rightarrow J/\psi R(\rightarrow \pi^+\pi^-))$	$f_0(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$\rho(770)$	$(2.58_{-0.25-0.38-0.04}^{+0.27+0.53+0.06}) \times 10^{-5}$	$57.9_{-4.5-9.7-1.5}^{+4.0+10.1+0.6}$	$22.9_{-2.2-6.0-0.4}^{+2.4+5.3+0.5}$	$19.2_{-1.8-4.1-0.2}^{+2.1+4.4+1.0}$
LHCb [3]	$(2.49_{-0.13-0.23}^{+0.20+0.16}) \times 10^{-5}$
LHCb [4]	$(2.50 \pm 0.10_{-0.15}^{+0.18}) \times 10^{-5}$	$57.4 \pm 0.2_{-3.1}^{+1.3}$	$23.4 \pm 1.7_{-1.3}^{+1.0}$	$19.2 \pm 1.7_{-1.2}^{+3.8}$
LHCb [39]	$(2.60 \pm 0.10) \times 10^{-5}$ ^a	56.7 ± 1.8	23.5 ± 1.5	19.8 ± 1.7
$\rho(1450)$	$(3.0_{-0.1-0.6-0.0}^{+0.2+1.1+0.1}) \times 10^{-6}$	$46_{-1-11-4}^{+3+12+1}$	$29_{-2-10-1}^{+1+9+2}$	25_{-2-2-0}^{+1+3+1}
LHCb [3]	$(2.1_{-0.6-0.4}^{+1.0+2.2}) \times 10^{-6}$
LHCb [4]	$(4.6 \pm 1.1 \pm 1.9) \times 10^{-6}$	$58 \pm 10_{-23}^{+14}$	$27 \pm 13_{-11}^{+7}$	$15 \pm 7_{-10}^{+28}$
LHCb [39]	$(3.6 \pm 0.7) \times 10^{-6}$ ^a	47 ± 11	39 ± 12	14 ± 8
$\rho(1700)$	$(1.8_{-0.0-0.5-0.0}^{+0.1+0.9+0.1}) \times 10^{-6}$	$31_{-0-9-0}^{+2+12+2}$	$38_{-1-14-1}^{+0+9+0}$	31_{-0-0-0}^{+1+2+1}
LHCb [4]	$(2.0 \pm 0.5 \pm 1.2) \times 10^{-6}$	$40 \pm 11_{-23}^{+13}$	$24 \pm 14_{-10}^{+7}$	$36 \pm 14_{-9}^{+28}$
LHCb [39]	$(1.2 \pm 0.3) \times 10^{-6}$ ^a	29 ± 12	42 ± 15	29 ± 15

^aThe fit fractions determined from the Dalitz plot analysis have been converted into the branching fraction measurements.

$$\begin{aligned}
 M &= 5.28, & m_b &= 4.8, & m_c &= 1.275, & m_\rho &= 0.775, \\
 m_{\pi^\pm} &= 0.140, & m_{\pi^0} &= 0.135, & m_{J/\psi} &= 3.097, & m_{\psi(2S)} &= 3.686.
 \end{aligned}
 \tag{29}$$

The Cabibbo-Kobayashi-Maskawa (CKM) parameters in the Wolfenstein parametrization are set to $\lambda = 0.22537$, $A = 0.814$, $\bar{\rho} = 0.117$, and $\bar{\eta} = 0.355$ [32]. The decay constants (in units of GeV) and the B meson lifetimes (in units of ps) are chosen as [32,45,46]

$$f_B = 0.19, \quad f_{J/\psi} = 0.405, \quad f_{\psi(2S)} = 0.296, \quad f_\rho = 0.216, \quad f_\rho^T = 0.184, \quad \tau_{B^0} = 1.519, \quad \tau_{B^\pm} = 1.638.
 \tag{30}$$

The resultant branching ratios \mathcal{B} and the polarization fractions f_λ together with the available experimental measurements from the LHCb Collaboration for the J/ψ involved modes are summarized in Table I, and the corresponding ones for $\psi(2S)$ are listed in Table II. Since the charged and neutral B meson decays differ only in the lifetimes and the isospin factor in our formalism, one can derive the branching ratios for the B^+ meson by multiplying those for the B^0 meson by the ratio $2\tau_{B^+}/\tau_{B^0}$.

The theoretical errors in Tables I and II are from some typical sources, namely, the two Gegenbauer moments in the twist-2 two-pion DAs, $a_2^0 = 0.2 \pm 0.2$ and $a_2^T = 0.5 \pm 0.5$, and the variation of the hard scales

t from $0.75t$ to $1.25t$, which characterize the energy release in decay processes (see the factorization formulas in the Appendix). It is worthwhile to mention that the hard kernels are evaluated only up to leading order plus the vertex corrections in this work, so the theoretical accuracy still needs to be improved. This is the case especially for B meson decays into charmonia, whose energy release may not be high enough for justifying the leading-order calculation. It is then expected that the hadronic parameters extracted from the data in the present framework should suffer larger theoretical uncertainty. Therefore, we have considered a wide range for the variation of the Gegenbauer moment $a_2^0 = 0.2 \pm 0.2$, which covers the central value

TABLE II. PQCD results for the branching ratios and the polarization fractions of the P -wave resonance channels in the $B^0 \rightarrow \psi(2S)\pi^+\pi^-$ decay.

R	$\mathcal{B}(B^0 \rightarrow \psi(2S)R(\rightarrow \pi^+\pi^-))$	$f_0(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$\rho(770)$	$(1.0_{-0.1-0.2-0.0}^{+0.1+0.2+0.0}) \times 10^{-5}$	50_{-2-8-0}^{+3+9+1}	26_{-2-7-1}^{+1+5+0}	24_{-1-3-1}^{+1+3+0}
$\rho(1450)$	$(8.2_{-0.0-1.5-0.2}^{+0.1+2.3+0.4}) \times 10^{-7}$	$46_{-0-10-3}^{+1+11+3}$	$28_{-1-10-2}^{+0+9+2}$	26_{-0-1-1}^{+0+1+0}

$a_2^0 = 0.3$ extracted from the data for charmless B meson decays in Ref. [32]. Eventually, we will improve the accuracy of our analysis and perform a global fit to all relevant data, when determining the involved hadronic parameters.

One can see that the errors from the two Gegenbauer moments are comparable and contribute to the major uncertainties as shown in Tables I and II, while the last one from the hard scales is only of a few percents due to the inclusion of the vertex corrections. We have also examined the sensitivity of our results to the choice of other Gegenbauer moments in the twist-3 two-pion DAs, a_2^s , a_2^t , a_2^a , and a_2^v , in Eq. (25). The first two give a comparable effect on the longitudinal branching ratio as a_2^0 does. With the increase (decrease) of a_2^s (a_2^t), the total branching ratios and the longitudinal polarization fractions become larger. On the contrary, the last two have a little impact on the total branching ratios, but can modify the relative importance of the parallel and perpendicular polarization amplitudes. As we set $a_2^a = a_2^v = 0$, the polarization fractions f_{\parallel} and f_{\perp} are roughly equal. When a_2^a and a_2^v are changed in the opposite direction, as indicated in Eq. (25), the difference between f_{\parallel} and f_{\perp} is enhanced and matches the data. It can be understood from the factorization formulas presented in the Appendix: the contribution from ϕ^a to the parallel polarization amplitudes plays a role similar to that from ϕ^v to the perpendicular polarization amplitudes, so the inputs of a_2^a and a_2^v opposite in sign increase the difference between the two amplitudes. It is also found that the coefficients $|c_i|$ in F_{\perp} cause a significant effect on the branching ratios for the $\rho(1450)$ and $\rho(1700)$ channels. The variation of $|c_i|$ by 20% results in the change of the branching ratios by 40%–50%. The uncertainties from other parameters in our formalism, such as the decay constants and the CKM matrix elements, are not discussed here. The polarization fractions are not sensitive to these parameters, because they mainly yield an overall effect, which cancels in the ratios defined by Eq. (28).

It is obvious that both our branching ratio and three polarization fractions for the $\rho(770)$ channel agree well with the high-precision LHCb data [3,4,39] in Table I. Although the central values of the measured branching ratios for the $\rho(1450)$ resonance vary in a wide range $(2.1\text{--}4.6) \times 10^{-6}$, their PDG weighted average leads to $2.9^{+1.6}_{-0.7} \times 10^{-6}$ [47], in good consistency with our prediction. For the $\rho(1700)$ channel, the LHCb Collaboration got $\mathcal{B}(B^0 \rightarrow J/\psi\rho''(\rightarrow\pi^+\pi^-)) = (2.0 \pm 0.5 \pm 1.2) \times 10^{-6}$ [4], while the subsequent measurement gave $(1.2 \pm 0.3) \times 10^{-6}$ [39] with the statistical uncertainty only. Our prediction $(1.8^{+0.9}_{-0.5}) \times 10^{-6}$ is in between, and matches both data within errors. For the $\psi(2S)$ involved modes, although the LHCb Collaboration [8] also observed a dominant contribution to the $B^0 \rightarrow \psi(2S)\pi^+\pi^-$ decay from the

$\rho(770)$ resonance, the detailed partial wave analysis for determining its fraction is still missing due to a limited number of events.

Summing over all the contributing P -wave resonances in the $\pi\pi$ invariant mass spectra $[2m_{\pi}, M - m]$, we have the total branching ratios

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow J/\psi(\pi^+\pi^-)_P) &= (3.1^{+0.4+0.8+0.2}_{-0.2-0.5-0.0}) \times 10^{-5}, \\ \mathcal{B}(B^0 \rightarrow \psi(2S)(\pi^+\pi^-)_P) &= (1.2^{+0.1+0.3+0.0}_{-0.1-0.2-0.0}) \times 10^{-5}, \end{aligned} \quad (31)$$

where the sources of the errors have been interpreted before. The former amounts up to 78% of the total three-body branching ratio $\mathcal{B}(B^0 \rightarrow J/\psi\pi^+\pi^-) = (3.96 \pm 0.17) \times 10^{-5}$ [47]. As noticed in [3], the S -wave $f_0(500)$ and D -wave $f_2(1270)$ resonances, besides the P -wave ones, were also produced significantly in the $J/\psi\pi^+\pi^-$ final states. The best fit model in [3] implies that one full $\rho(770)$ meson width contains 11.9% S -wave component and 0.72% D -wave component. Therefore, it is reasonable to leave the remaining 22% to the S -wave and D -wave contributions, as well as the nonresonant one and their interference in the entire invariant mass range. We estimate from Eq. (31) the ratio of the branching fractions,

$$\frac{\mathcal{B}(B^0 \rightarrow \psi(2S)(\pi^+\pi^-)_P)}{\mathcal{B}(B^0 \rightarrow J/\psi(\pi^+\pi^-)_P)} = 0.39^{+0.01}_{-0.03}, \quad (32)$$

in which all the uncertainties have been added in quadrature. The value is slightly lower than the LHCb measurement [8]

$$\begin{aligned} \frac{\mathcal{B}(B^0 \rightarrow \psi(2S)\pi^+\pi^-)}{\mathcal{B}(B^0 \rightarrow J/\psi\pi^+\pi^-)} &= 0.56 \pm 0.07(\text{stat}) \\ &\pm 0.05(\text{syst}) \pm 0.01(\mathcal{B}), \end{aligned} \quad (33)$$

where the third uncertainty corresponds to the one from the dilepton branching fractions of the J/ψ and $\psi(2S)$ charmonium decays. The minor discrepancy may be resolved by including other partial wave contributions.

The resonant decay rate obeys a simple factorization relation under the narrow width approximation,

$$\mathcal{B}(B^0 \rightarrow \psi R(\rightarrow\pi^+\pi^-)) = \mathcal{B}(B^0 \rightarrow \psi R)\mathcal{B}(R \rightarrow \pi^+\pi^-), \quad (34)$$

from which we extract the two-body $B \rightarrow \psi R$ branching ratios, given the input of $\mathcal{B}(R \rightarrow \pi^+\pi^-)$. Combining the experimental fact $\mathcal{B}(\rho \rightarrow \pi\pi) \sim 100\%$ [47] and the estimates of $\mathcal{B}(\rho' \rightarrow \pi\pi) = 10.04^{+5.23}_{-2.61}\%$ and $\mathcal{B}(\rho'' \rightarrow \pi\pi) = 8.11^{+2.22}_{-1.47}\%$ in Ref. [33], we obtain the central values

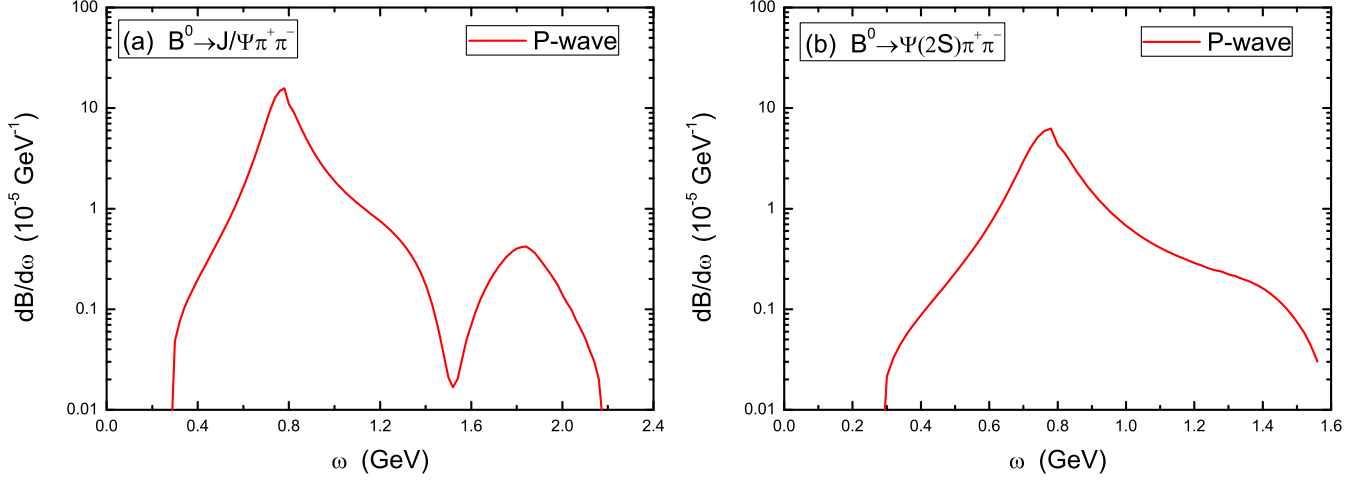


FIG. 2. P -wave contribution to the differential branching fractions of the modes (a) $B^0 \rightarrow J/\psi\pi^+\pi^-$ and (b) $B^0 \rightarrow \psi(2S)\pi^+\pi^-$.

$$\begin{aligned}
 \mathcal{B}(B^0 \rightarrow J/\psi\rho) &= 2.58 \times 10^{-5}, \\
 \mathcal{B}(B^0 \rightarrow J/\psi\rho') &= 3.0 \times 10^{-5}, \\
 \mathcal{B}(B^0 \rightarrow J/\psi\rho'') &= 2.2 \times 10^{-5}, \\
 \mathcal{B}(B^0 \rightarrow \psi(2S)\rho) &= 1.0 \times 10^{-5}, \\
 \mathcal{B}(B^0 \rightarrow \psi(2S)\rho') &= 8.2 \times 10^{-6}.
 \end{aligned} \tag{35}$$

It is seen that both $\mathcal{B}(B^0 \rightarrow J/\psi\rho)$ and $\mathcal{B}(B^0 \rightarrow \psi(2S)\rho)$ are consistent with those derived in the PQCD framework for two-body decays [48].

We plot in Fig. 2 the total differential branching fractions in the P -wave $\pi^+\pi^-$ invariant mass for the considered decays. The curve for the $B^0 \rightarrow J/\psi\pi^+\pi^-$ mode is similar to those for the charmless $B \rightarrow P\pi\pi$ decays [33], since the same time-like form factors for the two-pion DAs, fitted by the *BABAR* Collaboration via the e^+e^- annihilation process

[14], have been adopted. One finds a dip appearing at the invariant mass around 1.5–1.6 GeV in Fig. 2(a), that is usually interpreted as the destructive interference between the $\rho(1450)$ and $\rho(1700)$ channels [14,49]. In fact, the best fit model also shows that the destructive interference between $\rho(1450)$ and $\rho(1700)$ is comparable with their individual fit fractions (see Tables VII and IX in Ref. [4]). However, the dip is not observed in Fig. 2(b), because the $\rho(1700)$ state is beyond the dipion invariant mass spectra for the $B^0 \rightarrow \psi(2S)\pi^+\pi^-$ mode. Both cases exhibit a clear ρ - ω interference pattern in the ρ peak region. The individual resonance contributions are displayed in Fig. 3, where the red solid, green dashed, and blue dotted curves represent those from $\rho(770)$, $\rho(1450)$, and $\rho(1700)$, respectively. The different shapes among these individual channels are mainly governed by the corresponding BW functions and parameters c_i in Eq. (24). As expected,

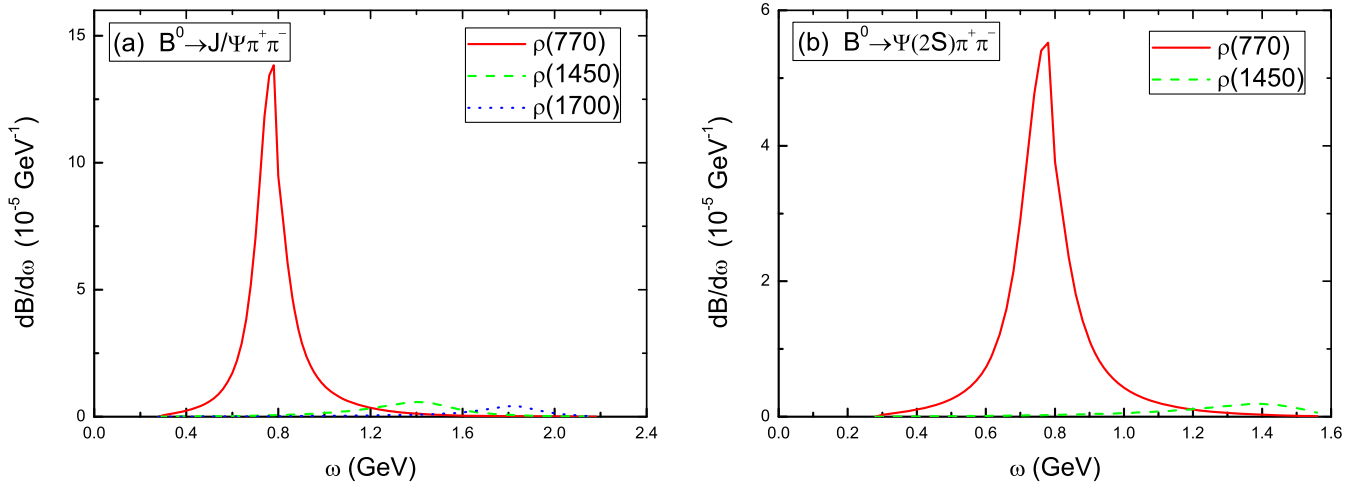


FIG. 3. $\rho(770)$, $\rho(1450)$, and $\rho(1700)$ resonance contributions to the differential branching fractions of (a) $B^0 \rightarrow J/\psi\pi^+\pi^-$ and (b) $B^0 \rightarrow \psi(2S)\pi^+\pi^-$, which are displayed by the solid red, dashed green, and dotted blue curves, respectively.

the $\rho(770)$ production is apparently dominant. Comparing Tables I and II with Eq. (31), the $\rho(770)$ resonance accounts for 83% of the total P -wave branching fractions in both the $B^0 \rightarrow J/\psi\pi^+\pi^-$ and $\psi(2S)\pi^+\pi^-$ decays, while the higher $\rho(1450)$ and $\rho(1700)$ resonances contribute less than 10%. The obtained distributions in the P -wave $\pi\pi$ mass as well as the individual resonance contributions agree fairly well with the LHCb data shown in Fig. 13 of Ref. [4] and in Fig. 4 of Ref. [8].

IV. CONCLUSION

In this paper we have performed the analysis of the $B \rightarrow \psi\pi\pi$ decays under the quasi-two-body approximation in the PQCD framework by introducing the two-pion DAs. Since both the charmonium and the P -wave pion pair in the final state carry the spin degrees of freedom, the two-pion DAs corresponding to both the longitudinal and transverse polarizations are the necessary nonperturbative inputs, and were constructed through a perturbative evaluation of the associated hadronic matrix elements as a timelike process. It was observed that the total momentum and the orbital angular momentum of the P -wave dipion system mimics its longitudinal and transverse polarizations, respectively. The two-pion DAs for various spin projectors were then decomposed in terms of the Gegenbauer polynomials that depend on parton momentum fractions, and the Legendre polynomials that depend on meson momentum fractions up to twist 3. The timelike form factors, normalizing the two-pion DAs, were parametrized to consist of a linear combination of the ρ , ρ' , and ρ'' resonant contributions together with the ρ - ω interference.

We have determined the hadronic parameters involved in the two-pion DAs from a global fit to the data of the $B^0 \rightarrow J/\psi\rho(\rightarrow\pi^+\pi^-)$ branching ratios and polarization fractions with good consistency. In particular, the resultant differential branching fractions in the P -wave di-pion invariant mass and individual resonance contributions match the LHCb data. We have also predicted the branching ratios and the polarization fractions of the $B^0 \rightarrow \psi(2S)\rho(\rightarrow\pi^+\pi^-)$ decays, which can be confronted with future measurements. As a by-product, we extracted the two-body $B^0 \rightarrow \psi\rho$ branching ratios from the results for the corresponding quasi-two-body modes by employing the narrow width approximation. The predictions for the $\rho(770)$ channels are in accordance with our previous PQCD calculations performed for two-body decays. The consistency between the three-body and two-body analyses

supports the PQCD approach to exclusive charmonium B meson decays. The predictions for the higher excited intermediate states still need to be tested at the ongoing and forthcoming experiments.

ACKNOWLEDGMENTS

We acknowledge Wen-Fei Wang and Chao Wang for helpful discussions. This work was supported in part by the National Natural Science Foundation of China under Grants No. 11605060 and No. 11547020, by the Program for the Top Young Innovative Talents of Higher Learning Institutions of Hebei Educational Committee under Grant No. BJ2016041, and the Ministry of Science and Technology of R. O. C. under Grant No. MOST-107-2119-M-001-035-MY3.

APPENDIX: DECAY AMPLITUDES

Before presenting the explicit factorization formula for each $B^0 \rightarrow \psi\pi\pi$ decay amplitude in this Appendix, we make a remark on the factorization theorem for hadronic B meson decays into charmonia. It has been argued [50] that the QCD factorization (QCDF) approach is applicable to exclusive B meson decays into J/ψ , since the transverse size of J/ψ becomes small in the heavy quark limit. On the other hand, the k_T factorization theorem also holds for B meson decays containing charmonia in the heavy quark limit under the power counting m_c/m_b , $\Lambda_{\text{QCD}}/m_c \ll 1$, with the QCD scale Λ_{QCD} , as elaborated in [51]. Because we focus on the resonant region of the di-pion system, what we studied here are basically quasi-two-body decays, and the reasoning in [51] for their factorization still applies. That is, the PQCD approach is expected to be suitable for describing the $B^0 \rightarrow \psi\pi\pi$ decays.

The contributions from the longitudinal polarization, the normal polarization, and the transverse polarization are labelled by the subscripts L , N and T , respectively. The contributions from the $(V-A) \otimes (V-A)$, $(V-A) \otimes (V+A)$, and $(S-P) \otimes (S+P)$ operators are labelled by the superscripts LL , LR , and SP , respectively. The total decay amplitude is decomposed into

$$A = \mathcal{A}_L + \mathcal{A}_N \epsilon_T \cdot \epsilon_{3T} + i \mathcal{A}_T \epsilon_{\alpha\beta\rho\sigma} n_+^\alpha n_-^\beta \epsilon_T^\rho \epsilon_{3T}^\sigma, \quad (\text{A1})$$

where the three individual polarization amplitudes are written as

$$\begin{aligned} \mathcal{A}_{L,N,T} = & \frac{G_F}{\sqrt{2}} \left\{ V_{cb}^* V_{cs} \left[\left(C_1 + \frac{1}{3} C_2 \right) \mathcal{F}_{L,N,T}^{LL} + C_2 \mathcal{M}_{L,N,T}^{LL} \right] - V_{tb}^* V_{ts} \left[\left(C_3 + \frac{1}{3} C_4 + C_9 + \frac{1}{3} C_{10} \right) \mathcal{F}_{L,N,T}^{LL} \right. \right. \\ & \left. \left. + \left(C_5 + \frac{1}{3} C_6 + C_7 + \frac{1}{3} C_8 \right) \mathcal{F}_{L,N,T}^{LR} + (C_4 + C_{10}) \mathcal{M}_{L,N,T}^{LL} + (C_6 + C_8) \mathcal{M}_{L,N,T}^{SP} \right] \right\}, \quad (\text{A2}) \end{aligned}$$

with the CKM matrix elements V_{ij} and the Fermi coupling constant G_F . The Wilson coefficients C_i encode the hard dynamics of weak decays. The above amplitudes are related to those in Eq. (26) via

$$\mathcal{A}_0 = \mathcal{A}_L, \quad \mathcal{A}_{\parallel} = \sqrt{2}\mathcal{A}_N, \quad \mathcal{A}_{\perp} = \sqrt{2}\mathcal{A}_T. \quad (\text{A3})$$

The explicit amplitudes $\mathcal{F}(M)$ from the factorizable (nonfactorizable) diagrams in Fig. 1 read as

$$\begin{aligned} \mathcal{F}_L^{LL} = & \frac{8\pi C_F f_{\psi} M^4}{\sqrt{1-\eta}} \int_0^1 dx_B dz \int_0^{\infty} b_B db_B b db \phi_B(x_B, b_B) \{ [-\phi^0(r^2(-2\eta z + 2z + 1) + (\eta - 1)(z + 1)) \\ & - \sqrt{\eta(1-r^2)}(\phi^s(\eta + r^2(2(\eta - 1)z + 1) - 2\eta z + 2z - 1) + \phi^t(\eta + r^2(2(\eta - 1)z - 1) - 2\eta z + 2z - 1))] E_e(t_a) h_a(x_B, z, b_B, b) \\ & + [2\phi^s(\sqrt{\eta(1-r^2)}(-\eta + r^2 x_B - r^2 + 1)) - \phi^0(-\eta^2 + \eta + \eta^2 r^2 - 2\eta r^2 + r^2 x_B)] E_e(t_b) h_b(x_B, z, b_B, b) \}, \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \mathcal{M}_L^{LL} = & -\frac{32\pi C_F M^4}{\sqrt{6(1-\eta)\eta(1-r^2)}} \int_0^1 dx_B dz dx_3 \int_0^{\infty} b_B db_B b_3 db_3 \phi_B(x_B, b_B) [\phi^0(\eta + r^2 - 1)\sqrt{\eta(1-r^2)} - 2\eta(r^2 - 1)\phi^t] \\ & \times [r^2 \psi^L(2(\eta - 1)x_3 + x_B - \eta z + z) - 2(\eta - 1) r r_c \psi^t + (\eta - 1) z \psi^L] E_n(t_d) h_d(x_B, z, x_3, b_B, b_3), \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \mathcal{F}_N^{LL} = & 8\pi C_F f_{\psi} M^4 r \int_0^1 dx_B dz \int_0^{\infty} b_B db_B b db \phi_B(x_B, b_B) \{ [\sqrt{\eta(1-r^2)}(\phi^a(r^2 z - z - 2) - (r^2 - 1)z\phi^v) \\ & + \phi^T(r^2 - 1 + \eta(-2r^2 z + 2z - 1))] E_e(t_a) h_a(x_B, z, b_B, b) + \sqrt{\eta(1-r^2)}[\phi^a(-\eta + r^2 + x_B - 1) \\ & + \phi^v(\eta + r^2 - x_B - 1)] E_e(t_b) h_b(x_B, z, b_B, b) \}, \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} \mathcal{F}_T^{LL} = & 8\pi C_F f_{\psi} M^4 r \int_0^1 dx_B dz \int_0^{\infty} b_B db_B b db \phi_B(x_B, b_B) \{ [\sqrt{\eta(1-r^2)}(\phi^v(r^2 z - z - 2) - (r^2 - 1)z\phi^a) \\ & + \phi^T(r^2 - 1 - \eta(-2r^2 z + 2z - 1))] E_e(t_a) h_a(x_B, z, b_B, b) + \sqrt{\eta(1-r^2)}[\phi^v(-\eta + r^2 + x_B - 1) \\ & + \phi^a(\eta + r^2 - x_B - 1)] E_e(t_b) h_b(x_B, z, b_B, b) \}, \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} \mathcal{M}_N^{LL} = & -\frac{64\pi C_F M^4}{\sqrt{6}} \int_0^1 dx_B dz dx_3 \int_0^{\infty} b_B db_B b_3 db_3 \phi_B(x_B, b_B) \{ \phi^T[r\psi^V(-\eta x_3 + x_3 - x_B + \eta z) + (\eta - 1)r_c \psi^T] \\ & - \sqrt{\eta(1-r^2)}\phi^a[r\psi^V(-\eta x_3 + x_3 - x_B + z) + (\eta - 1)r_c \psi^T] \} E_n(t_d) h_d(x_B, z, x_3, b_B, b_3), \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} \mathcal{M}_T^{LL} = & -\frac{64\pi C_F M^4}{\sqrt{6}} \int_0^1 dx_B dz dx_3 \int_0^{\infty} b_B db_B b_3 db_3 \phi_B(x_B, b_B) \{ \phi^T[r\psi^V(-\eta x_3 + x_3 - x_B - \eta z) + (\eta - 1)r_c \psi^T] \\ & - \sqrt{\eta(1-r^2)}\phi^v[r\psi^V(-\eta x_3 + x_3 - x_B + z) + (\eta - 1)r_c \psi^T] \} E_n(t_d) h_d(x_B, z, x_3, b_B, b_3), \end{aligned} \quad (\text{A9})$$

$$\mathcal{F}_{L,N,T}^{LR} = \mathcal{F}_{L,N,T}^{LL}, \quad (\text{A10})$$

$$\mathcal{M}_{L,N,T}^{SP} = -\mathcal{M}_{L,N,T}^{LL}, \quad (\text{A11})$$

with $r_c = m_c/M$, m_c being the charm quark mass, the color factor $C_F = 4/3$, and the decay constant f_{ψ} of the charmonium. The expressions for the evolution functions E , the hard kernels h , and the hard scales $t_{a,b,c,d}$ can be found in the Appendix of Ref. [28]. We point out that the amplitudes \mathcal{F} correspond to the $B \rightarrow \pi\pi$ transition form factors, which have been computed in QCD light-cone sum rules [52,53].

In addition, the vertex corrections to the factorizable diagrams in Fig. 1 are included through the modification of the Wilson coefficients as done in the QCDF approach [54–56], according to the argument in [57]. Note that the first step of the factorization of these diagrams is the same in the QCDF and PQCD approaches, at which the Wilson coefficients are factorized out of the exclusive B meson decays. The difference of the two approaches stems from whether the

remaining hadronic matrix elements of effective operators, namely, the soft form factors, are factorizable. Due to the different power counting on parton transverse momenta, these soft form factors are not factorizable in QCDF, but are in PQCD. Once the factorization is established, one can calculate radiative corrections to each involved piece separately. Since the Wilson coefficients are the same in the two

approaches, the vertex corrections to this piece obtained in QCDF can be applied to PQCD. Moreover, the infrared divergences in the vertex corrections cancel, when they are summed over, as stated in Ref. [50]. Therefore, it is not necessary to introduce parton transverse momenta into the evaluation of these corrections [57], and the QCDF results can be adopted directly and consistently.

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