Open spin chains from determinant like operators in ABJM theory

Hui-Huang Chen,[1,*] Hao Ouyang,[2,†] and Jun-Bao Wu[3,4,‡]

[1]*College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, China*

[2]*Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden*

[3]*Center for Joint Quantum Studies and Department of Physics, School of Science, Tianjin University, 135 Yaguan Road, Tianjin 300350, China*

[4]*Center for High Energy Physics, Peking University, 5 Yiheyuan Road, Beijing 100871, China*

(Received 4 October 2018; published 13 November 2018)

We study the mixing problem of the determinantlike operators in ABJM theory to two-loop order in the scalar sector. The gravity duals of these operators are open strings attached to the maximal giant graviton, which is a D4-brane wrapping a $\mathbb{CP}^2$ inside $\mathbb{CP}^3$ in our case. The anomalous dimension matrix of these operators can be regarded as an open spin chain Hamiltonian. We provide strong evidence of its integrability based on coordinate Bethe ansatz method and boundary Yang-Baxter equations.

DOI: 10.1103/PhysRevD.98.106012

I. INTRODUCTION

In recent years, a lot of progress has been made in applying techniques of integrability to planar $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence between IIB superstring theory on $\mathrm{AdS}_5 \times S^5$ and four-dimensional $\mathcal{N} = 4$ super Yang-Mills (SYM) theory; see [1] for a collection of reviews. Among all these notable progresses, spin chains or strings with periodic boundary condition are mostly studied and understood very well. People are also interested in nonperiodic cases, including twisted boundary conditions; see, e.g., [2,3] and open boundary conditions[4–8]. See [9,10] for reviews of these interesting topics.

In 2008, another example of AdS/CFT was proposed in [11], where the authors gave very strong evidence that type IIA string theory on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ background is dual to $\mathcal{N} = 6$ superconformal Chern-Simons matter theory (also known as Aharony-Bergman-Jafferis-Maldacena (ABJM) theory) in three dimensional spacetime with gauge group $U(N) \times U(N)$ and Chern-Simons levels $(k, -k)$. The 't Hooft coupling of ABJM theory turns out to be $\lambda = N/k$. People usually call this dual as $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence or $\mathrm{ABJM}/\mathrm{AdS}_4 \times \mathbb{CP}^3$ correspondence. The integrable structure in this setup was also extensively studied [12].

Along a similar path, many studies on nonperiodic integrable cases reemerged in the context of ABJM theory [13–17]. However, there are still some potential integrable setups which have not been investigated in the $\mathrm{AdS}_4/\mathrm{CFT}_3$ case, such as integrable Wilson loops [18–20] and integrability from giant gravitons[8,21] found in the $\mathcal{N} = 4$ SYM theory. In the SYM context, determinantlike operators are dual to open strings attached to D-branes wrapping cycles in S^5. On the gravity side, such D-branes wrapping some cycles and carrying some angular momentum are usually called giant gravitons. In the context of $\mathcal{N} = 4$ SYM, the integrability of the open chain from giant gravitons has been studied extensively [8,21–25]. However, such an integrable structure from the giant gravitons in the $\mathrm{AdS}_4/\mathrm{CFT}_3$ [26–28] case has not been explored as far as we know, though the plane wave limit in both sides was studied in [29]. In this paper, we would like to take a first step to fill these gaps. We study the anomalous dimension matrix of the determinantlike scalar operators in ABJM theory up to two-loop order in the scalar sector. The anomalous dimension matrix can be viewed as the Hamiltonian of an open spin chain. Using the coordinate Bethe ansatz method, we calculate the reflection matrix for fundamental excitations of this open chain. Based on the known bulk two body S-matrix, it is not hard to verify that the boundary Yang-Baxter equations (reflection equations) are satisfied, hinting that this open spin chain is integrable.

The outline of this paper is as follows. In Sec. II, we introduce the determinantlike scalar operators in ABJM theory. To study the mixing problem, we calculate their two-pointfunctions to two-loop order, giving the

*chenhh@jxnu.edu.cn
†hao.ouyang@su.se
‡junbao.wu@tju.edu.cn

Hamiltonian of an open spin chain. In Sec. III, we compute the reflection matrix of this open spin chain through the coordinate Bethe ansatz method. Borrowing the two body S-matrix in the bulk previously computed in [30], we confirm that the boundary Yang-Baxter equations (reflection equations) are satisfied. In the last section, we conclude and briefly discuss some possible problems for further studies.

II. OPEN SPIN CHAIN IN ABJM THEORY

A. Determinantlike operators in ABJM theory

We begin with a very brief review of determinantlike operators in ABJM theory. In ABJM theory, the scalar fields $(A_1, A_2, B_1^\dagger, B_2^\dagger)$ transform in the fundamental representation of the $SU(4)$ R-symmetry group. We make the following identification,

$$(A_1, A_2, B_1^\dagger, B_2^\dagger) = (Y^1, Y^2, Y^3, Y^4). \tag{2.1}$$

Using the conventions of [31], the action of ABJM theory can be written as

$$\begin{aligned}
S &= \int d^3x (L_{CS} + L_k - V_F - V_B),\\
L_{CS} &= \frac{k}{4\pi}\varepsilon^{\mu\nu\rho}\mathrm{tr}\bigg(A_\mu\partial_\nu A_\rho + \frac{2\mathrm{i}}{3}A_\mu A_\nu A_\rho\\
&\quad - \hat{A}_\mu\partial_\nu\hat{A}_\rho - \frac{2\mathrm{i}}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho\bigg),\\
L_k &= \mathrm{tr}(-D_\mu Y_I^\dagger D^\mu Y^I + \mathrm{i}\Psi^{\dagger I}\gamma^\mu D_\mu \Psi_I),\\
V_F &= \frac{2\pi i}{k}\mathrm{tr}(Y_I^\dagger Y^I \Psi^{\dagger J}\Psi_J - 2Y_I^\dagger Y^J \Psi^{\dagger I}\Psi_J + \epsilon^{IJKL}Y_I^\dagger\Psi_J Y_K^\dagger \Psi_L\\
&\quad - Y^I Y_I^\dagger \Psi_J \Psi^{\dagger J} - 2Y^I Y_J^\dagger \Psi_I \Psi^{\dagger J} + \epsilon_{IJKL}Y^I\Psi^{\dagger J}Y^K\Psi^{\dagger L}),\\
V_B &= -\frac{4\pi^2}{3k^2}\mathrm{tr}(Y_I^\dagger Y^J Y_J^\dagger Y^K Y_K^\dagger Y^I + Y_I^\dagger Y^I Y_J^\dagger Y^J Y_K^\dagger Y^K\\
&\quad + 4Y_I^\dagger Y^J Y_K^\dagger Y^I Y_J^\dagger Y^K - 6Y_I^\dagger Y^I Y_J^\dagger Y^K Y_K^\dagger Y^J).
\end{aligned}\tag{2.2}$$

Covariant derivatives are defined as

$$\begin{aligned}
D_\mu Y^I &= \partial_\mu Y^I + iA_\mu Y^I - iY^I\hat{A}_\mu,\\
D_\mu Y_I^\dagger &= \partial_\mu Y_I^\dagger + i\hat{A}_\mu Y_I^\dagger - iY_I^\dagger A_\mu\\
D_\mu \Psi_I &= \partial_\mu \Psi_I + iA_\mu\Psi_I - i\Psi_I\hat{A}_\mu.
\end{aligned}\tag{2.3}$$

In this paper, we focus on the determinantlike operators

$$O_W = \epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N}(A_1B_1)^{a_1}_{b_1}\ldots(A_1B_1)^{a_{N-1}}_{b_{N-1}}W^{a_N}_{b_N}, \tag{2.4}$$

with

$$W = Y^{I_1}Y_{J_1}^\dagger \cdots Y^{I_L}Y_{J_L}^\dagger. \tag{2.5}$$

It was suggested in [29] that the dual descriptions of these operators are open strings attached to the giant graviton D4-brane wrapping a $\mathbb{CP}^2$ inside $\mathbb{CP}^3$. The operator with $W = A_1B_1$ is dual to the D4-brane itself.

As discussed in [8], an open spin chain corresponding to determinantlike operators in $\mathcal{N} = 4$ SYM has nontrivial boundary conditions. One may expect that there are similar boundary conditions in the case of open spin chain in ABJM theory. To show this, we compute the tree-level two-point function. The operator O_W and its conjugate $\bar{O}_W$ can be rewritten as

$$\begin{aligned}
O_W &= \frac{1}{(N-1)!}\epsilon^{[I]_{N-1}c}_{[J]_{N-1}a}\epsilon^{[K]_{N-1}b}_{[L]_{N-1}c}(A_1)^{[J]_{N-1}}_{[I]_{N-1}}(B_1)^{[L]_{N-1}}_{[K]_{N-1}}W^a_b,\\
\bar{O}_W &= \frac{1}{(N-1)!}\epsilon^{[M]_{N-1}f}_{[S]_{N-1}d}\epsilon^{[Q]_{N-1}e}_{[P]_{N-1}f}(A_1^\dagger)^{[P]_{N-1}}_{[Q]_{N-1}}(B_1^\dagger)^{[S]_{N-1}}_{[M]_{N-1}}\bar{W}^d_e.
\end{aligned}\tag{2.6}$$

Here, we use the shorthand notations

$$[I]_{N-1} = I_1\ldots I_{N-1}, \qquad (A_1)^{[J]_{N-1}}_{[I]_{N-1}} = (A_1)^{J_1}_{I_1}\ldots(A_1)^{J_{N-1}}_{I_{N-1}}. \tag{2.7}$$

In the 't Hooft limit of large N with a fixed ratio $\lambda = N/k$, we need to distinguish two cases. When $Y^{I_1} \neq A_1$ and $Y_{J_L}^\dagger \neq B_1$ ($I_1 \neq 1$ and $J_L \neq 3$), we get

$$\begin{aligned}
\langle O_W\bar{O}_W\rangle &\sim \frac{1}{(N-1)!^2}(N-1)!^2\\
&\quad\times \epsilon^{[I]_{N-1}c}_{[J]_{N-1}a}\epsilon^{[K]_{N-1}b}_{[L]_{N-1}c}\epsilon^{[L]_{N-1}f}_{[K]_{N-1}d}\epsilon^{[J]_{N-1}e}_{[I]_{N-1}f}\langle W^a_b\bar{W}^d_e\rangle\\
&= (N-1)!^4N\langle\mathrm{tr}(W\bar{W})\rangle\\
&\sim (N-1)!^4N^{2L+2}.
\end{aligned}\tag{2.8}$$

Here, we have omitted the spacetime dependence explicitly because they can be easily put back at the end of the calculation. When $Y^{I_1} = A_1$ or $Y_{J_L}^\dagger = B_1$ ($I_1 = 1$ or $J_L = 3$), the operator factorizes [32,33], so the combinatorics of contractions is different. For instance, when $Y^{I_1} = A_1$, we have $W = A_1V$ and

$$O_W = \det A_1\epsilon^{[K]_{N-1}b}_{[L]_{N-1}c}(B_1)^{[L]_{N-1}}_{[K]_{N-1}}V^c_b, \tag{2.9}$$

and then

$$\langle O_W\bar{O}_W\rangle \sim N!(N-1)!^3N^{2L} = (N-1)!^4N^{2L+1}. \tag{2.10}$$

A similar analysis applies to the case when $Y_{J_L}^\dagger = B_1$. Therefore, the mixing between factorizing operators and nonfactorizing operators is suppressed in the large-N limit.[1] In this paper, we only consider operators with $Y^{I_1} \neq A_1$ and $Y_{J_L}^\dagger \neq B_1$.

B. Two-loop open spin-chain Hamiltonian

We now derive the two-loop anomalous dimension matrix for determinantlike operators in the 't Hooft limit. We need to consider the mixing of two operators

[1] This can be checked at two-loop order by a simple large-N counting.

$$W = Y^{I_1} Y^\dagger_{J_1} \cdots Y^{I_L} Y^\dagger_{J_L}, \qquad \bar{\tilde{W}} = Y^{M_L} Y^\dagger_{N_L} \cdots Y^{M_1} Y^\dagger_{N_1} \tag{2.11}$$

where $Y^{I_1} \neq A_1$, $Y^\dagger_{N_1} \neq A^\dagger_1$, $Y^\dagger_{J_L} \neq B_1$ and $Y^{M_L} \neq B^\dagger_1$. Keeping one A_1 and one B_1 uncontracted with the corresponding $A^\dagger_1$ and $B^\dagger_1$, we get

$$\begin{aligned}\langle O_W \bar{O}_{\tilde{W}} \rangle_{\text{2-loop}} &\sim (N-1)^2 \epsilon^{[I]_{N-2} ic}_{[J]_{N-2} ja} \epsilon^{[K]_{N-2} kb}_{[L]_{N-2} lc} \epsilon^{[L]_{N-2} mf}_{[K]_{N-2} sd} \epsilon^{[J]_{N-2} qe}_{[I]_{N-2} pf} \langle (A_1)^j_i (B_1)^l_k (A^\dagger_1)^p_q (B^\dagger_1)^s_m W^a_b \bar{\tilde{W}}^d_e \rangle_{\text{2-loop}} \\ &= (N-2)!^2 (N-1)!^2 \delta^{qe}_{ja} \delta^{ic}_{pf} \delta^{mf}_{lc} \delta^{kb}_{sd} \langle (A_1)^j_i (B_1)^l_k (A^\dagger_1)^p_q (B^\dagger_1)^s_m W^a_b \bar{\tilde{W}}^d_e \rangle_{\text{2-loop}}. \end{aligned} \tag{2.12}$$

Contractions of the generalized Kronecker deltas give

$$\begin{aligned} &\langle \delta^{qe}_{ja} \delta^{ic}_{pf} \delta^{mf}_{lc} \delta^{kb}_{sd} (A_1)^j_i (B_1)^l_k (A^\dagger_1)^p_q (B^\dagger_1)^s_m W^a_b \bar{\tilde{W}}^d_e \rangle_{\text{2-loop}} \\ &\quad = (N-2) \langle \mathrm{tr}(W\bar{\tilde{W}}) \mathrm{tr}(A_1 A^\dagger_1) \mathrm{tr}(B_1 B^\dagger_1) - \mathrm{tr}(\bar{\tilde{W}} W B^\dagger_1 B_1) \mathrm{tr}(A_1 A^\dagger_1) - \mathrm{tr}(A_1 A^\dagger_1 W \bar{\tilde{W}}) \mathrm{tr}(B_1 B^\dagger_1) \\ &\qquad + \mathrm{tr}(W B^\dagger_1 B_1 \bar{\tilde{W}} A_1 A^\dagger_1) \rangle_{\text{2-loop}} + \langle \mathrm{tr}(W\bar{\tilde{W}}) \mathrm{tr}(A_1 B_1 B^\dagger_1 A^\dagger_1) - \mathrm{tr}(W B^\dagger_1 A^\dagger_1 A_1 B_1 \bar{\tilde{W}}) \\ &\qquad - \mathrm{tr}(W \bar{\tilde{W}} A_1 B_1 B^\dagger_1 A^\dagger_1) + \mathrm{tr}(W B^\dagger_1 A^\dagger_1) \mathrm{tr}(A_1 B_1 \bar{\tilde{W}}) \rangle_{\text{2-loop}}. \end{aligned} \tag{2.13}$$

One can check that, in the large-N limit, the first, second, and third terms in the second line give bulk, right, and left boundary contributions, respectively, and the contributions from other terms are suppressed. For example, one part of the leading contribution from the second term corresponds to the contraction

$$-(N-2) \langle \mathrm{tr}(\bar{\tilde{W}} W B^\dagger_1 B_1) \rangle_{\text{connected, 2-loop}} \mathrm{tr}(\overbrace{A_1 A^\dagger_1}) \sim \frac{N^{2L+6}}{k^2}. \tag{2.14}$$

Note that the contraction between A_1 and $A^\dagger_1$ gives a factor $N^2 (N-1)^{-1}$, here the factor $(N-1)^{-1}$ is from avoiding repeatedly counting of contractions. The Hamiltonian of the bulk part the open chain is the same as that of the closed spin chain which was derived in [34,35]. We need to consider the boundary contributions. We first focus on the left boundary corresponding to the term

$$\langle -\mathrm{tr}(A_1 A^\dagger_1 W \bar{\tilde{W}}) \mathrm{tr}(B_1 B^\dagger_1) \rangle_{\text{2-loop}} \to N^{2L} \langle -\mathrm{tr}(A_1 A^\dagger_1 Y^{I_1} Y^\dagger_{J_1} Y^{M_1} Y^\dagger_{N_1}) \rangle_{\text{2-loop}}. \tag{2.15}$$

Contributions from wave function renormalization (self-interactions) are proportional to $\delta^{I_1}_{N_1}$ and thus flavor blind. To get contributions from gluon exchange and fermion exchange, one needs to contract Y^{M_1} with $Y^\dagger_{N_1}$ and, thus, get $(\ldots)\delta^{M_1}_{N_1}$. Because $I_1 \neq 1$ and $N_1 \neq 1$, the contributions must be proportional to $\delta^{I_1}_{N_1} \delta^{M_1}_{N_1}$. Therefore, contributions from gluon exchange and fermion exchange are also flavor blind. We only need to consider the following contribution from sextet scalar potential V_B given in (2.2)

$$\begin{aligned} &N^{2L} \langle -\mathrm{tr}(A_1 A^\dagger_1 Y^{I_1} Y^\dagger_{J_1} Y^{M_1} Y^\dagger_{N_1}) V_B \rangle \\ &\quad \propto -\frac{4\pi^2 N^{2L+6}}{3k^2} (-3\delta^{I_1}_{J_1} \delta^{M_1}_{N_1} - 12 \delta^{M_1}_1 \delta^1_{J_1} \delta^{I_1}_{N_1} + 6 \delta^{I_1}_{N_1} \delta^{M_1}_{J_1}), \end{aligned} \tag{2.16}$$

where $I_1 \neq 1$ and $N_1 \neq 1$ have been taken into account. Then we get

$$H'_{\text{left}} = \frac{\lambda^2}{2} \left(\frac{1}{2} \delta^{I_1}_{J_1} \delta^{M_1}_{N_1} + 2 \delta^{M_1}_1 \delta^1_{J_1} \delta^{I_1}_{N_1} - \delta^{I_1}_{N_1} \delta^{M_1}_{J_1} + C \delta^{I_1}_{N_1} \delta^{M_1}_{J_1} \right). \tag{2.17}$$

Here, the normalization is fixed by comparing with bulk Hamiltonian from sextet scalar potential. In fact, the first term in the second line of (2.13) leads to, among other terms,

$$\begin{aligned} &N^{2L} \langle \mathrm{tr}(Y^{I_1} Y^\dagger_{J_1} Y^{I_2} Y^\dagger_{N_2} Y^{M_1} Y^\dagger_{M_1}) V_B \rangle \\ &\quad \propto -\frac{4\pi^2 N^{2L+6}}{3k^2} (3 \delta^{I_1}_{N_1} \delta^{I_2}_{J_1} \delta^{M_1}_{N_2} + 3 \delta^{I_1}_{J_1} \delta^{I_2}_{N_2} \delta^{M_1}_{N_1} + 12 \delta^{I_1}_{N_2} \delta^{I_2}_{N_1} \delta^{M_1}_{J_1} \\ &\qquad - 6 \delta^{I_1}_{J_1} \delta^{I_2}_{N_1} \delta^{M_1}_{N_2} - 6 \delta^{I_1}_{N_1} \delta^{I_2}_{N_2} \delta^{M_1}_{J_1} - 6 \delta^{I_1}_{N_2} \delta^{I_2}_{J_1} \delta^{M_1}_{N_1}), \end{aligned} \tag{2.18}$$

and the contribution of this part to the spin chain Hamiltonian is [34,35]

$$\frac{\lambda^2}{2}\left(-\frac{1}{2}\delta^{I_1}_{N_1}\delta^{I_2}_{J_1}\delta^{M_1}_{N_2}-\frac{1}{2}\delta^{I_1}_{J_1}\delta^{I_2}_{N_2}\delta^{M_1}_{N_1}-2\delta^{I_1}_{N_2}\delta^{I_2}_{N_1}\delta^{M_1}_{J_1}+\delta^{I_1}_{J_1}\delta^{I_2}_{N_1}\delta^{M_1}_{N_2}+\delta^{I_1}_{N_1}\delta^{I_2}_{N_2}\delta^{M_1}_{J_1}+\delta^{I_1}_{N_2}\delta^{I_2}_{J_1}\delta^{M_1}_{N_1}\right). \tag{2.19}$$

The constant C in (2.17) comes from the contributions from gluon exchange, fermion exchange and self-interactions. An analogous discussion applies to the right boundary. We will show in the Appendix that the anomalous dimension of the operator with $W=(A_2B_2)^L$ is zero in the large-N limit, which allows us to determine the sum of the constant C and a similar constant from the right boundary. At the end, the total Hamiltonian is given by

$$\begin{aligned}H=&\lambda^2\sum_{l=2}^{2L-3}\left(\mathbb{I}-\mathbb{P}_{l,l+2}+\frac{1}{2}\mathbb{P}_{l,l+2}\mathbb{K}_{l,l+1}+\frac{1}{2}\mathbb{P}_{l,l+2}\mathbb{K}_{l+1,l+2}\right)Q_1^{A_1}Q_{2L}^{B_1}\\&+\lambda^2Q_1^{A_1}\left(\mathbb{I}+\frac{1}{2}\mathbb{K}_{1,2}-\mathbb{P}_{1,3}+\frac{1}{2}\mathbb{P}_{1,3}\mathbb{K}_{1,2}+\frac{1}{2}\mathbb{P}_{1,3}\mathbb{K}_{2,3}\right)Q_1^{A_1}Q_{2L}^{B_1}\\&+\lambda^2Q_{2L}^{B_1}\left(\mathbb{I}+\frac{1}{2}\mathbb{K}_{2L-1,2L}-\mathbb{P}_{2L-2,2L}+\frac{1}{2}\mathbb{P}_{2L-2,2L}\mathbb{K}_{2L-2,2L-1}+\frac{1}{2}\mathbb{P}_{2L-2,2L}\mathbb{K}_{2L-1,2L}\right)Q_1^{A_1}Q_{2L}^{B_1}\\&+\lambda^2(\mathbb{I}-Q_2^{A_1^\dagger})Q_1^{A_1}Q_{2L}^{B_1}+\lambda^2(\mathbb{I}-Q_{2L-1}^{B_1^\dagger})Q_1^{A_1}Q_{2L}^{B_1},\end{aligned} \tag{2.20}$$

where the trace operator $\mathbb{K}$ and permutation operator $\mathbb{P}$ are defined as

$$(\mathbb{K}_{ij})^{I_iI_j}_{J_iJ_j}=\delta^{I_iI_j}\delta_{J_iJ_j},\qquad (\mathbb{P}_{ij})^{I_iI_j}_{J_iJ_j}=\delta^{I_j}_{J_i}\delta^{I_j}_{J_i}, \tag{2.21}$$

and the Q operators are defined as [8]

$$Q^{\phi}|\phi\rangle=0,\qquad Q^{\phi}|\psi\rangle=|\psi\rangle,\quad \text{for } \psi\neq\phi. \tag{2.22}$$

Half of the $\frac{1}{2}\mathbb{K}_{1,2}$ ($\frac{1}{2}\mathbb{K}_{2L-1,2L}$) term in (2.20) comes from the third (second) term in (2.13), and another half comes from the first term in (2.13).

III. INTEGRABILITY FROM COORDINATE BETHE ANSATZ

In this section, we discuss the integrability of the above open spin chain in the framework of coordinate Bethe ansatz. The reflection equations are necessary conditions for the integrability of the open spin chain Hamiltonian. We want to know whether the boundary reflection matrices satisfy the reflection equations or not.

We chose A_2 and B_2 to be the vacuum flavors and, therefore, the vacuum of this open chain is chosen to be

$$W=(A_2B_2)\cdots(A_2B_2). \tag{3.1}$$

The fundamental one-particle excitations include replacing A_2 by A_1 or $B_1^\dagger$ at the odd site and replacing B_2 by B_1 or $A_1^\dagger$ at the even site. And we should keep in mind that the first A_2 cannot be replaced by A_1 and the last B_2 cannot be replaced by B_1. So the one-particle excitations include

$$\text{bulk odd site}\quad (A_2B_2)\cdots(A_1B_2)\cdots(A_2B_2) \tag{3.2}$$

$$(A_2B_2)\cdots(B_1^\dagger B_2)\cdots(A_2B_2) \tag{3.3}$$

$$\text{bulk even site}\quad (A_2B_2)\cdots(A_2B_1)\cdots(A_2B_2) \tag{3.4}$$

$$(A_2B_2)\cdots(A_2A_1^\dagger)\cdots(A_2B_2) \tag{3.5}$$

$$\text{left boundary}\quad (B_1^\dagger B_2)\cdots(A_2B_2) \tag{3.6}$$

$$\text{right boundary}\quad (A_2B_2)\cdots(A_2A_1^\dagger). \tag{3.7}$$

We denote the open chain as $(1)(2)\cdots(x)\cdots(L)$ with every site (x) containing two fields. Then the above excitations can be simply denoted as

$$\begin{aligned}&|x\rangle_{A_1},\, 2\leq x\leq L,\\&|x\rangle_{B_1^\dagger},\, 1\leq x\leq L,\\&|x\rangle_{B_1},\, 1\leq x\leq L-1,\\&|x\rangle_{A_1^\dagger},\, 1\leq x\leq L,\end{aligned} \tag{3.8}$$

where $|1\rangle_{B_1^\dagger}$ is the left boundary excitation state and $|L\rangle_{A_1^\dagger}$ is the right boundary excitation state while all others are bulk one-particle excitation state.

Let us begin with

$$|k\rangle_{B_1^\dagger}=\sum_{x=1}^{L}f_{B_1^\dagger}(x)|x\rangle_{B_1^\dagger}, \tag{3.9}$$

where

$$f_{B_1^\dagger}(x)=F_{B_1^\dagger}e^{ikx}+\tilde{F}_{B_1^\dagger}e^{-ikx}. \tag{3.10}$$

On the states $|x\rangle_{B_1^\dagger}$, the Hamiltonian acts as follows

$$H|x\rangle_{B_1^\dagger}=\lambda^2(2|x\rangle_{B_1^\dagger}-|x+1\rangle_{B_1^\dagger}-|x-1\rangle_{B_1^\dagger}), \tag{3.11}$$

when $2 \le x \le L-1$, and

$$H|1\rangle_{B_1^\dagger} = \lambda^2(|1\rangle_{B_1^\dagger} - |2\rangle_{B_1^\dagger}), \tag{3.12}$$

$$H|L\rangle_{B_1^\dagger} = \lambda^2(2|L\rangle_{B_1^\dagger} - |L-1\rangle_{B_1^\dagger}). \tag{3.13}$$

So we get

$$\begin{aligned} H|k\rangle_{B_1^\dagger} &= \lambda^2 \sum_{x=2}^{L-2} (2f_{B_1^\dagger}(x) - f_{B_1^\dagger}(x-1) - f_{B_1^\dagger}(x+1))|x\rangle_{B_1^\dagger} \\ &+ \lambda^2(f_{B_1^\dagger}(1) - f_{B_1^\dagger}(2))|1\rangle_{B_1^\dagger} \\ &+ \lambda^2(2f_{B_1^\dagger}(L) - f_{B_1^\dagger}(L-1))|L\rangle_{B_1^\dagger}. \end{aligned} \tag{3.14}$$

Then equation

$$H|k\rangle_{B_1^\dagger} = E(k)|k\rangle_{B_1^\dagger}, \tag{3.15}$$

leads to the following dispersion relation

$$E(k) = \lambda^2(2 - 2\cos k), \tag{3.16}$$

and

$$f_{B_1^\dagger}(1) = f_{B_1^\dagger}(0), \tag{3.17}$$

$$f_{B_1^\dagger}(L+1) = 0. \tag{3.18}$$

Since the reflections of $B_1^\dagger$ excitation at both sides are diagonal, we define the left reflection coefficient to be

$$K_{L,B_1^\dagger} = F_{B_1^\dagger}/\tilde{F}_{B_1^\dagger}, \tag{3.19}$$

and the right reflection coefficient to be[2]

$$K_{R,B_1^\dagger} = e^{2ik(L-1)} F_{B_1^\dagger}/\tilde{F}_{B_1^\dagger}. \tag{3.20}$$

They are determined by Eqs. (3.17) and (3.18), respectively. The results are

$$K_{L,B_1^\dagger} = e^{-ik}, \tag{3.21}$$

$$K_{R,B_1^\dagger} = -e^{-4ik}. \tag{3.22}$$

For the other three excitations, the computations are similar. So we only list the action of the Hamiltonian, obtained boundary conditions and reflection coefficients. For $|x\rangle_{A_1}, 2 \le x \le L$ we have

$$H|x\rangle_{A_1} = \lambda^2(2|x\rangle_{A_1} - |x+1\rangle_{A_1} - |x-1\rangle_{A_1}), \quad 3 \le x \le L-1 \tag{3.23}$$

$$H|2\rangle_{A_1} = \lambda^2(2|2\rangle_{A_1} - |3\rangle_{A_1}), \tag{3.24}$$

$$H|L\rangle_{A_1} = \lambda^2(|L\rangle_{A_1} - |L-1\rangle_{A_1}). \tag{3.25}$$

This gives

$$f_{A_1}(1) = 0, \qquad f_{A_1}(L+1) = f_{A_1}(L), \tag{3.26}$$

which leads to

$$K_{L,A_1} = -e^{-2ik}, \qquad K_{R,A_1} = e^{-3ik}. \tag{3.27}$$

For $|x\rangle_{B_1}, 1 \le x \le L-1$, we have

$$H|x\rangle_{B_1} = \lambda^2(2|x\rangle_{B_1} - |x+1\rangle_{B_1} - |x-1\rangle_{B_1}), \quad 2 \le x \le L-2 \tag{3.28}$$

$$H|1\rangle_{B_1} = \lambda^2(|1\rangle_{B_1} - |2\rangle_{B_1}), \tag{3.29}$$

$$H|L-1\rangle_{B_1} = \lambda^2(2|L-1\rangle_{B_1} - |L\rangle_{B_1}). \tag{3.30}$$

this leads to

$$f_{B_1}(1) = f_{B_1}(0), \qquad f_{B_1}(L) = 0, \tag{3.31}$$

then

$$K_{L,B_1} = e^{-ik}, \qquad K_{R,B_1} = -e^{-2ik}. \tag{3.32}$$

Finally for $|x\rangle_{A_1^\dagger}, 1 \le x \le L$, we have

$$H|x\rangle_{A_1^\dagger} = \lambda^2(2|x\rangle_{A_1^\dagger} - |x+1\rangle_{A_1^\dagger} - |x-1\rangle_{A_1^\dagger}), \quad 2 \le x \le L-1 \tag{3.33}$$

$$H|1\rangle_{A_1^\dagger} = \lambda^2(2|1\rangle_{A_1^\dagger} - |2\rangle_{A_1^\dagger}), \tag{3.34}$$

$$H|L\rangle_{A_1^\dagger} = \lambda^2(|L\rangle_{A_1^\dagger} - |L-1\rangle_{A_1^\dagger}). \tag{3.35}$$

This gives

$$f_{A_1^\dagger}(0) = 0, \qquad f_{A_1^\dagger}(L) = f_{A_1^\dagger}(L+1), \tag{3.36}$$

and

$$K_{L,A_1^\dagger} = -1, \qquad K_{R,A_1^\dagger} = e^{-3ik}. \tag{3.37}$$

With the order of the excitations as $A_1, B_1^\dagger, A_1^\dagger, B_1$, the left reflection matrix is

[2] We have taken into account that for every excitation, there are $L-1$ bulk sites.

$$K_L = \begin{pmatrix} -e^{-2ik} & & & \\ & e^{-ik} & & \\ & & -1 & \\ & & & e^{-ik} \end{pmatrix}, \tag{3.38}$$

and the right reflection matrix is

$$K_R = \begin{pmatrix} e^{-3ik} & & & \\ & -e^{-4ik} & & \\ & & e^{-3ik} & \\ & & & -e^{-2ik} \end{pmatrix}. \tag{3.39}$$

The two reflection matrices are diagonal in the chosen natural basis. This is quite different from the results in [17], where the reflection matrices are anti-diagonal in the same basis.[3] Also notice that each excitation always has Dirichlet boundary condition on one end of the open chain, and Neumann boundary condition on the other end. This is different from the SYM case [8,33] where the boundary conditions are always left-right symmetric. The S-matrix in ABJM theory can be found in [30]. It satisfies the Yang-Baxter equation

$$S_{12}(k_1,k_2)S_{13}(k_1,k_3)S_{23}(k_2,k_3) = S_{23}(k_2,k_3)S_{13}(k_1,k_3)S_{12}(k_1,k_2). \tag{3.40}$$

Now we are ready to check the reflection equations. It can be straightforward to verify that reflection equations are satisfied

$$K_{L2}(k_2)S_{12}(k_1,-k_2)K_{L1}(k_1)S_{21}(-k_2,-k_1) = S_{12}(k_1,k_2)K_{L1}(k_1)S_{21}(k_2,k_1)K_{L2}(k_2), \tag{3.41}$$

$$K_{R2}(-k_2)S_{21}(k_2,-k_1)K_{R1}(-k_1)S_{12}(k_1,k_2) = S_{21}(-k_2,-k_1)K_{R2}(-k_1)S_{12}(k_1,-k_2)K_{R2}(-k_2). \tag{3.42}$$

The $\frac{1}{2}\mathbb{K}_{1,2}$ and $\frac{1}{2}\mathbb{K}_{2L-1,2L}$ terms in the Hamiltonian (2.20) have no effect in the above calculation. To understand their role in the coordinate Bethe ansatz, one needs to consider impurities $A_2^\dagger$ and $B_2^\dagger$. These impurities can be described as bound states of the form $\phi\phi^\dagger, \phi = A_1, B_1^\dagger$. Although not shown here, we have checked that the $\frac{1}{2}\mathbb{K}_{1,2}$ and $\frac{1}{2}\mathbb{K}_{2L-1,2L}$ terms in the Hamiltonian are necessary in the construction of the eigenstates involving $\phi\phi^\dagger$ scattering and the above bound states using coordinate Bethe ansatz.

[3]A nonsupersymmetric flavored ABJM theory was constructed in [36], where the corresponding reflection matrices are diagonal.

IV. CONCLUSIONS AND DISCUSSIONS

We have obtained the two-loop Hamiltonian of the open spin chain corresponding to the determinantlike operators in ABJM theory which are dual to open strings attached to D4-branes wrapping cycles in $\mathbb{CP}^3$. The Hamiltonian is different from the periodic spin chain only in the boundary terms. Using the coordinate Bethe ansatz, we present strong evidence that the Hamiltonian may be integrable. In other words, the giant graviton may provide integrable boundary conditions for the open string. It is possible to go beyond the two-loop order to an all-loop prediction which is similar to previous studies in the SYM context [21,23] using symmetries as the guide, and even further to solve the full open string spectrum through boundary thermodynamical Bethe ansatz and/or Y-system which have already been done in the SYM case [24,25]. To have a more solid ground for integrability of our two-loop Hamiltonian, it would be better to have an algebraic Bethe ansatz construction [37] as people have done in the SYM theory [38].

ACKNOWLEDGMENTS

We would like to thank Nan Bai and Yunfeng Jiang for very helpful discussions. The work of J.-B. W. was supported by the National Natural Science Foundation of China, Grant No. 11575202. The work of H. O. was supported by the grant "Exact Results in Gauge and String Theories" from the Knut and Alice Wallenberg Foundation. H. O. and H.-H. C. gratefully acknowledge the kind hospitality of Tianjin University during the course of this work.

APPENDIX: VACUUM OF THE OPEN CHAIN

In this Appendix, we show that the anomalous dimension of the operator

$$O_0 = \epsilon_{a_1\ldots a_N}\epsilon^{b_1\ldots b_N}(A_1B_1)^{a_1}_{b_1}\ldots(A_1B_1)^{a_{N-1}}_{b_{N-1}}((A_2B_2)^L)^{a_N}_{b_N} \tag{A1}$$

is suppressed in the large-N limit with $\lambda = N/k$ fixed. As discussed in [39], at two-loop order the contributions from bosonic D-terms, gluon exchange, fermion exchange from fermionic D-terms and self-interactions cancel for operators in the $SU(2)\times SU(2)$ sector, and the fermionic F-terms do not contribute to the anomalous dimension. We only need to consider the contributions from bosonic F-terms [31]

$$V_F^{\text{bos}} = -\frac{16\pi^2}{k^2}\text{tr}(A_i^\dagger B_j^\dagger A_k^\dagger A_i B_j A_k - A_i^\dagger B_j^\dagger A_k^\dagger A_k B_j A_i + B_i^\dagger A_j^\dagger B_k^\dagger B_i A_j B_k - B_i^\dagger A_j^\dagger B_k^\dagger B_k A_j B_i). \tag{A2}$$

Using (2.13), one can check that the anomalous dimension of O_0 is subleading in $1/N$.

[1] N. Beisert *et al.*, Review of AdS/CFT integrability: An overview, Lett. Math. Phys. **99**, 3 (2012).
[2] N. Beisert and R. Roiban, Beauty and the twist: The Bethe ansatz for twisted $\mathcal{N}=4$ SYM, J. High Energy Phys. 08 (2005) 039.
[3] N. Beisert and R. Roiban, The Bethe ansatz for $\mathbb{Z}_S$ orbifolds of $\mathcal{N}=4$ super Yang-Mills theory, J. High Energy Phys. 11 (2005) 037.
[4] B. Chen, X. J. Wang, and Y. S. Wu, Integrable open spin chain in super Yang-Mills and the plane wave/SYM duality, J. High Energy Phys. 02 (2004) 029.
[5] B. Chen, X. J. Wang, and Y. S. Wu, Open spin chain and open spinning string, Phys. Lett. B **591**, 170 (2004).
[6] O. DeWolfe and N. Mann, Integrable open spin chains in defect conformal field theory, J. High Energy Phys. 04 (2004) 035.
[7] T. Erler and N. Mann, Integrable open spin chains and the doubling trick in $\mathcal{N}=2$ SYM with fundamental matter, J. High Energy Phys. 01 (2006) 131.
[8] D. Berenstein and S. E. Vazquez, Integrable open spin chains from giant gravitons, J. High Energy Phys. 06 (2005) 059.
[9] K. Zoubos, Review of AdS/CFT integrability, chapter IV.2: Deformations, orbifolds and open boundaries, Lett. Math. Phys. **99**, 375 (2012).
[10] S. J. van Tongeren, Integrability of the $\mathrm{AdS}_5\times S^5$ superstring and its deformations, J. Phys. A **47**, 433001 (2014).
[11] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, $\mathcal{N}=6$ superconformal Chern-Simons-matter theories, $M2$-branes and their gravity duals, J. High Energy Phys. 10 (2008) 091.
[12] T. Klose, Review of AdS/CFT integrability, Chapter IV.3:$\mathcal{N}=6$ Chern-Simons and strings on $\mathrm{AdS}_4\times CP^3$, Lett. Math. Phys. **99**, 401 (2012).
[13] S. He and J.-B. Wu, Note on integrability of marginally deformed ABJ(M) theories, J. High Energy Phys. 04 (2013) 012.
[14] H.-H. Chen, P. Liu, and J.-B. Wu, Y-system for γ-deformed ABJM theory, J. High Energy Phys. 03 (2017) 133.
[15] H. H. Chen and J. B. Wu, Finite-size effect for dyonic giant magnons in CP^3_β, Phys. Lett. B **769**, 90 (2017).
[16] N. Bai, H. H. Chen, X. C. Ding, D. S. Li, and J. B. Wu, Integrability of orbifold ABJM theories, J. High Energy Phys. 11 (2016) 101.
[17] N. Bai, H.-H. Chen, S. He, J.-B. Wu, W.-L. Yang, and M.-Q. Zhu, Integrable open spin chains from flavored ABJM theory, J. High Energy Phys. 08 (2017) 001.
[18] N. Drukker and S. Kawamoto, Small deformations of supersymmetric Wilson loops and open spin-chains, J. High Energy Phys. 07 (2006) 024.
[19] N. Drukker, Integrable Wilson loops, J. High Energy Phys. 10 (2013) 135.
[20] D. Correa, J. Maldacena, and A. Sever, The quark anti-quark potential and the cusp anomalous dimension from a TBA equation, J. High Energy Phys. 08 (2012) 134.
[21] D. M. Hofman and J. M. Maldacena, Reflecting magnons, J. High Energy Phys. 11 (2007) 063.
[22] D. Berenstein, D. H. Correa, and S. E. Vazquez, Quantizing Open Spin Chains with Variable Length: An Example from Giant Gravitons, Phys. Rev. Lett. **95**, 191601 (2005).
[23] C. Ahn and R. I. Nepomechie, The Zamolodchikov-Faddeev algebra for open strings attached to giant gravitons, J. High Energy Phys. 05 (2008) 059.
[24] Z. Bajnok, R. I. Nepomechie, L. Palla, and R. Suzuki, Y-system for $Y=0$ brane in planar AdS/CFT, J. High Energy Phys. 08 (2012) 149.
[25] Z. Bajnok, N. Drukker, A. Hegeds, R. I. Nepomechie, L. Palla, C. Sieg, and R. Suzuki, The spectrum of tachyons in AdS/CFT, J. High Energy Phys. 03 (2014) 055.
[26] D. Berenstein and D. Trancanelli, Three-dimensional $\mathcal{N}=6$ SCFT's and their membrane dynamics, Phys. Rev. D **78**, 106009 (2008).
[27] D. Giovannoni, J. Murugan, and A. Prinsloo, The Giant graviton on $\mathrm{AdS}_4\times CP^3$—another step towards the emergence of geometry, J. High Energy Phys. 12 (2011) 003.
[28] Y. Lozano, J. Murugan, and A. Prinsloo, A giant graviton genealogy, J. High Energy Phys. 08 (2013) 109.
[29] C. Cardona and H. Nastase, Open strings on D–branes from ABJM, J. High Energy Phys. 06 (2015) 016.
[30] C. Ahn and R. I. Nepomechie, Two-loop test of the $\mathcal{N}=6$ Chern-Simons theory S-matrix, J. High Energy Phys. 03 (2009) 144.
[31] M. Benna, I. Klebanov, T. Klose, and M. Smedback, Superconformal Chern-Simons theories and AdS(4)/CFT (3) correspondence, J. High Energy Phys. 09 (2008) 072.
[32] D. Berenstein, Shape and holography: Studies of dual operators to giant gravitons, Nucl. Phys. **B675**, 179 (2003).
[33] V. Balasubramanian, D. Berenstein, B. Feng, and M.-x. Huang, D-branes in Yang-Mills theory and emergent gauge symmetry, J. High Energy Phys. 03 (2005) 006.
[34] J. A. Minahan and K. Zarembo, The Bethe ansatz for superconformal Chern-Simons, J. High Energy Phys. 09 (2008) 040.
[35] D. Bak and S.-J. Rey, Integrable spin chain in superconformal Chern-Simons theory, J. High Energy Phys. 10 (2008) 053.
[36] J. B. Wu, Notes on integrable boundary interactions of open $SU(4)$ alternating spin chains, Sci. China Phys. Mech. Astron. **61**, 070011 (2018).
[37] N. Bai, H.-H. Chen, H. Ouyang, and J.-B. Wu (to be published).
[38] R. I. Nepomechie, Revisiting the $Y=0$ open spin chain at one loop, J. High Energy Phys. 11 (2011) 069.
[39] C. Kristjansen, M. Orselli, and K. Zoubos, Non-planar ABJM theory and integrability, J. High Energy Phys. 03 (2009) 037.