

Backreaction of super-Hubble cosmological perturbations beyond perturbation theory

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We discuss the effect of super-Hubble cosmological fluctuations on the locally measured Hubble expansion rate. We consider a large bare cosmological constant in the early Universe in the presence of scalar field matter (the dominant matter component), which would lead to a scale-invariant primordial spectrum of cosmological fluctuations. Using the leading-order gradient expansion, we show that the expansion rate measured by a (secondary) clock field which is not comoving with the dominant matter component obtains a negative contribution from infrared fluctuations, a contribution whose absolute value increases in time. This is the same effect that a decreasing cosmological constant would produce. This supports the conclusion that infrared fluctuations lead to a dynamical relaxation of the cosmological constant. Our analysis does not make use of any perturbative expansion in the amplitude of the inhomogeneities.

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I. INTRODUCTION

The cosmological constant problem (see [1] for reviews) is a key challenge for fundamental physics. Basic arguments imply that the vacuum energy of matter fields should act as a cosmological constant Λ and cause the Universe to accelerate. Assuming that an ultraviolet cutoff scale close to the Planck scale is used, the resulting value for Λ is about 120 orders of magnitude larger than the maximal one allowed by observations. The discovery of the acceleration of the Universe [2] has added a new perspective to this problem. If the observed acceleration is due to a small cosmological constant, then we do not only have to explain why the cosmological constant is not of the Planck scale, but also why it happens to be rearing its head at the present time in the cosmological history of the Universe. This is the

coincidence problem (see e.g., [3] for reviews of the dark energy problem).

It has been conjectured for some time that de Sitter space is unstable because of infrared effects [4,5] (see, however, [6] for arguments supporting the stability of de Sitter space), and that hence the bare cosmological constant in the Lagrangian would be invisible today.¹ Specifically, it was suggested by Tsamis and Woodard [10] that the backreaction of super-Hubble scale gravitational waves could give a negative contribution to the effective cosmological constant and cause the latter to relax. The problem was studied in perturbation theory, and it was found that one needs to go to two-loop order (fourth order in the amplitude of the gravitational waves) in order to obtain a nonvanishing effect. The study of the backreaction effect of long wavelength cosmological perturbations was initiated in [11] and it was found that at one-loop order (second order in the amplitude of the perturbations) super-Hubble cosmological perturbations lead to a negative contribution to the cosmological constant (see, also, [12–14]). Based on this analysis, it was then conjectured in [15] that this backreaction could lead to a late time scaling solution for

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¹See also [7–9] for a discussion on the difficulty of obtaining de Sitter space in string theory.

which the contribution of the cosmological constant tracks the contribution of matter to the total energy density.

The backreaction effect originates from the fact that the Einstein equations are highly nonlinear. Hence, if we consider only linear perturbations about a homogeneous and isotropic cosmological background metric, then the Einstein equations are not satisfied at quadratic order from a naive point of view. Then, specifically, each Fourier mode of the linear fluctuations yields a contribution to the background metric at quadratic order, and hence affects quantities such as the Hubble expansion rate.² Nonlinear effects also cause a correction to the fluctuations themselves, but initial studies [16] have shown that these effects are less important than the effects on the background.

The setup of the backreaction analysis of [11] was the following. A large bare cosmological constant Λ will lead to a phase of inflationary expansion of space. Quantum fluctuations in this quasi-de Sitter phase are continuously stretched beyond the Hubble radius, freeze out, are squeezed and will generate an increasing phase space of super-Hubble modes. In terms of comoving momenta, the phase space of these infrared modes runs from some value k_i which corresponds to the Hubble radius at the initial time t_i (and represents a physical infrared cutoff) to the Hubble scale $H e^{H(t-t_i)}$. The fact that new modes are continuously injected into the phase space of infrared modes is crucial for the backreaction to be effective. Once the backreaction effect of the long wavelength modes has built up sufficiently, it can cancel out the effects of the bare cosmological constant and terminate the phase of accelerated expansion.

Questions about the analysis of [11] were raised in [17] where the challenge was posed to show whether the effects predicted in [11] are in fact locally measurable. In fact, it was then shown in [18] (see also [19]) that in the case of pure adiabatic fluctuations the effects computed in [11] can be undone by a second-order time reparametrization. On the other hand, if there is a clock field present in addition to the matter which dominated the energy density, then, as shown in [20], the backreaction of cosmological fluctuations can be shown to influence the locally measured expansion rate in the sense that the locally measured expansion rate is smaller at a fixed value of the clock field if there are super-Hubble fluctuations present than if there are none (assuming that the clock field does not track the dominant component of matter, i.e., that entropy fluctuations are present). Considering two components of matter, a first dominant fluid (which sets up the cosmological

fluctuations) and a second sub-dominant clock field is very natural in the context of late time cosmology where we measure time in terms of the temperature of the sub-dominant radiation fluid. The analysis of [20] was extended in [21] following a new gauge-invariant approach introduced in [22] (and based on [23–25]). This approach was first applied to analyze at second order in the perturbative expansion single scalar field models, for which only expansion rates defined by isotropic observers experience a nontrivial negative quantum backreaction [26]. Moving to the above mentioned two field models, in [21], it was in fact confirmed that, at second order in perturbation theory, the backreaction of long wavelength cosmological perturbations leads to a decrease in the locally measured expansion rate (see also [27] for other studies demonstrating that backreaction effects are for real). Furthermore, in [28], it was shown physically how super-Hubble fluctuation modes can modify the parameters of a local Friedmann cosmology.

All analyses of the backreaction of cosmological fluctuations performed so far are, however, analyses in leading-order perturbation theory. Then, in almost all cases, backreaction effects of long wavelength fluctuations become important only when perturbation theory breaks down (see, e.g., [26]).³ Thus, while the fact that the leading-order perturbative backreaction effect leads to a negative contribution to the cosmological constant supports the possibility of an instability of (quasi-) de Sitter spacetime, it cannot give a definite answer since the effect may be undone by higher-order effects.

In this paper, we show that the backreaction effect of super-Hubble cosmological fluctuations on the local expansion rate persists beyond perturbation theory and that, given fluctuations in the clock field relative to those of the dominant matter field, the locally measured Hubble expansion rate obtains a negative contribution, a contribution whose amplitude grows in time. This supports the claim that (quasi-) de Sitter spacetime is unstable, and that it will lead to a dynamical relaxation of the cosmological constant.⁴

Although our analysis is nonperturbative in the amplitude of the cosmological perturbations, it is only a

²In this approach, second-order perturbations, which would have a nonzero average, are incorporated in the left-hand side of the Einstein equations producing, for example, an effective Hubble expansion. In this way, one takes into account the backreaction, which can be evaluated looking at the aforementioned effect of linear perturbations at quadratic order.

³See [29] for a case in which backreaction of tensor modes can be important within the perturbative regime, and [30,31] for a first study of the regime of validity of perturbation theory through gauge invariant variables. Tensor modes backreaction in a de Sitter background was also considered in [32].

⁴Our qualitative result has been obtained quite simply, but with a little price, since there is a residual gauge dependence of the result because of our choice of separating average null nonhomogeneous fluctuations. Given that our previous results in a fully gauge invariant framework are compatible with this assumption at second order in perturbation theory, we consider our analysis as a qualitative prediction at the nonperturbative level. We shall address the problem of a fully nonperturbative gauge invariant analysis in a future investigation.

leading-order analysis in the gradient expansion. Such an expansion should be reasonable for super-Hubble fluctuations, and the formalism we are using here was in fact suggested in [33] and applied to show that there is no parametric resonance of super-Hubble scale metric fluctuations during reheating (see e.g., [34] for a review) in the case of pure adiabatic perturbations. The analysis of [33] also shows that in the case of pure adiabatic fluctuations there can be no backreaction of super-Hubble fluctuation modes when the adiabatic field is the clock of the problem. However one obtains a nonzero backreaction from adiabatic fluctuation for an isotropic clock [26]. As said, applying the formalism of [33], we here demonstrate that in the presence of fluctuations of the clock field relative to the constant energy density hypersurfaces of the dominant matter there is a nonvanishing backreaction effect, and that this effect corresponds to a negative contribution to the locally measured Hubble expansion rate, a contribution whose absolute value increases in time.

The article is organized as follow. In Sec. II, we derive an expression for the local Hubble expansion rate in terms of the variables expressing the cosmological perturbations. We are interested in comparing the average of the expansion rate taken over a hypersurface of constant clock field between a manifold with cosmological perturbations and one without, at the same value of the clock field. In Sec. III, we evaluate this average expansion rate in the leading-order spatial gradient expansion (which will be a good approximation to study the effects of super-Hubble fluctuation modes) and show that the fluctuations result in a negative contribution ΔH to the expansion rate H which corresponds to a decrease in the effective cosmological constant. Let us also underline that, since $|\Delta H|$ is an increasing function of time, the backreaction effect corresponds in fact to an instability of (quasi-) de Sitter space and not just to a renormalization of the cosmological constant. Finally, we conclude in Sec. IV.

II. LOCAL HUBBLE EXPANSION RATE

In this section, we will derive an expression for the local Hubble expansion rate from the point of view of a clock field χ , extending the results of [21]. This is determined in terms of the normal vector n_μ to the constant χ hypersurfaces:

$$n_\mu = \frac{\partial_\mu \chi}{\sqrt{\xi}}, \quad (1)$$

with $\xi \equiv g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi$. This can then be used to determine the local expansion rate $\theta(x)$, where x denote the spatial coordinates, by

$$\theta = \nabla_\mu n^\mu = n^\mu \partial_\mu \log \sqrt{-g} + \partial_\mu n^\mu \quad (2)$$

Starting from this expression, and following [22], we want than to compute the average of $\theta/\sqrt{Z_\chi}$ over the constant χ hypersurface, which will be used to define our effective expansion rate including the contribution of the fluctuations. Such an average can be defined by [22]

$$\langle A \rangle_\chi = \frac{\int \sqrt{-\bar{\gamma}} \bar{A}}{\int \sqrt{-\bar{\gamma}}}, \quad (3)$$

where the barred coordinate system $\bar{x}^\mu = (\bar{t}, \bar{x})$ is the system where χ is homogeneous, and $\bar{\gamma}$ is the determinant of the induced metric on that hypersurface.

Note that we treat χ as a spectator field in the same way that the radiation field in late time cosmology can be viewed as a spectator field in the matter-dominated phase. The dominant matter component can be modelled as a different scalar field φ which sets up the cosmological fluctuations. In the same way that in current cosmology the dominant matter component is inhomogeneous from the point of view of the constant radiation temperature surfaces, we assumed that the dominant matter is inhomogeneous from the point of view of the constant χ surfaces. It is the dominant matter field which determines the metric fluctuations. To describe these fluctuations we will work in generalized longitudinal gauge (LG) (see e.g., [35] for an in-depth review of the theory of cosmological perturbations and [36] for a brief overview) in terms of which the metric is given by

$$g_{\mu\nu} = \text{diag}(e^{2\phi(t)}, -e^{-2\psi(t)}, -e^{-2\psi(t)}, -e^{-2\psi(t)}), \quad (4)$$

where ϕ and ψ are functions of space and time. In linear perturbation theory, $\psi = \phi$ in the absence of anisotropic stress. Beyond linear perturbation theory, however, ϕ and ψ must be treated as independent. Note that this is the unique gauge in which the metric has no off-diagonal components. If we want the metric to locally look like a Friedmann metric for long wavelength fluctuations, then this gauge is the preferred one (see also [26]).

The coordinate transformation from longitudinal gauge to the constant χ gauge is [21]

$$\begin{aligned} x^\mu &= (t, \vec{x}) \rightarrow \bar{x}^\mu = (\bar{t}, \bar{\vec{x}}) \\ &= (\chi(t, \vec{x}), \vec{x}) \equiv f^\mu(x^\nu), \end{aligned} \quad (5)$$

and the metric in these coordinates, expressed in function of LG variables, becomes

$$\bar{g}_{\mu\nu}(x) = \frac{1}{\left(\frac{\partial \chi}{\partial t}\right)^2} \begin{pmatrix} e^{2\phi} & -e^{2\phi} \partial_i \chi \\ -e^{2\phi} \partial_j \chi & e^{2\phi} \partial_i \chi \partial_j \chi - e^{-2\psi} \delta_{ij} \left(\frac{\partial \chi}{\partial t}\right)^2 \end{pmatrix}, \quad (6)$$

with its inverse being

$$\bar{g}^{\mu\nu}(x) = \begin{pmatrix} \left(\frac{\partial\chi}{\partial t}\right)^2 e^{-2\phi} - |\vec{\nabla}\chi|^2 e^{2\psi} & -e^{2\psi} \vec{\nabla}\chi \\ -e^{2\psi} (\vec{\nabla}\chi)^t & -e^{2\psi} \mathbb{I} \end{pmatrix} \Big|_{f^{-1}(x)} = \begin{pmatrix} e^{-2\phi} & -e^{2\psi} \vec{\nabla}\chi \\ -e^{2\psi} (\vec{\nabla}\chi)^t & -e^{2\psi} \mathbb{I} \end{pmatrix} \Big|_{f^{-1}(x)}, \quad (7)$$

where we made $\dot{\chi} = 1$. All the quantities on the right-hand sides are evaluated at $(f^{-1})^\mu(x^\nu)$. Above we have used the fact that

$$\xi = e^{-2\phi} \left(\frac{\partial\chi}{\partial t}\right)^2 - e^{2\psi} (\vec{\nabla}\chi)^2$$

under the gauge transformation becomes

$$\bar{\xi}(x) = e^{-2\phi(f^{-1}(x))}. \quad (8)$$

Hence, the induced metric on the constant χ surfaces becomes

$$ds^2 = e^{2\phi} \frac{(d\vec{x} \cdot \vec{\nabla}\chi)^2}{\dot{\chi}^2} - e^{-2\psi} d\vec{x}^2. \quad (9)$$

From (6) it follows that the determinant of the metric takes the form

$$\sqrt{-\bar{g}} = \left(\frac{1}{\frac{\partial\chi}{\partial t}} e^{\phi-3\psi}\right) \Big|_{f^{-1}(x)}. \quad (10)$$

We will now compute the local measure of expansion directly in the barred coordinates, i.e., using

$$\bar{\theta}(x) = \bar{\nabla}_\mu \bar{n}^\mu = \bar{n}^\mu \partial_\mu \log \sqrt{-\bar{g}} + \partial_\mu \bar{n}^\mu, \quad (11)$$

where \bar{n}^μ is the normal vector to the constant χ hypersurfaces in the barred coordinates. In the following, all quantities which define the barred variables are evaluated at $f^{-1}(x)$. We then have

$$\bar{n}_\mu(x) = \left[\frac{\partial x^\alpha}{\partial f^\mu}\right]_{f^{-1}(x)} n_\alpha(f^{-1}(x)) \quad (12)$$

which becomes

$$\bar{n}_\mu(x) = \frac{1}{\sqrt{\xi(f^{-1}(x))}} \begin{pmatrix} \dot{\chi} & \vec{\nabla}\chi \\ \vec{0}^t & \mathbb{I} \end{pmatrix} \cdot \begin{pmatrix} 1/\dot{\chi} & -\vec{\nabla}\chi/\dot{\chi} \\ \vec{0}^t & \mathbb{I} \end{pmatrix} \quad (13)$$

and, finally, gives

$$\bar{n}_\mu(x) = e^\phi (1, \vec{0}), \quad (14)$$

corresponding indeed [see Eq. (1)] to the case of a homogeneous $\bar{\chi}$ which labels the time coordinate. One also has

$$\bar{n}^\mu = \bar{g}^{\alpha\mu} \bar{n}_\alpha, \quad (15)$$

where the inverse metric is given in Eq. (7) so that

$$\bar{n}^\mu = (1, -e^{2\psi} (\vec{\nabla}\chi)^t) \Big|_{f^{-1}(x)}. \quad (16)$$

We also have from Eq. (10)

$$\log(\sqrt{-\bar{g}}) = \phi - 3\psi - \log \dot{\chi} \quad (17)$$

which implies

$$\partial_\mu [\log(\sqrt{-\bar{g}})]_{f^{-1}(x)} = \left(\partial_\mu (\phi - 3\psi) + \frac{\partial_\mu \dot{\chi}}{\dot{\chi}}\right). \quad (18)$$

In the leading-order gradient expansion, the spatial derivative terms in the above are negligible. Making use of $\bar{n}^0 = \exp(-\phi)$ we then get

$$\begin{aligned} \bar{\theta}(x) &= \bar{g}^{00} \left(\dot{\phi} - 3\dot{\psi} - \frac{\dot{\chi}}{\dot{\chi}}\right) \\ &\quad + \bar{g}^{0i} \partial_i (\phi - 3\psi - \log \dot{\chi}) + \partial_\mu \bar{g}^{0\mu}, \end{aligned} \quad (19)$$

which then yields

$$\bar{\theta}(x) = -3e^{-\phi} \dot{\psi}. \quad (20)$$

In order to compute the spatial average of θ over the constant χ hypersurfaces, we need the determinant of the induced metric $\bar{\gamma}_{ij}$ on these surfaces [see (3)]. This metric is obtained from the spatial part of (6)

$$\bar{\gamma}_{ij} = \frac{e^{2\phi}}{\dot{\chi}^2} (\vec{\nabla}\chi)^t \vec{\nabla}\chi - e^{-2\psi} \mathbb{I} \quad (21)$$

so that the measure factor is given by

$$\sqrt{-\bar{\gamma}} = e^{-3\psi} \sqrt{1 - 3e^{2(\phi+\psi)} \frac{|\vec{\nabla}\chi|^2}{\dot{\chi}^2}}. \quad (22)$$

In the leading-order gradient expansion, we can neglect the spatial gradient terms, and hence the above expression reduces to

$$\sqrt{-\bar{\gamma}} = e^{-3\psi}. \quad (23)$$

We want now to compute the effective expansion rate H_{eff} . As introduced before, this can be defined as

$$H_{\text{eff}} = \dot{\chi}^{(0)} \frac{1}{3} \left\langle \frac{\theta}{\sqrt{\bar{\xi}}} \right\rangle. \quad (24)$$

Noting that $\bar{\xi} = e^{-2\phi}$, we would have at a classical level

$$H_{\text{eff}} = \frac{1}{\int dx \sqrt{\bar{\gamma}(x)}} \int dx \sqrt{\bar{\gamma}(x)} e^{\phi(f^{-1}(x))} \bar{\theta}(x). \quad (25)$$

where, in the equation above, we have considered $\dot{\chi} = 1$.

III. EVALUATION OF THE LOCAL HUBBLE EXPANSION RATE

As we have seen in the previous section, the effective Hubble parameter is given by

$$H_{\text{eff}} = \frac{1}{3} \frac{\langle -3e^{-3\psi_T} \dot{\psi}_T \rangle}{\langle e^{-3\psi_T} \rangle}, \quad (26)$$

where ψ_T is what we called ψ in the previous section. The reason for this change in notation is that in the following we want to denote by ψ the fluctuating part of ψ_T .

We can separate the background contribution from the total ψ_T by writing

$$\psi_T = -\ln(a/a_0) + \psi. \quad (27)$$

There are two ways to set up this separation. In the first, we take $a(t)$ to be a solution to the Friedmann equations in the absence of fluctuations. In this case, the spatial average of the fluctuation ψ only vanishes at linear order in perturbation theory, but not at higher order. This is the view which was taken in [21]. Here, on the other hand, we consider all contributions to the metric which are homogeneous in space to be part of $a(t)$, or, more generally, to be part of the observable that we want to study including the backreaction of the metric perturbation which have a nonzero average. In this manuscript, we consider as our observable the one defined in (26). Hence, the spatial average of ψ vanishes even beyond linear order in perturbation theory. On the other hand, the scale factor $a(t)$ will not naively satisfy the Friedmann equations if fluctuations are present. The clock field χ evolves according to the full metric, not according to the metric which obeys the Friedmann equations in the absence of fluctuations. Hence, we will consider the separation where the spatial average of ψ vanishes.⁵

⁵Note that this separation between background and perturbation has a subtle gauge dependence, so that, using this definition, one cannot make fully gauge-invariant statements. On the other hand, as we shall see, this choice has the advantage of giving easily access to some qualitative results at a nonperturbative level in the leading order of the gradient expansion and we believe that this approach can give a first indication of what the nonperturbative backreaction is. A more rigorous, fully gauge invariant, calculation is left for future work.

We then write Eq. (26) as

$$\begin{aligned} H_{\text{eff}} &= \frac{1}{3} \frac{\langle +3a^3 e^{-3\psi} (H_{\text{hom}} - \dot{\psi}) \rangle}{\langle a^3 e^{-3\psi} \rangle} \\ &= \frac{\langle e^{-3\psi} H_{\text{hom}} \rangle}{\langle e^{-3\psi} \rangle} - \frac{\langle e^{-3\psi} \dot{\psi} \rangle}{\langle e^{-3\psi} \rangle}, \end{aligned} \quad (28)$$

where H_{hom} is the Hubble expansion rate in the absence of perturbations, from which it is evident that in order to obtain the contribution to H_{eff} from the cosmological fluctuations we must compute the quantity

$$\Delta H_{\text{eff}} \equiv - \frac{\langle e^{-3\psi} \dot{\psi} \rangle}{\langle e^{-3\psi} \rangle}. \quad (29)$$

In the following, we will evaluate the above expression for the cosmological background of interest to us, namely an inflationary phase driven by a bare cosmological constant but in presence of a scalar field φ supporting gauge invariant fluctuations. In the absence of fluctuations, the expansion is characterized by a constant Hubble expansion rate H . Like in inflationary cosmology, we assume that the phase of accelerated expansion begins at some time t_i , and we denote by k_i the comoving momentum whose wavelength is equal to the Hubble radius at the initial time. Causal dynamics of the accelerated phase cannot determine anything about fluctuations on larger length scales, and we will introduce a physical infrared cutoff by setting any initial super-Hubble fluctuations to zero.

Let us begin by evaluating ΔH_{eff} to leading order in perturbation theory. By expanding the term $e^{-3\psi}$ in the expression of ΔH_{eff} , we obtain

$$\Delta H_{\text{eff}} \approx - \frac{\langle (1 - 3\psi) \dot{\psi} \rangle}{\langle (1 - 3\psi) \rangle}. \quad (30)$$

The term linear in $\dot{\psi}$ vanishes when taking the average. Hence, we obtain

$$\Delta H_{\text{eff}} \approx - \langle -3\psi \dot{\psi} \rangle. \quad (31)$$

Let us note that, according to our definition, in Eq. (25), $\bar{\theta}(x) = \theta(f^{-1}(x))$ with f defined in Eq. (5). Writing $f^{-1}(x) = (h(t, \vec{x}), \vec{x})$ with $h(t, \vec{x}) = h(t, 0) + h^{(1)}(t, \vec{x}) + \dots$, one finds that in the leading order in the gradient expansion the spatial dependence of ψ and $\dot{\psi}$ is unaffected by the change of variable, since corrections induced by $h^{(1)}$ are suppressed for large scales.

We can spatially Fourier expand the fluctuation ψ

$$\psi = \int d^3k \epsilon_{\mathbf{k}} \psi_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x} + \alpha(k)} \quad (32)$$

where ψ_k represent the amplitudes of the modes (and hence are positive), $\alpha(\mathbf{k})$ are phases, and the ϵ_k are independent Gaussian random variables, i.e.,

$$\langle \epsilon_{\mathbf{k}} \epsilon_{\mathbf{k}'} \rangle = \delta(\mathbf{k} - \mathbf{k}'). \quad (33)$$

Since the fluctuations produced during the inflationary phase have a roughly scale-invariant spectrum [37], we have (neglecting the tilt)

$$P(k) = |\psi_k|^2 k^3 = \text{const}. \quad (34)$$

Therefore,

$$\psi_k \sim k^{-3/2}. \quad (35)$$

To obtain the backreaction effect of super-Hubble modes, we must integrate over all values of k with $k \equiv |\mathbf{k}|$ between the Hubble crossing scale and the infrared cutoff k_i described above. We obtain

$$3\langle \psi(t, x) \dot{\psi}(t, x) \rangle = 3 \int d^3k \psi_k \dot{\psi}_k. \quad (36)$$

To evaluate this term we use the results of the theory of cosmological fluctuations which tells us (see e.g., Eq. (6.56) of [35]) that ψ_k is constant on super-Hubble scales modulo a decaying mode. Thus, we can write ψ_k as

$$\psi_k = A_{0k} + \delta A_k(t), \quad (37)$$

where A_{0k} is the constant mode, and $\delta A_k(t)$ is the decaying mode which in an inflationary background scales as

$$\delta A_k = C_7 A_{0k} e^{-H(t-t_H(k))}, \quad (38)$$

where $t_H(k)$ is the time when the mode k crosses the Hubble radius, and H is H_{hom} . From Eq. (6.56) of [35] it also can be seen that both modes have equal strength at Hubble radius crossing, the coefficient C_7 is positive and its absolute value is of order one.⁶ In particular, this implies that the spectrum of both A_{0k} and $C_7 A_{0k}$ is scale-invariant. Note that the fact that for a super-Hubble fluctuation the amplitude of the adiabatic mode is a constant plus a decaying piece is valid also beyond the perturbative regime (see e.g., [38]). Above, A_{0k} is the contribution considered previously (35), namely

$$A_{0k} \sim k^{-3/2}, \quad (39)$$

⁶The result that the coefficients of both the constant and the decaying modes are of the same order of magnitude at Hubble radius crossing can also be seen by matching the sub-Hubble oscillatory solutions for ψ_k with the corresponding super-Hubble modes at Hubble radius crossing, equating both the amplitude and the time derivative of the modes.

which was obtained since the spectrum of fluctuations produced during the inflationary phase is (almost) scale invariant spectrum. From (38) it immediately follows that

$$\dot{\delta A} = -C_7 A_{0k} H e^{-H(t-t_H)}. \quad (40)$$

Consequently,

$$\begin{aligned} A_{0k} \dot{\delta A}_k &= -C_7 A_{0k}^2 H e^{-H(t-t_H)} \\ &= -C_8 k^{-3} H e^{-H(t-t_H)}, \end{aligned} \quad (41)$$

where C_8 is another positive constant. Integrating this contribution over all super-Hubble modes, therefore, yields

$$\langle \psi(t, x) \dot{\psi}(t, x) \rangle = -C_8 H e^{-Ht} \int_{k_i}^{H e^{Ht}} dk k^2 k^{-3} e^{Ht_H(k)}. \quad (42)$$

Since $e^{Ht_H} = k/H$ we obtain

$$\begin{aligned} \langle \psi(t, x) \dot{\psi}(t, x) \rangle &= -C_8 e^{-Ht} H e^{Ht} \left(1 - \frac{k_i}{H} e^{-Ht} \right) \\ &= -C_8 H (1 - f(t)), \end{aligned} \quad (43)$$

where $f(t)$ is a positive decreasing function of time (always smaller than 1).

Therefore, substituting the above equation into the expression for ΔH_{eff} , given by (31), we obtain

$$\Delta H_{\text{eff}} \approx -3C_8 H (1 - f(t)), \quad (44)$$

where C_8 is positive and $f(t)$ is a positive decreasing function of time. We can see that in the perturbative regime the backreaction effect of super-Hubble cosmological fluctuations yields an increasingly negative contribution to the Hubble parameter.

Now let us analyze the effective Hubble parameter beyond perturbation theory, but in leading order in the gradient expansion. We begin with the previously derived general expression of H_{eff} in Eq. (28). We consider the term

$$\Delta H_{\text{eff}} \equiv - \frac{\langle e^{-3\psi} \dot{\psi} \rangle}{\langle e^{-3\psi} \rangle}. \quad (45)$$

At any time we can Fourier expand the fluctuation field $\psi(x, t)$

$$\psi(x, t) = A(t)g(x, t) \quad (46)$$

where $A(t)$ characterizes the amplitude of the fluctuation and $g(x, t)$ is a function of unit amplitude whose spatial average vanishes (since ψ is a fluctuation whose spatial average vanishes). Neglecting, for a moment, the fact that modes cross the Hubble radius, it would then conclude from the conservation of adiabatic fluctuations on

super-Hubble scales (see e.g., [38]) that the amplitude $A(t)$ has a constant component A_0 and a decaying piece $A_1(t)$, i.e.,

$$A(t) = A_0 + A_1(t), \quad (47)$$

both multiplying the same function $g(x, t)$. Recall that for each Fourier mode of the fluctuation, the two modes have comparable amplitude when the mode exits the Hubble radius, and the second mode afterwards decreases as $\exp(-H(t - t_e))$, where t_e is the time when the mode exits the Hubble radius. If we neglect the fact that new modes cross the Hubble radius (i.e., neglecting the increase of the phase space of super-Hubble modes) the function $g(t, x)$ would be independent of time. At first, we will work in an adiabatic approximation in which we neglect the time-dependence of g . In evaluating (45), we make use of the fact that only the second mode in (47) depends on time, and that the overall amplitude of this mode remains constant when we take into account that new modes are continuously exiting the Hubble radius. Hence, $\dot{\psi} = -HA_1g$, and

$$\Delta H_{\text{eff}} \equiv H \frac{\langle e^{-3A(t)g(x,t)} A_1 g(x,t) \rangle}{\langle e^{-3\psi} \rangle}. \quad (48)$$

The factor $e^{-3A(t)g(x,t)}$ acts as a weighting function. It gives larger weight to values of x where $g(x, t)$ is negative. Hence, the expectation value in the numerator of (48) is negative, and we conclude that

$$\Delta H_{\text{eff}} < 0. \quad (49)$$

Since the phase space of super-Hubble modes is increasing, the effective overall value of A will increase in time (because the phase space of infrared modes is increasing). Hence, the absolute value of $\Delta H_{\text{eff}}^{(1)}$ will be increasing in time, i.e.,

$$\frac{d}{dt} \Delta H_{\text{eff}} < 0. \quad (50)$$

Note that (49) and (50) are the same conclusions obtained in the perturbative analysis.

The inequalities in (49) and (50) are the main results of our analysis. They demonstrate that, to leading order in the gradient expansion, the backreaction of super-Hubble cosmological fluctuations leads to a decrease in the expansion rate which an observer described by a clock field χ (which has a negligible contribution to the energy density) measures. The absolute magnitude of the

backreaction effect increases in time as more modes become super-Hubble. The main new feature of the present analysis (compared to previous work) is that our analysis does not make use of a perturbative expansion in the amplitude of the cosmological perturbations. We stress that a residual gauge dependence is present in this approach, which, we believe, is nevertheless catching at a qualitative level, with really minimal efforts, the right phenomenological behavior of the effect of the backreaction.

We also remind the reader that we have used a *quasi-adiabatic* approximation in which we at any time t consider the phase space of modes which are super-Hubble at time t , and treat it as a time-independent phase space. In this approximation, we compute the magnitude of the change in the Hubble expansion rate, finding $\Delta H_{\text{eff}} < 0$. In a second step, we ask how the modes crossing the Hubble radius change the position space amplitude of the fluctuation, and then reach the conclusion that the absolute value of ΔH_{eff} increases in time. It would be nice to find an analysis which avoids having to make this approximation.

IV. DISCUSSION

We have studied the effect of super-Hubble cosmological fluctuations on the locally measured Hubble expansion rate. We worked in the leading-order gradient expansion, but nonperturbatively in the amplitude of the fluctuations as a novel step in our approach. We consider a large bare cosmological constant which leads to accelerated expansion which in turn generates an (almost) scale-invariant spectrum of cosmological fluctuations on super-Hubble scales. We have shown that the expansion rate measured by a clock field which is not comoving with the dominant matter component obtains a negative contribution from infrared fluctuations, a contribution whose absolute value increases in time. This is the same effect which a decreasing cosmological constant would produce. This supports the conclusion that infrared fluctuations lead to a dynamical relaxation of the cosmological constant [15].

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