Wavefunction for the Universe close to its beginning with dynamically and uniquely determined initial conditions

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(Received 24 July 2018; published 13 November 2018)

In this paper I will first outline an effective field theory for cosmology (EFTC) that is based on the standard model coupled to general relativity and improved with Weyl symmetry. There are no new physical degrees of freedom in this theory, but what is new is an enlargement of the domain of the existing physical fields and of spacetime via the larger symmetry, thus curing the geodesic incompleteness of the traditional theory. Invoking the softer behavior of an underlying theory of quantum gravity, I further argue that it is reasonable to ban higher curvature terms in the effective action, thus making this EFTC mathematically well behaved at gravitational singularities, as well as geodesically complete, thus able to make new physics predictions. Using this EFTC, I show some predictions of surprising behavior of the universe at singularities including a unique set of big-bang initial conditions that emerge from a dynamical attractor mechanism. I will illustrate this behavior with detailed formulas and plots of the classical solutions and the quantum wavefunction that are continuous across singularities for a cosmology that includes the past and future of the big bang. The solutions are given in the geodesically complete global minisuperspace that is similar to the extended spacetime of a black hole or extended Rindler spacetime. The analytic continuation of the quantum wavefunction across the horizons describes the passage through the singularities. This analytic continuation solves a long-standing problem of the singular $(-1/r^2)$ potential in quantum mechanics that dates back to Von Neumann. The analytic properties of the wavefunction also reveal an infinite stack of universes sewn together at the horizons of the geodesically complete space. Finally a comparison with recent papers using the path integral approach in cosmology is given.

DOI: 10.1103/PhysRevD.98.103510

I. INTRODUCTION

This paper presents an extension of work I started ten years ago in the context of 2T-Physics [1,2], and pursued in a series of papers on cosmology and black holes in collaboration with Chen, Steinhardt, Turok, Araya, and James, where the role of Weyl symmetry, in the geodesically complete form that emerges from 2T-physics, was emphasized [2–15]. By now, foundational ideas are better understood and in this paper applied to the quantum wavefunction for the universe. The current paper highlights the main concepts and new results on classical cosmological solutions, the quantum wavefunction, and associated propagator.

The paper is organized as follows. Section II introduces the geodesically complete fundamental theory and its attractive features, while Sec. III discusses its minisuperspace, its geometrical structure and the transformation between systems of minisuperspace coordinates that highlights a global system analogous to the Kruskal-Szekeres global coordinates for a black hole. In Sec. V explicit analytic classical solutions of the minisuperspace are given; these display an attractor mechanism leading to unique dynamically determined initial conditions at the big bang, and help establish a theorem on the behavior of all the degrees of freedom (d.o.f.) at cosmological singularities. The Wheeler de Witt equation (WdWe) that also leads to the same attractor mechanism is solved analytically in three stages. First, in Sec. IV the WdWe is setup using geodesically complete global coordinates, quantum ordering is settled globally, a 2-step approximation scheme is devised, and the general physical behavior of the wavefunction is qualitatively determined through an effective potential in a Schrödinger-like equation. Second, in Sec. VI the continuity of the wavefunction is determined across the horizons in the global minisuperspace. Third, in Sec. VII the full solution for the wavefunction containing no unknown parameters is explicitly given, and its predicted form at the big bang is displayed. Finally an overall discussion is given in Sec. VIII; this highlights the results of this paper, outlines areas for future progress, and contrasts this work to other recent papers that discuss the quantization of minisuperspace in the path integral approach, including the quantum wavefunction and propagators.

II. THE FUNDAMENTAL THEORY

The Lagrangian for the standard model coupled to general relativity and improved with Weyl symmetry to obtain the *geodesically complete* version of this theory, without adding new *physical degrees of freedom*, is [8]

$$\mathcal{L}(x) = \sqrt{-g}\begin{pmatrix} L_{\mathrm{SM}}(A_\mu^{\gamma,W,Z,g},\psi_{q,l},\nu_R,\chi) + g^{\mu\nu}(\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - D_\mu H^\dagger D_\nu H) \\ -(\frac{\lambda}{4}(H^\dagger H - w^2\phi^2)^2 + \frac{\lambda'}{4}\phi^4) + \frac{1}{12}(\phi^2 - 2H^\dagger H)R(g) \end{pmatrix}. \tag{1}$$

In the first line, L_{SM} contains the usual d.o.f. of the extended standard model minimally coupled to gravity, namely, gauge bosons $A_\mu^{\gamma,W,Z,g}$, quarks and leptons $\psi_{q,l}$, right handed neutrinos ν_R, some candidate(s) for dark matter χ, and their $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ invariant interactions with the Higgs doublet H, as well as the additional singlet boson ϕ that can couple only to ν_R,χ because of the electroweak gauge symmetry. The remaining terms in (1) give the kinetic terms for the conformally coupled scalars (ϕ, H), their renormalizable and scale invariant potential energy capable of dynamically generating the Higgs mass [16], and their locally scale invariant unique nonminimal couplings to curvature $R(g)$. Under the local $\lambda(x)$ scale transformations, $g_{\mu\nu}\to\lambda^{-2}g_{\mu\nu}$ $\phi\to\lambda\phi$, $H\to\lambda H$, $\psi_{q,l}\to\lambda^{3/2}\psi_{q,l}$, $A_\mu^{\gamma,W,Z,g}\to$ unchanged, the Lagrangian (1) transforms to a total derivative and therefore the action is invariant. Because the Weyl symmetry can remove one gauge d.o.f. This version of the standard model coupled to gravity has *no new physical degrees of freedom although there is new physics because the domain of the physical fields are considerably enlarged*. This action cannot contain any dimensionful constants. Weyl invariant renormalization[1] of the nongravitational part of this action maintains the local Weyl symmetry by taking the renormalization scale to be the field ϕ, thus allowing only those counterterms that run as a function of Weyl invariant logarithms such as $\ln(H^\dagger H/\phi^2)$ [8,9].

Although there exist in the literature other forms of Weyl invariant field couplings to gravity (in particular using the "Stuckelberg trick"), usually those are geodesically incomplete. Incompleteness is a sign of unwittingly suppressing physical effects and should be considered to be a serious problem looking for a cure. As discussed in [2–15], in the case of only two scalar fields, the form in (1) is unique and geodesically complete, furthermore all other incomplete forms can be obtained from this one by field redefinitions [8] and artificially deleting patches of field space. In ([8]) it is shown how more scalar fields can be included in the geodesically complete theory. In the current paper, I continue to explore the possibility that the minimal case (1) may be sufficient.

One of the virtues of this formalism is that it explains how the dimensionful constants that fill the universe emerge from the same source. This is seen by choosing a Weyl gauge, dubbed "c-gauge" [1,8] that fixes $\phi(x)=\phi_0$ (a constant) for all x^μ. Although several other gauge choices [11] are convenient for various computations of *gauge invariants*, the c-gauge is most convenient to recognize the low energy physics. In the c-gauge, the usual standard model with *no additional degrees of freedom*, containing all low energy dimensionful parameters, is seen to arise from interactions with the scalars (ϕ, H). In particular, the gravitational constant G, the Higgs vacuum value, and cosmological constant Λ, are

$$(16\pi G)^{-1} = \phi_0^2/12, \qquad \langle H^\dagger H\rangle = w^2\phi_0^2,$$
$$(16\pi G)^{-1}2\Lambda = \frac{\lambda'}{4}\phi_0^4. \tag{2}$$

Universe-filling constants such as these raise the question whether these are independent or related to each other. There is no literature that analyzes this question of cosmological significance. It is hard to imagine three different mechanisms that would generate such an outcome. In the current formalism, although the hierarchy of scales (which is achieved through dimensionless parameters) is not explained, a unique source for all universe-filling dimensionful parameters is identified. That such universal parameters are not independent but are actually related to the same source, resolves a long-standing puzzle for this author, thus providing more credence to the current approach with Weyl symmetry.

Another significant feature introduced by the Weyl symmetry is the coefficient of curvature, $\frac{1}{12}(\phi^2 - 2H^\dagger H)R(g)$, or a gauge fixed version such as the c-gauge, $((16\pi G)^{-1} - \frac{2}{12}H^\dagger(x)H(x))R(g(x))$. This relative sign is obligatory and cannot be altered (otherwise a positive gravitational constant is not possible) [8]. The question arises whether the dynamics of the theory forces the sign to flip in some regions of spacetime x^μ. It was found through analytic solutions of the equations of motion that in fact such a sign flip is the generic behavior [2–15]. In patches of spacetime where the sign is negative, gravity is repulsive, hence antigravity rules in those regions of spacetime.

[1]In this Weyl invariant renormalization scheme, the usual trace anomaly, of the energy momentum tensor of all matter except ϕ, is still present, but it is cancelled by an equal anomaly due to the additional term in the full energy momentum tensor containing the extra field ϕ [17]. Thus, the local Weyl symmetry survives in the quantized theory.

The *sign flip from positive to negative can occur only at gravitational singularities* [see explanation in Eq. (7)], therefore from the perspective of observers like us in the gravity sector(s), antigravity occurs only on the other side of cosmological or black hole type singularities. For geodesic completeness, all gravity and antigravity patches must be included.

This is the structure predicted by the symmetries of 2T-physics for relativistic 1T-physics [1] and it was one of the main reasons to start an investigation of this topic in 2008. It turns out that in addition to 2T-physics there are other cherished symmetries in 1T field theory that require the same structure, so this is not just an isolated weird field theory. It was later noted that Weyl-symmetric supergravity [8,18,19], as well as usual supergravity [20], also predict a similar sign-changing structure due to the Kaehler potential, but this was swept under the rug in investigations of supergravity [20]. Thus, the possibility of a sign flip from gravity to antigravity, that geodesically completes the spacetime, remained unknown until the work in [2–15]. The sign-changing feature of the curvature term is an essential part of geodesic completeness in both spacetime as well as in field space [2–15]. For answers to questions raised about unitarity or instability due to this sign flip see [13]. In short, by now there remains no concerns about unitarity, instability, or the physical meaning of this setup, although more work is welcome to better understand the interesting physics as indicated in [13].

A new feature introduced formally in the current paper (carried out casually in [2–15]) is how to take into account the smoothing effects of a quantum theory of gravity as part of an effective field theory for cosmology (EFTC). The EFTC would also be applicable to black holes, black strings etc. [12]. Although currently there is no universally accepted theory of quantum gravity (QG), one of its universally expected features is that gravitational singularities are softer or even possibly nonexistent in a successful QG. Assuming that this softer behavior is true in principle, in attempts to capture general effects of QG in the form of an EFTC, the effective theory would be physically wrong if the EFTC is too singular. In an EFTC that is compatible with the smoother behavior of QG there should be some restriction on which singular curvatures (or their powers) may appear in the equations of motion when it is being applied close to singularities.[2] One reasonable way to insure this, is the following proposal which is based on some past success: namely, define the EFTC to be given by Eq. (1) that includes the $R(g)$ term, with the additional condition of not admitting any other higher curvature terms *in the effective action* when it is being applied near singularities. This restriction may seem ad-hoc, but the fact that, in practice, it produces just the desired smoother mathematical properties of a workable model including singularities may be taken as its temporary justification. Namely, this EFTC turns out to be sufficiently well-behaved mathematically, as well as being geodesically complete, despite the presence of curvature singularities in the form of $R(g)$ and $R_{\mu\nu}(g)$ that do appear in its equations of motion. Higher nontrivial curvatures and/or their powers exists in the relevant manifolds but *these terms do not appear in the action or equations of motion* derived from the proposed EFTC. Thanks to the underlying Weyl symmetry that is still present, and that can transform curvatures to less singular expressions in various gauges, the singular terms turn out to be mathematically manageable in solving equations, computing *gauge invariant physical quantities*, and establishing geodesic completeness, as already demonstrated amply in [2–15]. More along these lines will become apparent in the remainder of this paper.

[2]For example, string theory makes definite predictions of higher curvature terms. Those are applicable only at low energies, and not at all close to the singularities. At the Planck scale string theory provides a totally different and nonsingular description of the physics, but this is not yet well understood. In any case, the high curvature terms are absent near gravitational singularities.

III. GEODESICALLY COMPLETE MINISUPERSPACE

The Friedmann equation, as parametrized in the context of the ΛCDM model [21–24] provides an approximate phenomenological parametrization of the evolution of the universe in terms of some constant *dimensionless* measured parameters Ω_i

$$\frac{H^2(x^0)}{H_0^2} = \Omega_\Lambda + \frac{\Omega_K}{a_E^2(x^0)} + \frac{\Omega_m}{a_E^3(x^0)} + \frac{\Omega_r}{a_E^4(x^0)} + \frac{\Omega_\sigma + \Omega_\alpha}{a_E^6(x^0)} + \cdots, \tag{3}$$

where $a_E(x^0)$ is the scale factor in the *Einstein frame*, $H(x^0)$ is the Hubble parameter, H_0 is the Hubble constant, the $[\Omega_\Lambda, \Omega_K, \Omega_m, \Omega_r, \Omega_\sigma, \Omega_\alpha]$ are associated to the energy densities per unit volume respectively for [dark energy, curvature, massive matter (dark and baryonic), radiation (massless relativistic matter), scalar field, anisotropy]. According to data, $\Omega_\Lambda = 0.692 \pm 0.012$, $\Omega_m = 0.308 \pm 0.012$ show that dark energy and dark matter dominate the energy balance today (i.e., when $a_E(x^0_{\text{today}}) = 1$). Radiation is small such that $\Omega_m + \Omega_r \simeq 0.31$, $\Omega_K = (1 - \sum_{i\neq K}\Omega_i) = 0.0002 \pm 0.0026$ is computed from all the other Ω_i, finally $(\Omega_\sigma, \Omega_\alpha)$ are no greater than the error bars set on the other parameters.

As the universe expands $a_E(x^0) \to \infty$, Ω_Λ will dominate the future accelerated expansion of the universe. On the other hand, in the early universe, as $a_E(x^0) \to 0$, no matter how small the parameters $(\Omega_\sigma, \Omega_\alpha)$ may be, the dominant term is $(\Omega_\sigma + \Omega_\alpha) a_E^{-6}(x^0)$, and next are the terms in (3) in reverse order, with Ω_Λ the least influential. Hence a scalar field and anisotropy combined denominate the d.o.f. that govern the evolution of the universe close to cosmological

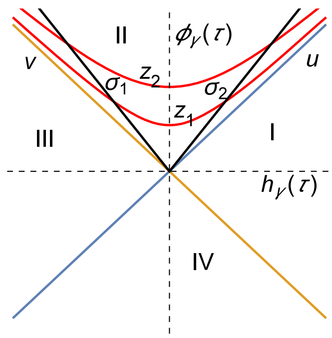

FIG. 1. Minkowski (ϕ_γ, h_γ) versus Rindler (z, σ) coordinates. In region II, the parabolas are at fixed values of z, $0 < z_1 < z_2 < \infty$, and the rays are at fixed values of σ, $-\infty < \sigma_1 < \sigma_2 < \infty$. Similarly in regions I–IV.

singularities (big bang, big crunch), and these cannot be neglected in any approach that attempts to understand the very beginning. The typical cosmologist gives up at that point, however in this paper I will show that persisting in this study leads to unique initial conditions.

I emphasize that Eq. (3) *for* a_E *is in the Einstein frame*. The spacetime in this frame is geodesically incomplete but, via local scale (Weyl) invariance, it can be extended to the complete spacetime shown in Fig. 1, where the era after the big bang described by Eq. (3), occupies only the patch labeled as the future quadrant II, as explained below.

The EFTC model in (1) describes the evolution of the universe consistently with (3), but in more detail, in terms of the so-called "minisuperspace" d.o.f. These consist of the scale factor $a(\tau)$, anisotropy d.o.f. $\alpha_{1,2}(\tau)$ in the metric below (Bianchi I or VIII or IX), and the Higgs field $h(\tau)$ in the unitary gauge $H = (0, h/\sqrt{2})$,

$$ds^2 = a^2(\tau)(-(d\tau)^2 e^2(\tau) + ds_3^2)$$
$$ds_3^2 = e^{2\alpha_1(\tau)}(e^{2\sqrt{3}\alpha_2(\tau)} d\sigma_x^2 + e^{-2\sqrt{3}\alpha_2(\tau)} d\sigma_y^2) + e^{-4\alpha_1(\tau)} d\sigma_z^2. \tag{4}$$

Then the minisuperspace action, $S_{\rm mini} = \int d\tau \mathcal{L}_{\rm mini}$, follows directly [3] from the EFTC in (1) by dimensional reduction, keeping only the τ-dependence of fields

$$\begin{aligned}\mathcal{L}_{\rm mini} = &\frac{1}{2e}[-(\partial_\tau(\phi a))^2 + (\partial_\tau(ha))^2 \\ &+ a^2(\phi^2 - h^2)((\partial_\tau\alpha_1)^2 + (\partial_\tau\alpha_2)^2)] - \frac{e}{2}V \\ V = &\, a^4(\phi^2 - h^2)^2 f\left(\frac{h}{\phi}\right) + a^2(\phi^2 - h^2)V_K(\alpha_1, \alpha_2) + \Omega_c\end{aligned} \tag{5}$$

The different parts of the potential energy, $\Omega_c, f(h/\phi)$, $V_K(\alpha_1, \alpha_2)$ come from the following sources. The parameter Ω_c is related to the energy density of all the conformally invariant matter described by the standard model term $L_{\rm SM}$ in (1), when this matter is approximated by a "conformal dust" energy momentum tensor. The T_{00} component for $L_{\rm SM}$ then has the form Ω_c/a^4, just like conformally invariant radiation appears in the Freedman equation (3). So, the coefficient $\Omega_c = \Omega_m + \Omega_r \simeq 0.31$, includes dark matter, baryonic matter, as well as radiation. The Higgs potential that appears in (1) is written as, $V(\phi, h) = (\phi^2 - h^2)^2 f(\frac{h}{\phi})$, and the anisotropy potential that arises from the metric (4) is written as $(\phi^2 - h^2)V_K(\alpha_1, \alpha_2)$, where $V_K(\alpha_1, \alpha_2)$ was computed by Misner [25]. These are given by[3]

$$\begin{aligned} f(h/\phi) &\equiv 2\frac{\Omega_\lambda((h/\phi)^2 - w^2)^2 + \Omega_\Lambda}{(1-(h/\phi)^2)^2}, \\ &\begin{cases}\Omega_\Lambda = \frac{\lambda'}{4}\frac{3}{4\pi}\left(\frac{m_P t_P}{H_0 t_P}\right)^2 \simeq 0.692 \\ \Omega_\lambda = \frac{\lambda}{4}\frac{3}{4\pi}\left(\frac{m_P t_P}{H_0 t_P}\right)^2 \simeq 10^{120}\end{cases}, \\ V_K(\alpha_1, \alpha_2) &\equiv \frac{k|\Omega_K|}{4k-1}\begin{pmatrix} e^{-8\alpha_1} + 4e^{4\alpha_1}\sinh^2(2\sqrt{3}\alpha_2) \\ -4ke^{-2\alpha_1}\cosh(2\sqrt{3}\alpha_2)\end{pmatrix}, \\ &|\Omega_K| \simeq 0.0002, \end{aligned} \tag{6}$$

where m_P, t_P are the Planck mass and time. In $V_K(\alpha_1, \alpha_2)$ the parameter $k = (0, -1, +1)$ is used to distinguish the 3-dimensional anisotropic (flat, open, closed)-metrics, Bianchi I,VIII, IX respectively. Note that, in the chosen units explained in footnote (3), $\Omega_\lambda \sim 10^{120}$ is huge. However, this term in the Higgs potential is suppressed because, just after the electroweak phase transition (EW), the Higgs sits at the minimum of its potential, $|h(\tau)/\phi(\tau)| \to |h_0/\phi_0| = w \sim 10^{-17}$, during most of the cosmological evolution of the universe.

A. Mini Weyl symmetry, gauges and transformations among them

$S_{\rm mini}$ is invariant under local rescaling (Weyl) transformations using the arbitrary time dependent gauge parameter $\lambda(\tau)$, namely $a \to \lambda^{-1}a, \phi \to \lambda\phi, h \to \lambda h, \alpha_{1,2} \to \alpha_{1,2}$. There are three gauge dependent minisuperspace d.o.f. (a, ϕ, h)

[3]In (5) I choose units such that, the time parameter τ is the conformal time x^0 in (3) rescaled by the Hubble time, $\tau \equiv H_0 x^0$; the dimensionful scalar d.o.f. ϕ, h in (1) are rescaled by a factor of $\phi_0 = \sqrt{12/16\pi G}$ defined in (2), $(\phi, h) = \phi_0(\bar{\phi}, \bar{h})$, so that the corresponding symbols appearing in the cosmological analysis below are the dimensionless $(\bar{\phi}, \bar{h})$. However, to avoid a proliferation of symbols, instead of $(\bar{\phi}, \bar{h})$ the same symbols (ϕ, h) will be understood to mean $(\bar{\phi}, \bar{h})$ when there is no confusion. Similarly, the dimensionless anisotropy d.o.f. $\alpha_{1,2}$ in (4) are rescaled by ϕ_0 as compared to previous publications [2–15]. With this choice of units the minisuperspace action below contains the same dimensionless parameters Ω_i that appear in the phenomenological parametrization (3) of the Friedmann equation.

while $\alpha_{1,2}$ are scale invariant. Other scale invariants include $(a\phi, ah, h/\phi)$. One may choose a Weyl gauge in which some combination of (a, ϕ, h) is gauge fixed for all τ.

The "γ-gauge" is defined by setting the scale factor to 1 for all τ, $a_\gamma(\tau) = 1$, while $\phi_\gamma(\tau)$, $h_\gamma(\tau)$ along with $\alpha_{1,2}(\tau)$ are the remaining dynamical d.o.f. The label γ emphasizes that the d.o.f. are defined in this gauge, however (ϕ_γ, h_γ) are actually gauge invariants since $(a\phi, ah) = (1\phi_\gamma, 1h_\gamma)$.[4] As will be clarified below, (ϕ_γ, h_γ) turn out to be global d.o.f. that cover all the patches of the geodesically complete mini-superspace partly shown in Fig. 1. It is useful to define $z(\tau) \equiv (\phi_\gamma^2 - h_\gamma^2)$ and the $\mathrm{sign}(z(\tau)) \equiv \varepsilon_z(\tau)$. The sign of $(\phi^2 - h^2)$ cannot be changed under local Weyl rescalings, therefore $\varepsilon_z = \mathrm{sign}(\phi_\gamma^2 - h_\gamma^2) = \mathrm{sign}(\phi^2 - h^2)$ is gauge invariant, and $\varepsilon_z(\tau) = \pm 1$ distinguishes between gravity/antigravity sectors at any given τ as seen from Eq. (1).

By contrast, the Einstein frame with its own $a_E(\tau)$ that appears in phenomenological equations such as (3), emerges in the "E-gauge" which is defined by, $(\phi_E^2 - h_E^2) = (\phi_E - h_E)(\phi_E + h_E) = \varepsilon_z(\tau)$, for all τ, and parametrized by $(\phi_E + h_E) = \pm' e^{\sigma(\tau)}$, and $(\phi_E - h_E) = \pm' \varepsilon_z e^{-\sigma(\tau)}$, where $\pm'$ is an additional set of signs that distinguish various regions in Fig. 1. The traditional Einstein-Hilbert theory corresponds to taking only the patch $(\phi_E + h_E) > 0$ and $(\phi_E - h_E) > 0$, which corresponds to $\pm' \to +$ and also $\varepsilon_z(\tau) \to +1$, so that the conventional theory is defined in the geodesically incomplete future quadrant shown in Fig. 1.

By comparing gauge invariants in these two gauges, such as $a^2(\phi^2 - h^2) = a_E^2 \varepsilon_z = 1(\phi_\gamma^2 - h_\gamma^2) = z$, one learns

$$a_E^2 = |z| = |\phi_\gamma^2 - h_\gamma^2|. \tag{7}$$

So the a_E in the Friedmann equation is $a_E(\tau) = +|z(\tau)|^{1/2} = +|\phi_\gamma^2 - h_\gamma^2|^{1/2}$, noting that z can be positive (gravity sectors II ad IV in Fig. 1) or negative (antigravity sectors I&III in Fig. 1). From (7) it is clear that the singularity in the Einstein frame, $a_E^2 = 0$, occurs only when $(\phi_\gamma^2 - h_\gamma^2)$ vanishes, but when this vanishes in the γ-gauge, $(\phi^2 - h^2)$ in any gauge must also vanish since the sign of this quantity is Weyl gauge invariant. The same argument holds for all gravitational singularities (including black holes) in the Einstein frame, hence these occur precisely when the coefficient of R in the original Weyl invariant action (1) changes sign.

Similarly, by considering another set of gauge invariants, $(a\phi, ah) = (a_E\phi_E, a_E h_E) = (\phi_\gamma, h_\gamma)$, one finds the following transformation between the global coordinates (ϕ_γ, h_γ) and the patchy E-frame coordinates (z, σ), both sets being Weyl invariants,

$$\begin{aligned} &u = \phi_\gamma + h_\gamma = \pm' \sqrt{|z|} e^{\sigma}, \\ &v = \phi_\gamma - h_\gamma = \pm' \sqrt{|z|} e^{-\sigma} \varepsilon_z, \qquad -\infty < \phi_\gamma, h_\gamma < \infty, \\ &z = \phi_\gamma^2 - h_\gamma^2 = uv, \\ &\sigma = \frac{1}{2} \ln \left| \frac{\phi_\gamma + h_\gamma}{\phi_\gamma - h_\gamma} \right| = \frac{1}{2} \ln \left| \frac{u}{v} \right|; \qquad -\infty < z, \sigma < \infty. \end{aligned} \tag{8}$$

For low energy physics, one should also keep track of the c-gauge, $\phi_c(x^\mu) = \phi_0 = 1$ (in the units of footnote 3), that was used to identify the universal constants (2) and all low energy physics d.o.f. Using the Weyl gauge invariants h/ϕ and $a\phi$ one obtains $h/\phi = h_E/\phi_E = h_\gamma/\phi_\gamma = h_c/1$ and $a\phi = a_E\phi_E = 1\phi_\gamma = a_c 1$. Hence the Weyl invariant low energy Higgs field h_c and scale factor a_c are written in terms of the Weyl invariant cosmologically global fields (ϕ_γ, h_γ), and the Weyl invariant patchy fields (z, σ) of the E-gauge (related to a_E used in cosmological phenomenology as in (3)), as follows

$$\begin{aligned} h_c &= \frac{h_\gamma}{\phi_\gamma} = \frac{\varepsilon_z e^{2\sigma} - 1}{\varepsilon_z e^{2\sigma} + 1}, \\ a_c &= \phi_\gamma = \pm' \frac{1}{2} \sqrt{|z|} (e^{\sigma} + e^{-\sigma} \varepsilon_z). \end{aligned} \tag{9}$$

Note that (h_c, a_c) are also global variables (i.e., not patchy). At the observed low energies in today's era, $\varepsilon_z(\tau) = +1$, in the future patch $\pm' \to +$, in Fig. 1, we have $\sigma \simeq h_c \simeq \frac{240\ \mathrm{GeV}}{10^{19}\ \mathrm{GeV}} \lll 1$ and $a_E(\tau) \simeq a_c(\tau) = \phi_\gamma(\tau)$. However, cosmologically none of these quantities are small or close to each other numerically, so their distinct meanings as given in (8), (9) should be kept in mind when discussing physics at various energy regimes and various cosmological eras.

The transformation of coordinates displayed in (8) is precisely the same as the transformation between 2-dimensional flat Minkowski coordinates (ϕ_γ, h_γ) and *extended* Rindler coordinates (z, σ) as used recently in [14], but now understood as part of the d.o.f. in minisuperspace

$$ds_{\mathrm{mini}}^2 = -du dv = -d\phi_\gamma^2 + dh_\gamma^2 = -(4z)^{-1} dz^2 + (z) d\sigma^2. \tag{10}$$

As shown in Fig. 1, the γ-frame (ϕ_γ, h_γ) or (u, v) cover globally all four quadrants of extended Rindler space (see [14] for more detail) with an unambiguous identification of timelike (ϕ_γ) and spacelike (h_γ) coordinates is a geodesically complete minisuperspace. The curvature singularity

[4]The γ-gauge is also available in the full spacetime x^μ. It amounts to fixing the determinant of the metric $g_{\mu\nu}(x^\mu)$ to one for all x^μ, i,e, $(-g(x^\mu)) = 1$. So the γ-gauge may also be called the unimodular gauge for gravity.

that occurs in the E-frame, when $a_E^2 = 0$, corresponds to $z = 0$ which translates to either $u = 0$ or $v = 0$ in the flat global space of Eq. (10). So the cosmological bang or crunch singularities of the E-frame can occur only at the horizons of the γ-frame that form the boundaries of the four Rindler quadrants in Fig. 1.

This globally flat 2D-Minkowski geometry is the intrinsic geometrical property of the scale invariant minisuperspace in any frame, including the geodesically completed E-frame written in terms of z as in (10). This is the underlying reason for how it is possible to go through cosmological singularities—that amount to horizons in global coordinates—to complete geodesics in complete field space in minisuperspace, as well as space-time x^μ, as explored extensively in [2–15].

IV. QUANTUM WAVEFUNCTION—1

The minisuperspace action (5) can now be expressed in the γ-gauge in terms of the Weyl invariant (ϕ_γ, h_γ) d.o.f. by setting $a_\gamma = 1$. From this point on, the γ label will be suppressed for simplicity and (ϕ, h) will be understood to mean (ϕ_γ, h_γ) when there is no confusion.

$$\begin{aligned} \mathcal{L}^\gamma_{\rm mini} &= \frac{1}{2e}[-\dot\phi^2 + \dot h^2 + (\phi^2 - h^2)(\dot\alpha_1^2 + \dot\alpha_2^2)] - \frac{e}{2}V, \\ V &= (\phi^2 - h^2)^2 f\left(\frac{h}{\phi}\right) + (\phi^2 - h^2)V_K(\alpha_1, \alpha_2) + \Omega_c, \\ \mathcal{H} &= \left[-\pi_\phi^2 + \pi_h^2 + \frac{1}{\phi^2 - h^2}(\pi_1^2 + \pi_2^2) + V\right] = 0. \end{aligned} \tag{11}$$

The last line is the constraint that follows from the e equation of motion, $\mathcal{H} = \partial S_{\rm mini}/\partial e(\tau) = 0$. This is the vanishing Hamiltonian $\mathcal{H}$ expressed in terms of the canonical momenta $(\pi_\phi = -\dot\phi/e, \ldots, \pi_2 = (\phi^2 - h^2)\,\dot\alpha_2/e)$ for any $e(\tau)$. Note that there is no need to gauge fix the lapse function $e(\tau)$ due to τ reparametrization symmetry since the properties of the canonical phase space in $\mathcal{H}$ is insensitive to a gauge choice for $e(\tau)$. Straightforward quantization rules applied to this system, and applying the constraint on physical states, $\mathcal{H}\Psi = 0$, produces the Wheeler deWitt equation (WdWe) that follows from $\mathcal{L}^\gamma_{\rm mini}$,

$$\left[\partial_\phi^2 - \partial_h^2 - \frac{1}{\phi^2 - h^2}(\partial_1^2 + \partial_2^2) + V\right]\Psi = 0. \tag{12}$$

There is no ambiguity of quantum ordering problems in the quantum phase space as it appears in $\mathcal{H}$ above in contrast to other choices of minisuperspace parametrizations such as $(z, \sigma, \alpha_1, \alpha_2)$. Choosing the (ϕ, h) global coordinates [which are the ones naturally appearing in the full action (1)], as the preferred d.o.f. in the definition of the quantum theory, resolves once and for all this long-standing annoying quantum ambiguity [26,27].[5]

Having resolved the quantum ordering, the WdWe can now be rewritten in the (z, σ) basis in the Einstein frame (i.e., in terms of a_E used by phenomenologists) by using the coordinate transformation (8) and noting the nontrivial ordering that is uniquely predicted in the z variable, $\partial_\phi^2 - \partial_h^2 = 4\partial_u\partial_v = 4z\partial_z^2 + 4\partial_z - z^{-1}\partial_\sigma^2$, while the rest is straightforward. The WdWe in the (z, σ) basis is then manipulated to the following nonrelativistic Schrödinger-type equation form

$$\begin{aligned} &\left[-\partial_z^2 - \frac{1}{4z^2}(1 - \partial_1^2 - \partial_2^2 - \partial_\sigma^2) - \frac{\Omega_c}{4z} \right. \\ &\left. \quad - \frac{1}{4}V_K(\alpha_1, \alpha_2) - \frac{z}{4}V(\sigma, \varepsilon_z)\right](\sqrt{z}\Psi) = 0 \end{aligned} \tag{13}$$

where $V(\sigma, \varepsilon_z) = f(h/\phi)$ after using (8). The term $1/4z^2$ arises from rewriting $(4z\partial_z^2 + 4\partial_z)\Psi = \sqrt{z}(4\partial_z^2 + z^{-2})(\sqrt{z}\Psi)$.

Thinking of z as a "time" variable, (13) can be viewed as a time-dependent Hamiltonian problem in Schrödinger-equation-type quantum mechanics for which well known time-dependent methods exist to make progress and interpret the physics. Nevertheless, this is a difficult partial differential equation in the presence of the Higgs and anisotropy potentials $V(\sigma, \varepsilon_z)$, $V_K(\alpha_1, \alpha_2)$, so numerical methods will be needed to analyze it fully. However, there is no substitute for analytic approximations that can guide such numerical efforts. This provides an incentive to look for circumstances that make it possible to find approximate analytic methods to solve (13). I suggest the following approach.

It was noted following (6) that, the large term $\Omega_\lambda \sim 10^{120}$ in the Higgs potential is suppressed because, just after the electroweak phase transition (EW), the Higgs sits at the minimum of its potential, $|h(\tau)/\phi(\tau)| \to |h_0/\phi_0| = w \sim 10^{-17}$, during most of the cosmological evolution of the universe. Furthermore before EW and close to the singularity $z \simeq 0$ in the very early universe, the Higgs and anisotropy potential terms in (13) are subdominant due to the factors of a vanishing z. Therefore, the terms involving $V_K(\alpha_1, \alpha_2)$, $V(\sigma, \varepsilon_z)$ in (13) can be neglected in the

[5]The ambiguity in the ordering prescription proposed in [27] is to write the kinetic terms in (12) in the form of the Klein-Gordon operator with an added curvature term with an unknown ξ coefficient, $(\nabla^2 + \xi R(g) + V)\Phi = 0$, where $\nabla^2\Phi = (-g)^{-1/2}\partial_\mu((-g)^{1/2}g^{\mu\nu}\partial_\nu\Phi)$. In the current case the metric is conformally flat, $ds^2 = -d\phi^2 + dh^2 + (\phi^2 - h^2)(d\alpha_1^2 + d\alpha_2^2) = -dudv + uv(d\alpha_1^2 + d\alpha_2^2)$, and its curvature is $R(g) = 6(\phi^2 - h^2)^{-1}$. In this expression replacing Φ by $\Phi = (\phi^2 - h^2)^{-1/2}\Psi$ and also fixing $\xi = -1/6$, reproduces precisely Eq. (12) for Ψ. This shows that the straightforward no-need-to-order prescription applied to obtain (12) is in agreement with [27] but only when $\xi = -1/6$, indicating that the ambiguity in [27] is fully resolved by the preferred quantum *global coordinates*.

computation of the wavefunction near $z \simeq 0$ before EW, as well as well as after EW because the Higgs settles down to the bottom of the potential.

Based on the comments in the previous paragraph, I observe that, as the system moves away from singularities, the d.o.f. $\vec{s} = (\alpha_1, \alpha_2, \sigma)$ will quickly descend to the ground state in their respective potential energies, and stay there during most of the evolution of the universe. This observation is consistent with general physical behavior of d.o.f. subjected to attractive time dependent potentials, which is the case in the current problem. It is also consistent both with cosmological data as well as the behavior of classical solutions of these d.o.f. as studied in the past, both analytically [3–6] and numerically [9]. I use these facts to devise the following 2-step strategy to approximate the effects of the potentials in cosmological calculations. Briefly,

(1) The first step of this strategy is an approximation that replaces the functions $V(\sigma, \varepsilon_z), V_K(\alpha_1, \alpha_2)$ by constant values at their lowest energy configuration

$$V_K(\alpha_1, \alpha_2) \to -k|\Omega_K|, \qquad V(\sigma, \varepsilon_z) \to 2\Omega_\Lambda. \quad (14)$$

In this step, (13) turns into the following much simpler second order ordinary differential equation that has analytic solutions,

$$(-\partial_z^2 + V(z))(\sqrt{z}\Psi) = 0,$$
$$V(z) = -\left(\frac{\vec{p}^2+1}{4z^2} + \frac{\Omega_c}{4z} - \frac{\Omega_K}{4} + \frac{\Omega_\Lambda}{2}z\right). \quad (15)$$

In this form Ψ is taken to momentum space thus diagonalizing the operator $(-\partial_1^2 - \partial_2^2 - \partial_\sigma^2) \to \vec{p}^2$, where $\vec{p} = (p_1, p_2, p_3)$ are the canonical conjugates to $\vec{s} = (\alpha_1, \alpha_2, \sigma)$. Note also in (15) there is an accidental SO(3) symmetry that rotates the vectors $(\vec{p}, \vec{s})$. Then $\Psi_{\pm|\vec{p}|}(z)$ are the two linearly independent solutions of (15). The general solution is the superposition of the complete set of states in momentum space, namely

$$\Psi(z, \vec{s}) = \int d^3p e^{-i\vec{p}\cdot\vec{s}}(A_+(\vec{p})\Psi_{+|\vec{p}|}(z) + A_-(\vec{p})\Psi_{-|\vec{p}|}(z)), \quad (16)$$

and this needs to be continuous in the geodesically complete superspace in Fig. 1. The latter is not trivial as discussed in Sec. VI.

(2) The second step of the strategy is to ensure that the momenta $\vec{p}$ are limited in magnitude because these d.o.f. will be sitting in their ground state, so their kinetic energy cannot exceed the total energy of the respective ground states. The limit set on the size of p_3^2 has the physical interpretation of the cosmological parameter Ω_σ that measures the energy density of the scalar field in the Friedmann equation (3). Similarly, the limit set on $(p_1^2 + p_2^2)$ has the interpretation of Ω_α that measures the energy density of anisotropy. This limitation will be taken into account by requiring the wavepacket coefficients to behave like Gaussians (or something of that form) controlled by the parameters $(\Omega_\sigma, \Omega_\alpha)$

$$A_\pm(\vec{p}) \sim e^{-(p_1^2+p_2^2)/2\Omega_\alpha} e^{-p_3^2/2\Omega_\sigma}. \quad (17)$$

Before an analytic solution of (15), it is valuable to understand the qualitative physical behavior of the wavepacket $(\sqrt{z}\Psi(z, \vec{s}))$ by examining the potential $V(z)$ in (15) that is plotted in Fig. 2. The values of the parameters $(|\vec{p}|, \Omega_c, \Omega_K, \Omega_\Lambda)$ in Fig. 2 are not the measured ones, but are taken in a range that pictorially emphasize the essential physical features of the potential $V(z)$. In addition, Fig. 3 is included for the closely related potential (the solid curve), $\tilde{V}(z) = -(\frac{\vec{p}^2+1}{4z^2} + \frac{\Omega_\Lambda}{2}z)$, where Ω_c, Ω_K are set to 0. The parameters $|\vec{p}|, \Omega_\Lambda$ in $\tilde{V}$ are those in V that are the most dominant at $z = 0$ (i.e., $\vec{p}^2 + 1$) and the most dominant at large z (i.e., Ω_Λ). The dashed curve in Fig. 3 corresponds to $V(z)$ including all the parameters, so comparing the solid and dashed curves in Fig. 3 shows the qualitative effect of the additional parameters (Ω_c, Ω_K). I remark that the leading terms of the basic solutions $\Psi_{\pm|\vec{p}|}(z)$, as $z \to 0^\pm$

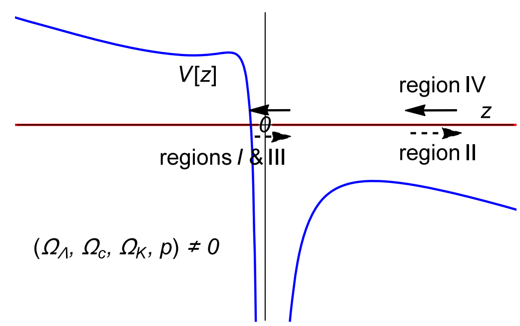

FIG. 2. $V(z)$ with all parameters $\Omega_i \neq 0$.

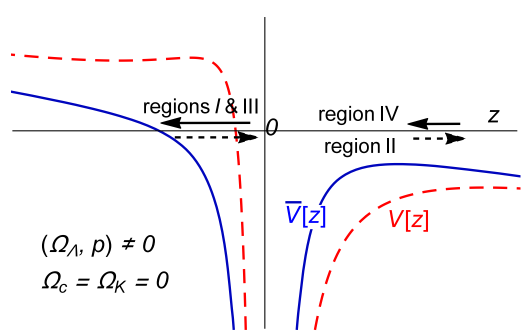

FIG. 3. $\tilde{V}(z)$. only dominant. $|\vec{p}|$, Ω_Λ.

and as $z \to \pm\infty$, that will be needed shortly, are the same for V or $\tilde{V}$.

The physical behavior of the wavepacket $(\sqrt{z}\Psi)$ can be read off directly from Fig. 2 or Fig. 3 as follows. The Schrödinger energy eigenvalue in (15) is zero, so the energy level of the corresponding eigenstate coincides with the horizontal axis in Figs. 2 and 3. The wavepacket spreads from $z \sim +\infty$ as indicated by the left pointing arrow in Figs. 2 and 3. This corresponds to a contracting universe evolving from the asymptotic past in quadrant IV in Fig. 1. The wave packet in this region must then behave like an incoming scattering state oscillatory solution because the energy level is higher than the potential. The wave packet passes through the singularity at $z = 0$, meaning it propagates continuously through the past horizons in Fig. 1 where the universe experiences a big crunch, so it reaches into the antigravity regions I and III where $z < 0$. The wave packet cannot go deep into antigravity in regions I and III because of the potential barrier in Figs. 2 and 3, so it gets reflected toward $z = 0$, meaning it turns around within regions I and III and propagates toward to future horizons in Fig. 1. The part of the wave packet that tunnels under the mountain in Figs. 2 and 3 cannot be an oscillatory solution and it must exponentially decay away as z becomes more negative—this is because the potential is higher than the total energy in that region and the probability must vanish in the asymptotic parts in antigravity regions I and III. The reflected wave packet is oscillatory, it passes through the singularity at $z = 0$ again to come back to $z > 0$ as shown by the right pointing arrows in Figs. 2 and 3, meaning the universe propagates through the future horizons with a big bang into region II in Fig. 1. After this, the wave packet propagates to larger values of z, meaning the universe expands in the future region II in Fig. 1.

The qualitative physical properties in this account will be encoded in the analytic expressions for time dependent classical solutions for $(z(\tau), \vec{s}(\tau))$ as well as in analytic wave packets $\Psi(z, \vec{s})$ given in the following sections.

V. DYNAMICAL ATTRACTOR AND INITIAL CONDITIONS

It is helpful to begin with the classical solution version of the physical scenario at the end of last section. The classical action $S_{\rm mini}$ is then defined by inserting the approximation (14) in (11)

$$S_{\rm mini} \simeq \int d\tau \left\{ \frac{1}{2e}\left[-\frac{1}{4z}(\partial_\tau z)^2 + z(\partial_\tau \vec{s})^2 \right] - \frac{e}{2}[2\Omega_\Lambda z^2 - |\Omega_K| z + \Omega_c] \right\}. \tag{18}$$

The equations of motion for $z(\tau)$ and $\vec{s}(\tau)$ that follow from this action are reduced to first order differential equations

$$z(\partial_\tau \vec{s}) = \vec{p},$$
$$(\partial_\tau z)^2 = [4\vec{p}^2 + 8\Omega_\Lambda z^3 - 4|\Omega_K| z^2 + 4z\Omega_c]. \tag{19}$$

Here $\vec{p}$ is the canonical conjugate to $\vec{s}$ which is a constant $\partial_\tau \vec{p} = 0$ due to the Euler-Lagrange equations of motion that follow from $S_{\rm mini}$. The first order differential equation of motion for $z(\tau)$ is a rewriting of the constraint that follows from $\partial S_{\rm mini}/\partial e = 0$. This is a first integral of the second order differential equation of motion for $z(\tau)$ that can be derived from $S_{\rm mini}$.

I have obtained the general solution of (19) with arbitrary initial conditions for $z(\tau)$ and $\vec{s}(\tau)$. The reader can verify that $z(\tau)$ is given analytically in terms of the doubly periodic JacobiCN$[u|m]$ elliptic function usually denoted as $\mathrm{cn}(u|m)$, as follows

$$z(\tau) = -|z_0| + z_2 \frac{1 - \mathrm{cn}(\sqrt{8\Omega_\Lambda z_2}\tau|m)}{1 + \mathrm{cn}(\sqrt{8\Omega_\Lambda z_2}\tau|m)}, \quad m \equiv \frac{1}{2} + \frac{|z_0| + z_1}{2z_2},$$
$$z_1 \equiv \frac{1}{2}\left(|z_0| + \frac{\Omega_K}{2\Omega_\Lambda} \right) > 0,$$
$$z_2 \equiv \sqrt{z_0^2 + 2|z_0| z_1 + \frac{\vec{p}^2}{2\Omega_\Lambda |z_0|}} > (z_1 + |z_0|). \tag{20}$$

Here z_0, z_1, z_2 are constants determined by $(\vec{p}^2, \Omega_\Lambda, \Omega_K, \Omega_c)$ as given below. In particular z_0 is the value of $z(\tau)$ where the bracket in (19) vanishes, $[\cdots] = 0$, which occurs at the instant $\dot{z}(\tau) = 0$. Due to the τ-translation symmetry of this system, one may choose this instant to be $\tau = 0$. Note that z_0 is negative since it corresponds to the location of the barrier in Figs. 2 and 3, so it is determined as the only real finite root of the cubic equation $[4\vec{p}^2 + 8\Omega_\Lambda z^3 - 4|\Omega_K| z^2 + 4z\Omega_c] = 0$. Thus, $z_0 \equiv z(\tau = 0)$, is $z(\tau)$ at the instant $\tau = 0$ when it reaches its most negative *classical* (as opposed to quantum) value in the antigravity regime. The explicit solution for the relevant root z_0 is written as follows (the other two roots are complex because of the physical values of the parameters $(\vec{p}^2, \Omega_\Lambda, \Omega_K, \Omega_c)$ given is Sec. III).

$$z_0 = -\frac{1}{6\Omega_\Lambda}((R + R\cos\phi)^{\frac{1}{3}} - (R - R\cos\phi)^{\frac{1}{3}} - \Omega_K) < 0,$$
$$R \equiv [(\Omega_K^3 - 54\Omega_\Lambda^2 \vec{p}^2 - 9\Omega_\Lambda \Omega_c \Omega_K)^2 + (6\Omega_\Lambda \Omega_c - \Omega_K^2)^3]^{\frac{1}{2}},$$
$$\cos\phi \equiv \frac{1}{R}(54\Omega_\Lambda^2 \vec{p}^2 + 9\Omega_\Lambda \Omega_c \Omega_K - \Omega_K^3) > 0. \tag{21}$$

The reader can verify analytically that (20)–(21) is the general solution of (19) by using properties of the Jacobi elliptic functions, namely $\partial_u \mathrm{cn}(u|m) = -\mathrm{sn}(u|m) \times \mathrm{dn}(u|m)$, $\mathrm{sn}^2(u|m) + \mathrm{cn}^2(u|m) = 1$ and $\mathrm{dn}(u|m) = 1 - m\,\mathrm{sn}^2(u|m)$.

The exact solution for $\vec{s}(\tau)$ that follows from $z(\partial_\tau \vec{s}) = \vec{p}$ is,

$$\begin{aligned}\vec{s}(\tau) &= \vec{s}_0 + \vec{p}\int_0^{\tau}\frac{d\tau'}{z(\tau')} = \vec{s}_0 + \vec{p}\int_{-|z_0|}^{z(\tau)}\frac{dz'}{z'\dot{z}'}\\ &= \vec{s}_0 + \vec{p}\int_{-|z_0|}^{z(\tau)}\frac{dz'}{z'\sqrt{4\vec{p}^2 + 8\Omega_\Lambda z'^3 - 4|\Omega_K|z'^2 + 4z'\Omega_c}}\\ &= \vec{s}_0 + \vec{p}f(z(\tau)), \qquad (22)\end{aligned}$$

Here, $\vec{s}_0 = \vec{s}(\tau = 0)$ is an integration constant chosen as the value of $\vec{s}(\tau)$ at the instant $\tau = 0$ when $z(\tau)$ is most negative in the antigravity regime. The positive square root is used in determining $\dot{z}$ from (19) because of the choice of integration region $0 < \tau$. The function $f(z(\tau))$ that is common to all directions of $\vec{p}$, can be written explicitly in terms of the Jacobi Π function and has the same double periodicity properties as $z(\tau)$. The result $f(z(\tau))$ is valid not only for $\tau > 0$ but also for $\tau < 0$ because the solution for $z(\tau)$ is symmetric under τ-reversal, $z(-\tau) = z(\tau)$ as seen in the figures below.

These functions are plotted in Figs. 4–6 using nonrealistic values of the phenomenological parameters to emphasize the important physical features of $(z(\tau), \vec{s}(\tau))$. Figure 4 shows a universe $z(\tau)$ (recall $a_E^2(\tau) = |z(\tau)|$) that begins to contract from infinite size $(z(-T_+/2) \sim \infty)$, passes through zero size $(z(-T_0) = 0)$, reaches a maximum size in antigravity $(z(0) = -|z_0|)$, contracts back to zero size $(z(T_0) = 0)$ and then expands up to infinity $(z(T_+/2) \sim \infty)$. This classical behavior of $z(\tau)$ is in line with the qualitative description of a wave packet given following Figs. 2 and 3.

An interesting feature in Fig. 4 is the periodicity seen in conformal time for $z(\tau + T_+) = z(\tau)$; This is because of the periodicity of the Jacobi elliptic function. Cosmic observers that use cosmic t as time (not conformal time τ), can detect

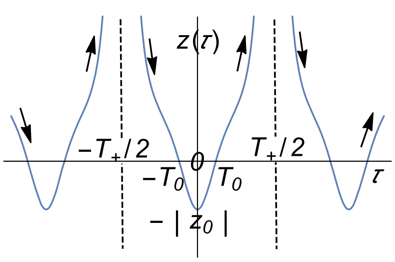

FIG. 4. $z(\tau)$, crunch → antigravity → bang.

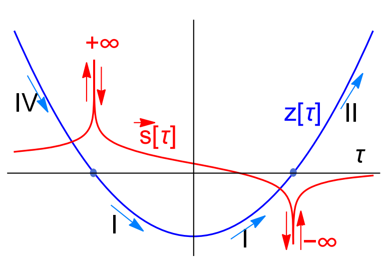

FIG. 5. $\vec{s}(\tau)$ near $z(\tau) \simeq 0$.

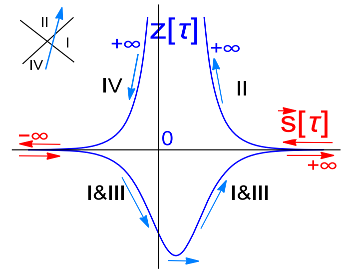

FIG. 6. Parametric $(\vec{s}(\tau), z(\tau))$.

only one period of the periodic Jacobi elliptic functions. But it is intriguing that more generally the plot in conformal time in Fig. 4 shows a universe that gets renewed periodically an infinite number of times, like a cyclic universe.[6]

The spiking behavior of $\vec{s}(\pm T_0) = \pm\infty$ when $z(\pm T_0) = 0$ at the cosmological singularities, as seen in Fig. 5 is quite general. This occurs for all values of the integration constants $(\vec{s}_0, \vec{p})$, so the simultaneous divergence of the scalar $\sigma(\tau)$ and anisotropy $(\alpha_1(\tau), \alpha_2(\tau))$ when $z(\tau)$ hits zero cannot be avoided. The constant $\vec{s}_0$ is represented in Fig. 5 as the point the vertical axis intersects the curve $\vec{s}(\tau)$. To emphasize the spiking property of $\vec{s}(\tau)$, I also produce a parametric plot $(z(\tau), \vec{s}(\tau))$ that amounts to a plot of $\vec{s}$ as a function of z, or vice-versa, as seen in Fig. 6.

It is revealing to reorganize the four d.o.f. $(z, \vec{s})$ into the form $(\phi_{\hat{p}}, h_{\hat{p}})$ defined as follows

$$\begin{aligned}u_{\hat{p}} &\equiv \sqrt{|z|}e^{\hat{p}\cdot\vec{s}}, v_{\hat{p}} \equiv \sqrt{|z|}e^{-\hat{p}\cdot\vec{s}}\\ \phi_{\hat{p}}(\tau) &= \frac{1}{2}(u_{\hat{p}}(\tau) + \varepsilon_z v_{\hat{p}}(\tau)),\\ h_{\hat{p}}(\tau) &= \frac{1}{2}(u_{\hat{p}}(\tau) - \varepsilon_z v_{\hat{p}}(\tau)), \qquad (23)\end{aligned}$$

where $\hat{p} = \vec{p}/|\vec{p}|$ is a unit vector. These generalize (ϕ, h) by including anisotropy through the angles $\hat{p}$. A parametric plot of these functions is given in Fig. 7, where the trajectory along the arrows show the progress as τ increases from $-T_+/2$ to $T_+/2$. The motion is in the plane slicing the touching cones and containing the vector $\hat{p}$ perpendicular to the vertical axis. A revolution of this figure around the vertical axis is equivalent to changing the direction of the vector $\hat{p}$. The interior of the cones correspond to the gravity regions II and IV while the exterior of the cones correspond to the antigravity regions I and III. The trajectory of the universe in this figure is equivalent to the solution for $(z(\tau), \vec{s}(\tau))$ plotted in Fig. 6 but now re-plotted in these new

[6]This is similar in spirit but different in detail than [9] where λ' was taken artificially negative to imitate tunneling-like effects in the metastable Higgs potential. Here, since $\lambda' \sim \Omega_\Lambda > 0$ is positive, the renewal in regions $|\tau| > T_+/2$ has no relation to the possible tunneling of the Higgs in a metastable renormalized Higgs potential. A physical interpretation of the reason behind this cyclic-universe type periodicity remains open.

coordinates. The figure visually shows that the universe contracts in region IV, passes in all directions $\hat{p}$ through a 4-dimensional pin hole, $\phi_{\hat{p}}(\tau_c) = h_{\hat{p}}(\tau_c) = 0$ at the crunch time $\tau_c = -T_0$, expands a little into antigravity, and turns around during antigravity to pass again through a pin hole, $\phi_{\hat{p}}(\tau_b) = h_{\hat{p}}(\tau_b) = 0$ at the bang time $\tau_b = +T_0$, and then expands in region II.

This plot is a generalization of a similar one in [5], but shows more dramatically the pin hole effect in four dimensions including anisotropy. This emphasizes the theorem stated below that the scalar and anisotropy d.o.f. spike to infinity at the singularity. In the reorganized pin hole version this makes it evident that the dynamics is one of an unavoidable attractor to the tips of the cones. The trajectory of the universe cannot cross the cosmological horizons represented by the walls of the cones in Fig. 7. The trajectory can evolve to the neighboring Rindler regions only by going through the "pin hole" and tangentially to the walls of the cones. This result is a remarkable unique cosmological prediction for the initial conditions of the universe at the "beginning." This is a sufficiently important cosmological prediction of the EFTC introduced in Sec. II, that I will highlight it as a theorem:

Theorem: The attractor mechanism displayed in Fig. 7 produces dynamically unique initial values for all d.o.f. at cosmological singularities $z = 0$. Quantitatively, at both the bang/crunch, the fields $(\phi_{\hat{p}}(\tau), h_{\hat{p}}(\tau))$ must vanish simultaneously for every $\hat{p}$ while their ratio must be plus or minus one,

$$\phi_{\hat{p}}(\tau_{b/c}) = h_{\hat{p}}(\tau_{b/c}) = 0, \qquad \frac{h_{\hat{p}}(\tau_{b/c})}{\phi_{\hat{p}}(\tau_{b/c})} = \pm 1. \tag{24}$$

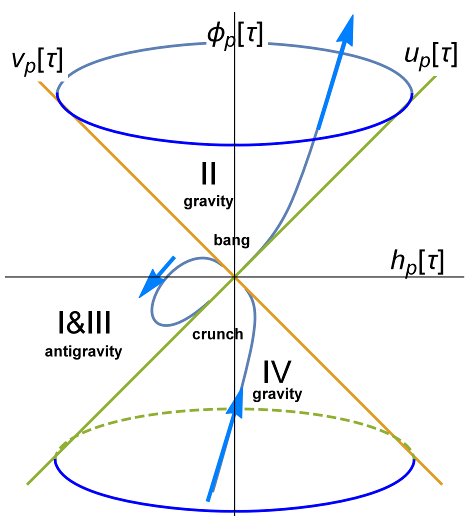

FIG. 7. Evolution of the universe must pass through all gravity and antigravity regions but only by shrinking to zero size, IV → 0 → (I & II) → 0 → II. At zero size $\phi_{\hat{p}} = h_{\hat{p}} = 0$ and their ratio is ± 1 as seen by the tangential slopes to the horizons $u_{\hat{p}} = 0$ or $v_{\hat{p}} = 0$.

Thus, the universe must pass through a "pin hole" while matching all d.o.f. on both sides of the pin hole in all directions $\hat{p}$. An alternative description of the "pin hole" initial values in the 4D variables, $(z(\tau_{b/c}), \vec{s}(\tau_{b/c})) = (0, \pm\vec{\infty})$, is given in Fig. 6. This theorem emerges not only in the classical analysis given above, but also in the quantum analysis given in Sec. VI.

A remark about Misner's "mixmaster universe" problem [25] is in order. The classical solution for $(z(\tau), \vec{s}(\tau))$ that includes anisotropy, as given in (20), (22) and plotted in Figs. 4–7, clearly did not suffer from this problem even in the case of Bianchi-IX metric ($k = 1$), so it invites an explanation. This solution was obtained under the assumption that $V_K(\alpha_1, \alpha_2)$ was replaced by a constant as in (14), so naturally the rattling around that causes Misner's mixmaster did not happen in the constant potential. Therefore one must reexamine if the approximation of neglecting the details of the potential is valid. I have performed the following computation: I inserted the solution above into the full action (11) without approximating $V_K(\alpha_1, \alpha_2)$ and asked under what conditions the potential energy *for this generic solution as a function of* τ can be neglected as compared to its kinetic energy as a function of τ while satisfying $\mathcal{H} = 0$. The answer to this question is that, it is true that the kinetic terms *for the solution* do dominate and the potential term $z(\tau)V_K(\alpha_1(\tau), \alpha_2(\tau))$ vanishes as $\tau \to \tau_{b/c}$. In more detail, based on this analysis, the solutions (20), (22) are legitimate approximations as long as the conserved momenta satisfy the inequality, $0 < 4|p_{1,2}| \lesssim |p_3| \lesssim |\vec{p}|$. It is significant that without a nonzero $|p_3|$ it is not possible to satisfy this inequality, which means the Higgs is crucial. This simple result for the avoidance of mixmaster is consistent with the considerably more complicated Belinskii-Khalatnikov-Lifshitz analysis [28] that concludes the mixmaster is avoidable when there are scalars in the theory.

In the current EFTC the presence of the Higgs is essential for avoiding the mixmaster, as well as for creating the dynamical attractor that predicts initial conditions. So the Higgs in this EFTC has important cosmological roles in shaping our universe.

VI. QUANTUM WAVEFUNCTION—2

I now turn to the solution of the WdWe (13). First I analyze its full form before the suggested approximation strategy in Eqs. (14)–(17) and confront the problem of continuity in general. The partial differential equation (12) or (13), in any of the (ϕ, h) or (u, v) or (z, σ) bases, is a mathematically well-defined quantum problem that includes continuous passage through singularities at $u = (\phi + h) = 0$ or $v = (\phi - h) = 0$ (which means E-frame singularity at $z = 0$ or $a_E = 0$). Continuity of the wavefunction through the global coordinates at $u = 0$ or $v = 0$ is not trivial and poses a main challenge whose full resolution is given below in Eq. (28). The tricky technical problem is outlined as follows. The total potential in (12), (13) is dominated by the Ω_c-term close to the singularity $z \to 0$. Since the potentials become immaterial in this limit, one may go to momentum space and solve the two basic solutions of (12) in (u, v) basis, $(4\partial_u\partial_v - \frac{1}{uv}(\partial_1^2 + \partial_2^2) + \Omega_c + \cdots)\Psi = 0$, near the singularity $z = uv \sim 0$. These are similar to (16)

$$\Psi\colon\ A_{\pm}(\vec{p})e^{-ip_1\alpha_1 - ip_2\alpha_2}u^{-i(p_3\pm|\vec{p}|)/2}v^{i(p_3\mp|\vec{p}|)/2}S_{\pm|\vec{p}|}(uv), \tag{25}$$

with the leading terms in $S_{\pm|\vec{p}|}(uv) \simeq (1 + O(\Omega_c uv))$, and any real $\vec{p} \equiv (p_1, p_2, p_3)$. The complete solution Ψ near $z = uv = 0$ is the general linear superposition over these two basic solutions summed up as in (16), although now this is without approximating the potentials, but examining only the vicinity of $z = 0$. So, the form (25) near the singularity persists despite the potentials.

The problem with continuity is that, at either $u \to 0$ when v is finite, or at $v \to 0$ when u is finite, i.e., at any point on the horizons in Fig. 1, the basic solutions (25) seem to oscillate wildly so that their values on either side of the horizons appear to be undeterminable. However, under the integral $\int d^3p$, with sufficiently smooth $A_{\pm}(\vec{p})$, such wildly oscillating integrands produce a definite vanishing value for the integral, thus giving $\Psi \to 0$ at the horizons. If this were true, the coefficients $A_{\pm}(\vec{p})$ would be chosen independently on either side of each horizon in Fig. 1, so that a solution Ψ within each of the four quadrants would vanish at the horizons, and be independent from the solutions within all other quadrants in Fig. 1.

A discontinuous wavefunction due to uncontrollable oscillations is rejected since it is not a solution of (12) and furthermore gives problems with the Hermiticity of the Hamiltonian just as in the case of the singular $(-1/r^2)$ potential in ordinary quantum mechanics. Until now this remained an unsettled problem[7] that goes back to Von Neumann. Various suggested solutions are nonunique and differ physically in different physical approximations to a regulated $(-1/r^2)$ [29]. A resolution seems to be in sight by using wavepacket solutions as just outlined above, because the wild oscillations are controlled in a complete set of normalizable wave packets and continuity appears to be satisfied with $\Psi \to 0$ at both sides of all horizons. However, this is problematic for a geodesically complete cosmology advanced in this paper, because there seems to be no relation between the wavefunctions for the past (region IV), the future (region II) or antigravity (regions I and III), and therefore no information from the past seems to survive to the future of the big bang.

The very tricky subtlety that resolves this problem is that the story for vanishing wave packets $\Psi \to 0$ at all horizons is not true. This is because, according to (8) one can rewrite the basic solutions near the singularity (25) as follows

$$e^{-ip_1\alpha_1 - ip_2\alpha_2}u^{-i(p_3\pm|\vec{p}|)/2}v^{i(p_3\mp|\vec{p}|)/2} \sim e^{-i\vec{p}\cdot\vec{s}}\sqrt{|z|}^{\mp i|\vec{p}|}$$
$$= (\sqrt{|z|}e^{\pm\hat{p}\cdot\vec{s}})^{\mp i|\vec{p}|}, \tag{26}$$

where $\hat{p} = \vec{p}/|\vec{p}|$. Then, when $|z| \to 0$, there are regions of the integral $\int d^3p$ that do not oscillate wildly by having $|\hat{p}\cdot\vec{s}| \to \infty$ in tandem with $\sqrt{|z|} \to 0$ so that either $\sqrt{|z|}e^{\hat{p}\cdot\vec{s}}$ or $\sqrt{|z|}e^{-\hat{p}\cdot\vec{s}}$ remains finite while the other goes to zero. Defining $u_{\hat{p}} \equiv \sqrt{|z|}e^{\hat{p}\cdot\vec{s}}$, $v_{\hat{p}} \equiv \sqrt{|z|}e^{-\hat{p}\cdot\vec{s}}$, as in (23), the two solutions near $z = 0$ in (26) take the form

$$u_{\hat{p}}^{-i|\vec{p}|} \quad \text{or} \quad v_{\hat{p}}^{i|\vec{p}|}, \tag{27}$$

such that $u_{\hat{p}}^{-i|\vec{p}|}$ remains finite when $v_{\hat{p}}^{i|\vec{p}|}$ oscillates wildly and vice versa. So there is a part of the superposition in Ψ at each horizon in which the wild oscillations do not occur, thus making a finite contribution to Ψ. This region is always in the minisuperspace region where $|\hat{p}\cdot\vec{s}| \to +\infty$ as $z \to 0$ for all available directions $\hat{p}$. This finite part of Ψ has to be the same on either side of each horizon in order to have a continuous wavefunction between neighboring quadrants in the full 4D minisuperspace $(z, \vec{s})$. This then resolves the problem of continuity which is now achieved simply by analytic continuation in the complex $u_{\hat{p}}$, $v_{\hat{p}}$ planes. The details of this continuity mechanism was discussed recently in great detail in [14] for the case $\hat{p} = (0, 0, \pm 1)$. The techniques are the same and it amounts to a simple generalization of [14] to the general $\hat{p}$.

The outcome is that, close to the singularity $z = 0$ the probability amplitude Ψ gets its support from the region where the scalar and anisotropy d.o.f. $\vec{s}$ diverge, in agreement with Fig. 6. Equivalently, the support for nonzero Ψ close to the singularity is found in the equivalent variables $(u_{\hat{p}}, v_{\hat{p}})$ in the neighborhood of the pin hole as in Fig. 7. Hence the *theorem for unique initial conditions* emerges both in classical and quantum dynamics.

With this information one can now carry out the task of imposing continuity of the wavefunction as in [14], and find that the past, antigravity and future wavepacket coefficients are related to each other. This leads to the complete general analytic solution of the full continuous wavefunction Ψ, with *only one set of independent wave-packet coefficients* $(a(\vec{p}), b^{\dagger}(\vec{p}))$ that appear in the solutions in the various quadrants, as displayed in Eq. (28).

[7]The connection to $(-1/r^2)$ becomes evident in Eq. (15) that displays the attractive $(-1/z^2)$ potential as the leading singular term of the potential, corresponding precisely to the cosmological singularity of order a_E^{-6} in the Friedmann equation (3). The unique solution to this problem in the setting of this paper, using the complete set of normalized and continuous wave packets, given in the next paragraph was not attempted before.

$$
\begin{aligned}
\Psi_{II}(z,\vec{s}) &\overset{z\geq 0}{=} \int_{-\infty}^{\infty} d^3p \left[a(\vec{p})\,\frac{e^{-i\vec{p}\cdot\vec{s}}\sqrt{z}^{-i|\vec{p}|}}{\sqrt{16\pi^3|\vec{p}|}} S_{|\vec{p}|}(z) + b^{\dagger}(\vec{p})\,\frac{e^{i\vec{p}\cdot\vec{s}}\sqrt{z}^{i|\vec{p}|}}{\sqrt{16\pi^3\pi|\vec{p}|}} S_{-|\vec{p}|}(z) \right], \\
\Psi_{I\&III}(z,\vec{s}) &\overset{z\leq 0}{=} \int_{-\infty}^{\infty} d^3p \left[\begin{array}{l} a(\vec{p})\left\{ \frac{e^{-i\vec{p}\cdot\vec{s}}\sqrt{-z}^{-i|\vec{p}|}S_{|\vec{p}|}(z)}{\sqrt{16\pi^3|\vec{p}|}} + \frac{e^{-i\vec{p}\cdot\vec{s}}\sqrt{-z}^{i|\vec{p}|}S_{-|\vec{p}|}(z)}{\sqrt{16\pi^3|\vec{p}|}} \frac{(\frac{\Omega_\Lambda}{18})^{i\frac{|\vec{p}|}{6}}\Gamma(-i\frac{|\vec{p}|}{3})}{\Gamma(i\frac{|\vec{p}|}{3})} \right\} \\ +b^{\dagger}(\vec{p})\left\{ \frac{e^{i\vec{p}\cdot\vec{s}}\sqrt{-z}^{i|\vec{p}|}S_{-|\vec{p}|}(z)}{\sqrt{16\pi^3\pi|\vec{p}|}} + \frac{e^{i\vec{p}\cdot\vec{s}}\sqrt{-z}^{-i|\vec{p}|}S_{|\vec{p}|}(z)}{\sqrt{16\pi^3|\vec{p}|}} \frac{(\frac{\Omega_\Lambda}{18})^{-i\frac{|\vec{p}|}{6}}\Gamma(i\frac{|\vec{p}|}{3})}{\Gamma(-i\frac{|\vec{p}|}{3})} \right\} \end{array} \right] \\
\Psi_{IV}(z,\vec{s}) &\overset{z\geq 0}{=} \int_{-\infty}^{\infty} d^3p \left[a(\vec{p})\,\frac{e^{-i\vec{p}\cdot\vec{s}}\sqrt{z}^{i|\vec{p}|}S_{-|\vec{p}|}(z)}{\sqrt{16\pi^3|\vec{p}|}} \frac{(\frac{\Omega_\Lambda}{18})^{i\frac{|\vec{p}|}{6}}\Gamma(-i\,\frac{|\vec{p}|}{3})}{\Gamma(i\,\frac{|\vec{p}|}{3})} + b^{\dagger}(\vec{p})\,\frac{e^{i\vec{p}\cdot\vec{s}}\sqrt{z}^{-i|\vec{p}|}S_{|\vec{p}|}(z)}{\sqrt{16\pi^3|\vec{p}|}} \frac{(\frac{\Omega_\Lambda}{18})^{-i\frac{|\vec{p}|}{6}}\Gamma(i\,\frac{|\vec{p}|}{3})}{\Gamma(-i\,\frac{|\vec{p}|}{3})} \right]. \qquad (28)
\end{aligned}
$$

The form of (28) is consistent with the form (25) if taken only near $z \sim 0$ without approximating the potentials. It is also consistent with the form (16) if the suggested approximation strategy of Sec. IV is applied. In the latter case the functions $S_{\pm|\vec{p}|}(z)$ that appear in these expressions is computed analytically and given below; in that case the solution for the wavefunction above is valid in all regions of the geodesically complete superspace depicted in Fig. 7. The labels II, I and III, IV on the wavefunctions in (28) refer to the corresponding regions in Fig. 7, namely the inside and outside regions of the cones. As expected from the qualitative discussion of Figs. 2 and 3, the $\Psi_{\mathrm{II}}(z,\vec{s})$ and $\Psi_{\mathrm{IV}}(z,\vec{s})$, that are the future and past positive-z gravity sectors respectively, are oscillatory (scattering-type wave packets), while $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ that are at negative-z have fast decaying asymptotic behavior as $z \to -\infty$ deep into the antigravity region. The asymptotic behavior of the functions $S_{\pm|\vec{p}|}(z)$ as $z \to \pm\infty$ were used in order to fix all relative coefficients in these expressions so that the correct physical behavior is obtained (oscillatory in II and IV and decay in I and III). So, the probability of the universe to spend time in the antigravity regions I and III during its evolution is limited by the fast decay of $\Psi_{I\&III}(z,\vec{s})$ away from the $z = 0$ singularity. The simple explanation for this behavior is easily understood from the shape of the potential barrier as discussed above in relation to Figs. 2 and 3.

How continuity works requires some guidance. Continuity at the future horizons in Fig. 7 requires the wavefunction $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ and $\Psi_{\mathrm{II}}(z,\vec{s})$ to share the same $(a(\vec{p}), b^{\dagger}(\vec{p}))$ coefficients as follows. The part proportional to $a(\vec{p})$ in $\Psi_{\mathrm{II}}(z,\vec{s})$ at $z > 0$ is directly related to *the first term* in $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ at $z < 0$, as written in (28). The second term proportional to $a(\vec{p})$ in $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ vanishes at the future horizon due to the fast oscillations. The part proportional to $b^{\dagger}(\vec{p})$ in comparing $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ and $\Psi_{\mathrm{II}}(z,\vec{s})$ at the future horizons in Fig. 7 works exactly the same way. For the past horizon, the arguments are quite similar, and in this way the *second terms* in $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ proportional $(a(\vec{p}), b^{\dagger}(\vec{p}))$ are analytically continued from $z < 0$ to $z > 0$ thus connecting to $\Psi_{\mathrm{II}}(z,\vec{s})$ as written in (28), while the first terms in $\Psi_{\mathrm{I\&III}}(z,\vec{s})$ proportional to $(a(\vec{p}), b^{\dagger}(\vec{p}))$ vanish at $z = 0$ for the past horizon due to the fast oscillations. This continuity of the overall $\Psi_{\mathrm{II,I\&III,IV}}$ can be rephrased in terms of the $(u_{\hat{p}}, v_{\hat{p}})$ basis, as genuine analytic continuation around cuts in the complex planes of the variables $(u_{\hat{p}}, v_{\hat{p}})$ in every direction $\hat{p}$. This is explained in great detail in [14] for the special direction $\hat{p} = (0, 0, \pm 1)$.

The form (28) of the general solution is complete because it is based on the complete set of solutions for the basis functions and can be applied to all possible physical circumstances. Like in field theory, the wavepacket coefficients $(a(\vec{p}), b^{\dagger}(\vec{p}))$ can be interpreted as analogs of creation/annihilation operators for positive/negative frequency massless plane waves $e^{-i\vec{p}\cdot\vec{s}\pm i|\vec{p}|\ln\sqrt{|z|}}$ at the horizons, since $S_{|\vec{p}|}(z) \overset{z\to 0}{\to} 1$, while interpreting $\ln\sqrt{|z|}$ as the analog of time close to the horizons in Fig. 7. This is similar to the case of the plane wave basis at black hole horizons. One may set up a Bogoliubov transformation between these and creation/annihilation operators for plane waves at the asymptotics of region II, similar to the case of black holes or similar to the one given in [14] for the special direction $\hat{p} = (0, 0, \pm 1)$.

For the physical application of interest in this paper, namely the wavefunction for the universe, additional boundary conditions are required in the asymptotic regions of II and IV. Namely, the wave packet $\Psi_{\mathrm{IV}}(z,\vec{s})$ must have *only incoming asymptotic waves* (not horizon waves), where incoming is defined in region IV at $z \to +\infty$ as the leading oscillatory behavior $e^{i\omega z^{3/2}}$ with positive ω. Once this is imposed in momentum space on $\Psi_{\mathrm{IV}}(z,\vec{s})$ it turns out the wave packet $\Psi_{\mathrm{II}}(z,\vec{s})$ automatically has only outgoing asymptotic waves because of the relations of the wavepacket coefficients in the different regions as displayed in (28). This asymptotic boundary condition determines $b^{\dagger}(-\vec{p})$ as a function of $a(\vec{p})$

$$
b^{\dagger}(-\vec{p}) = a(\vec{p})\,\frac{\Gamma(-i\,\frac{|\vec{p}|}{3})}{\Gamma(i\,\frac{|\vec{p}|}{3})} \left(\frac{\Omega_\Lambda}{18}\right)^{i\frac{|\vec{p}|}{3}} e^{-\frac{\pi|\vec{p}|}{3}}. \qquad (29)
$$

To get this result, the asymptotic form of $S_{|\vec{p}|}(z)$ given in Eq. (33) below is used.

In addition, $(a(\vec{p}), b^{\dagger}(\vec{p}))$ must be limited to Gaussians as explained in (17) as part of the approximation strategy in Sec. IV, therefore

$$a(\vec{p}) = Ae^{-(p_1^2+p_2^2)/2\Omega_\alpha} e^{-p_3^2/2\Omega_\sigma}, \tag{30}$$

and similarly for $b^{\dagger}(\vec{p})$ related by (29). The wavefunction in this form has no remaining unknown parameters since the overall A is just a normalization factor.

Major conclusions of this discussion are

(1) Anisotropy and Higgs $|\hat{p}\cdot\vec{s}|$ must be at infinity for all available directions $\hat{p}$ when $a_E = 0$, or equivalently at the pin hole of Fig. 7 in terms of the $(u_{\hat{p}}, v_{\hat{p}})$ basis.

(2) The general solution for the wavefunction for all physical applications as given in (28) depends only on one set of wavepacket coefficients $(a(\vec{p}), b^{\dagger}(\vec{p}))$, analogous to the creation/annihilation operators of a field, that are shared in all patches of the geodesically complete minisuperspace. From this it is evident that, if the $(a(\vec{p}), b^{\dagger}(\vec{p}))$ were quantized, the resultant Fock space would be complete, and would describe the physics for all gravity and antigravity patches, in a *unitary Hilbert space*. This is in agreement with the complementary discussion given in [13] on unitarity and stability.

(3) The wavefunction for the universe is given by the restricted form of $(a(\vec{p}), b^{\dagger}(\vec{p}))$ in Eqs. (29) and (30). In this completely fixed form the wavefunction has a unique dynamically generated initial value at the "beginning." This predicted last form is a topic of discussion in Sec. VII.

VII. QUANTUM WAVEFUNCTION—3

The wavefunction for the universe derived in the previous section is simplified as follows. It has no unspecified attributes and depends only on the phenomenologically measured parameters $(\Omega_\Lambda, \Omega_c, \Omega_K, \Omega_\sigma, \Omega_\alpha)$ that appear in the Friedmann equation (3)

$$\begin{aligned}
\Psi_{\mathrm{II}}(z,\vec{s}) &\overset{z\geq 0}{=} A\int_{-\infty}^{\infty}\frac{d^3p}{\sqrt{(2\pi)^3 2|\vec{p}|}} e^{-\frac{p_3^2}{2\Omega_\sigma}} e^{-\frac{p_1^2+p_2^2}{2\Omega_\alpha}} e^{-i\vec{p}\cdot\vec{s}} H^+_{|\vec{p}|}(z)\\
\Psi_{\mathrm{I\&III}}(z,\vec{s}) &\overset{z\leq 0}{=} A\int_{-\infty}^{\infty}\frac{d^3p}{\sqrt{(2\pi)^3 2|\vec{p}|}} e^{-\frac{p_3^2}{2\Omega_\sigma}} e^{-\frac{p_1^2+p_2^2}{2\Omega_\alpha}}\\
&\quad\times e^{-i\vec{p}\cdot\vec{s}}(H^+_{|\vec{p}|}(z)+H^-_{|\vec{p}|}(z))\\
\Psi_{\mathrm{IV}}(z,\vec{s}) &\overset{z\geq 0}{=} A\int_{-\infty}^{\infty}\frac{d^3p}{\sqrt{(2\pi)^3 2|\vec{p}|}} e^{-\frac{p_3^2}{2\Omega_\sigma}} e^{-\frac{p_1^2+p_2^2}{2\Omega_\alpha}} e^{-i\vec{p}\cdot\vec{s}} H^-_{|\vec{p}|}(z)
\end{aligned} \tag{31}$$

where the overall A is fixed by normalization, and the functions $H^{\pm}_{|\vec{p}|}(z)$ are

$$\begin{aligned}
H^+_{|\vec{p}|}(z) &= \Bigg(\sqrt{|z|}^{-i|\vec{p}|}S_{|\vec{p}|}(z)\\
&\quad + \sqrt{|z|}^{i|\vec{p}|}S_{-|\vec{p}|}(z)e^{-\frac{\pi|\vec{p}|}{3}}\frac{(\frac{\Omega_\Lambda}{18})^{i\frac{|\vec{p}|}{3}}\Gamma(-i\frac{|\vec{p}|}{3})}{\Gamma(i\frac{|\vec{p}|}{2})}\Bigg)\\
H^-_{|\vec{p}|}(z) &= \Bigg(\sqrt{z}^{i|\vec{p}|}S_{-|\vec{p}|}(z)\frac{\Gamma(-i\frac{|\vec{p}|}{3})}{\Gamma(i\frac{|\vec{p}|}{3})}\\
&\quad + \sqrt{z}^{-i|\vec{p}|}S_{|\vec{p}|}(z)e^{-\frac{\pi|\vec{p}|}{3}}\Bigg)\left(\frac{\Omega_\Lambda}{18}\right)^{i\frac{|\vec{p}|}{6}}
\end{aligned} \tag{32}$$

Finally there remains to give an explicit $S_{\pm|\vec{p}|}(z)$ that depends on $(|\vec{p}|, \Omega_\Lambda, \Omega_c, \Omega_K)$. The $S_{\pm|\vec{p}|}(z)$ that solves the WdWe (15) for the potential V in Fig. 2 with all parameters $(|\vec{p}|, \Omega_\Lambda, \Omega_c, \Omega_K)$ nonzero is not known analytically at this time, although I think this could be obtained. It can certainly be determined numerically or other approximations, such as the Wentzel-Kramers-Brillouin approximation. However, I have constructed analytic solutions for all the cases listed below in which some of these parameters $(|\vec{p}|, \Omega_\Lambda, \Omega_c, \Omega_K)$ are set to zero.

The most useful approximation is case-1 given below. This case captures best the physical features of the full $S_{\pm|\vec{p}|}(z)$ because the potential $\tilde{V}(z)$ shown in Fig. 3 agrees with the leading terms of the full $V(z)$ at both limits $z\to 0^{\pm}$ and $z\to\pm\infty$

$$\begin{aligned}
&\text{Case-1}: (\Omega_c,\Omega_K)\to 0,\quad V(z)\to\tilde{V}(z) = -\left(\frac{\vec{p}^2+1}{4z^2}+\frac{\Omega_\Lambda}{2}z\right),\\
&\mathcal{H}\Psi = \left(\partial_\phi^2-\partial_h^2-\frac{\partial_1^2+\partial_2^2}{\phi^2-h^2}+2\Omega_\Lambda(\phi^2-h^2)^2\right)\Psi = 0,\\
&S_{|\vec{p}|}(z) = \sum_{n=0}^{\infty}\frac{(\frac{-\Omega_\Lambda}{18}z^3)^n\Gamma(1-i\frac{|\vec{p}|}{3})}{n!\Gamma(n+1-i\frac{|\vec{p}|}{3})} = {}_0F_1\left(1-i\frac{|\vec{p}|}{3}, \frac{-\Omega_\Lambda z^3}{18}\right)\\
&S_{|\vec{p}|}(z)\overset{z\to\pm\infty}{\to}\frac{\Gamma(1-i\frac{|\vec{p}|}{3})e^{\frac{\pi|\vec{p}|}{6}}}{2\sqrt{\pi}(-1)^{-1/4}}\left(\frac{\Omega_\Lambda z^3}{18}\right)^{-\frac{1}{4}+i\frac{|\vec{p}|}{6}}\\
&\times\left[e^{\frac{i\pi}{4}+\frac{\pi|\vec{p}|}{6}-2i\sqrt{\frac{\Omega_\Lambda z^3}{18}}}+e^{-\frac{i\pi}{4}-\frac{\pi|\vec{p}|}{6}+2i\sqrt{\frac{\Omega_\Lambda z^3}{18}}}\right]+\cdots
\end{aligned} \tag{33}$$

The hypergeometric function ${}_0F_1(a, W)$, is an entire function in the finite complex W plane for any complex a. The asymptotic property of this $S_{|\vec{p}|}(z)$ was used in order to fix the correct relative coefficients in Eqs. (29), (31), and (32).

Other analytic solutions of interest in various limits of the Ω_i include the following

$$\begin{aligned}
&\text{Case-2}:\ (\Omega_\Lambda, \Omega_K, p_1, p_2) \to 0,\\
&V(z) \to \tilde{V}(z) = -\left(\frac{p_3^2+1}{4z^2} + \frac{\Omega_c}{4z}\right),\\
&\mathcal{H}\Psi = (\partial_\phi^2 - \partial_h^2 + \Omega_c)\Psi = 0,\\
&S_{|p_3|}(z) = \sum_{n=0}^{\infty} \frac{(-\frac{\Omega_c}{4}z)^n \Gamma(1-i|p_3|)}{n!\Gamma(n+1-i|p_3|)}\\
&\quad = {}_0F_1\left(1-i|p_3|, \frac{-\Omega_c z}{4}\right). \qquad (34)
\end{aligned}$$

This is the case with $\hat{p} = (0,0,\pm 1)$ that was studied in [14] (no anisotropy in this case). The WdWe (12) for case-2 simplifies a great deal in the global basis (ϕ, h) as above. This "massive" Klein-Gordon equation in $1+1$ dimensions is easily solved. The alternative bases, namely Minkowski (ϕ, h) versus Rindler (z, σ), are fully equivalent to each other in the classical theory. However, in the quantum theory there is a nontrivial analyticity property in connecting $(\phi, h) \leftrightarrow (z, \sigma)$ due to branch points and branch cuts in $\Psi(u, v)$. The unique definition of $\Psi(u, v)$ introduces an infinite number of sheets in the complex u and v planes. This is interpreted as a new *multiverse for which an extensive discussion* is given in [14]. The new multiverse is present in the general case all $\Omega_i \neq 0$.

A case with vanishing $\Omega_\Lambda \to 0$, but all other parameters nonvanishing, is

$$\begin{aligned}
&\text{Case 3}:\ \Omega_\Lambda \to 0, \qquad V(z) \to \tilde{V}(z) = -\left(\frac{\vec{p}^2+1}{4z^2} + \frac{\Omega_c}{4z} - \frac{\Omega_K}{4}\right),\\
&\mathcal{H}\Psi = \left(\partial_\phi^2 - \partial_h^2 - \frac{\partial_1^2+\partial_2^2}{\phi^2-h^2} - \Omega_K(\phi^2-h^2) + \Omega_c\right)\Psi = 0.\\
&S_{|\vec{p}|}(z) = \begin{cases} = e^{-\frac{1}{2}\sqrt{\Omega_K}z} \sum_{n=0}^{\infty} \frac{\Gamma\left(\frac{1-i|\vec{p}|}{2} - \frac{\Omega_c}{4\sqrt{\Omega_K}} + n\right)\Gamma(1-i|\vec{p}|)}{\Gamma\left(\frac{1-i|\vec{p}|}{2} - \frac{\Omega_c}{4\sqrt{\Omega_K}}\right)\Gamma(1-i|\vec{p}|+n)} \frac{(\sqrt{\Omega_K}z)^n}{n!} \\ = e^{-\frac{1}{2}\sqrt{\Omega_K}z} {}_1F_1\left[\left(\frac{1-i|\vec{p}|}{2} - \frac{\Omega_c}{4\sqrt{\Omega_K}}\right), (1-i|\vec{p}|), \sqrt{\Omega_K}z\right] \end{cases}, \qquad (35)
\end{aligned}$$

Here ${}_1F_1(a, b, W)$ is another hypergeometric function that is entire in the complex W plane. The asymptotic behavior of the potential $\tilde{V}(z)$ is now dominated by the curvature constant Ω_K.

The classical solution analogous to Fig. 7, namely $(z(\tau), \vec{s}(\tau))$ turns out to be completely periodic in this case, and gives the generic trajectory in Fig. 8 as compared to Fig. 7.

The analytic solution for this trajectory is given in terms of ordinary periodic functions, and of course agrees with the $\Omega_\Lambda \to 0$ limit of Eqs. (20)–(22)

$$\begin{aligned}
z(\tau) &= -\frac{2\vec{p}^2}{\sqrt{\Omega_c^2 + 4\Omega_K\vec{p}^2} + \Omega_c} + \frac{\sqrt{\Omega_c^2 + 4\Omega_K\vec{p}^2}}{\Omega_K}\sin^2(\sqrt{\Omega_K}\tau),\\
\vec{s}(\tau) &= \vec{s}_0 + \frac{\vec{p}}{2|\vec{p}|}\ln\left|\frac{(\sqrt{\Omega_c^2 + 4\Omega_K\vec{p}^2} + \Omega_c)\tan(\sqrt{\Omega_K}\tau) - 2|\vec{p}|\sqrt{\Omega_K}}{(\sqrt{\Omega_c^2 + 4\Omega_K\vec{p}^2} + \Omega_c)\tan(\sqrt{\Omega_K}\tau) + 2|\vec{p}|\sqrt{\Omega_K}}\right|. \qquad (36)
\end{aligned}$$

Another case is the specialized version of case-3 with vanishing anisotropy,

$$\begin{aligned}
&\text{Case 4}:\ (\Omega_\Lambda, p_1, p_2) \to 0,\\
&V(z) \to \tilde{V}(z) = -\left(\frac{p_3^2+1}{4z^2} - \frac{\Omega_K}{4}\right),\\
&[\partial_\phi^2 - \partial_h^2 - \Omega_K(\phi^2-h^2) + \Omega_c]\Psi = 0. \qquad (37)
\end{aligned}$$

Of course, $S_{|\vec{p}|}(z)$ is just the $(p_1, p_2) \to 0$ limit of (35) and $(z(\tau), \sigma(\tau))$ are just the limits of (36) when $p_1 = p_2 = s_{10} = s_{20} = 0$. However, just like case 2, the

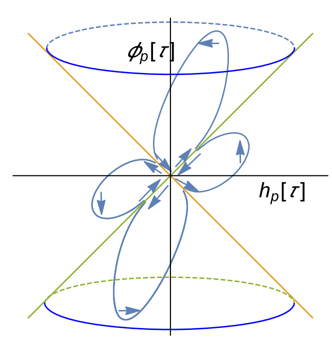

FIG. 8. Periodic trajectory driven by curvature.

WdWe has a nice interpretation in the global (ϕ, h) space in Fig. 1: the WdWe (12) reduces to the $1+1$ dimensional relativistic harmonic oscillator Hamiltonian that is constrained to a single energy eigenvalue fixed to $-\Omega_c$, as indicated in (37). The complete ghostfree unitary analysis of this equation was first given in [30], its application to cosmology was outlined in a footnote in [3], and given again in detail in [13] where it was noted Ω_c must be quantized in this setting. A path integral quantization was also applied in [31] that is in complete agreement with [13] but misses on the quantization of Ω_c.

Exactly solvable cases (1–4) are of course only limits of the full case discussed in this paper. These limits are helpful in better understanding the structure and meaning of the full solution. But it must be noted that in some of these limits, in particular those dominated by curvature (cases 3,4), and those lacking anisotropy, the solution is qualitatively different.

Finally, I would like to present the wavefunction very close to the "beginning," i.e., at the big bang. This is not an input, but rather it is a unique prediction driven by the dynamics that attracts to the pin hole in Fig. 7 discussed earlier. $\Psi(z, \vec{s})$ close to the singularity is obtained by taking the $z \to 0$ limit of the general solution (31). In this expression there is no need to know the details of $S_{|\vec{p}|}(z)$ because at $z=0$ it is exactly 1 in all cases, including the full case with all nonzero parameters, $S_{|\vec{p}|}(0)=1$. The next to the leading terms,

$$S_{|\vec{p}|}(z) = 1 - \frac{\Omega_c}{4(1-i|\vec{p}|)} z + \frac{(\Omega_c^2 + 4\Omega_K(1-i|\vec{p}|))}{32(1-i|\vec{p}|)(2-i|\vec{p}|)} z^2 + O(z^3), \tag{38}$$

are obtained from the series expansion in (35), while at order $O(z^3)$ the parameter Ω_Λ also contributes as seen from (33). I will concentrate only on the first term $S_{|\vec{p}|}(z) \to 1$ and evaluate the integral $\Psi_{\rm II}(z, \vec{s})$ in the proximity of the tip of the upper cone in Fig. 7 (or 8)

$$\Psi_{\rm II}(z, \vec{s}) \overset{z\sim 0}{=} \int_{-\infty}^{\infty} d^3p \frac{A e^{-\frac{p_3^2}{2\Omega_\sigma} - \frac{p_1^2+p_2^2}{2\Omega_\alpha} - i\vec{p}\cdot\vec{s}}}{\sqrt{(2\pi)^3 2|\vec{p}|}} \times \left(\sqrt{|z|}^{-i|\vec{p}|} + \sqrt{|z|}^{i|\vec{p}|} \frac{e^{-\frac{\pi|\vec{p}|}{3}} (\frac{\Omega_\Lambda}{18})^{i\frac{|\vec{p}|}{3}} \Gamma(-i\frac{|\vec{p}|}{3})}{\Gamma(i\frac{|\vec{p}|}{2})} \right). \tag{39}$$

I will approximate $\Omega_\sigma = \Omega_\alpha \equiv \Omega_{\sigma,\alpha}$ to have a rotationally symmetric integrand that is simpler to evaluate. This is sufficient to get the general idea. Then the angular integration over $\hat{p}$ yields $\int d^2\hat{p} e^{-i\vec{p}\cdot\vec{s}} = 4\pi e^{-i|\vec{p}||\vec{s}|}$, while the remaining radial integral is a function of only (z, s), with $s = \sqrt{\alpha_2^2 + \alpha_2^2 + \sigma^2} > 0$. The second term in the parenthesis oscillates wildly as $z \to 0$, and it vanishes, as expected in the discussion of Eq. (28) near $z=0$. However, the first term has a stationary region when $z \to 0$ and $s \to \infty$ in tandem so that $e^s\sqrt{|z|}$ is finite. This is the attractor mechanism at work, showing once again that anisotropy and scalar d.o.f. diverge while the scale factor vanishes. I find,

$$\Psi_{\rm II}(z, s) \overset{z\sim 0}{=} \frac{4\pi A}{\sqrt{(2\pi)^3}} \int_0^\infty \frac{p^2 dp}{\sqrt{2p}} e^{-\frac{p^2}{2\Omega_{\sigma,\alpha}}} (e^s\sqrt{|z|})^{-ip} = \frac{A\pi^2(-1)^{\frac{3}{4}}}{\sqrt{2(2\pi)^3 \Omega_{\sigma,\alpha}^{7/2}}} \phi(x). \tag{40}$$

The integral is performed exactly. It is written in terms of Bessel functions $I_\nu(-\frac{x^2}{4})$, and expressed as a function $\phi(x)$ of a finite and positive x,

$$\phi(x) \equiv \sqrt{x} e^{-\frac{x^2}{4}} \left[\begin{array}{c} (x^2-1) I_{-\frac{1}{4}}(-\frac{x^2}{4}) + (x^2-3) I_{\frac{1}{4}}(-\frac{x^2}{4}) \\ + x^2 (I_{\frac{3}{4}}(-\frac{x^2}{4}) + I_{\frac{5}{4}}(-\frac{x^2}{4})) \end{array} \right], \quad x \equiv \frac{e^{|\vec{s}|}\sqrt{|z|}}{\sqrt{\Omega_{\sigma,\alpha}}}. \tag{41}$$

The x parameter is proportional to some average of the $(u_{\hat{p}}, v_{\hat{p}})$ parameters in Fig. 7. This $\phi(x)$ is a complex function whose absolute value and phase are plotted in Fig. 9. The upper curve in Fig. 9, $|\phi(x)|$, represents the probabilistic distribution of the wavefunction near the tip of the cone in Fig. 7. It shows that the probability is larger for smaller values of $(u_{\hat{p}}, v_{\hat{p}})$. This means larger probability when close to the pin hole, consistent with the classical solution in Fig. 7 that shows horizons are crossed *classically only at the pin hole*. Figure 9 conveys a fuzzy quantum version of the same result, confirming that anisotropy and Higgs d.o.f. must get larger ($|\vec{s}| \to \infty$) as the universe gets smaller ($z \to 0$) so that probability continuously propagates through the *neighborhood of the pin hole*. This further softening of singularities is due to quantum mechanics.

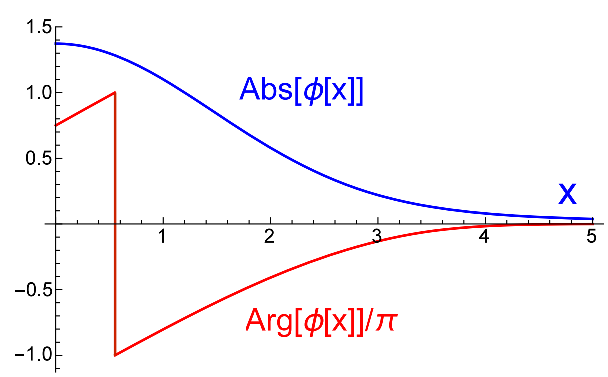

FIG. 9. Wavefunction near pin hole.

Predicted corrections to this result near $z \simeq 0$ follow directly from (38). The wavefunction for all $(z, \vec{s})$ throughout the geodesically complete superspace is also determined in Eq. (31), and can be plotted in a similar way.

VIII. DISCUSSION AND OUTLOOK

In this Sec. I will highlight the main results in this paper, comment on open questions and problems, and compare to other discussions in recent literature [32–40] that relate to the quantum treatment of minisuperspace.

A. Overview of results

I defined an EFTC that is mathematically sufficiently well behaved at gravitational singularities. Part of its quantitative definition includes the suppression of higher curvature terms in the effective action by relying on the softening effects of some underlying theory of quantum gravity (QG). The remaining singular terms are mathematically controllable with a local scale symmetry. Quantum mechanics makes it even softer, since passage through a singularity occurs in a *neighborhood of a point* rather than only at a point, as demonstrated quantitatively in Fig. 9 for the *predicted initial conditions of the wavefunction of the universe at the big bang*.

Advocating that higher curvature terms be banned in an effective field theory for cosmological applications is a new point of view that is motivated by notions of QG. Undoubtedly, this blurry point that seems reasonable at the outset, needs more discussion, and I hope it will be a starting point for future investigations and improvements. The temporary justification for this EFTC is that it provides a practically working formalism to investigate quantitatively singular spacetimes, including cosmology and black holes, and be able to make predictions that were not available before. Furthermore, at the fundamental level, this EFTC is grounded in the successful standard model and general relativity, with only a modest improvement to achieve geodesic completeness of its spacetime through a local scale (Weyl) symmetry. Combined with the softer classical and quantum mathematical properties, this provides a physically strong basis for new progress whose results can be compared to other attempts of QG when those can produce comparable computations.

This paper is focused on quantizing the d.o.f. of minisuperspace, including scale, Higgs, anisotropy, dark matter and dark energy, radiation and curvature, that are expected to play the main roles in shaping the very early universe and its later development. One aim was extracting from them the prediction of this EFTC for the wavefunction of the universe. This was fulfilled in this paper, culminating in the prediction of an explicit wavefunction of the universe that contains no parameters, has dynamically produced unique initial values, and is continuous in a geodesically complete universe that includes gravity as well as antigravity patches.

It is straightforward to compute the Feynman propagator from any point to any other point in the geodesically complete superspace as follows

$$G(\phi', h', \alpha'_{1,2}; \phi, h, \alpha_{1,2}) = \langle \phi', h', \alpha'_{1,2} \Big| \frac{i}{\mathcal{H} + i\varepsilon} \Big| \phi, h, \alpha_{1,2} \rangle = i \int d\lambda (\lambda + i\varepsilon)^{-1} \Psi_\lambda(\phi', h', \alpha'_{1,2}) \times \Psi^*_\lambda(\phi, h, \alpha_{1,2}), \tag{42}$$

where $\mathcal{H}\Psi_\lambda = \lambda\Psi_\lambda$ is explicitly given by the WdWe differential operator in either (12) or (13) and by replacing Ω_c by $(\Omega_c - \lambda)$. A quick (but not always the best version) solution for Ψ_λ is, $\Psi_\lambda = \Psi_{\lambda=0}(\Omega_c - \lambda)$, where $\Psi_{\lambda=0}$, is the complete set of solutions of the WdWe already obtained in the previous sections. The result of (42) can be expressed in either the global coordinates $\Psi_\lambda(\phi, h, \alpha_{1,2})$ or the patchy coordinates $\Psi_\lambda(z, \vec{s})$. In the latter case, the choice of Ψ_{II} or $\Psi_{\mathrm{I\&III}}$ or Ψ_{IV} depends on the location of the corresponding "points" $(\phi', h', \alpha'_{1,2}; \phi, h, \alpha_{1,2})$ in the geodesically complete minisuperspace. Although these details are not fully carried out here for all values of the parameters $(|\vec{p}|, \Omega_\Lambda, \Omega_c, \Omega_K)$, the complete solution is already available in the literature for some subcases. This is thanks to the recognition emphasized in Eq. (10), that the global minisuperspace (z, σ) has the geometry of Rindler space geodesically completed to $1+1$ dimensional flat Minkowski space. Then some computations become very simple. Namely, the following propagators,

(a) Case-2 in Eq. (34) is equivalent to the massive Klein-Gordon (KG) equation in $1+1$ dimensions, so the associated propagator is simply the massive KG propagator in the full space in Fig. 1.

$$G(\phi', h'; \phi, h) = i \int \frac{dp_\phi dp_h}{(2\pi)^2} \exp(-i(\phi' - \phi)p_\phi + i(h' - h)p_h) \times (-p_\phi^2 + p_h^2 + \Omega_c + i\varepsilon)^{-1}. \tag{43}$$

This can be expressed in terms of the patchy coordinates $G(z', \sigma'; z, \sigma)$ by the coordinate transformation (8). It is harder to compute the propagator directly in the (z, σ) basis using the z-version of $\mathcal{H}$ in Eq. (34) including the potential $\tilde{V}(z)$. However, with some labor involving Bogoliubov transformations between Rindler waves and Minkowski waves, given in [14], the propagator can be brought to this form.

(b) Case 4 in Eq. (37), is equivalent to the relativistic harmonic oscillator, whose unitary infinite dimensional quantum basis is given in [30]. The associated propagator is worked out explicitly in [13]. This can again be easily rewritten in terms of (z, σ), and in that form the propagator is generalized to case-3 in the $(z, \vec{s})$ basis. The case 3 propagator will appear in a separate paper.

The computations in this paper were possible thanks to the control provided at cosmological singularities by the underlying local scaling (Weyl) gauge symmetry in this EFTC. The new computations at both classical and quantum levels revealed surprizing dominant behavior of some d.o.f. at the very beginning of the universe. Specifically, this led to a theorem that states: *anisotropy and Higgs (or another scalar) degrees of freedom must keep growing indefinitely as the scale factor of the universe keeps getting smaller when crunch or bang type singularities are approached.* This striking conclusion was derived in the quantum treatment of the wavefunction through the steps of Secs. IV, VI, and VII. Specifically, it is because of this behavior that the wavefunction manages to be continuous in propagating through cosmological singularities that separate gravity and antigravity patches. This quantum conclusion strengthens an earlier similar result in the classical treatment [5] of the relevant d.o.f. With the inclusion of the cosmological constant that was missing in [5], this paper presents a more complete unique classical solution in Sec. V that displays a spectacular attractor mechanism and passage through the singularity, as represented by the pin hole in Fig. 7. Furthermore, the computations also predicted the mathematically unavoidable *multiverse aspects of the wavefunction* (more thoroughly discussed in [14]); the multiverse continues to be under study to understand its physical significance.

It should not go unnoticed that the Higgs in this EFTC has important cosmological roles in shaping the very early universe. These include providing geodesic completeness, participating in the attractor mechanism and continuity of the wavefunction from gravity to antigravity patches, and in the avoidance of the mixmaster chaos (last part of Sec. V).

B. Open problems

There are open questions that deserve further investigation:

(i) Anisotropy is predicted to be huge at the beginning, then how does it become miniscule in today's universe? Some would advocate inflation as a possible mechanism, but inflation has not yet been considered as an added feature to this EFTC, although such a modification of the EFTC may be considered as an option. However, it is interesting that a very different and rather natural mechanism has also emerged in this paper for how anisotropy can evolve from huge at the bang to tiny today. The basic idea is the observation enunciated just before Eqs. (14)–(16) that motivated the 2-step strategy for taking into account approximately the effect of the potentials $V_K(\alpha_1, \alpha_2)$, $V(\sigma, \varepsilon_z)$. Namely, in a time dependent Hamiltonian, d.o.f. that are subjected to attractive potentials, will quickly descend to the ground state. In the case of anisotropy, the time (i.e., z) dependent potential is $(\phi^2 - h^2)V_K(\alpha_1, \alpha_2) = zV_K(\alpha_1, \alpha_2)$, as seen in (6), (11). A plot of V_K [25] shows that this is an infinite potential well, of the approximate shape of an upside down infinite triangular pyramid, whose strength $(\phi^2 - h^2) = z$ keeps growing as the universe expands. The progressively stronger attractive potential will bind anisotropy more and more tightly in its ground state, thus driving $\alpha_{1,2} \to 0$. This seems like a perfect natural mechanism to explain why the average *homogeneous anisotropy* is so small in the later universe even though it is infinitely large at the bang. It should be mentioned that in discussions of dynamics in this potential [28,41], it is claimed that not only average anisotropy but also average inhomogeneity (if included in the equations in the first place) would tend to get smaller as the universe expands. Renewed vigorous investigations, on whether this scenario actually produces sufficient suppression of anisotropy as well as inhomogeneities, and the extent to which this supports the 2-step strategy applied in this paper, would be useful.

(ii) A similar investigation regarding the 2-step strategy with the Higgs potential is in order. The reasoning was that at the electroweak phase transition the Higgs should settle to the minimum of its potential, $|h/\phi| \to w \sim 10^{-17}$, and remain there for the subsequent evolution. This seems reasonable in the gravity sectors where $|h/\phi| < 1$. However during evolution in the antigravity sector where $|h/\phi| > 1$, the Higgs potential $V(\phi, h) = (\phi^2 - h^2)^2 f(h/\phi)$ given in (6) is far from the minimum; then the huge $\Omega_\lambda \sim 10^{120}$ term can play an important role. The modification of Ψ can be assessed qualitatively as follows: The huge Ω_λ term creates a very strong potential that prevents h from getting large during antigravity; then the loops in the antigravity sectors in Fig. 7 (and correspondingly the probability amplitude $|\Psi_{\mathrm{I\&III}}|$ at large $|z|$) will be considerably smaller since $|z| = h^2|1 - (\phi/h)^2|$ is prevented more strongly from growing in antigravity. There may be other interesting effects during antigravity that are hard to guess without an explicit computation. In addition, recall that any function $f(h/\phi)$ is consistent with Weyl symmetry; even sticking with the standard model form in (6), the renormalized dimensionless parameters $(\Omega_\lambda, w, \Omega_\Lambda)$ as functions of $\ln(h/\phi)$, are not known beyond lowest order in perturbation theory. This remaining unknown in the effective theory can be a source of speculation, including the possibility of a metastable Higgs [42] with additional dramatic consequences in our understanding of cosmology, as discussed in [9].

Further studies that address such remaining questions related to the Higgs would be of interest.

(iii) The next goal is to include *inhomogeneous* small fluctuations of the metric and Higgs in the minisuperspace action (5) and treat them as perturbations to the unique homogeneous background solution for $\Psi_{\mathrm{II,I\&III,IV}}$ given in Sec. VII. The results would eventually be confronted with available data and phenomenology of the observed properties of the cosmic microwave background (CMB). The background wavefunction reported in this paper $\Psi_{\mathrm{II,I\&III,IV}}(z,\vec{s})$ already has built in nonperturbative dominant parts of the metric such as anisotropy, as well as matter such as the Higgs, as part of the $\vec{s}$ dependence. Therefore the suggested expansion in small fluctuations of the metric and Higgs is very different than previous attempts, either in the usual classical approach, or the few quantum versions attempted in recent literature. This is because the huge nonperturbative dynamical effects, such as ($|\vec{s}| \to \infty$ when $z \to 0$), were not known or taken into account in previous computations. Because of this, I expect that the *previous computations, that as part of their setup, assumed only perturbative small fluctuations, should have internal inconsistencies*.

C. Comparing quantum approaches

The last remark provides an introduction to a comparison of the current work to other recent path integral approaches [32–40] that have discussed the quantum minisuperspace. Because there is some confusing debate still brewing in this topic, it would be useful to readers to clarify where the present work stands relative to this controversy. A main message is that the other approaches lack some of the important and essential features in the current paper and there is room for improvement of the path integral computations if these features can (I believe with some difficulty) be incorporated:

(1) The authors in [32–40] use path integral quantization as opposed to WdWe method to compute the wavefunction of the universe or a related propagator. In principle all such methods should agree, so different approaches are welcome. As in item (*b*) above, I find agreement for the *propagator* in case-4 Eq. (37) in the WdWe method [13] versus the counterpart in the path integral method [31]. This is a good sign, but beyond this, so far, there is little available in the other approaches to compare with the results in the current paper. As a next easy comparison, I would suggest the propagator in Eq. (43) for case 2 in (34), that is not available yet in the path integral formalism.

(2) This paper presents exact quantum solutions for the WdWe and its propagators. By contrast, the path integral results are only semiclassical. Sharp disagreements between competing groups, [32–35] versus [32–39], doing path integral computations remain unsettled. Part of the controversy is over the fundamental correctness of Lorentzian versus Euclidean path integration in the computation of the wavefunction for the universe. On that score, I side with Lorentzian as a principle, but also the agreement of propagators in [13] versus [31] noted in item (1) lends support to Lorentzian. A second, more subtle technical part of the controversy, involves which path is the correct integration path, to define the quantum theory—this should be settled by comparing to the WdWe work in this paper. A third part of the controversy is that one group claims to compute a wavefunction while the other group insists on propagators. The current paper based on the WdWe approach produces exact quantum results for both quantities. Future semiclassical path integral results that may disagree with the exact quantum results of the current paper would, in my opinion, be suspect.

(3) The path integral teams have been working with a geodesically incomplete mini-superspace that covers only region II in Fig. 1 or Fig. 7. Signals of the incompleteness arises in their computations; specifically, their parameter $q > 0$, that is related directly to $q = a_E^2 = |z|$, runs into contradictions with the mathematical properties of their equations because imposing $q > 0$ (half space of my z) at the quantum level is problematic; but they sweep this problem under the rug. For the purpose of comparing to their results, it is possible to narrow the results of the present work to only region II, and therein there are fundamental differences of principle. In particular initial conditions *at the big bang* is really an input in their case (even though it is called "no boundary proposal"), but it is an output and a prediction in the current paper as seen in Fig. 9 and related equations. This difference is connected to the attractor mechanism in Figs. 6 and 7 that is completely lacking in their approach because the drivers of this mechanism are absent in their simplified model.

(4) Most importantly, the path integral approaches do not include some of the minisuperspace d.o.f. Specifically, anisotropy, scalar field, conformal dust matter Ω_c, are hugely dominant in the early universe as compared to the cosmological constant Ω_Λ. Yet, in the models investigated in the recent path integral papers, the cosmological constant is the main ingredient driving the evolution. Leaving out certain terms in the action produces a much more manageable integral, but unfortunately this misses the *dominant nonperturbative effects of anisotropy and Higgs* emphasized repeatedly from different classical and quantum perspectives in the current paper.

(5) In the Lorentzian path integral approach, inconsistencies concerning small *inhomogeneous metric*

fluctuations were discovered [32–35]. As reported, computation shows that the fluctuations come out larger than the *homogeneous* background; however, this is contrary to the setup of the computation in which *linearized fluctuations* were assumed to be smaller than the background to begin with. As noted at the end of item (iii) above, this is to be expected since, as this paper demonstrated, there are very large nonperturbative effects in the homogeneous metric and scalar field, namely anisotropy and Higgs, that should be part of the background. Inhomogeneous metric and scalar fluctuations, on top on this nonperturbative background, would be expected to remain consistently small and overcome this problem.

Meaningful comparison between the results of the current paper and the path integral approach will be possible, for both the wavefunction and the propagator, when the listed differences in the approaches are ironed out. These include the choice of models and d.o.f. they contain, inclusion of potentials and/or implementation of the 2-step strategy for approximating their effects, semiclassical versus exact quantum computation, and the inclusion of nonperturbative effects in the homogeneous background solution. Given the encouraging agreement for the propagator in one of the simplest cases [case 4 in Eq. (37)] as reported in [13,31], I expect full agreement when the computations of various groups focus on the same system and the same physical quantities.

D. Toward an ultraviolet completion of the EFTC

The geodesically complete EFTC promoted in this paper is capable of providing detailed quantitative description of passage through cosmological singularities at both the classical and quantum levels. This kind of prediction is also possible for black holes by using the same EFTC as suggested in [12]. In this way it is demonstrated that events in the spacetime on the other side of singularities (such as far past boundary conditions) affect the properties of the physics in the spacetime past the singularities. So, the geodesically complete spacetime must be taken into account for cosmology. When this is done, as in this paper, initial conditions at the big bang are *predicted* not guessed. This in itself is remarkable about this EFTC because other approaches in cosmology (including stringy approaches) have not been able to provide comparable detail.

As emphasized earlier in this paper there are three crucial ingredients in this EFTC: (i) a Weyl symmetry and associated geodesic completeness, (ii) a ban on higher derivatives at high energies, and (iii) close connection to the standard model at low energies, including the Higgs field. I now address the question of "how could these ingredients be compatible with an ultraviolet complete approach, such as string theory?"

The ban on higher derivatives is in fact attributed to a softening provided by quantum gravity, such as string theory. This is expected just on the basis that the description of the physics in the strongly interacting regime is given in terms of stringy configurations involving string fields (including stringy modes) as compared to pointlike fields in the low energy approximation. Past experience with string theory shows that perturbative string amplitudes expanded in powers of α' (string tension of Planck scale) are reproduced in the low energy effective theory by including higher derivative terms (such as higher curvatures) that are multiplied by powers of α'. But these terms are valid only at small momenta (small derivatives) or small energies E when $\alpha' E^2 \ll 1$. The string amplitudes at high energies $\alpha' E^2 \gtrsim 1$ cannot be reproduced by using the higher derivative terms of the low energy theory. Furthermore, the stringy description at high energy does not involve higher curvatures, but instead it involves stringy modes that provide a much softer behavior of the theory even in a strongly interacting regime. Therefore, it is completely wrong to include higher derivative terms in the low energy theory if the purpose is to describe a phenomenon such as the transitions through a singularity. This is the justification for banning higher derivatives in the EFTC.

The EFTC is of course not a substitute for an ultraviolet complete theory. At best, the EFTC is expected to correctly describe the physics up to some fraction of the Planck energy and possibly be inaccurate at higher energies. Nevertheless it is gratifying that the EFTC in this paper does provide a mathematically self-consistent answer to questions at the scale of Planck energies, including passage through singularities. It presently stands as the only tool that provides quantitative answers to questions at the Planck scale. Until a more reliable tool becomes available, I believe this is at least an answer to think about. To be certain of the physical correctness or inaccuracies of this EFTC description at the Planck scale one must construct and then analyze an appropriate string theory and then compare answers.

Unfortunately string cosmology is not an easy task. Past attempts have encountered a number of difficulties, including those described in Sec. III.4 of [43] and references therein. Part of the problem is that most attempts rely on the perturbative setup of string theory for strings that propagate on a cosmological background. However, near a singularity stringy interactions become strong so that a perturbative stringy approach cannot work. For instance near null cosmological singularities (which allow for detailed analysis), strings are known to become highly excited (nontrivial oscillator modes) suggesting that backreaction is important. Likewise in discussions of cosmological singularities and gauge/gravity duality, it can be argued in certain cases (with spacelike singularities) that the dual gauge theory (which might have been naively hoped to lead to a controlled weakly coupled description) also breaks down, implying that continuing past the bulk big crunch singularity is unclear (the AdS Kasner singularities of [44] have been revisited in [45] and subsequent work). If a smooth gauge theory description exists allowing continuation past the singularity (e.g., as certain null singularities suggest), it would amount to the bulk gravitational description necessarily being strongly coupled. These and related investigations remain inconclusive themselves but point to some difficulties of the perturbative setup in string cosmology and showing that the perturbative setup to string cosmology is useless. A more useful, but quite difficult, approach could be string field theory, where nonperturbative solutions in terms of string fields in an appropriate cosmological background are possible, thus possibly providing a better nonperturbative tool.

In any case, I believe that, in both perturbative and nonperturbative string theory, to capture the correct physics one must use geodesically complete backgrounds that include all patches of a complete spacetime on both sides of singularities. That this is essential has been demonstrated in this paper in the context of the EFTC. However, the notion of a geodesically complete space is totally missing in all previous attempts in string cosmology. Connected to the same fact, the notion of a stringy background that is also Weyl symmetric in target space has also been missing in overall string theory because string theory has a fundamental length α' (the string torsion related to the Newton constant G_N). On the other hand, since the Weyl symmetric EFTC does exist as in this paper, in which G_N is generated by spontaneous breakdown, and geodesic completeness is built in, one should wonder which string background could yield it as a low energy approximation?

The previous paragraph poses a challenge for all quantum gravity (QG) attempts, not only string theory. The low energy EFTC in the present paper is geodesically complete, and the Weyl symmetry is crucial. On the other hand all known attempts for QG, including string theory, have a dimensionful parameter that is equivalent to the gravitational constant G_N, so they are not Weyl symmetric in target spacetime, and do not have an effective gravitational function that could change sign so that geodesic completeness of the backgrounds is built in. Clearly, such inherently geodesically incomplete QG theories could not generate the EFTC suggested in this paper. However, it is possible to improve string theory with Weyl symmetry on target space to make it consistent with the properties of this EFTC. This is possible by replacing the string tension in string theory to be a background field that can change sign, as shown in [11]. How this can be incorporated in the BRST operator in string field theory has also been discussed briefly in [13].

The Weyl-improved string theory is of course difficult to analyze, but at least it has the right properties to be the ultraviolet completion of the EFTC discussed in this paper. Future work may reach a stage that provides stringy results to be compared to those obtained in this paper, thus showing the level of success or shortcomings of the EFTC.

ACKNOWLEDGMENTS

I thank the Perimeter Institute for its generous support and hospitality during my sabbatical. Research at the Perimeter Institute is supported by the Government of Canada and the Province of Ontario through the Ministry of Research and Innovation.

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