Determining the dependence on the energy and atmospheric depth of the angular distribution of muons using the CORSIKA simulation code

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With the CORSIKA code, 3×10^5 extensive air showers with proton, alpha, and carbon particles as the primary particles have been simulated. The primary energy range was chosen between 1 GeV and 1 PeV with a differential flux given by $dN/dE \propto E^{-2.7}$ and zenith angles of 0° to 60°. Using the muons produced by the extensive air showers, the angular distribution of the muons has been investigated at various depths. A distribution of $\sin\theta \cos^{k(X_v,E)}(\theta)$ for the zenith angle of muons has been obtained such that the values of $k(X_v, E)$ are found for different atmospheric vertical depths, X_v, and energies, E, using the simulation data. We obtained $k(X_v; E) = (0.4494 \pm 0.0046) \ln(X_v/\mathrm{g\,cm^{-2}}) + (-0.4903 \pm 0.0079) \ln(E/\mathrm{GeV})$ with a regression of 95%.

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I. INTRODUCTION

When a high-energy cosmic ray comes into the atmosphere, secondary particles are produced by interacting with the air nuclei as targets. By repeating the interaction of these secondary particles with air nuclei, more particles are generated, which is called an extensive air shower (EAS). One of the most important particles in these reactions is the muon. The most important primary particles of cosmic rays are proton (P), alpha (α), and carbon (C) particles, which are the main sources of atmospheric muons. In fact, with the interaction of the cosmic rays with air nuclei, pions and kaons are produced ($A_{\mathrm{CR}} + A_{\mathrm{Air}} \to \pi^\pm,\ \pi^0,\ k^\pm,$ other hadrons). The prevalence of decay or interaction depends on which of the values is greater, the energy of the meson, E, or $\epsilon = \frac{h_0 mc^2}{c\tau}$ [1]. Where $h_0 = 6.4$ km, m and τ are, respectively, the mass and mean lifetime of the meson, and c is the speed of light. For $E \ll \epsilon$, interaction is negligible [1]. The rest energy and lifetime of pions and kaons are (139.570 MeV, 26 ns) and (493.688 MeV, 12 ns) respectively, so we obtain $\epsilon_\pi \simeq 115$ GeV and $\epsilon_K \simeq 850$ GeV. Therefore, for energies below $E = 100$ GeV, the condition ($E < \epsilon$) is valid and the interaction can be ignored. When charged pions and kaons decay via the weak interaction, muons are created as

$$\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu); \qquad \sim 100\%, \qquad \tau_0 = 26 \text{ ns} \tag{1}$$

$$k^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu); \qquad \sim 63.5\%, \qquad \tau_0 = 12 \text{ ns.} \tag{2}$$

Generally, muons produced in air showers do not multiply, but gradually lose their energy due to ionization in the atmosphere. Due to the effects of special relativity, muons can penetrate the sea level. Muons have rest energy $m_\mu c^2 = 105.7$ MeV and rest frame lifetime $\tau \approx 2.2\ \mu$s. Experimental evidence indicates, however, that muons are produced at a high altitude in the atmosphere, with travel time to sea level much greater than τ in the rest frame of the Earth. According to relativity, a muon with $E = 2.0$ GeV has the Lorentz factor $\Gamma \approx 18.9$. Hence, the lifetime of the muon, as seen from the Earth, is $\tau' = \Gamma\tau \approx 41.6\ \mu$s, allowing 2.0 GeV muons to reach the Earth's surface from a height of over $h = c(\Gamma^2 - 1)^{0.5}\tau \approx 12.5$ km. But muons lose their energy by almost $dE/dX \simeq -2\ \mathrm{MeV}/(\mathrm{g\,cm^{-2}})$ due to ionization loss in matter. Thus, muons lose about 2 GeV before they reach sea level (with vertical depth $X_v \simeq 1000\ \mathrm{g\,cm^{-2}}$). Hence, 2.0 GeV muons, which have a decay length of 12.5 km, are reduced to 6.2 km due to energy loss. Consequently, the energy and angular distribution of the muons at each depth of the atmosphere are a convolution of the production spectrum, decay, and energy loss in the atmosphere.

The energy spectrum of primary cosmic rays follows the power law $E_{\mathrm{pri}}^{-\gamma}$. The distribution of pions and muons also complies with this law, but is modified due to the energy loss. The distribution of the muons can be expressed as follows:

$$\frac{dN(E,\theta)}{dEd\Omega} = a(E_\theta + E)^{-\gamma}. \tag{3}$$

Taking into account the total number of muons in all energies, N_0, the value of a is $(\gamma - 1)N_0 E_\theta^{\gamma-1}$, and E_θ is the energy loss of a muon (both the hadronic and electromagnetic interaction with air nuclei) in the inclined direction θ.

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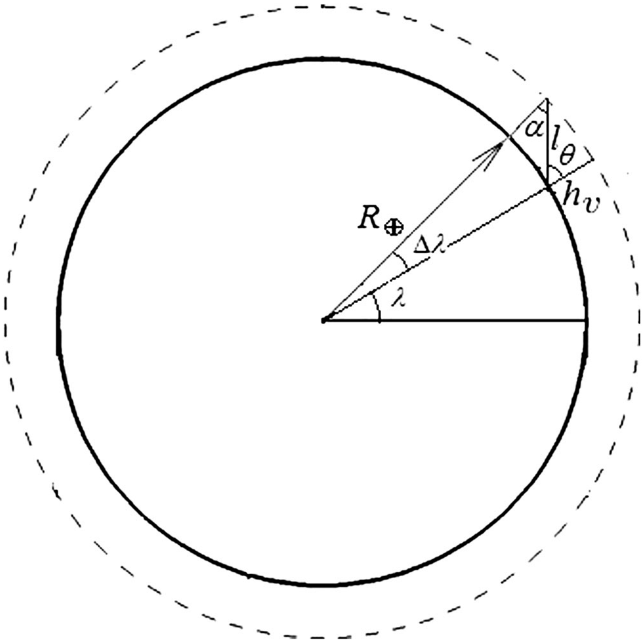

FIG. 1. A schematic of the Earth and atmosphere for showing the angles and notations used in the text.

The intensity of the muons decreases with increasing zenith angle due to the path length. According to Fig. 1 the inclined distance, l, in terms of vertical distance, h_v, the zenith angle, θ, and the Earth's radius, $R_\oplus$, is

$$l = (h_v + R_\oplus)\frac{\sin(\Delta\lambda)}{\sin\theta}, \tag{4}$$

with $\Delta\lambda = \theta - \alpha$. Also we have

$$\sin\alpha = \frac{R_\oplus}{h_v + R_\oplus}\sin\theta. \tag{5}$$

By combining Eqs. (4) and (5) we obtain

$$\begin{aligned} L(\theta) &= \frac{l}{h_v} = \left(1 + \frac{R_\oplus}{h_v}\right)\frac{\sin(\theta-\alpha)}{\sin\theta} \\ &= \frac{R_\oplus}{h_v}\cos\theta\left(\sqrt{1 + 2\frac{h_v}{R_\oplus\cos^2\theta} + \frac{h_v^2}{R_\oplus^2\cos^2\theta}} - 1\right). \end{aligned} \tag{6}$$

The ratio of the integrated muon intensity at the zenith angle θ to that at $\theta = 0$ can be obtained from Eq. (3) as

$$\frac{\frac{dN}{d\Omega}(\theta)}{\frac{dN}{d\Omega}(0)} = \frac{\int_0^\infty (E_\theta + E)^{-\gamma} dE}{\int_0^\infty (E_0 + E)^{-\gamma} dE} = \left(\frac{E_\theta}{E_0}\right)^{1-\gamma}, \tag{7}$$

where E_θ and E_0 are the energy loss of a muon in the inclined and vertical directions, respectively. Since the energy loss is proportional to the path length of the muon through the atmosphere, the ratio in the parentheses of Eq. (7) is the same as the ratio of $L(\theta) = \frac{l}{h_v}$. So,

$$\frac{dN}{d\Omega}(\theta) = \frac{dN}{d\Omega}(0) L^{1-\gamma}(\theta). \tag{8}$$

In the limit $\frac{R_\oplus}{h_v} \gg 1$, from Eq. (6) we obtain $L(\theta) = \frac{1}{\cos\theta}$, which by substituting in Eq. (8), we have

$$\frac{dN}{d\Omega}(\theta) = \frac{dN}{d\Omega}(0)\cos^{\gamma-1}(\theta). \tag{9}$$

So the zenith angle distribution of atmospheric muons at different atmospheric depths and different energies can be described almost as follows:

$$\frac{dN}{d\theta}(\theta) \propto \sin\theta\cos^{k(X_v,E)}(\theta), \tag{10}$$

where X_v is the vertical depth traveled by the muon; E is the energy of the muon; θ is the polar angle with respect to the vertical or the same zenith angle; $\sin\theta$ comes from the solid angle; and $k(X_v, E) = \gamma - 1$ is an empirical constant. The main objective of this paper is to study the muons produced from air showers derived from primary particles including proton, alpha, and carbon particles at different energies using the CORSIKA simulation code. In this work, the effect of atmospheric thickness on the angular distribution of the muons at different energies has been investigated.

II. GENERATION OF AIR SHOWERS SIMULATED WITH CORSIKA

The Monte Carlo air shower simulation program CORSIKA (version 74000) has been used for the simulated data. The CORSIKA (Cosmic Ray Simulations for Kascade) package [2] simulates EAS initiated by photon and cosmic ray particles, such as the proton, alpha, and so on. Different hadronic interaction models are available in CORSIKA. In this work we used the QGSJET (qgsjet01.f package) [3] and GHEISHA (Gamma Hadron Electron Interaction Shower code) [4] models, for hadronic interactions above and below $E_{\mathrm{lab}} > 80$ GeV, respectively. In the coordinates of the experimental site (Tehran, 1200 m above sea level $\equiv 897\ \mathrm{g\,cm^{-2}}$), the components of the geomagnetic field are $B_x = 27.97\ \mu$T, $B_z = 39.45\mu$T, which are taken from NOAA's National Centers for Environmental Information (NCEI) [5]. We assume that the geomagnetic field is constant and independent of the angular direction of the muon. To explain this point, consider Fig. 1. Simply we have

$$h_v \simeq l\cos\theta + \frac{l^2}{2R_\oplus}\sin^2\theta \quad (l/R_\oplus \ll 1), \tag{11}$$

where h_v is the vertical altitude of atmosphere and $l(\theta)$ is the length of a slant trajectory. For θ less than 60° we can neglect the second term in Eq. (11). If, according to Fig. 1, the latitude of the observation site is λ, the variation in the latitude value of the starting point of the muon, $\Delta\lambda$, with the zenith angle θ is given by the following equation:

$$\sin(\Delta\lambda) = l(\theta)\frac{\sin\theta}{h_v + R_\oplus}. \tag{12}$$

With approximation $l \simeq h_v/\cos\theta$, for values $\theta = 60°$, $h_v \simeq 20$ km, and $R_\oplus = 6400$ km, we obtain $\Delta\lambda \simeq 0.3°$. With this value $\Delta\lambda$, the relative variation of the geomagnetic field components at the location of Tehran is calculated. With the first order approximation we take the geomagnetic field as a dipole, i.e., $\mathbf{B} = (3(\mathbf{b}\cdot\mathbf{x})\mathbf{x} - x^2\mathbf{b})/x^5$, where b has intensity $b \approx 8 \times 10^{15}$ Tm^3. So, we get $|\frac{\Delta B_x}{B_x}| = 0.003$, and $|\frac{\Delta B_z}{B_z}| = 0.007$. With these small errors, it can be concluded that a fixed geomagnetic field is a fairly good presumption. The energy of the primary particles was considered between 1 GeV and 1 PeV with a differential flux distribution as $dN/dE \propto E^{-2.7}$. A total of 3×10^5 EASs were simulated in all azimuthal angles and for zenith angles of up to 60°. The cutoff energy of the air shower muons is 0.3 GeV. Proton, alpha, and carbon particles are considered the primary particles with fractions of 87%, 12%, and 1%, respectively, for the production of muons in the simulation. The spatial distribution of primary particles is taken as isotropic at the top of the atmosphere.

III. SIMULATION RESULTS AND DISCUSSION

A. Energy distribution of muons

With 3×10^5 simulated air showers, the energy distributions of the muons produced in the EAS simulations at different depths are shown in Fig. 2. Some of the depths selected are the depths of some of the existing observatories in the world. At all levels of observation, more than 97% of the muons have an energy of less than 100 GeV; therefore, the energy distribution is shown to this energy. The energy spectrum is almost flat below 1 GeV but gradually steepens, indicating a primary distribution in the 10 to 100 GeV range. It is observed that the number of muons in energies of less than 10 GeV decreases with increasing depth of the atmosphere, which is due to the muon decay. We use Eq. (3) for the energy distribution function of the muons by adding the coefficient $(1 + E/\epsilon)^{-1}$, which modifies the power of the distribution function in the high-energy part and should be considered for the lifetime of pions and kaons, i.e.,

$$\frac{dN(E,\theta)}{dEd\Omega} = N_0\frac{(E_\theta + E)^{-\gamma}}{(1 + \frac{E}{\epsilon})}. \tag{13}$$

Here, N_0 is the normalization constant that can be calculated numerically. According to Eq. (9), the general distribution function in terms of a function of both the energy and the zenith angle can be written as follows:

$$\frac{dN(E,\theta)}{dEd\Omega} = N_0\frac{(E_0 + E)^{-\gamma}}{1 + \frac{E}{\epsilon}}\cos^{\gamma-1}(\theta). \tag{14}$$

Gaisser [1] gives the following formula for muon energy distribution which is valid for high energy ($E_\mu > 100$ GeV):

$$\frac{dN(E,\theta)}{dEd\Omega} = \frac{0.14E^{-2.7}}{\text{cm}^2.\text{s.sr. GeV}}\left(\frac{1}{1 + 1.1\frac{E\cos\theta}{\epsilon_\pi}} + \frac{0.054}{1 + 1.1\frac{E\cos\theta}{\epsilon_K}}\right). \tag{15}$$

The first and second fractional terms of Eq. (15) are, respectively, related to the contribution of the pions and kaons, while in the distribution function (14), only one parameter ϵ is used, which is determined by fitting to the data obtained from the simulations. Since the distributions of Fig. 2 are integrated over all azimuthal angles and up to zenith angle 60°, we use the following function for fitting:

$$\frac{dN(E)}{dE} = b\frac{(E_0 + E)^{-\gamma}}{1 + \frac{E}{\epsilon}}, \tag{16}$$

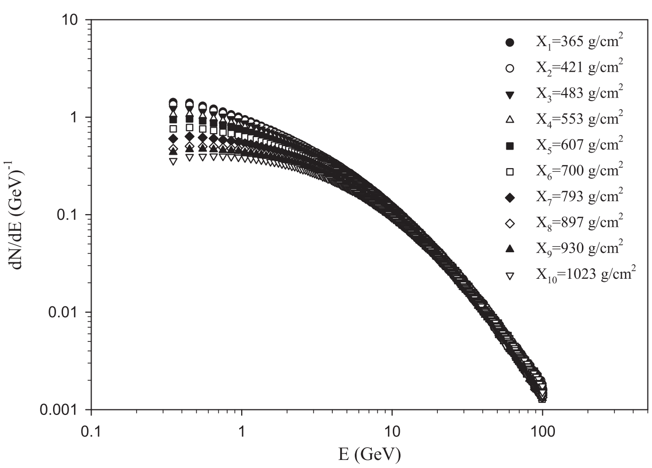

FIG. 2. Energy distributions of the muons at different depths. The depths X_5 to X_{10} are the depths of the Tibet, GAMA, GRAPES-3, ALBORZ, AGASA, and KASCADE observatories, respectively.

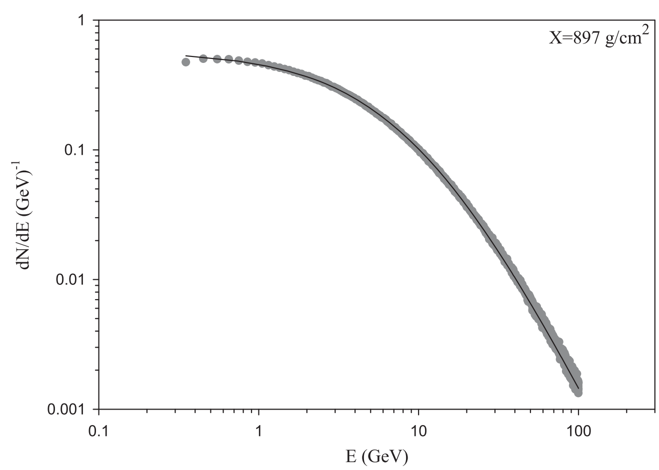

FIG. 3. Energy distributions of the muons at Tehran's level. The solid line shows the fitting of Eq. (16).

TABLE I. The values of parameters b, E_0, γ, and ϵ for different depths.

Altitude (m)	X_v (g cm^{-2})	b (GeV)$^{\gamma-1}$	E_0 (GeV)	γ	ϵ (GeV)	Regression (r^2)
8000	365	18.8 ± 2.6	3.3 ± 0.1	2.0 ± 0.1	$(8 \pm 2) \times 10^3$	0.997
7000	421	20.0 ± 2.7	3.6 ± 0.1	2.0 ± 0.1	$(8 \pm 2) \times 10^3$	0.997
6000	483	21.3 ± 2.6	3.9 ± 0.1	2.0 ± 0.1	$(8 \pm 1) \times 10^3$	0.998
5000	553	22.7 ± 2.5	4.3 ± 0.1	2.0 ± 0.1	$(8 \pm 1) \times 10^3$	0.999
4300 (TIBET)	607	23.8 ± 2.2	4.6 ± 0.1	2.0 ± 0.1	$(8 \pm 1) \times 10^3$	0.999
3200 (GAMA)	700	26.5 ± 2.1	5.4 ± 0.1	2.0 ± 0.1	$(8 \pm 1) \times 10^3$	0.999
2200 (GRAPES-3)	793	50.5 ± 5.5	7.1 ± 0.1	2.2 ± 0.1	$(8 \pm 1) \times 10^3$	0.999
1200 (ALBORZ)	897	166.8 ± 37.9	9.8 ± 0.3	2.5 ± 0.1	$(8 \pm 2) \times 10^3$	0.999
900 (AGASA)	930	283.5 ± 72.5	10.7 ± 0.4	2.6 ± 0.1	$(8 \pm 1) \times 10^3$	0.999
110 (KASCADE)	1023	643.8 ± 378.9	13.3 ± 0.7	2.8 ± 0.1	$(8 \pm 2) \times 10^3$	0.998

where $b = 2\pi N_0 \int_0^{\pi/2} \sin\theta \cos^{\gamma-1}(\theta) d\theta$, and the parameters b, E_0, γ, and ϵ are found by fitting. Figure 3 shows the fitting of function (16) to the muon spectrum at the level of Tehran. The values of parameters b, E_0, γ, and ϵ for different depths are listed in Table I. The value of the parameter E_0 varies between 3.3 and 13.3 GeV for depths between 365 and 1023 g cm^{-2}, which indicates that its value increases due to a longer path length in the atmosphere. The value of power γ increases with increasing depth due to the muon interaction processes in the atmosphere. The parameter ϵ is almost constant, which has a good agreement with the distribution given by Gaisser [1].

B. Zenith angle distribution of muons at different atmosphere depths

Figure 4 shows the zenith angle distributions of muons at all energies, $dN(\theta; X_v)/d\theta$, for 10 atmospheric depths. These distributions are different, which is related to the effect of atmosphere. By calculating the integral $N(X_v) = \int (dN(\theta; X_v)/d\theta) d\theta$ in 29 depths of the atmosphere, the average number of muons per shower, produced from proton, alpha, and carbon particles with fractions of 87%, 12%, and 1%, respectively, are obtained at different atmospheric depths. The $N(X_v)$ as a function of X_v is shown in Fig. 5. This distribution can be fitted by a log-normal function as

$$N(X_v) = \frac{a}{X_v} e^{-\frac{1}{2}\left(\frac{\ln \frac{X_v}{X_0}}{b}\right)^2}, \tag{17}$$

where $a = 35234 \pm 1002$, $b = 1.114 \pm 0.023$, and $X_0 = 1320 \pm 76$, with a regression of 99%. As is seen we found that a log-normal function is a convenient description of the distribution. In fact, the left tail of this function, which is rapidly increasing, stems from the rapid production of muons at the start of the production of air showers. The right tail of the distribution changes softly as a result of the slow decrease in the number of muons due to their low interaction with the atmosphere. The maximum number of muons is also at $X_{\max} = 367$ g cm^{-2}.

It is also possible to fit the distribution of the muons at different depths, $dN(\theta; X_v)/d\theta$, with the function $\sin\theta \cos^{k(X_v)}(\theta)$. Figure 6 shows the fitting of this function

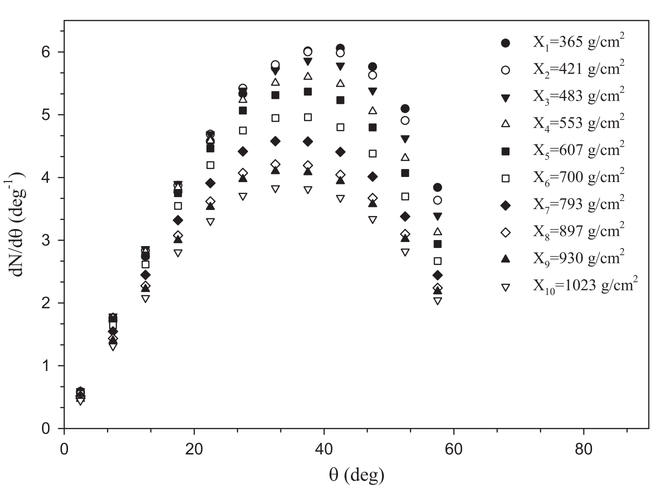

FIG. 4. Zenith angle distribution of atmospheric muons for 10 different depths.

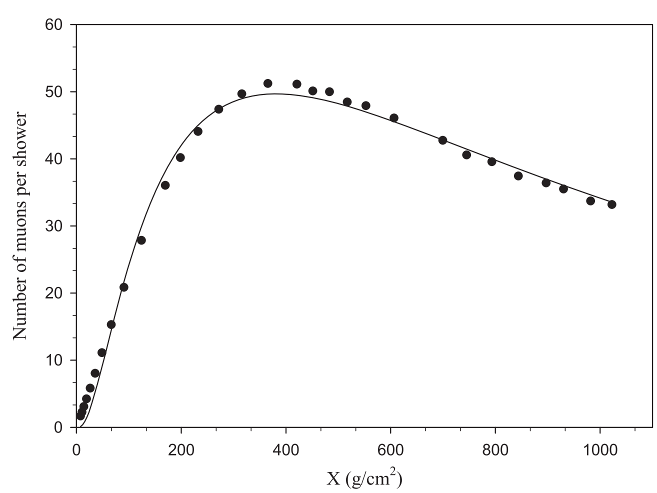

FIG. 5. The average number of muons per shower at different atmospheric depths, produced from proton, alpha, and carbon particles with fractions of 87%, 12%, and 1%, respectively, and with energy 1 GeV to 1 PeV.

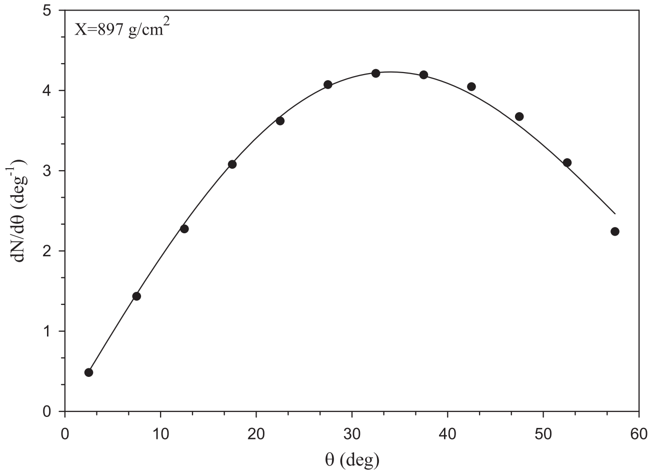

FIG. 6. Muon distribution at Tehran's level. The solid line shows the fitting of Eq. (10).

to the data obtained at the level of Tehran. The different values of $k(X_v)$ in terms of different depths are shown in Fig. 7. These values can be represented by the function $k(X_v) = k_0 + k_1 \ln(X_v/\mathrm{g\,cm^{-2}}) + k_2 \ln^2(X_v/\mathrm{g\,cm^{-2}})$, where $k_0 = -24.43 \pm 1.06$, $k_1 = 7.67 \pm 0.33$, and $k_2 = -0.55 \pm 0.03$, and with a regression of 99.9%. It is seen that the value of the parameter $k(X_v)$ in the zenith angle distribution function of muons increases by increasing the depth of the atmosphere, indicating the interaction of low-energy muons in the atmosphere. A comparison of the results of this simulation with that of various experiments is presented in Fig. 7 [6–9]. Since zenith angles greater than $\theta = 0°$ are equivalent to greater vertical depths, it is possible to use muon observations at nonzero zenith angles to determine the parameter $k(X_v)$ at vertical depths greater than the observation location. Although the atmospheric depth is related to the distribution of mass in the atmosphere, the mass density changes with the height of the atmosphere, h, as $\rho = \rho_0 e^{-h/H}$, with a scale height (for middle latitudes) in

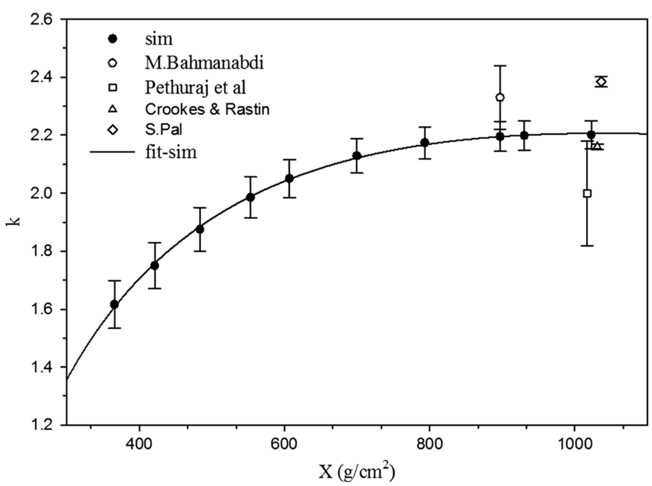

FIG. 7. The values k in terms of the atmosphere depth. The experimental data are from Refs. [6–9].

the atmosphere $H = 8.4$ km. The large value of the scale height indicates that the mass density changes slowly and at the heights we desire; with a good approximation, the slant depths will be equivalent to greater vertical depths. In the next section we will provide a model for this work.

C. Finding the parameter $k(X_v)$ at a depth greater than the observation point

When particles of an air shower reach a ground level under a zenith angle θ at an observed vertical depth X_0, it can be assumed that the air shower is like an air shower with a zenith angle $\theta = 0°$, which reaches at the vertical depth of $X = X_0/\cos\theta$. If we go to an observational surface with a vertical depth of X, it is clear that an event with a zenith angle Θ at this depth is equivalent to an event at the depth of X_0 with a zenith angle θ' (Fig. 8). So, as shown in Fig. 8, we have $X' = X_0/\cos\theta' = X/\cos\Theta = X_0/(\cos\theta\cos\Theta)$. Hence we obtain

$$\cos\theta' = \cos\theta\cos\Theta. \tag{18}$$

From the simulation data we have the distribution function $dN(\theta'; X_0)/d\theta' = N(X_0)\sin\theta'\cos^{k(X_0)}\theta'$. To construct the function $dN(\Theta; X)/d\Theta = N(X)\sin\Theta\cos^{k(X)}\Theta$ at the vertical depth X, we use the distribution function at depth X_0 as follows. The default primary intensity distribution in CORSIKA is as $I \propto \sin\theta\cos\theta$, where the sine term respects the solid angle element of the sky, while the cosine term takes the geometrical efficiency of a flat horizontal detector into account. With this assumption, we split $dN(\theta'; X0)/d\theta'$ as follows:

$$\frac{dN(\theta'; X_0)}{d\theta'} = f(\theta', X_0)\sin\theta'\cos\theta', \tag{19}$$

with $f(\theta', X_0) = N(X_0)\cos^{k(X_0)-1}(\theta')$. $f(\theta', X_0)$ depends only on the track length which is passed by muon particles, i.e., $X_0/\cos\theta'$. So $f(\Theta; X) = f(\theta'; X_0)$, and the distribution function $dN(\Theta; X)/d\Theta$ is calculated by the following formula:

$$\frac{dN(\Theta; X)}{d\Theta} = f(\theta', X_0)\sin\Theta\cos\Theta, \tag{20}$$

where $\theta' = \cos^{-1}(\cos\theta\cos\Theta)$. So, in order to find the muon distribution at other vertical depths, we perform

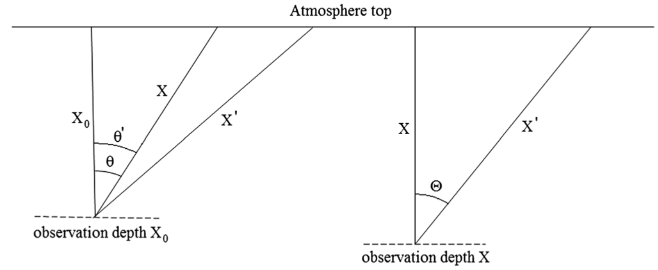

FIG. 8. Geometry of slant depth in vertical depths X_0 and X.

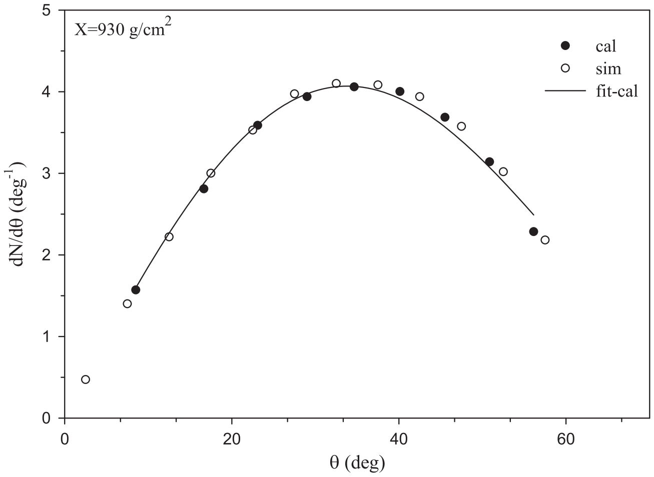

FIG. 9. Zenith angle distribution of atmospheric muons at $X_v = 930\ \mathrm{g\,cm^{-2}}$ both by simulation and calculated with the data at $X_v = 897\ \mathrm{g\,cm^{-2}}$.

the following steps in the order below: (a) By dividing $dN(\theta', X_0)/d\theta'$ by $\sin\theta'\cos\theta'$, we find $f(\theta', X_0)$, (b) The value Θ is calculated by using $\cos\Theta = \cos\theta'/\cos\theta$, (c) And finally, by multiplying the $\sin\Theta\cos\Theta$ by $f(\theta'; X_0)$, the distribution function $dN(\Theta; X)/d\Theta$ is obtained. For example, Fig. 9 shows the zenith angle distribution of atmospheric muons at a vertical depth of $X_v = 930\ \mathrm{g\,cm^{-2}}$ using directly simulation and also by calculating from the simulation data at Tehran's level ($X_v = 897\ \mathrm{g\,cm^{-2}}$). The relative difference, $2(k_{\mathrm{cal}} - k_{\mathrm{sim}})/(k_{\mathrm{cal}} + k_{\mathrm{sim}})$, is less than 1.4%, where $k_{\mathrm{cal}} = 2.23 \pm 0.07$ and $k_{\mathrm{sim}} = 2.20 \pm 0.05$ are, respectively, the values of $k(X)$ obtained from direct simulation and calculation. Therefore, it can be seen that the proposed model is a self-consistent approach.

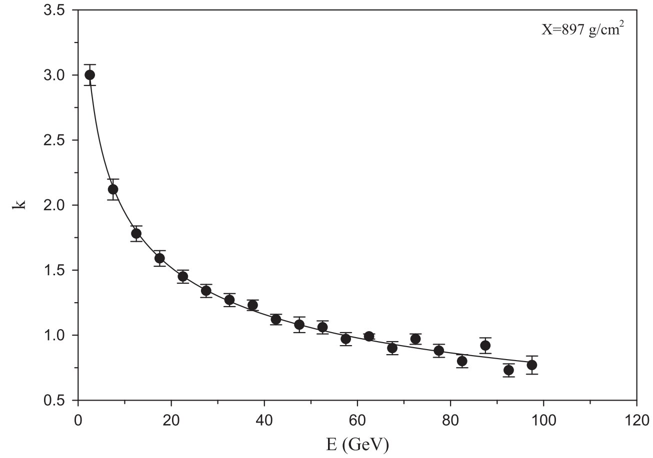

FIG. 10. The value of the parameter k for different energies at Tehran's level.

D. Energy dependence of muon distribution

If, at each atmospheric depth, the zenith angle distribution of muons is obtained for different energy intervals, the value of $k(X_v, E)$ can be obtained by fitting the function of Eq. (10) over this distribution. Figure 10 shows the values of this parameter for different energies, with a bin width of 5 GeV, at Tehran's level. The values of this parameter at Tehran's level can be displayed by the function $k(X_{\mathrm{Tehran}}, E) = k_0 + k_1 \ln(E/\mathrm{GeV}) + k_2 \ln^2(E/\mathrm{GeV})$, with $k_0 = 3.8162 \pm 0.0751$, $k_1 = -0.9676 \pm 0.0522$, and $k_2 = 0.0671 \pm 0.0086$,

TABLE II. The values of the parameter $k(X_v, E)$ at different depths and energies.

	X_v ($\mathrm{g\,cm^{-2}}$)									
E (GeV)	365	421	483	553	607	700	793	897	930	1023
2.5 ± 2.5	2.05	2.22	2.39	2.56	2.67	2.82	2.93	3.00	3.02	3.05
7.5 ± 2.5	1.43	1.56	1.69	1.81	1.88	1.98	2.06	2.12	2.14	2.18
12.5 ± 2.5	1.20	1.32	1.43	1.52	1.59	1.67	1.74	1.78	1.79	1.83
17.5 ± 2.5	1.06	1.18	1.28	1.37	1.41	1.49	1.54	1.59	1.60	1.64
22.5 ± 2.5	1.02	1.11	1.20	1.27	1.32	1.38	1.42	1.45	1.46	1.49
27.5 ± 2.5	0.94	1.03	1.10	1.17	1.21	1.27	1.30	1.34	1.35	1.38
32.5 ± 2.5	0.89	0.98	1.04	1.10	1.13	1.18	1.23	1.27	1.28	1.31
37.5 ± 2.5	0.88	0.96	1.02	1.09	1.12	1.19	1.21	1.23	1.23	1.26
42.5 ± 2.5	0.81	0.88	0.95	1.01	1.04	1.07	1.10	1.12	1.14	1.14
47.5 ± 2.5	0.75	0.82	0.89	0.94	0.97	1.01	1.05	1.08	1.09	1.12
52.5 ± 2.5	0.79	0.86	0.93	0.98	1.00	1.03	1.06	1.06	1.06	1.06
57.5 ± 2.5	0.71	0.77	0.81	0.85	0.88	0.92	0.93	0.97	0.97	0.98
62.5 ± 2.5	0.72	0.78	0.85	0.90	0.93	0.96	0.99	0.99	0.99	1.00
67.5 ± 2.5	0.67	0.73	0.77	0.78	0.80	0.83	0.87	0.90	0.92	0.96
72.5 ± 2.5	0.70	0.76	0.79	0.88	0.90	0.92	0.94	0.97	0.96	0.97
77.5 ± 2.5	0.64	0.70	0.77	0.80	0.82	0.84	0.86	0.88	0.89	0.90
82.5 ± 2.5	0.63	0.68	0.71	0.75	0.76	0.79	0.80	0.80	0.80	0.81
87.5 ± 2.5	0.64	0.68	0.77	0.81	0.84	0.87	0.89	0.92	0.93	0.94
92.5 ± 2.5	0.60	0.62	0.62	0.69	0.70	0.73	0.75	0.73	0.73	0.79
97.5 ± 2.5	0.52	0.62	0.69	0.70	0.73	0.73	0.74	0.77	0.77	0.76

and with a regression of 99.5%. As can be seen, the parameter k decreases with increasing energy. This indicates that with increasing energy, the zenith angle distribution of the muons becomes softer. At large zenith angles, reducing the number of muons at low energies is greater than high energy. Table II shows the values of the parameter $k(X_v, E)$ at different depths and energies, and the function $k(X_v; E) = k_1 \ln(X_v/\mathrm{g\,cm^{-2}}) + k_2 \ln(E/\mathrm{GeV})$, with $k_1 = 0.4494 \pm 0.0046$ and $k_2 = -0.4903 \pm 0.0079$, shows a fairly good fit with a regression of 95%. The value $k(X_v; E)$ is between 0.52 and 3.05 at the atmospheric depths and muon energies ($365\ \mathrm{g\,cm^{-2}}$, 97.5 ± 2.5 GeV) and ($1023\ \mathrm{g\,cm^{-2}}$, 2.5 ± 2.5 GeV), respectively.

IV. CONCLUSION

The study of the propagation of muons in the atmosphere is important because we can obtain their propagation properties and their attenuation length at different energies. This, in turn, is important for the performance of the muonic detectors and their efficiency. Using the CORSIKA simulation code, we obtained the zenith angle distribution of muons at different atmospheric depths and energies. A distribution as $\sin\theta \cos^{k(X_v,E)}(\theta)$ for the zenith angle of muons has been obtained. These values can be displayed by the function $k(X_v; E) = (0.4494 \pm 0.0046) \ln(X_v/\mathrm{g\,cm^{-2}}) + (-0.4903 \pm 0.0079) \ln(E/\mathrm{GeV})$ with a regression of 95%.

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