$a_0(980)$ physics in semileptonic D^0 and D^+ decays

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The decays $D^0 \to d\bar{u}e^+\nu \to a_0^-(980)e^+\nu \to \pi^-\eta e^+\nu$ and $D^+ \to d\bar{d}e^+\nu \to a_0^0(980)e^+\nu \to \pi^0\eta e^+\nu$ (and the charge conjugated ones) are the direct probe of the constituent two-quark components in the $a_0^\pm(980)$ and $a_0^0(980)$ wave functions. The recent BESIII experiment is the first step in the experimental study of these decays. We suggest adequate formulas for the data analysis and present a variant of $\eta\pi$ invariant mass distribution when $a_0(980)$ has no constituent two-quark component at all.

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I. INTRODUCTION

The $a_0(980)$ and $f_0(980)$ mesons are well-established parts of the assumed light scalar meson nonet [1]. From the beginning, the $a_0(980)$ and $f_0(980)$ mesons became one of the central problems of nonperturbative QCD, as they are important for understanding the way chiral symmetry is realized in the low-energy region and, consequently, for understanding confinement. Many experimental and theoretical papers have been devoted to this subject.

There is much evidence that supports the four-quark model of light scalar mesons [2,3].

The suppression of the $a_0^0(980)$ and $f_0(980)$ resonances in the $\gamma\gamma \to \eta\pi^0$ and $\gamma\gamma \to \pi\pi$ reactions, respectively, was predicted in the four-quark model in 1982 [4], $\Gamma_{a_0^0\gamma\gamma} \approx \Gamma_{f_0\gamma\gamma} \approx 0.27$ keV, and confirmed by experiment [1]. The high quality Belle data [5,6] allowed one to elucidate the mechanisms of the $\sigma(600)$, $f_0(980)$, and $a_0^0(980)$ resonance production in $\gamma\gamma$ collisions [7,8]. Light scalar mesons are produced in $\gamma\gamma$ collisions mainly via rescatterings, that is, via the four-quark transitions. As for $a_2(1320)$ and $f_2(1270)$ (the well-known two-quark states), they are produced mainly via the two-quark transitions (direct couplings with $\gamma\gamma$).

The argument in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ is the fact that the $\phi(1020) \to a_0^0\gamma$ and $\phi(1020) \to f_0\gamma$ decays go through the kaon loop: $\phi \to K^+K^- \to a_0^0\gamma$, $\phi \to K^+K^- \to f_0\gamma$, i.e., via the four-quark transition [9–13]. The kaon-loop model was suggested in Ref. [9] and confirmed by experiment ten years later [14–16].

It was shown in Ref. [10] that the production of $a_0^0(980)$ and $f_0(980)$ in $\phi \to a_0^0\gamma \to \eta\pi^0\gamma$ and $\phi \to f_0\gamma \to \pi^0\pi^0\gamma$ decays is caused by the four-quark transitions, resulting in strong restrictions on the large-N_C expansions of the decay amplitudes. The analysis showed that these constraints give new evidence in favor of the four-quark nature of the $a_0(980)$ and $f_0(980)$ mesons.

In Refs. [17,18] it was shown that the description of the $\phi \to K^+K^- \to \gamma a_0^0(980)/f_0(980)$ decays requires virtual momenta of $K(\bar{K})$ greater than 2 GeV, while in the case of loose molecules with a binding energy about 20 MeV, they would have to be about 100 MeV. Besides, it should be noted that the production of scalar mesons in the pion-nucleon collisions with large momentum transfers also points to their compactness [19].

It was also shown in Refs. [20,21] that the linear $S_L(2) \times S_R(2)$ σ model [22] reflects all of the main features of low-energy $\pi\pi \to \pi\pi$ and $\gamma\gamma \to \pi\pi$ reactions up to energy 0.8 GeV and agrees with the four-quark nature of the σ meson. This allowed for the development of a phenomenological model with the right analytical properties in the complex s plane that took into account the linear σ model, the $\sigma(600) - f_0(980)$ mixing, and the background [23]. This background has a left cut inspired by crossing symmetry, and the resulting amplitude agrees with results obtained using the chiral expansion, dispersion relations, and the Roy equation [24], as well as with the four-quark nature of the $\sigma(600)$ and $f_0(980)$ mesons. This model well describes the experimental data on $\pi\pi \to \pi\pi$ scattering up to 1.2 GeV.

Moreover, the suppression of $J/\psi \to \gamma f_0(980)$, $\rho a_0(980)$, $\omega f_0(980)$ decays in the presence of intense $J/\psi \to \gamma f_2(1270)$, $\gamma f_2'(1525)$, $\rho a_2(1320)$, $\omega f_2(1270)$ decays is at variance with the P-wave two-quark structure of $a_0(980)$ and $f_0(980)$ resonances [25].

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It is shown in Ref. [26] that the recent data on the $K_S^0 K^+$ correlation in Pb-Pb interactions Ref. [27] agree with the data on the $\gamma\gamma \to \eta\pi^0$ and $\phi \to \eta\pi^0\gamma$ reactions and support the four-quark model of the $a_0(980)$ meson. It is shown that the data do not contradict the validity of the Gaussian assumption.

In Refs. [28,29] the program of studying light scalars in semileptonic D and B decays was suggested. We studied production of scalars $\sigma(600)$ and $f_0(980)$ in the $D_s^+ \to \pi^+\pi^- e^+\nu$ decays, the conclusion was that the percentage of the two-quark components in $\sigma(600)$ and $f_0(980)$ is small. This is the direct evidence in favor of the exotic nature of these particles. Unfortunately, at the moment the statistics is rather poor, and thus new high-statistics data are highly desirable.

It was noted in Refs. [28,29] that no less interesting is the study of semileptonic decays of D^0 and D^+ mesons —$D^+ \to d\bar{d}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$, $D^0 \to d\bar{u}e^+\nu \to a_0^- e^+\nu \to \pi^-\eta e^+\nu$, and $D^+ \to d\bar{d}e^+\nu \to a_0^0 e^+\nu \to \pi^0\eta e^+\nu$ (or the charged-conjugated ones) which had not been investigated. It is very tempting to study light scalar mesons in semileptonic decays of B mesons [29]: $B^0 \to d\bar{u}e^+\nu \to a_0^- e^+\nu \to \pi^-\eta e^+\nu$, $B^+ \to u\bar{u}e^+\nu \to a_0^0 e^+\nu \to \pi^0\eta e^+\nu$, $B^+ \to u\bar{u}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^- e^+\nu$.

Recently BES Collaboration measured the decays $D^0 \to d\bar{u}e^+\nu \to a_0^- e^+\nu \to \pi^-\eta e^+\nu$ and $D^+ \to d\bar{d}e^+\nu \to a_0^0 e^+\nu \to \pi^0\eta e^+\nu$ for the first time [30]. In this paper we discuss the Ref. [28] program in light of these measurements taking into account the contribution of the a_0' meson with mass about 1400 MeV.

A variant when $a_0(980)$ has no constituent two-quark component at all is presented. That is, $a_0^-(980)$ is produced as a result of mixing $a_0'^- \to a_0^-(980)$, $D^0 \to d\bar{u}e^+\nu \to a_0'^- e^+\nu \to a_0^- e^+\nu \to \pi^-\eta e^+\nu$, and correspondingly for the D^+ decay.

This variant describes the set of experimental data considered in Ref. [26]. Moreover, in comparison with that paper we take into account high-statistical KLOE data on the $\phi \to \eta\pi^0\gamma$ decay of Ref. [31] (instead of Ref. [16]). To describe this precise data we change parametrization of the $K\bar{K}$ scattering background phase, which changes the module of the $\phi \to K^+K^- \to (a_0^0 + a_0'^0)\gamma \to \eta\pi^0\gamma$ amplitude below the $K\bar{K}$ threshold. We also take into account this phase in the $K_S^0 K^+$ correlation and introduce the $m_{a_0^+} - m_{a_0^0}$ mass difference.

II. D DECAYS INVOLVING SCALARS AND PSEUDOSCALARS

The amplitude of the $D^0 \to S(\text{scalar})e^+\nu$ decay is of similar form to the D_s^+ decay [28]

$$M[D^0(p) \to S(p_1)W^+(q) \to S(p_1)e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cd} A_\alpha L^\alpha, \tag{1}$$

where G_F is the Fermi constant, V_{cd} is the Cabibbo-Kobayashi-Maskava matrix element,

$$A_\alpha = f_+^S(q^2)(p + p_1)_\alpha + f_-^S(q^2)(p - p_1)_\alpha,$$
$$L_\alpha = \bar{\nu}\gamma_\alpha(1 + \gamma_5)e, \qquad q = (p - p_1). \tag{2}$$

The influence of the $f_-^S(q^2)$ form factor is negligible because of the small mass of the positron.

The decay rate into the stable S state is

$$\frac{d\Gamma(D^0 \to Se^+\nu)}{dq^2} = \frac{G_F^2 |V_{cd}|^2}{24\pi^3} p_1^3(q^2) |f_+^S(q^2)|^2, \tag{3}$$

$$p_1(q^2) = \frac{\sqrt{m_{D^0}^4 - 2m_{D^0}^2(q^2 + m_S^2) + (q^2 - m_S^2)^2}}{2m_{D^0}}. \tag{4}$$

For the $f_+^S(q^2)$ form factor we use the vector dominance model

$$f_+^S(q^2) = f_+^S(0)\frac{m_A^2}{m_A^2 - q^2} = f_+^S(0) f_A(q^2), \tag{5}$$

where $A = D_1(2420)^\pm$ [1].

Following Fig. 1 we write $f_+^S(0)$ in the form

$$f_+^S(0) = g_{D^0 c\bar{u}} F_S g_{d\bar{u}S}, \tag{6}$$

where $g_{D^0 c\bar{u}}$ is the $D^0 \to c\bar{u}$ coupling constant, $g_{d\bar{u}S}$ is the $d\bar{u} \to S$ coupling constant, and F_S is the loop integral assumed to be constant in the region of interest.

The amplitude of the $D^0 \to d\bar{u}e^+\nu \to [a_0^-(980) + a_0'^-]e^+\nu \to \eta\pi^- e^+\nu$ decay is

$$M(D^0 \to d\bar{u}e^+\nu \to \eta\pi^- e^+\nu)$$
$$= \frac{G_F}{\sqrt{2}} V_{cd} L^\alpha (p + p_1)_\alpha g_{D^0 c\bar{u}} f_A(q^2) \frac{1}{\Delta(m)}$$
$$\times \Big(F_{a_0^-} g_{d\bar{u}a_0^-} D_{a_0'^-}(m) g_{a_0\eta\pi} + F_{a_0^-} g_{d\bar{u}a_0^-} \Pi_{a_0^- a_0'^-}(m) g_{a_0'\eta\pi}$$
$$+ F_{a_0'^-} g_{d\bar{u}a_0'^-} \Pi_{a_0'^- a_0^-}(m) g_{a_0\eta\pi} + F_{a_0'^-} g_{d\bar{u}a_0'^-} D_{a_0^-}(m) g_{a_0'\eta\pi} \Big), \tag{7}$$

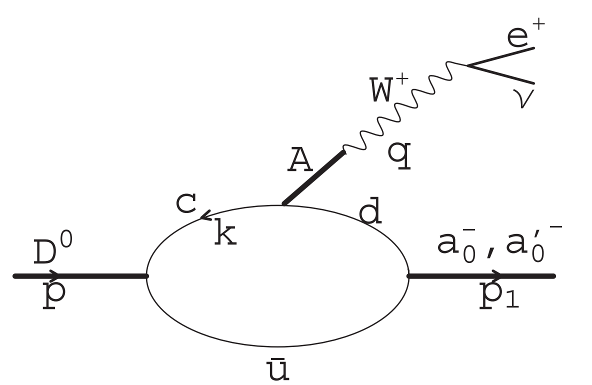

FIG. 1. Model of the $D^0 \to (a_0^-, a_0'^-)e^+\nu$ decay.

where m is the invariant mass of the $\eta\pi^-$ system, $\Delta(m) = D_{a_0'^-}(m)D_{a_0^-}(m) - \Pi_{a_0'^-a_0^-}(m)\Pi_{a_0^-a_0'^-}(m)$, $D_{a_0^-}(m)$ and $D_{a_0'^-}(m)$ are the inverted propagators of the a_0^- and $a_0'^-$ mesons, and $\Pi_{a_0^-a_0'^-}(m) = \Pi_{a_0'^-a_0^-}(m)$ is the nondiagonal element of the polarization operator, which mixes the a_0^- and $a_0'^-$ mesons. All the details can be found in Appendix A.

The double differential rate of the $D^0 \to d\bar{u}e^+\nu \to [a_0^-(980) + a_0'^-]e^+\nu \to \eta\pi^- e^+\nu$ decay taking into account the a_0' scalar meson is

$$\frac{d^2\Gamma(D^0 \to \eta\pi^- e^+\nu)}{dq^2 dm} = \frac{G_F^2|V_{cd}|^2}{24\pi^3} g^2_{D^0 c\bar{u}} |f_A(q^2)|^2 p_1^3(q^2, m) \times \frac{1}{8\pi^2} m\rho_{\eta\pi^-}(m) \left|\frac{1}{\Delta(m)}\right|^2 \Big| F_{a_0^-} g_{d\bar{u}a_0^-} D_{a_0'^-}(m) g_{a_0\eta\pi} + F_{a_0^-} g_{d\bar{u}a_0^-} \Pi_{a_0^- a_0'^-}(m) g_{a_0'\eta\pi} + F_{a_0'^-} g_{d\bar{u}a_0'^-} \Pi_{a_0'^- a_0^-}(m) g_{a_0\eta\pi} + F_{a_0'^-} g_{d\bar{u}a_0'^-} D_{a_0^-}(m) g_{a_0'\eta\pi}\Big|^2, \tag{8}$$

where $\rho_{\eta\pi^-}(m) = \sqrt{(1-(m_\eta + m_{\pi^-})^2/m^2)(1-(m_\eta - m_{\pi^-})^2/m^2)}$.

The $D^+ \to d\bar{d}e^+\nu \to Se^+\nu$ and $D^+ \to \eta\pi^0 e^+\nu$ decays are described in the same way; see Fig. 2. It is enough to substitute in Eqs. (1)–(8) $D^0 \to D^+$, $d\bar{u} \to d\bar{d}$, $a_0^- \to a_0^0$, $a_0'^- \to a_0'^0$, and $\pi^- \to \pi^0$. The coupling $g_{d\bar{d}a_0'^0} = g_{d\bar{u}a_0'^-}/\sqrt{2}$.

The key question is the size of the a_0' contribution. In Ref. [30] fits take into account only the $a_0(980)$ contribution, but one can see from Fig. 3(a) that the Ref. [30] curve lies below the data in the interval $m \equiv M_{\eta\pi} = 1.1$–$1.3$ GeV (though within large errors). It may be a manifestation of a sizable a_0' contribution.

In Ref. [26] we simultaneously described the data on the $\gamma\gamma \to \eta\pi^0$ reaction in Ref. [6], the $\phi \to \eta\pi^0\gamma$ decay [16], and the recent data on the $K_S^0 K^+$ correlation in Pb-Pb interactions in Ref. [27].

In this article we present for the first time to our knowledge a variant of data descriptions when $a_0(980)$ has no constituent two-quark component at all: the $a_0^0(980)$ direct two-quark transition coupling to the $\gamma\gamma$ channel $g^{(0)}_{a_0\gamma\gamma} = 0$ and $g_{d\bar{u}a_0^-} = g_{d\bar{d}a_0^0} = 0$. The results are shown in Figs. 4 and 5 and in Tables I and II.

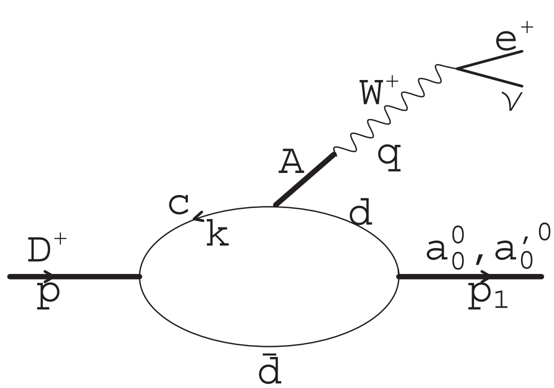

FIG. 2. Model of the $D^+ \to (a_0^0, a_0'^0)e^+\nu$ decay.

Fitting the data on Fig. 3(a) with obtained parameters gives the histograms plotted in Figs. 6 and 7. Only normalization is a free parameter in this fitting. The point on 1.225 GeV was omitted in fitting, and the background was extracted from Fig. 3(a) approximately. The optimal integral is 28.0 events in the experimental region 0.7–1.3 GeV, and the signal branching $1.45^{+0.43}_{-0.40} \times 10^{-4}$—one can compare it with Ref. [30] result $(1.33^{+0.33}_{-0.29}(\text{stat}) \pm 0.09(\text{syst})) \times 10^{-4}$. Of course, all this consideration is very preliminary due to large experimental errors.

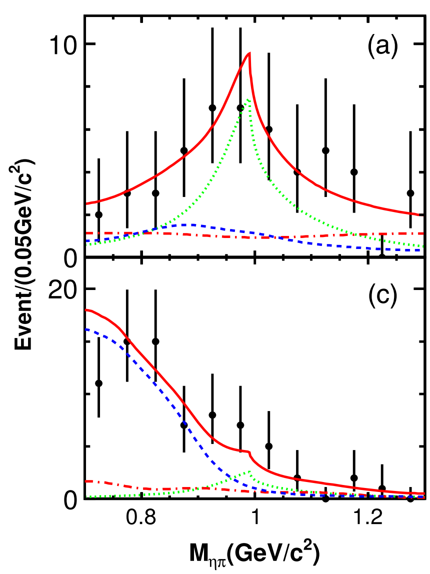

FIG. 3. Experimental data on (a) $D^0 \to (a_0^-, a_0'^-)e^+\nu \to \eta\pi^- e^+\nu$ and (c) $D^+ \to (a_0^0, a_0'^0)e^+\nu \to \eta\pi^0 e^+\nu$ decays. Direct copy of Figs. 2(a) and 2(c) in Ref. [30]. Dotted curves are signals, solid ones represent total contribution, and the other ones represent backgrounds.

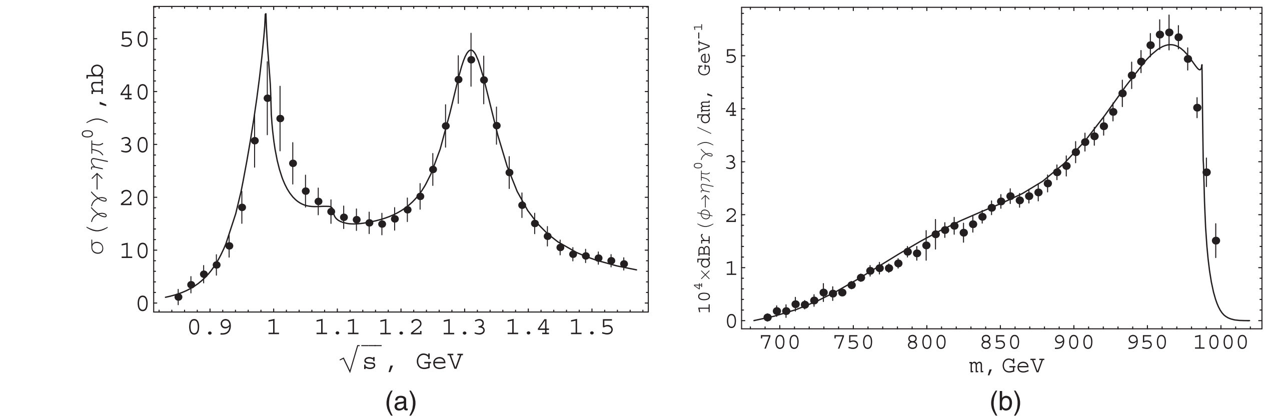

FIG. 4. Results of our fit (see Tables I and II) on (a) the Belle data on the $\gamma\gamma \to \eta\pi^0$ cross section [6], and (b) the KLOE data on the $\phi \to \eta\pi^0\gamma$ decay [31], where m is the invariant $\eta\pi^0$ mass.

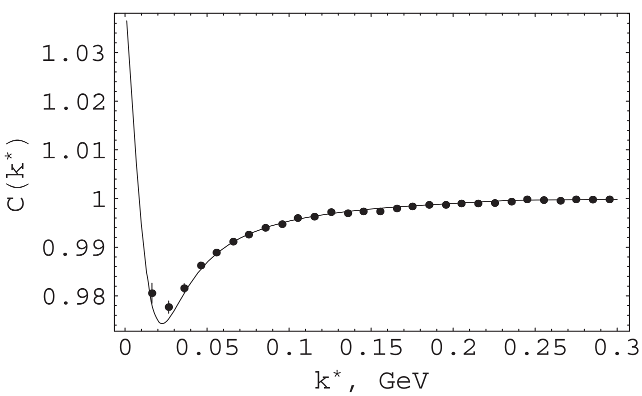

FIG. 5. The $K_S^0 K^+$ correlation $C(k^*)$; see Ref. [26] and references therein. The solid line represents our fit, and points are experimental data [27].

The corresponding $d\text{Br}(D^0 \to d\bar{u}e^+\nu \to (a_0^-, a'^-_0)e^+\nu \to \pi^-\eta e^+\nu)/dm$ and $d\text{Br}(D^+ \to d\bar{d}e^+\nu \to (a_0^0, a'^0_0)e^+\nu \to \pi^0\eta e^+\nu)/dm$ curves are shown in Figs. 8 and 9. The line shapes of these curves differ from the signal curve on Figs. 3(a) and 3(c).

Some details and parameters of the fit are placed in Appendix B and Table II therein.

The KLOE data on the $\phi \to \eta\pi^0\gamma$ decay of Ref. [31] are so precise that one should take into account even small effects to describe them. One of the important features is the background phase of the $K\bar{K}$ scattering $\delta^{bg}_{K\bar{K}}(s)$ for isospin $I = 1$, defined in Eqs. (25) and (27) of Ref. [32]. Analytical continuation of this phase under the $K\bar{K}$ threshold changes the absolute value of the $\phi \to K^+K^- \to a_0\gamma \to \eta\pi^0\gamma$ amplitude. Unfortunately, the $K\bar{K}$ scattering phase is poorly known.

The influence of the analytical continuation of the $K\bar{K}$ phase is not large near the resonance peak situating near the $K\bar{K}$ threshold. In the current work we upgrade the $K\bar{K}$ scattering phase parametrization:

$$e^{2i\delta^{bg}_{K\bar{K}}(s)} = \frac{1 + iF_{K\bar{K}}(s)}{1 - iF_{K\bar{K}}(s)}, \qquad e^{2i\delta^{bg}_{\bar{K}^0K^+}(s)} = \frac{1 + iF_{\bar{K}^0K^+}(s)}{1 - iF_{\bar{K}^0K^+}(s)}, \tag{9}$$

where

$$F_{K\bar{K}}(s) = f_{K\bar{K}} \frac{\sqrt{s - 4m^2_{K^+}} + \sqrt{s - 4m^2_{K^0}}}{2} + g_{K\bar{K}} \frac{\sqrt{1 - 4m^2_{K^+}/s} + \sqrt{1 - 4m^2_{K^0}/s}}{2}, \tag{10}$$

TABLE I. Properties of the resonances and the description quality.

$m_{a_0^0}$, MeV	988.3	$m_{a'_0}$, MeV	1423.9	R, fm	6.3
$g_{a_0^0K^+K^-}$, GeV	4.06	$g_{a'^0_0K^+K^-}$, GeV	4.19	λ	1
$g_{a_0\eta\pi}$, GeV	3.99	$g_{a'_0\eta\pi}$, GeV	0.80	$\chi^2_{\gamma\gamma}/36$ points	13.8
$g_{a_0\eta'\pi}$, GeV	−4.24	$g_{a'_0\eta'\pi}$, GeV	1.27	$\chi^2_{sp}/49$ points	65.5
$g^{(0)}_{a_0^0\gamma\gamma}$	0	$g^{(0)}_{a'^0_0\gamma\gamma}$, 10^{-3} GeV^{-1}	−12.90	$\chi^2_{\text{corr}}/29$ points	28.4
$m_{a_0^+}$, MeV	997.6	$C_{a_0a'_0}$, GeV^2	−0.163	$(\chi^2_{\gamma\gamma} + \chi^2_{sp} + \chi^2_{\text{corr}})$/n.d.f.	107.8/99

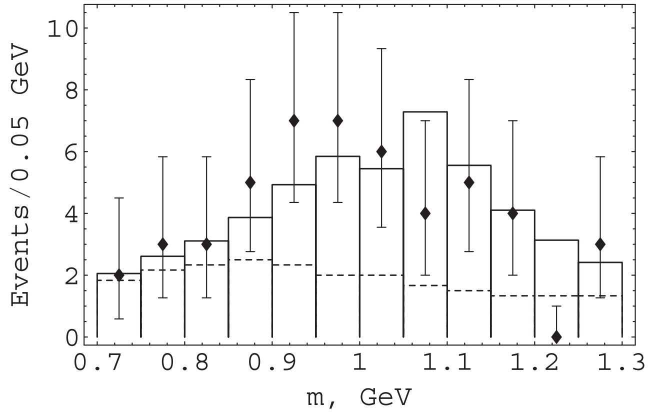

FIG. 6. The data on the $D^0 \to (a_0^-, a_0'^-)e^+\nu \to \eta\pi^- e^+\nu$ decay and the fit corresponding to 28.0 events and the signal branching 1.45×10^{-4}. The solid histogram is the total contribution, and the dashed histogram represents the sum of backgrounds from Fig. 3.

$$F_{\bar{K}^0K^+}(s) = f_{K\bar{K}} \frac{\sqrt{(s-(m_{K^0}+m_{K^+})^2)(s-(m_{K^0}-m_{K^+})^2)}}{\sqrt{s}} + g_{K\bar{K}} \frac{\sqrt{(s-(m_{K^0}+m_{K^+})^2)(s-(m_{K^0}-m_{K^+})^2)}}{s}. \quad (11)$$

Compared with parametrization used in [32] and later, we add to $F_{K\bar{K}}(s)$ a term proportional to velocity and take into account the kaon mass difference. The phase $\delta^{bg}_{K\bar{K}}$ is used in the $\gamma\gamma \to \eta\pi^0$ and $\phi \to \eta\pi^0\gamma$ reactions, and $\delta^{bg}_{\bar{K}^0K^+}$ is used to study the $K^0_S K^+$ correlation.

We also upgrade Eq. (6) in Ref. [26] describing the amplitude of the $\bar{K}^0K^+$ scattering,

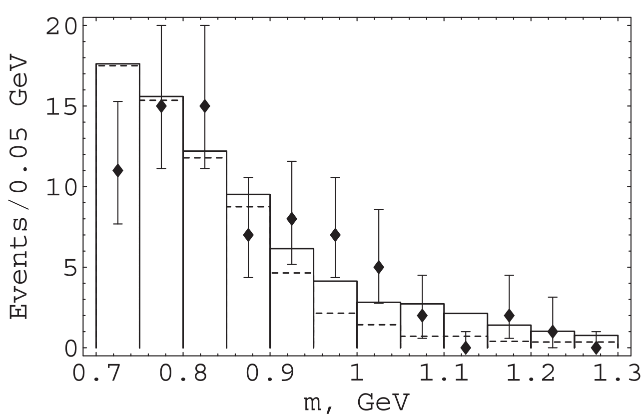

FIG. 7. The data on the $D^+ \to (a_0^0, a_0'^0)e^+\nu \to \eta\pi^0 e^+\nu$ decay and the signal corresponding to fit shown in Fig. 6; signal branching is 1.94×10^{-4}. The solid histogram is the total contribution, and the dashed histogram represents the sum of backgrounds from Fig. 3.

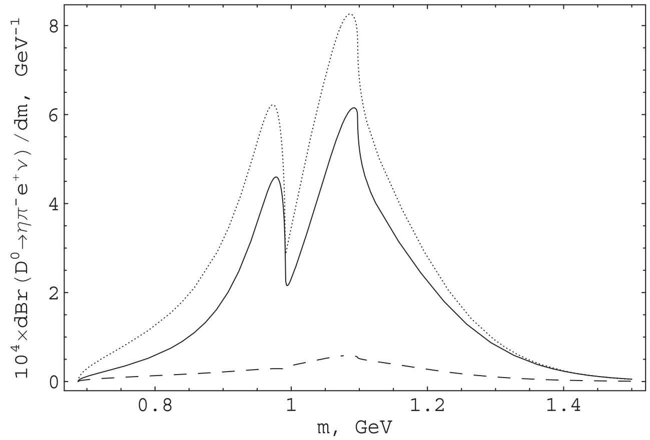

FIG. 8. The plot of $D^0 \to (a_0^-, a_0'^-)e^+\nu \to \eta\pi^- e^+\nu$ spectrum with parameters of our fit (with $g_{d\bar{u}a_0^-} = 0$). The solid line is the total contribution, the dotted line is the term $\sim F_{a_0'^-} g_{d\bar{u}a_0'^-} \Pi_{a_0'^- a_0^-}(m) g_{a_0\eta\pi}$ contribution, and the dashed line is the term $\sim F_{a_0'^-} g_{d\bar{u}a_0'^-} D_{a_0^-}(m) g_{a_0'\eta\pi}$ contribution; see Eq. (8).

$$f(k^*) = \frac{e^{2i\delta^{bg}_{\bar{K}^0K^+}(s)} - 1}{2i\rho_{K^0K^+}} + e^{2i\delta^{bg}_{\bar{K}^0K^+}(s)} \frac{4}{\sqrt{s}} \times \sum_{S,S'} \frac{g_{SK^0_SK^+} G^{-1}_{SS'} g_{S'K^0_SK^+}}{16\pi}, \quad (12)$$

where $S, S' = a_0^+, a_0'^+$, the constants $g_{SK^0_SK^+} = -g_{SK^0_LK^+} = g_{SK^+K^-}$, and k^* is the kaon momentum in the kaon pair rest frame,

$$k^* = \frac{\sqrt{(s-(m_{K^0_S}-m_{K^+})^2)(s-(m_{K^0_S}+m_{K^+})^2)}}{2\sqrt{s}}. \quad (13)$$

In comparison with Eq. (6) of Ref. [26] we take the $\bar{K}^0K^+$ scattering phase into account and fix a misprint: $\frac{2}{\sqrt{s}}$ was written instead of $\frac{4}{\sqrt{s}}$. The calculations in Ref. [26] were done with the correct formula.

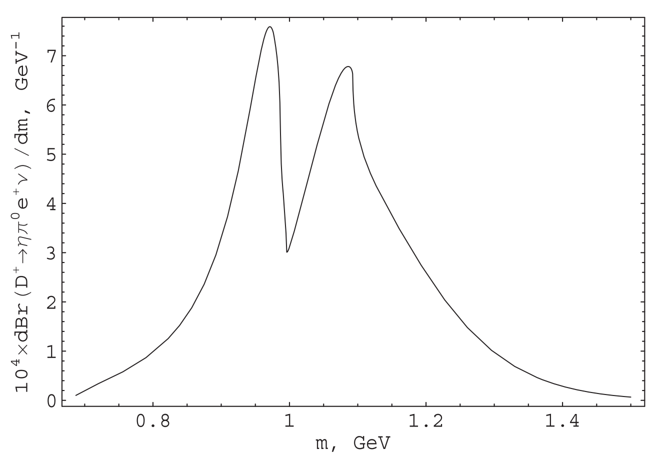

FIG. 9. The plot of $D^+ \to (a_0^0, a_0'^0)e^+\nu \to \eta\pi^0 e^+\nu$ spectrum with parameters of our fit.

Remember that the $K_S^0K^+$ correlation reads [27,33]

$$C(k^*) = 1 + \frac{\lambda}{2}\left(\frac{1}{2}\left|\frac{f(k^*)}{R}\right|^2 + 2\frac{\mathrm{Re}f(k^*)}{\sqrt{\pi}R}F_1(2k^*R) - \frac{\mathrm{Im}f(k^*)}{R}F_2(2k^*R)\right), \tag{14}$$

where R is the radius parameter from the spherical Gaussian source distribution, λ is the correlation strength, and

$$F_1(z) = \frac{e^{-z^2}}{z}\int_0^z e^{x^2}dx; \qquad F_2(z) = \frac{1-e^{-z^2}}{z}. \tag{15}$$

To fit the data we use the χ^2 function with the addition of terms providing some restrictions, including terms that guarantee being close to the four-quark model relations; see Appendix 3 in Ref. [32] for details. Finally there are 15 effective free parameters of the fit, including several parameters that are softly restricted by terms $\sim(P-P_0)^2$, where P is the parameter and P_0 is its notably desired value. So results in Tables I and II are not obtained by pure χ^2 method, and we present a possible scenario.

For the data on $\phi\to\eta\pi^0\gamma$ we use a modified χ^2 function stressing on the resonant region $m>800$ MeV and with poor weight of the low m region. One can see in Fig. 4(b) that the description is close to experimental data for all m.

We faced several minima of the resulting function to minimize; they are rather close to each other. We show the best one. $\chi^2_{\gamma\gamma}$, χ^2_{sp}, and χ^2_{corr} shown in Table I are pure χ^2 values built on $\gamma\gamma\to\eta\pi^0$ data [6], the data on the $\phi\to\eta\pi^0\gamma$ decay [31], and the $K_S^0K^+$ correlation data [27] correspondingly.

Since we use a different model (including different parametrization of $\delta^{bg}_{K\bar{K}}$ and $\delta^{bg}_{\bar{K}^0K^+}$) and different data set (newer KLOE data on $\phi\to\eta\pi^0\gamma$ decay), and, besides, consider the case when $a_0(980)$ has no constituent two-quark component, the results shown in Tables I and II differ from the results in Ref. [26]. One can treat this difference as an error estimation.

III. CONCLUSION

The first measurement of $D^0\to d\bar{u}e^+\nu\to[a_0^-(980)+a_0'^-]e^+\nu\to\pi^-\eta e^+\nu$ and $D^+\to d\bar{d}e^+\nu\to[a_0^0(980)+a_0'^0]e^+\nu\to\pi^0\eta e^+\nu$ decays [30] is an important step for the investigation of the nature of light scalar mesons.

The data description with $g^{(0)}_{a_0^0\gamma\gamma}=0$ is presented for the first time to our knowledge, and it means that $a_0(980)$ has no constituent two-quark component. The data are described well, and the $a_0(980)$ coupling constants agree with the four-quark model scenario: they obey (or almost obey) the relations [9]

$$g_{a_0\eta\pi^0} = \sqrt{2}\sin(\theta_p+\theta_q)g_{a_0K^+K^-} = (0.85\text{–}0.98)g_{a_0K^+K^-},$$
$$g_{a_0\eta'\pi^0} = -\sqrt{2}\cos(\theta_p+\theta_q)g_{a_0K^+K^-} = -(1.13\text{–}1.02)g_{a_0K^+K^-}, \tag{16}$$

where $g_{a_0\eta\pi^0}=0.85g_{a_0K^+K^-}$ and $g_{a_0\eta'\pi^0}=-1.13g_{a_0K^+K^-}$ for $\theta_p=-18°$ and $g_{a_0\eta\pi^0}=0.98g_{a_0K^+K^-}$ and $g_{a_0\eta'\pi^0}=-1.02g_{a_0K^+K^-}$ for $\theta_p=-11°$. The $\theta_q=54.74°$.

The corresponding prediction of $D^0\to d\bar{u}e^+\nu\to[a_0^-(980)+a_0'^-]e^+\nu\to\pi^-\eta e^+\nu$ and $D^+\to d\bar{d}e^+\nu\to[a_0^0(980)+a_0'^0]e^+\nu\to\pi^0\eta e^+\nu$ decays is presented and does not contradict the data [30]. An experiment on higher statistics could check this prediction.

The experiment on $D_s^+\to s\bar{s}e^+\nu\to[\sigma(600)+f_0(980)+f_0']e^+\nu\to\pi^+\pi^-e^+\nu$ with higher precision than in Ref. [34] is also strongly interesting.

Let us repeat that no less interesting is to probe the light scalars in semileptonic $D^+\to d\bar{d}e^+\nu\to[\sigma(600)+f_0(980)+f_0']e^+\nu\to\pi^+\pi^-e^+\nu$, $B^0\to d\bar{u}e^+\nu\to[a_0^-(980)+a_0'^-]e^+\nu\to\pi^-\eta e^+\nu$, $B^+\to u\bar{u}e^+\nu\to[a_0^0(980)+a_0'^0]e^+\nu\to\pi^0\eta e^+\nu$, and $B^+\to u\bar{u}e^+\nu\to[\sigma(600)+f_0(980)+f_0']e^+\nu\to\pi^+\pi^-e^+\nu$ decays which have not yet been investigated.

The approach of this paper is valid for the B mesons decay $B^0\to\pi^-\eta e^+\nu$ and $B^+\to\pi^0\eta e^+\nu$. It is enough to make obvious changes $V_{cd}\to V_{ub}$, $g_{D^0c\bar{u}}\to g_{B^0d\bar{b}}$, and $g_{D^+c\bar{d}}\to g_{B^+u\bar{b}}$. In Eq. (5) $A=D_1(2420)^\pm\to B_1(5721)^+$.

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APPENDIX A: SCALAR PROPAGATORS AND POLARIZATION OPERATORS

The matrix of the inverse propagators is

$$G_{SS'}(m) = \begin{pmatrix} D_{a_0'}(m) & -\Pi_{a_0'a_0}(m) \\ -\Pi_{a_0'a_0}(m) & D_{a_0}(m) \end{pmatrix}, \tag{A1}$$

$$\Pi_{a_0'a_0}(m) = \sum_{a,b}\frac{g_{a_0'ab}}{g_{a_0ab}}\Pi^{ab}_{a_0}(m) + C_{a_0'a_0}, \tag{A2}$$

where $m=\sqrt{s}$, and the constant $C_{a_0'a_0}$ incorporates the subtraction constant for the transition $a_0(980)\to(0^-0^-)\to a_0'$ and effectively takes into account the contributions of multiparticle intermediate states to the $a_0\leftrightarrow a_0'$ transition. The inverse propagator of the scalar meson S [9,13,32,35] is

$$D_S(m) = m_S^2 - m^2 + \sum_{ab}[\mathrm{Re}\Pi_S^{ab}(m_S^2) - \Pi_S^{ab}(m^2)], \quad (A3)$$

where $\sum_{ab}[\mathrm{Re}\Pi_S^{ab}(m_S^2) - \Pi_S^{ab}(m^2)] = \mathrm{Re}\Pi_S(m_S^2) - \Pi_S(m^2)$ takes into account the finite-width corrections of the resonance which are the one-loop contributions to the self-energy of the S resonance from the two-particle intermediate ab states. We take into account the intermediate states $\eta\pi^+$, $K\bar{K}$, and $\eta'\pi^+$ in the $a_0^+(980)$ and $a_0'^+$ propagators,

$$\Pi_S = \Pi_S^{\eta\pi^+} + \Pi_S^{K_S^0 K^+} + \Pi_S^{K_L^0 K^+} + \Pi_S^{\eta'\pi^+}, \quad (A4)$$

and $\eta\pi^0$, $K\bar{K}$, and $\eta'\pi^0$ in the $a_0^0(980)$ and $a_0'^0$ propagators.

For pseudoscalar mesons a, b and $m_a \geq m_b$, $m \geq m_+$, one has

$$\Pi_S^{ab}(m^2) = \frac{g_{Sab}^2}{16\pi}\left[\frac{m_+ m_-}{\pi m^2}\ln\frac{m_b}{m_a} + \rho_{ab}\left(i + \frac{1}{\pi}\ln\frac{\sqrt{m^2 - m_-^2} - \sqrt{m^2 - m_+^2}}{\sqrt{m^2 - m_-^2} + \sqrt{m^2 - m_+^2}}\right)\right], \quad (A5)$$

where $\rho_{ab}(s) = 2p_{ab}(s)/\sqrt{s} = \sqrt{(1 - m_+^2/s)(1 - m_-^2/s)}$, and $m_\pm = m_a \pm m_b$. Analytical continuation to other energy regions could be found, e.g., in Ref. [26] and references therein.

The constants g_{Sab} are related to the width as

$$\Gamma_S(m) = \sum_{ab}\Gamma(S \to ab, m) = \sum_{ab}\frac{g_{Sab}^2}{16\pi m}\rho_{ab}(m). \quad (A6)$$

TABLE II. Parameters not mentioned in Table I.

c_0	−0.34	$f_{K\bar{K}}$, GeV^{-1}	−2.14
c_1, GeV^{-2}	−9.04	$g_{K\bar{K}}$	2.37
c_2, GeV^{-4}	1.40	$f_{\pi\eta'}$, GeV^{-1}	−0.50
δ, °	−128.3		

APPENDIX B: OTHER PARAMETERS AND DETAILS

For completeness, we show in Table II the background parameters and the parameters that are not described above. One can find all of the details in Ref. [32].

In this paper we take the form factor $G_\omega(s,t) = G_\rho(s,t)$,

$$G_\omega(s,t) = G_\rho(s,t) = \exp[(t - m_\omega^2)b_\omega(s)], \quad (B1)$$

differently from Refs. [8,32]. We take

$$b_\omega(s) = b_\omega^0 + \alpha'_\omega \ln[1 + (s/s_0)] \quad (B2)$$

and obtain $b_\omega^0 = 2.3\times10^{-3}$ GeV^{-2}, and $s_0 = 1.005$ GeV^2. $\alpha'_\omega = 0.8$ GeV^{-2} is the same. Form factors for the K^* exchange are modified the same way. Besides, we obtain $r_{a_2} = 1.2$ GeV^{-1} instead of $r_{a_2} = 1.9$ GeV^{-1} in Refs. [8,32].

The $\pi\eta$ scattering length agrees with the estimates based on current algebra and chiral perturbation theory, according to which $a_0^1 \approx 0.005$–0.01 (in units of m_π^{-1}); see Ref. [8].

[1] M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018).
[2] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); **15**, 281 (1977).
[3] S. Weinberg, Phys. Rev. Lett. **110**, 261601 (2013).
[4] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **108B**, 134 (1982); Z. Phys. C **16**, 55 (1982).
[5] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. D **78**, 052004 (2008).
[6] S. Uehara *et al.* (Belle Collaboration), Phys. Rev. D **80**, 032001 (2009).
[7] N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988).
[8] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **77**, 074020 (2008); **81**, 094029 (2010); Usp. Fiz. Nauk **54**, 799 (2011); [Sov. Phys. Usp. **181**, 827 (2011)].
[9] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).
[10] N. N. Achasov, Nucl. Phys. **A728**, 425 (2003).
[11] N. N. Achasov and V. V. Gubin, Phys. Rev. D **63**, 094007 (2001); N. N. Achasov and A. V. Kiselev, Phys. Rev. D **73**, 054029 (2006).
[12] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **68**, 014006 (2003).
[13] N. N. Achasov and V. V. Gubin, Phys. Rev. D **56**, 4084 (1997).
[14] M. N. Achasov *et al.* (SND Collaboration), Phys. Lett. B **438**, 441 (1998); M. N. Achasov *et al.*, Phys. Lett. B **479**, 53 (2000).
[15] M. N. Achasov *et al.* (SND Collaboration), Phys. Lett. B **440**, 442 (1998); M. N. Achasov *et al.*, Phys. Lett. B **485**, 349 (2000); R. R. Akhmetshin *et al.* (CMD-2 Collaboration),

Phys. Lett. B **462**, 380 (1999); A. Aloisio *et al.* (KLOE Collaboration), Phys. Lett. B **537**, 21 (2002); C. Bini, P. Gauzzi, S. Giovanella, D. Leone, and S. Miscetti, KLOE Note 173 06/02, http://www.lnf.infn.it/kloe/.

[16] A. Aloisio *et al.* (KLOE Collaboration), Phys. Lett. B **536**, 209 (2002).

[17] N. N. Achasov, V. V. Gubin, and V. I. Shevchenko, Phys. Rev. D **56**, 203 (1997).

[18] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **76**, 077501 (2007); **78**, 058502 (2008).

[19] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **58**, 054011 (1998).

[20] N. N. Achasov and G. N. Shestakov, Phys. Rev. D **49**, 5779 (1994).

[21] N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. **99**, 072001 (2007).

[22] M. Gell-Mann and M. Levy, Nuovo Cimento **16**, 705 (1960).

[23] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **83**, 054008 (2011); **85**, 094016 (2012).

[24] I. Caprini, G. Colangelo, and H. Leutwyler, Phys. Rev. Lett. **96**, 132001 (2006).

[25] N. N. Achasov, Usp. Fiz. Nauk **168**, 1257 (1998); [Phys. Usp. **41**, 1149 (1998)]; Yad. Fiz. **65**, 573 (2002); [Phys. At. Nucl. **65**, 546 (2002)].

[26] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **97**, 036015 (2018).

[27] S. Acharya *et al.* (ALICE Collaboration), Phys. Lett. B **774**, 64 (2017).

[28] N. N. Achasov and A. V. Kiselev, Phys. Rev. D **86**, 114010 (2012).

[29] N. N. Achasov and A. V. Kiselev, Int. J. Mod. Phys. Conf. Ser. **35**, 1460447 (2014).

[30] M. Ablikim *et al.* (BESIII Collaboration), Phys. Rev. Lett. **121**, 081802 (2018).

[31] F. Ambrosino *et al.* (KLOE Collaboration), Phys. Lett. B **681**, 5 (2009).

[32] N. N. Achasov, A. V. Kiselev, and G. N. Shestakov, Phys. At. Nucl. **79**, 397 (2016).

[33] R. Lednicky and V. L. Lyuboshits, Yad. Fiz. **35**, 1316 (1982); [Sov. J. Nucl. Phys. **35**, 770 (1982)].

[34] K. M. Ecklund *et al.* (CLEO Collaboration), Phys. Rev. D **80**, 052009 (2009).

[35] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979).