

What becomes of semilocal non-Abelian strings in $\mathcal{N} = 1$ supersymmetric QCD

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We study non-Abelian vortex strings in $\mathcal{N} = 2$ supersymmetric QCD with the gauge group $U(N)$ deformed by the mass μ of the adjoint matter. This deformation breaks $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$ and in the limit of large μ the theory flows to $\mathcal{N} = 1$ QCD. Non-Abelian strings in addition to translational zero modes have orientation moduli. In the $\mathcal{N} = 2$ limit of small μ the dynamics of orientational moduli is described by the two-dimensional $CP(N - 1)$ model for QCD with $N_f = N$ flavors of quark hypermultiplets. For the case of $N_f > N$ the non-Abelian string becomes semilocal developing additional size moduli that modify the effective two-dimensional σ -model on the string making its target space noncompact. In this paper we consider the μ -deformed theory with $N_f > N$ eventually making μ large. We show that size moduli develop a potential that forces the string transverse size to shrink. Eventually in the large μ limit size moduli decouple and the effective theory on the string reduces to the $CP(N - 1)$ model. We also comment on physics of confined monopoles.

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I. INTRODUCTION

Searches for a non-Abelian generalization of the Seiberg-Witten scenario of quark confinement [1,2] lead to the discovery of non-Abelian vortex strings in $\mathcal{N} = 2$ supersymmetric QCD [3–6]; see also [7–10] for reviews. They are formed in the Higgs phase of $U(N)$ gauge theory due to the condensation of scalar quarks and are responsible for the confinement of monopoles. In the strong coupling regime the theory is in the “instead-of-confinement” phase. Particularly rich structure with non-Abelian dual gauge group appears in a theory with number of quark flavors $N_f > N$; see [11] for a review.

The $\mathcal{N} = 2$ supersymmetric quantum chromodynamics (SQCD) is a nice theoretical laboratory to study non-perturbative non-Abelian dynamics. However, since we wish to learn more about the “real world,” we are interested in studying more realistic theories. $\mathcal{N} = 1$ supersymmetric QCD is one of the most promising examples. Much in the same way as the real world QCD it has no adjoint scalars

and no Abelianization of the theory can occur due to their condensation.

To study this theory we start with $\mathcal{N} = 2$ SQCD deformed by the mass μ of the adjoint matter. This deformation breaks $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$. In the limit of large μ the adjoint matter decouples and the theory flows to $\mathcal{N} = 1$ QCD. It was shown that the non-Abelian instead-of-confinement phase survives for $N_f > N$; see review [11] and references therein.

Motivated by these results in this paper we study non-Abelian confining strings in μ -deformed $\mathcal{N} = 2$ SQCD with $N_f > N$. In particular, we consider the limit of large μ when the theory flows to $\mathcal{N} = 1$ SQCD.

The case $N_f = N$ was studied earlier. To the leading order in μ the mass term for the adjoint matter reduces to the Fayet-Iliopoulos (FI) F -term, which does not break $\mathcal{N} = 2$ supersymmetry [12,13]. In the quark vacuum squark condensate is determined by $\sqrt{\mu m}$, where m is a quark mass. In this setup non-Abelian strings were first found [3–6] and their dynamics was well studied; see [9] for a review. In addition to the translational zero modes typical for Abelian Abrikosov-Nielsen-Olesen (ANO) vortex strings [14], non-Abelian strings have orientational moduli associated with rotations of their fluxes inside the non-Abelian $SU(N)$ group. The dynamics of the orientational moduli in $\mathcal{N} = 2$ QCD is described by the two-dimensional $CP(N - 1)$ model living on the world sheet of the non-Abelian string.

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The μ deformation of $\mathcal{N} = 2$ SQCD was considered recently in [15]. It turns out that the non-Abelian string ceases to be Bogomol'nyi-Prasad-Sommereld (BPS), and world sheet supersymmetry is completely lost. The fermionic sector of the low energy world sheet theory decouples at large μ , while the bosonic sector is given by the two-dimensional $\text{CP}(N-1)$ model. It was also shown that in the case of equal quark masses confined monopoles seen in the world sheet theory as kinks [5,6] survive μ deformation and present in the limit of $\mathcal{N} = 1$ SQCD. The potential in two-dimensional world sheet theory induced by quark mass differences was also found.

Non-Abelian strings in $\mathcal{N} = 2$ SQCD with ‘‘extra’’ quark flavors ($N_f > N$) were also well studied. In this setting the string develops size moduli and becomes semilocal. In particular, in the Abelian case these strings interpolate between ANO local strings and sigma-model lumps [16–20]. World sheet theory on the semilocal non-Abelian string was first considered from a D-brane perspective [3,6], and later from a field theory side [21–24]. In particular, in [24] it was found that the world sheet theory is the so-called $\mathcal{N} = (2, 2)$ supersymmetric zn model.

In this paper we continue studies of non-Abelian strings in SQCD with additional quark flavors, $N_f > N$, and consider μ -deformed theory. In particular, we study what becomes of semilocal non-Abelian strings as we increase μ and take the large μ limit where the theory flows to $\mathcal{N} = 1$ SQCD. First we found that much in the same way as for $N_f = N$ case [15] the string is no longer BPS and the world sheet supersymmetry is lost.

Moreover, as we switch on the deformation parameter μ the string itself ceases to be semilocal. Considering the world sheet theory at small μ we show that string size moduli develop a potential that forces them to shrink. Eventually in the large μ limit size moduli decouple and the effective theory on the string reduces to the $\text{CP}(N-1)$ model.

We also briefly discuss the physics of confined monopoles.

The paper is organized as follows. In Sec. I we briefly outline the underlying bulk theory and calculate its mass spectrum. In Sec. II we consider the non-Abelian semilocal string in the μ deformed theory and study the world sheet theory for this string. We summarize our results in Sec. III.

II. THEORETICAL SETUP

A. Bulk theory

In this section we briefly describe our initial theory in the bulk. The basic model is four-dimensional $\mathcal{N} = 2$ supersymmetric QCD with the gauge group $\text{SU}(N) \times \text{U}(1)$. The field content of the theory is as follows. The matter consists of $N_f = N + \tilde{N}$ flavors of quark hypermultiplets in the fundamental representation, scalar components being

q^{kA} and \tilde{q}_{Ak} . Here, $A = 1, \dots, N_f$ is the flavor index and $k = 1, \dots, N$ is the color index. The vector multiplets consist of $\text{U}(1)$ gauge field A_μ and $\text{SU}(N)$ gauge field A_μ^a , complex scalar fields a and a^a in the adjoint representation of the color group, and their Weyl fermion superpartners. Index a runs from 1 to $N^2 - 1$, and the spinorial index $\alpha = 1, 2$.

Superpotential of the $\mathcal{N} = 2$ SQCD is

$$\mathcal{W}_{\mathcal{N}=2} = \sqrt{2} \left\{ \frac{1}{2} \tilde{q}_A \mathcal{A}^{\text{U}(1)} q^A + \tilde{q}_A \mathcal{A}^a T^a q^A \right\} + m_A \tilde{q}_A q^A, \quad (2.1)$$

which includes adjoint matter chiral $\mathcal{N} = 1$ multiplets $\mathcal{A}^{\text{U}(1)}$ and $\mathcal{A}^{\text{SU}(N)} = \mathcal{A}^a T^a$, and the quark chiral $\mathcal{N} = 1$ multiplets q^A and \tilde{q}_A (here we use the same notation for the quark superfields and their scalar components). The μ deformation considered in this paper is given by the superpotential

$$\mathcal{W}_{\mathcal{N}=1} = \sqrt{\frac{N}{2}} \frac{\mu_1}{2} (\mathcal{A}^{\text{U}(1)})^2 + \frac{\mu_2}{2} (\mathcal{A}^a)^2. \quad (2.2)$$

We assume the deformation parameters to be of the same order, $\mu_1 \sim \mu_2 \sim \mu$. When we increase $\mu \rightarrow \infty$, $\mathcal{N} = 2$ supersymmetry becomes broken, and the theory flows to $\mathcal{N} = 1$ SQCD. Instead in the limit of small μ this superpotential does not break the $\mathcal{N} = 2$ supersymmetry and reduces to a FI F -term [12,13].

In order to control the theory and stay at weak coupling as we take this limit, we require the product $\sqrt{\mu m}$ to stay fixed and well above $\Lambda_{\mathcal{N}=1}$, which is the scale of the $\text{SU}(N)$ sector of $\mathcal{N} = 1$ QCD.

The bosonic part of the action is given by

$$\begin{aligned} S_{\text{bos}} = \int d^4x & \left(\frac{1}{2g_2^2} \text{Tr}(F_{\mu\nu}^{\text{SU}(N)})^2 + \frac{1}{4g_1^2} (F_{\mu\nu}^{\text{U}(1)})^2 \right. \\ & + \frac{2}{g_2^2} \text{Tr}|\nabla_\mu a^{\text{SU}(N)}|^2 + \frac{1}{g_1^2} |\partial_\mu a^{\text{U}(1)}|^2 + |\nabla_\mu q^A|^2 \\ & \left. + |\nabla_\mu \tilde{q}^A|^2 + V(q^A, \tilde{q}_A, a^{\text{SU}(N)}, a^{\text{U}(1)}) \right). \quad (2.3) \end{aligned}$$

Here ∇_μ is the covariant derivative in the corresponding representation,

$$\nabla_\mu^{\text{adj}} = \partial_\mu - i[A_\mu^a T^a, \cdot],$$

$$\nabla_\mu^{\text{fund}} = \partial_\mu - \frac{i}{2} A_\mu^{\text{U}(1)} - iA_\mu^a T^a,$$

with the $\text{SU}(N)$ generators normalized as $\text{Tr}(T^a T^b) = (1/2)\delta^{ab}$. Superpotentials (2.1) and (2.2) contribute to the scalar potential V , which is given by the sum of F and D terms,

$$\begin{aligned}
V(q^A, \tilde{q}_A, a^{\text{SU}(N)}, a^{\text{U}(1)}) &= \frac{g_2^2}{2} \left(\frac{1}{g_2^2} f^{abc} \bar{a}^b a^c + \bar{q}_A T^a q^A - \tilde{q}_A T^a \tilde{q}^A \right)^2 \\
&+ \frac{g_1^2}{8} (\bar{q}_A q^A - \tilde{q}_A \tilde{q}^A)^2 + 2g_2^2 \left| \tilde{q}_A T^a q^A + \frac{1}{\sqrt{2}} \frac{\partial \mathcal{W}_{N=1}}{\partial a^a} \right|^2 \\
&+ \frac{g_1^2}{2} \left| \tilde{q}_A q^A + \sqrt{2} \frac{\partial \mathcal{W}_{N=1}}{\partial a^{\text{U}(1)}} \right|^2 \\
&+ 2 \sum_{A=1}^{N_f} \left\{ \left| \left(\frac{1}{2} a^{\text{U}(1)} + \frac{m_A}{\sqrt{2}} + a^a T^a \right) q^A \right|^2 \right. \\
&\left. + \left| \left(\frac{1}{2} a^{\text{U}(1)} + \frac{m_A}{\sqrt{2}} + a^a T^a \right) \tilde{q}^A \right|^2 \right\}, \quad (2.4)
\end{aligned}$$

where summation is implied over the repeated flavor indices A (and over omitted color indices, too).

Consider the case when we have one extra flavor, $N_f = N + 1$. Scalar potential (2.4) has a set of supersymmetric vacua, but in this paper we concentrate on a particular vacuum where the maximal number of squarks equal to the rank of the gauge group N condense. Up to a gauge transformation, the squark vacuum expectation values are given by

$$\langle q^{kA} \rangle = \langle \tilde{q}^{kA} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\xi_1} & 0 & 0 & 0 & 0 \\ 0 & \ddots & \vdots & \vdots & \vdots \\ \vdots & \dots & \sqrt{\xi_{N-1}} & 0 & 0 \\ 0 & \dots & 0 & \sqrt{\xi_N} & 0 \end{pmatrix}, \quad (2.5)$$

where we write quark fields as rectangular matrices $N \times N_f$ and ξ_P are defined as

$$\xi_P = 2 \left(\sqrt{\frac{2}{N}} \mu_1 \hat{m} + \mu_2 (m_P - \hat{m}) \right), \quad (2.6)$$

$$\hat{m} = \frac{1}{N} \sum_{A=1}^N m_P. \quad (2.7)$$

If we define a scalar adjoint matrix as

$$\Phi = \frac{1}{2} a + T^a a^a, \quad (2.8)$$

then the adjoint fields vacuum expectation values (VEVs) are given by

$$\langle \Phi \rangle = -\frac{1}{\sqrt{2}} \begin{pmatrix} m_1 & \dots & 0 \\ \dots & \dots & \dots \\ 0 & \dots & m_N \end{pmatrix}. \quad (2.9)$$

The vacuum field (2.5) results in the spontaneous breaking of both gauge $\text{U}(N)$ and flavor $\text{SU}(N)$. However, in the equal mass limit $m_A \equiv m$, $A = 1, \dots, N_f$ all parameters ξ become equal, $\xi_P \equiv \xi$, $P = 1, \dots, N$ and a diagonal global $\text{SU}(N)_{C+F}$ survives, or, more exactly,

$$\text{U}(N)_{\text{gauge}} \times \text{SU}(N)_{\text{flavor}} \rightarrow \text{SU}(N)_{C+F} \times \text{SU}(\tilde{N})_F \times \text{U}(1). \quad (2.10)$$

Thus, a color-flavor locking takes place in the vacuum. The presence of the color-flavor $\text{SU}(N)_{C+F}$ global symmetry is the reason for the formation of non-Abelian strings; see [9] for a review.

In the special case when

$$\mu_2 = \mu_1 \sqrt{2/N} \equiv \mu, \quad (2.11)$$

superpotential (2.2) is simplified and becomes a single-trace operator

$$\mathcal{W}_{N=1} = \mu \text{Tr}(\Phi^2). \quad (2.12)$$

B. Mass spectrum

In this section we review the mass spectrum of our bulk SQCD taking all quark masses equal, cf., [9,13,25]. Because of squark condensation, the gauge bosons acquire masses¹

$$\begin{aligned}
m_{\text{U}(1)} &= g_1 \sqrt{\frac{N}{2}} \xi, \\
m_{\text{SU}(N)} &= g_2 \sqrt{\xi}. \quad (2.13)
\end{aligned}$$

Scalar state masses are to be read off from the potential (2.4). Expanding and diagonalizing the mass matrix one can find $N^2 - 1$ real scalars with the masses $m_{\text{SU}(N)}$ and one scalar with the mass $m_{\text{U}(1)}$. These are $\mathcal{N} = 1$ superpartners of $\text{SU}(N)$ and $\text{U}(1)$ gauge bosons. Other N^2 components are eaten by the Higgs mechanism. Another $2 \times 2N^2$ real scalars (adjoint scalars a^a , a and the half of squarks) become scalar components of the following $\mathcal{N} = 1$ chiral multiplets: one with mass

$$m_{\text{U}(1)}^+ = g_1 \sqrt{\frac{N}{2}} \xi \lambda_1^+, \quad (2.14)$$

and another one with mass

$$m_{\text{U}(1)}^- = g_1 \sqrt{\frac{N}{2}} \xi \lambda_1^-. \quad (2.15)$$

¹Here we assume for simplicity that ξ , μ_1 , μ_2 are real.

The remaining $2(N^2 - 1)$ chiral multiplets have masses

$$m_{\text{SU}(N)}^+ = g_2 \sqrt{\xi \lambda_2^+}, \quad (2.16)$$

$$m_{\text{SU}(N)}^- = g_2 \sqrt{\xi \lambda_2^-}. \quad (2.17)$$

Here λ_i^\pm are roots of the quadratic equation [9,13]

$$\lambda_i^2 - \lambda_i(2 + \omega_i^2) + 1 = 0 \quad (2.18)$$

with

$$\omega_1 = \frac{g_1 \mu_1}{\sqrt{\xi}}, \quad \omega_2 = \frac{g_2 \mu_2}{\sqrt{\xi}}. \quad (2.19)$$

Once $N_f > N$ apart from the above massive scalars, we also have $4N(N_f - N)$ scalars that come from the extra squark flavors q^K and \tilde{q}_K , $K = (N + 1), \dots, N_f$. In the equal mass limit these extra scalars are massless, and the theory enjoys a Higgs branch

$$\mathcal{H} = T^* \text{Gr}^C(N_f, N) \quad (2.20)$$

of real dimension

$$\dim \mathcal{H} = 4N(N_f - N). \quad (2.21)$$

In the large μ limit, states with masses $m_{\text{U}(1)}^+$ and $m_{\text{SU}(N)}^+$ become heavy with masses $\sim g^2 \mu$ and decouple. They correspond to the adjoint matter multiplets. Instead states with masses $m_{\text{U}(1)}^-$ and $m_{\text{SU}(N)}^-$ become light with masses $\sim \xi/\mu$. Scalar components of these multiplets are Higgs scalars. They develop VEVs (2.5). In the opposite limit of small μ their masses are given by

$$\begin{aligned} m_{\text{U}(1)}^- &= g_1 \sqrt{\frac{N}{2} \xi} \left(1 - \frac{g_1 \mu_1}{2\sqrt{\xi}} + \dots \right), \\ m_{\text{SU}(N)}^- &= g_2 \sqrt{\xi} \left(1 - \frac{g_2 \mu_2}{2\sqrt{\xi}} + \dots \right). \end{aligned} \quad (2.22)$$

As we already mentioned $\mathcal{N} = 2$ supersymmetry is not broken in our theory to the leading order at small μ [12,13]. The leading order corresponds to sending parameters ω in (2.19) to 0 while keeping FI parameter $\xi \sim \mu m$ fixed. One can see that in the $\mathcal{N} = 2$ limit Higgs scalars are degenerate with the gauge fields,² but become lighter as we switch on the μ -deformation.

The ratio of squares of Higgs and gauge boson masses β is an important parameter in the theory of superconductivity. Type I superconductors correspond to $\beta < 1$, while

²They belong to the same long vector $\mathcal{N} = 2$ supermultiplet [13].

type II superconductors correspond to $\beta > 1$. BPS strings arise on the border at $\beta = 1$. We see that in our theory both parameters β ,

$$\beta_{\text{U}(1)} = \left(\frac{m_{\text{U}(1)}^-}{m_{\text{U}(1)}} \right)^2, \quad \beta_{\text{SU}(N)} = \left(\frac{m_{\text{SU}(N)}^-}{m_{\text{SU}(N)}} \right)^2, \quad (2.23)$$

are less than unity, and thus our theory is in the type I superconducting phase at nonzero μ . This turns out to be important later.

III. SEMILOCAL NON-ABELIAN VORTICES

In this section we study a vortex string solution in the equal quark mass limit. First we review previous results [24] for the BPS semilocal non-Abelian vortex string and then consider a small μ -deformation. We derive the world sheet effective theory for the string moduli fields in this case. For simplicity we consider the theory with one extra quark flavor, $N_f = N + 1$.

A. BPS semilocal non-Abelian string

We start by reviewing the semilocal non-Abelian string in the $\mathcal{N} = 2$ limit [24]. Once the number of flavors exceeds the number of colors vortices have no longer the conventional exponentially small tails of the profile functions. The presence of the Higgs branch and associated massless fields in the bulk makes them semilocal; see a detailed review of the Abelian case in [20]. The semilocal strings have a power falloff at large distances from the string axis. For example, the semilocal Abelian BPS string interpolates between the ANO string [14] and two-dimensional O(3) sigma-model instanton uplifted to four dimensions (also known as the lump). For one extra flavor the semilocal string possesses two additional zero modes parametrized by the complex modulus ρ . The string's transverse size is associated with $|\rho|$. In the limit $|\rho| \rightarrow 0$ in the Abelian case we recover the ANO string while at $|\rho| \gg 1/m_{\text{U}(1)}$ it becomes a lump.

Consider an infinite static string stretched along the x_3 axis using the following ansatz:

$$q^{kA} = \bar{q}^{kA} = \frac{1}{\sqrt{2}} \varphi^{kA}, \quad (3.1)$$

$$\varphi = (\phi_2(r) + n\bar{n}(\phi_1(r) - \phi_2(r)) | n\phi_3(r) e^{-i\alpha}) \quad (3.2)$$

for quarks, while the gauge fields are given by

$$\begin{aligned} A_i^{\text{SU}(N)} &= \varepsilon_{ij} \frac{x^j}{r^2} f_G(r) (n\bar{n} - 1/N), \\ A_i^{\text{U}(1)} &= \frac{2}{N} \varepsilon_{ij} \frac{x^j}{r^2} f(r). \end{aligned} \quad (3.3)$$

Index i runs $i = 1, 2$; all other components are 0; α, r are the polar angle and radius in the (x_1, x_2) plane, respectively.

The complex parameters $n^l, l=1, \dots, N$ obey the $CP(N-1)$ constraint $\bar{n}n = 1$. They parametrize the orientational zero modes of the non-Abelian string, which appear due to the presence of the color-flavor group (2.10); see [9] for a review.

The string profile functions entering (3.2) and (3.3) satisfy first order BPS equations. For the case

$$\frac{g_1^2}{2} = \frac{g_2^2}{N} \equiv \frac{g^2}{N} \quad (3.4)$$

the solution is particularly simple [24]. It is parametrized by a complex size modulus ρ ,

$$\begin{aligned} \phi_1 &\approx \sqrt{\xi} \frac{r}{\sqrt{r^2 + |\rho|^2}}, \\ \phi_2 &\approx \sqrt{\xi}, \\ \phi_3 &= \frac{\rho}{r} \phi_1 \approx \sqrt{\xi} \frac{\rho}{\sqrt{r^2 + |\rho|^2}}, \\ f &= f_G \approx \frac{|\rho|^2}{r^2 + |\rho|^2}. \end{aligned} \quad (3.5)$$

This solution is valid in the limit $|\rho| \gg 1/(g_2 \sqrt{\xi} |\rho|)$, i.e., when the scalar fields approach the vacuum manifold (Higgs branch). Tension of the BPS string is given by

$$T_{\text{BPS}} = 2\pi\xi. \quad (3.6)$$

To obtain the low energy effective two-dimensional theory living on the string world sheet, one should assume n^P and ρ to be slowly varying functions of the transversal coordinates t, z , and substitute the solution (3.5) into the action (2.3). This procedure yields the effective action

$$\begin{aligned} S_{\text{SUSY}}^{2d} &= \int d^2x \left\{ 2\pi\xi |\partial_k(\rho n_P)|^2 \ln \frac{L}{|\rho|} \right. \\ &\quad \left. + \frac{4\pi}{g^2} [|\partial_k n_P|^2 + (\bar{n}_P \partial_k n_P)^2] \right\}, \end{aligned} \quad (3.7)$$

where the integration is carried over the coordinates x_0, x_3 ; see the detailed derivation in [24]. Here $k=0, 3$, and L is an infrared (IR) cutoff introduced for the regularization of the logarithmic divergences of orientational and size zero modes of the string. More exactly we introduce the string of a large but finite length L . This also regularizes the spread of string profile functions in the transverse plane [21]. The IR divergences arise due to the slow (power) falloff of the string profile functions associated with the presence of the Higgs branch [21,24].

B. Deformed world sheet theory

When we take into account higher order μ -corrections, supersymmetry in the bulk reduces to $\mathcal{N}=1$, and as we

already explained our theory becomes that of the type I superconductor, cf., [13]. The string is no longer BPS saturated. To mimic this we consider a simplified version of our theory with the bosonic action given by

$$\begin{aligned} S_0 &= \int d^4x \left\{ \frac{1}{4g_2^2} (F_{\mu\nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + |\nabla_\mu \varphi^A|^2 \right. \\ &\quad \left. + \lambda_N (\bar{\varphi}_A T^a \varphi^A)^2 + \lambda_1 (|\varphi^A|^2 - N\xi)^2 \right\}. \end{aligned} \quad (3.8)$$

This model depends on two parameters—ratios of the squares of U(1) and SU(N) Higgs and gauge boson masses given by

$$\begin{aligned} \beta_{U(1)} &= \frac{8\lambda_1}{g_1^2}, \\ \beta_{\text{SU}(N)} &= \frac{2\lambda_N}{g_2^2}, \end{aligned} \quad (3.9)$$

which we identify with β -parameters (2.23) of our original theory. The model above is a non-Abelian generalization of the one considered in [26], where the scalar QED was studied; see also [20].

In $\mathcal{N}=2$ supersymmetric QCD parameters β are exactly equal to 1. In this case the Bogomol'nyi representation produces first order equations for the string profile functions. World sheet theory in this case is given by (3.7).

As we switch on μ -corrections parameters β are no longer equal to unity. Let us write the Bogomol'nyi representation for the tension of the string

$$\begin{aligned} T_\beta &= \int d^2x_\perp \left\{ \left[\frac{1}{\sqrt{2}g_2} F_{12}^a + \frac{g_2}{\sqrt{2}} (\bar{\varphi}_A T^a \varphi^A) \right]^2 \right. \\ &\quad + \left[\frac{1}{\sqrt{2}g_1} F_{12} + \frac{g_1}{2\sqrt{2}} (|\varphi^A|^2 - N\xi) \right]^2 \\ &\quad + |\nabla_1 \varphi^A + i\nabla_2 \varphi^A|^2 + \frac{N}{2} \xi F_3^* \\ &\quad + \frac{g_2^2}{2} (\beta_{\text{SU}(N)} - 1) (\bar{\varphi}_A T^a \varphi^A)^2 \\ &\quad \left. + \frac{g_1^2}{8} (\beta_{U(1)} - 1) (|\varphi^A|^2 - N\xi)^2 \right\}, \end{aligned} \quad (3.10)$$

where \vec{x}_\perp represents the coordinates in the transverse plane. Two extra terms written in the last line above appear. The Bogomol'nyi bound is no longer valid. But if the values $\beta_{U(1)}$ and $\beta_{\text{SU}(N)}$ only slightly differ from unity, then we can use the first order equations to rewrite expressions in these extra terms as follows:

$$\begin{aligned} g_2^2 (\bar{\varphi}_A T^a \varphi^A) &= -F_{12}^a, \\ \frac{g_1^2}{2} (|\varphi^A|^2 - N\xi) &= -F_{12}. \end{aligned} \quad (3.11)$$

In the case (3.4) we can use (3.5) to calculate the effective action. Substituting (3.2), (3.3), and (3.5) into (3.11) and (3.10) one arrives at the deformed world sheet theory,

$$S_{\beta}^{2d} = \int d^2x \left\{ 2\pi\xi |\partial_k(\rho n_P)|^2 \ln \frac{L}{|\rho|} + \frac{4\pi}{g^2} [|\partial_k n_P|^2 + (\bar{n}_P \partial_k n_P)^2] + \frac{\beta-1}{g^2} \frac{4\pi}{3|\rho|^2} + \dots \right\}, \quad (3.12)$$

where now $\beta \equiv \beta_{U(1)} = \beta_{\text{SU}(N)}$ and the dots represent corrections in powers of $1/g^2 \xi |\rho|^2$.

We see that for non-BPS string ρ is no longer a modulus. It develops a potential proportional to the deviation of β from unity. In particular, for the type I superconductor ($\beta < 1$) the size ρ tends to shrink, while for the type II superconductor ($\beta > 1$) the size ρ tends to expand making the vortex unstable, cf., [20].

In our case, the value of β is less than unity and is given by (2.22) at small μ , namely,

$$\beta = 1 - \frac{g\mu}{\sqrt{\xi}} + \dots \quad (3.13)$$

This gives the effective world sheet action on the string,

$$S_{\beta}^{2d} = \int d^2x \left\{ 2\pi\xi |\partial_k(\rho n_P)|^2 \ln \frac{L}{|\rho|} + \frac{4\pi}{g^2} [|\partial_k n_P|^2 + (\bar{n}_P \partial_k n_P)^2] - 4\pi \frac{\mu}{3g\sqrt{\xi}} \frac{1}{|\rho|^2} + \dots \right\}. \quad (3.14)$$

We see that the size of the semilocal string tends to shrink and at large μ we expect that the long-range tails of the string are not developed. The string becomes a local non-Abelian string with only orientational moduli n^l , whose world sheet dynamics is described by the $CP(N-1)$ model.

In fact we can argue on general grounds that as we turn on μ and make it large the semilocal string becomes unstable. The semilocal string solution (3.5) is “made” of massless fields associated with the Higgs branch of the theory. As we already mentioned, in the Abelian case this solution corresponds to the instanton of the two-dimensional $O(3)$ sigma model uplifted to four dimensions. The instanton is essentially a BPS solution and therefore it is natural to expect that it becomes unstable once we increase μ breaking the world sheet supersymmetry.

In particular, as we see from Bogomol’ny representation (3.10) extra terms arising at $\beta < 1$ reduce the tension of the string. This is forbidden for BPS lump (uplifted instanton) since its tension is exactly determined by the central charge and given by $2\pi\xi$; see (3.6). As we increase μ the string is no longer BPS; ρ develops instability and shrinks leading at large μ to much lower tension; see (4.1) below.

IV. SUMMARY OF RESULTS

In this paper we studied what happens to the non-Abelian semilocal string in $\mathcal{N} = 2$ supersymmetric QCD as we switch on the μ -deformation and go to the large μ limit. We showed that the size modulus ρ develops a potential and eventually decouples as the theory flows to the $\mathcal{N} = 1$ SQCD at large μ . Note that the Higgs branch is still there; just the string is no longer made of massless fields, so the long-range tails of the string disappear.

Thus, the semilocal string degenerates into the local one. Non-Abelian local strings in the large μ limit of $\mathcal{N} = 1$ SQCD were studied in [15], and now we see that results of this paper can be directly applied to our case $N_f > N$ as well. Below we briefly summarize these results.

In the large μ limit, the string tension is logarithmically suppressed [15],

$$T_{\text{local}} = \frac{4\pi|\xi|}{\ln \frac{g^2|\mu|}{|m|}}. \quad (4.1)$$

This should be contrasted with the BPS formula (3.6) valid to the leading order at small μ .

As usual the world sheet theory contains translational moduli but they decouple from the orientational sector. The orientational sector is described by the $CP(N-1)$ model with the action

$$S^{(1+1)} = \int dt dz \{ \gamma [(\partial_k \bar{n} \partial_k n) + (\bar{n} \partial_k n)^2] + V_{1+1} \}. \quad (4.2)$$

Note that orientational fermionic zero modes are all lifted [15] and do not enter the low energy world sheet theory. The above world sheet theory is purely bosonic.

Here two-dimensional inverse coupling constant γ is large, given by

$$\gamma \sim \frac{|\mu|}{|m|} \frac{1}{\ln^2 \frac{g^2|\mu|}{|m|}}. \quad (4.3)$$

At the quantum level the $CP(N-1)$ model is asymptotically free, so the coupling γ runs and at the energy E is given by

$$2\pi\gamma(E) = N \log \left(\frac{E}{\Lambda_{CP}} \right), \quad (4.4)$$

where the scale of the world sheet theory is given by

$$\Lambda_{CP} \approx \sqrt{\xi} \exp \left(-\text{const} \frac{|\mu|}{|m|} \frac{1}{\ln^2 \frac{g^2|\mu|}{|m|}} \right). \quad (4.5)$$

We see that the scale Λ_{CP} of the $CP(N-1)$ model above is exponentially small, so the world sheet theory is weakly

coupled in a wide region of energies $\gg \Lambda_{CP}$. This should be contrasted to non-Abelian string in $\mathcal{N} = 2$ QCD where world sheet theory has a scale Λ_{CP} equal to the scale $\Lambda_{\mathcal{N}=2}$ of the bulk SQCD [9].

In the case when the quark masses entering the Lagrangian (2.3) are nonidentical, a potential for n^P is generated. In the simplest case when all quark masses are positive, this potential is given by [15]

$$V_{1+1} \approx \frac{8\pi|\mu|}{\ln \frac{g^2|\mu|}{|m|}} \sum_{P=1}^N m_P |n^P|^2. \quad (4.6)$$

The potential (4.6) has only one minimum and one maximum at generic Δm_{AB} . Other $(N-2)$ extreme points are saddle points. For equal quark masses this potential reduces to the constant equal to the tension of the string (4.1).

Since our four-dimensional theory is in the Higgs phase for squarks, 't Hooft-Polyakov monopoles present in the theory in the $\mathcal{N} = 2$ limit of small μ are confined by non-Abelian strings and serve as junctions of two distinct strings [5,6,27]. In the effective world sheet theory on the non-Abelian string they are seen as kinks interpolating between different vacua of the $CP(N-1)$ model; see [9] for a review.

In the large μ limit adjoint fields decouple. Therefore we could expect quasiclassically that the confined monopoles disappear in this limit. This indeed happens for nonequal quark masses. If quark mass differences are nonzero, a potential (4.6) is generated. It does not have multiple local minima; therefore, kinks (confined monopoles of the bulk theory) become unstable and disappear.

However, in the equal quark mass case the potential (4.6) is absent and the bosonic $CP(N-1)$ model supports kinks. Thus, in this case confined monopoles do survive the large μ limit [15]. The monopoles are represented by kinks in the effective $CP(N-1)$ model on the non-Abelian string; see [9] for a detailed review.

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