Augmenting the gauge-gravity correspondence to include hadron polarizabilities

Pervez Hoodbhoy

Department of Physics, Forman Christian College, Lahore, Pakistan

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AdS/CFT models have achieved considerable success in describing hadronic properties such as masses and Regge trajectories. Even if the minimal vertex that couples photons to structureless spin-zero fields is used, one still ends up with electromagnetic form factors of hadrons that are in fair to good agreement with experiment. However, contradicting both experiment and naive expectation, this minimal model gives zero for hadronic electric and magnetic polarizabilities. We show here that if effective vertices are used, and axial and vector mesons are allowed to propagate as intermediate states, then the static polarizabilities can in principle be computed from AdS/CFT.

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Following the Maldacena revolution [1], hundreds of papers have been written over the last two decades in attempts to build a theory of hadronic structure based upon the correspondence between anti–de Sitter/conformal field theory (AdS/CFT) with strongly interacting systems described at a fundamental level by quantum chromodynamics (QCD). Also called the gauge-gravity correspondence, it allows extraction of information about four-dimensional strongly coupled gauge theories by mapping them onto gravitational theories in five dimensions where, because of the weak coupling, they may be solved much more easily. In the semiclassical approximation, the QCD generating functional of the quantum field theory is given by the minimum of the classical action of the 5D theory at the 4D asymptotic border of the 5D space. Thus, in principle one can compute physical observables in a strongly coupled gauge theory in terms of a weakly coupled classical gravity theory, which encodes information of the boundary theory. In the so-called bottom-up approach, a 5D holographic dual to QCD is constructed and quantitative predictions for soft hadronic quantities are deduced. A current state of the field and references can be found in the review by Brodsky *et al.* [2] in which the authors also discuss an interesting connection between light-front dynamics, its holographic mapping to gravity in a higher-dimensional AdS space, and conformal quantum mechanics. This approach sheds additional light on the confinement dynamics in QCD in the limit of massless quarks. A different set of topics can be found in the review by Kim and Yi [3].

Nevertheless, one must not forget the limitations of the AdS/CFT approach. This implicitly relies upon a large-N_c approach, which means all calculations in the bulk are at the tree level. These are easily performed, hence the attractiveness of the approach. On the other hand, the suppression of loops could lead to an inaccurate description of physical processes in certain circumstances. So, for example, tensor structures present in actual scattering amplitudes (as will be needed here) could potentially be generated by loops but are not seen at the tree level. Moreover, since the real holographic dual to QCD is unknown, one must fall back upon physically motivated prescriptions for the dual theory. This suggests that naive attempts to model hadronic quantities will inevitably fail at some level.

Hadronic polarizabilities seem to be where the canonical AdS/CFT approach fails. The simplest of these quantities are α_E and β_M which are, respectively, the static electric and magnetic dipole polarizabilities of the charged pions. These characterize the induced dipole moments of the pion during $\gamma\pi$ Compton scattering [4,5]. The moments are induced via the interaction of the photon's electromagnetic field with the quark substructure of the pion. The incident photon can be thought of as creating the polarizing fields and the outgoing photon carries information of the extent to which the hadron has been polarized. α_E is the proportionality constant between the γ's electric field and the induced electric dipole moment, while β_M is similarly related to the γ's magnetic field and the induced magnetic dipole moment. A more pointlike and strongly bound system will be less polarizable than an extended, weakly bound system. As such the polarizabilities are fundamental hadronic characteristics and in principle computable using the many approximate methods used to solve QCD. This should also be true for AdS/CFT but a calculation of pionic polarizabilities by Marquet *et al.* [6] in an arbitrary background dilaton field yielded exactly zero for both quantities. This is in contradiction with the observed experimental values [5]. An earlier calculation by Gao and Xiao [7] in an AdS/CFT hard-wall model also yielded zero (although these authors do not explicitly mention polarizability).

At one level this is surprising. Naively, a minimal model that couples photons to structureless spin-zero fields should not yield any structure information at all. However one quickly realizes that the fields in the bulk are allowed to oscillate in all possible ways subject to boundary conditions, and this endows the "shadow" in 4D endowed with structure. Earlier, in the work of Grigoryan and Radyushkin [8], the vertex was shown to be dressed in a manner that resembles a generalized vector meson dominance model. This and other efforts result in electromagnetic form factors of hadrons that are in fair to good agreement with experiment. A reasonable shape of the pion form factor suggests that the model correctly reflects an important aspect of the pion's quark substructure. One can go further: as shown by de Teramond *et al.* [9] exploring three-point functions in a different kinematic regime leads to generalized parton distributions (GPDs) that lie within the range of acceptable GPD parametrizations which are consistent with the data. Nevertheless, this minimal AdS/CFT model, whether or not VMD improved, can go only so far. Zero polarizability falsely suggests that the pion is an elementary particle; obviously its compositeness is inadequately reflected and one therefore needs to extend the model.

The goal of the present paper is to incorporate hadronic polarizabilities into AdS/CFT by suggesting additions to the basic QCD gravity dual. This will involve the introduction of effective vertices consistent with Lorentz and discrete symmetries. In principle the vertices are computable in approximations to QCD [10]. We shall also permit vector and axial vector mesons to propagate as intermediate states. As the simplest nontrivial possibility, consider the Compton scattering of purely real photons from a charged pion. Of course, more information could be gained using virtual photons since these allow for access to what are known as generalized polarizabilities [11,12]. The extraction of α_E and β_M, as well as higher multipole probabilities, depends crucially on using the Low–Gell-Mann–Goldberger low energy theorem (LET) [13]. In principle, any model respecting the symmetries entering the derivation of the LET should reproduce the constraints of the LET. It is only terms of second order which contain new information on the structure of the nucleon specific to Compton scattering. For a spinless target, with ω, ω' being the incident and scattered photon energies respectively in the laboratory frame, the theorem gives the form of the scattering amplitude,

$$f = -\frac{e^2}{m}\varepsilon'^* \cdot \varepsilon + \alpha_E \omega\omega' \varepsilon'^* \cdot \varepsilon + \beta_M \omega\omega' (\varepsilon'^* \times \hat{\mathbf{q}}') \cdot (\varepsilon \times \hat{\mathbf{q}}) + O(\omega^3). \tag{1}$$

This LET needs to be put into a Lorentz-invariant form, which calls for identifying a tensor basis with coefficients free from kinematical singularities. There have been several careful discussions of this in the literature [12,14]. An acceptable set that is model independent must be based only on the requirement of gauge invariance, Lorentz invariance, crossing symmetry, and the discrete symmetries. The simplest and most illuminating basis is that of L'vov *et al.* [12] who identify a tensor basis with appropriate Lorentz-invariant amplitudes that are free from kinematical singularities. A gauge-invariant separation is then made into a generalized Born term containing ground-state properties only and a residual contribution describing the model-dependent internal structure. More specifically, these authors show that for a spinless hadron, the non-Born terms from which polarizabilities can be extracted must lead to a real photon Compton scattering amplitude of the form

$$T = T_{\mathrm{Born}} + \frac{1}{2} b_1(0) f^{\mu\nu} f'_{\mu\nu} + b_2(0) P_\mu f^{\mu\nu} P^\rho f'_{\rho\nu}, \tag{2}$$

$$P_\mu = p_\mu + p'_\mu, \tag{3}$$

$$f_{\mu\nu} = -i(q_\mu \varepsilon_\nu - q_\nu \varepsilon_\mu), \tag{4}$$

$$f'_{\mu\nu} = i(q'_\mu \varepsilon'^*_\nu - q'_\nu \varepsilon'^*_\mu). \tag{5}$$

In the above, $b_1(0)$ and $b_2(0)$ are the $q^2 \to 0$ limits of scalar functions of kinematical invariants and are related to the static polarizabilities α_E and β_M in the small ω limit (as measured in the lab frame),

$$\alpha_E = -\frac{1}{2m} b_1(0) - \frac{m}{2} b_2(0) \tag{6}$$

$$\beta_M = \frac{1}{2m} b_1(0). \tag{7}$$

We now turn towards the AdS/CFT calculation of Compton scattering. With the notations and conventions as used by Marquet *et al.* [6], the bulk action with a scalar field Φ minimally coupled to the field A_m is

$$S_0 = \int d^4x dz \sqrt{-g} \left(-\frac{1}{4} F_{mn} F^{mn} + e^{-\chi} D^m \Phi^* D_m \Phi + e^{-\chi} \mu_S^2 \Phi^* \Phi \right), \tag{8}$$

$$D_m \Phi = \partial_m \Phi - ieA_m \Phi. \tag{9}$$

The dilaton field $\chi(z)$ breaks conformal symmetry, which is necessary for the 4D particles to have nonzero mass. A popular choice is $\chi(z) = k^2 z^2$. However we shall not commit to any particular choice in this paper. The 5D field $A_m(x,z)$ is dual to the 4D electromagnetic field $A_\mu(x)$. Since we are concerned here only with real photons, it is best to choose $A_z = 0$. This forces the condition, $\partial^\mu A_\mu = 0$. There is no dilatonic cutoff on the real photon; it may travel freely in the bulk. As per the usual convention, the latin indices m, $n = 0, 1, 2, 3, z$ and the greek indices μ, $\nu = 0$, 1, 2, 3. The Minkowski space metric used here is $\eta_{\mu\nu} = (-1,1,1,1)$ and $g_{mn}dx^m dx^n = (R^2/z^2)(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$.

The strategy of going from the action to the scattering amplitude is well known and will not be repeated here in detail. The main steps are (a) find the classical equations of motion and then perform the mode expansion, subject to defined values of the field on the 4D asymptotic border of the 5-D space; (b) reexpress the action in terms of the solutions thus found by expanding to the required order in the electric charge e; and (c) functionally differentiate with respect to the boundary values of the field after using the Gubser-Klebanov-Polyakov-Witten relation [15,16] $Z_{\rm QFT} = Z_{\rm Bulk}$. In calculating the four-point correlation function one needs the bulk-to-bulk scalar Green's function which, after Fourier transformation, has the mode expansion [6]

$$\hat{G}(z,z',k) = -\sum_{n=0}^{\infty} \frac{\Phi_n^*(z)\Phi_n(z')}{k^2 + m_n^2 - i\varepsilon}. \tag{10}$$

The eigenfunctions $\Phi_n(z)$ are normalized according to

$$\int_0^\infty dz \frac{e^{-\chi(z)}}{z^3} \Phi_n^*(z)\Phi_m(z) = \delta_{mn}. \tag{11}$$

Marquet *et al.* [6] give the explicit (and rather complicated) form of the scattering amplitude for the general case where both photons are virtual. We can easily check their result for the simpler situation where both photons are real. In that case close to the threshold only the $n = 0$ term in Eq. (10) contributes. After using the normalization Eq. (11), up to $O(\omega^2)$ one sees that only the Born contribution remains:

$$T = (2\pi)^4 \delta^4(p + q - p' - q')\mathcal{M}_{\rm Born}, \tag{12}$$

$$\mathcal{M}_{\rm Born} = e^2 \varepsilon'^{*\mu}\left(2\eta_{\mu\nu} - \frac{(2p+q)_\mu (2p'+q')_\nu}{s+m^2} - \frac{(2p'-q)_\mu(2p-q')_\nu}{u+m^2}\right)\varepsilon^\nu. \tag{13}$$

Thus the coefficients $b_1(0)$, $b_2(0)$ are identically zero here; the action in Eq. (8) gives zero polarizabilities in contradiction with both expectations and measurements. A hint towards the remedy comes from current algebra [4]. The Compton scattering amplitude can be related via current algebra/PCAC to that for radiative charged pion decay and involves both axial and vector currents. We shall take this phenomenological route, anticipating that correct additions to the AdS action will lead to desired symmetries for the scattering amplitudes. To this end we supplement "by hand" Eq. (8) with an action for charged axial vector fields,

$$S_a = -\int d^4x dz \sqrt{-g} e^{-\chi}\left(\frac{1}{2} a_{mn}^* a^{mn} + \mu_A^2 a_m^* a^m + \frac{1}{2} e g_A F^{mn}(a_{mn}^*\Phi + a_{mn}\Phi^*)\right), \tag{14}$$

$$a_{mn} = \partial_m a_n - \partial_n a_m. \tag{15}$$

One notes that there is another possible effective vertex $F^{mn}a_m^*\partial_n\Phi$ which is closely similar to $F^{mn}a_{mn}^*\Phi$ in Eq. (14). They are not identical in general, but close to the threshold the two vertices yield exactly the same scattering amplitudes and hence will not be considered separately. The gauge $a_5(x,z) = 0$ is the obvious choice. In the AdS/CFT correspondence, the field $a_\mu(x,z)$ is sourced by the axial current $A_\mu(x) = \bar{q}\gamma_5\gamma_\mu q$ on the boundary at $z = 0$. The classical equation of motion is obtained from Eq. (14) and it is easy to see that the Green's function $\hat{G}_{\mu\nu}(x,z,x',z')$ obeys

$$\left[\frac{z}{e^{-\chi}}\partial_z\left(\frac{e^{-\chi}}{z}\partial_z\right) + \eta^{\mu\nu}\partial_\mu\partial_\nu - \left(\frac{\mu_A^2 R^2}{z^2}\right)\right]\hat{G}_{\mu\nu} = \frac{g_{\mu\nu}}{\sqrt{-g}e^{-\chi}}\delta^4(x-x')\delta(z-z'). \tag{16}$$

Translational invariance on the $z = 0$ boundary allows for a Fourier transformation,

$$\hat{G}_{\mu\nu}(x,z,x',z') = g_{\mu\nu}(z')\int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-x')} h_A(z,z',k), \tag{17}$$

$$\left[\frac{z}{e^{-\chi}}\partial_z\left(\frac{e^{-\chi}}{z}\partial_z\right) - k^2 - \left(\frac{\mu_A^2 R^2}{z^2}\right)\right] h_A(z,z',k) = \frac{1}{\sqrt{-g}e^{-\chi}}\delta(z-z'). \tag{18}$$

The spectral decomposition of the axial propagator $h_A(z,z',k)$ is easily found:

$$h_A(z,z',k) = -\sum_{n=0}^{\infty}\frac{\Psi_{An}^*(z)\Psi_{An}(z')}{k^2 + M_{An}^2 - i\varepsilon}, \tag{19}$$

where Ψ_n obeys

$$H_A\Psi_n = -M_{An}^2\Psi_n, \tag{20}$$

$$H_A = \frac{z}{e^{-\chi}}\partial_z\left(\frac{e^{-\chi}}{z}\partial_z\right) - \left(\frac{\mu_A^2 R^2}{z^2}\right). \tag{21}$$

The normalization is

$$\int_0^\infty dz \frac{e^{-\chi(z)}}{z} \Psi^*_{An}(z)\Psi_{Am}(z) = \delta_{mn}. \tag{22}$$

For a given source term $K_\nu(x,z)$ the axial vector field, up to a free solution, is obtained from

$$a_\mu(x,z) = \int d^4x' dz' \sqrt{-g(z')} e^{-\chi(z')} \hat{G}_{\mu\nu}(x,z,x',z') K^\nu(x',z'). \tag{23}$$

For the effective coupling in Eq. (14) the relevant source term for $a_\mu(x,z)$ is

$$K_\nu(x,z) = e g_A F_{\nu\alpha} \eta^{\alpha\beta} \partial_\beta \Phi. \tag{24}$$

We can now insert Eq. (23) into the action in Eq. (14) to get the $O(e^2)$ term, pick out the Fourier components, and then differentiate with respect to the sources on the 4D boundary. Only terms up to second order in the photon energy are kept. After a straightforward calculation one obtains the following contribution to the scattering amplitude:

$$\mathcal{M}_A = \frac{1}{4} e^2 g_A^2 P_\mu f^{\mu\nu}(q) P^\rho f_{\rho\nu}(q') \int \frac{dz}{z^5} \frac{dz'}{z'^5} e^{-\chi(z)} e^{-\chi(z')} (z^4 + z'^4) \Phi_0(z) h_A(z,z',k) \Phi_0^*(z'). \tag{25}$$

The function $\Phi_0(z)$ is the pion wave function which was encountered earlier in the $n = 0$ term in Eq. (10). In Eq. (25) the propagator is evaluated at $k = p + q = p' + q'$, corresponding to the s channel Feynman diagram. There is also a crossed channel contribution with $k = p - q' = p' - q$ and with photon polarization vectors exchanged. At the quadratic level, the crossed contribution is equal to that in Eq. (25). Again, at the quadratic level, they are both of the form given in the second term of Eq. (2). Since $h_A(z,z',k) = h_A(z',z,k)$, the z^4 and z'^4 contribute equally in Eq. (25). Finally, using the expression for the Green's function in Eq. (19) with $k^2 = -m^2$ gives $b_2(0)$:

$$b_2(0) = e^2 g_A^2 \sum_{n=0}^{\infty} \frac{C_n D_n}{-m_0^2 + M_{An}^2}, \tag{26}$$

$$C_n = \int_0^\infty \frac{dz}{z} e^{-\chi(z)} \Phi_0(z) \Psi^*_{An}(z), \tag{27}$$

$$D_n = \int_0^\infty \frac{dz}{z^5} e^{-\chi(z)} \Psi_{An}(z) \Phi_0^*(z). \tag{28}$$

The sum in Eq. (26) is expected to converge rapidly with n, i.e., as the number of wave function nodes increases. From the equations of motion close to the UV boundary where $\chi(z) \to 0$, one finds that the scalar wave functions asymptotically behave as $\Phi_n(z) \sim z^a$ with $a = \frac{1}{2} + \sqrt{4 + \mu_S^2 R^2}$. Correspondingly, the axial vector wave functions have $\Psi_n(z) \sim z^b$ with $b = \frac{1}{2} + \sqrt{1 + \mu_A^2 R^2}$. Hence the integrals in Eq. (28) converge at the lower limit for $\mu_S^2 > 0$ or $\mu_A^2 > 0$. If $\mu_S^2 = \mu_A^2 = 0$ then B_n is logarithmically divergent and can be regulated by putting a cutoff at $z = \varepsilon$.

As yet we do not have a contribution to $b_1(0)$ as in Eq. (7). An effective interaction reflecting QCD symmetries that rectifies the problem calls for the exchange of a charged vector meson. Consider therefore a further supplement to the action that preserves the usual continuous and discrete symmetries,

$$S_\mathrm{v} = -\int d^4x dz \sqrt{-g} e^{-\chi} \left(\frac{1}{2} \mathrm{v}^*_{mn} \mathrm{v}^{mn} + \mu_\mathrm{v}^2 \mathrm{v}^*_m \mathrm{v}^m + \frac{1}{4} e g_\mathrm{v} \tilde{F}^{mn} (\mathrm{v}^*_{mn} \Phi + \mathrm{v}_{mn} \Phi^*) \right), \tag{29}$$

$$\mathrm{v}_{mn} = \partial_m \mathrm{v}_n - \partial_n \mathrm{v}_m, \qquad \tilde{F}^{mn} = \varepsilon^{mnpqz} F_{pq}. \tag{30}$$

Both $S_\mathrm{a}, S_\mathrm{v}$ have only U(1) symmetry. The A_1 and ρ have very different masses; in fact $m_{A_1}^2 \approx 2m_\rho^2$. Thus, we expect μ_v^2 [Eq. (29)] and μ_A^2 [Eq. (14)] to have quite different values. Let us note that if chiral $\mathrm{SU}(2) \times \mathrm{SU}(2)$ had been an exact symmetry of the ordinary sort, the ρ meson would have been exactly degenerate with the A_1. But, in the model of Erlich *et al.* [17] an additional condensate field can be introduced and this forces the masses to be different. On the other hand, in light front holographic QCD with zero mass quarks, the pion is massless but $m_{A_1} - m_\rho$ owes to the different values of light-front orbital angular momentum L for the A_1 and ρ [see Eq. (5.9) of Brodsky *et al.* [2]].

The leading order scattering amplitude coming from the vector meson part is

$$\mathcal{M}_V = e^2 g_A^2 m^2 \tilde{f}^{\mu\nu}(q)\tilde{f}_{\mu\nu}(q') \int \frac{dz}{z^5}\frac{dz'}{z'^5} e^{-\chi(z)} e^{-\chi(z')}(z^4 + z'^4)\Phi_0(z) h_V(z,z',k)\Phi_0^*(z'). \tag{31}$$

Using $\tilde{f}^{\mu\nu}(q)\tilde{f}_{\mu\nu}(q') = -4 f^{\mu\nu}(q) f_{\mu\nu}(q')$, and adding in the contribution from the crossed channel, we then compare with Eq. (7) to obtain

$$b_1(0) = -e^2 m^2 g_V^2 \sum_{n=0}^{\infty} \frac{E_n F_n}{-m_0^2 + M_{Vn}^2}, \tag{32}$$

$$E_n = \int_0^\infty \frac{dz}{z} e^{-\chi(z)} \Phi_0(z)\Psi_{Vn}^*(z), \tag{33}$$

$$F_n = \int_0^\infty \frac{dz}{z^5} e^{-\chi(z)} \Psi_{Vn}(z)\Phi_0^*(z). \tag{34}$$

The couplings g_A, g_V are determined by the charged meson decay amplitudes for the axial vector and vector mesons respectively. The lowest lying axial-vector meson is the A_1 and the corresponding vector meson is the ρ. From Eq. (29), after using translational invariance on the 4D boundary, one immediately sees that

$$\mathcal{M}_{A_1\to\gamma\pi} = (2\pi)^4 \delta^4(k-q-p) e g_A (\varepsilon'^* \cdot \varepsilon q \cdot k - \varepsilon \cdot q \varepsilon'^* \cdot k) C_0$$
$$\mathcal{M}_{\rho\to\gamma\pi} = (2\pi)^4 \delta^4(k-q-p) e g_V \varepsilon^{\mu\nu\rho\sigma} k_\mu \varepsilon_\nu q_\rho \varepsilon_\sigma'^* E_0.$$

C_0, E_0 are the overlaps in Eqs. (27) and (33). Knowing the electromagnetic decay widths of the A_1 and ρ one can find g_A, g_V for any dilaton profile and choice of parameters μ_S, μ_A, μ_V and hence the electric and magnetic polarizabilities from Eqs. (26) and (32) together with Eqs. (6) and (7). The widths $\Gamma_{A_1\to\gamma\pi}$ and $\Gamma_{\rho\to\gamma\pi}$ have been measured (albeit quite imprecisely) and the constants g_A, g_V can be related to them. We shall not pursue numerical possibilities here, having obtained a minimal extension of the minimal AdS/CFT model that can in principle accommodate hadronic polarizabilities.

It would certainly be interesting to see how more elaborate AdS/CFT QCD inspired models, whether top-down or bottom-up, might fare on hadronic polarizabilities and whether they too would need to be supplemented in some way. Of course, gauge-gravity duality has been rigorously established only for a certain supersymmetric theory (which QCD is definitely not) and one is still guessing at the QCD dual. The large N_c limit, wherein quark and gluon loops are suppressed, makes the approach attractive and has brought it much attention. This is well deserved but, at the same time, it is important to compare it against QCD phenomenology, such as in low energy inclusive scattering for which effective field theories have been developed. This will help towards determining the domain of applicability of the AdS/CFT approach.

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