Towards nonsingular rotating compact object in ghost-free infinite derivative gravity

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The vacuum solution of Einstein's theory of general relativity provides a rotating metric with a ring singularity, which is covered by the inner and outer horizons and an ergo region. In this paper, we will discuss how ghost-free, quadratic curvature, infinite derivative gravity (IDG) may resolve the ring singularity. In IDG the nonlocality of the gravitational interaction can smear out the delta-Dirac source distribution by making the metric potential finite everywhere including at $r = 0$. We show that the same feature also holds for a rotating metric. We can resolve the ring singularity such that no horizons are formed in the linear regime by smearing out a delta-source distribution on a ring. We will also show that the Kerr metric does not solve the full nonlinear equations of motion of ghost-free quadratic curvature IDG.

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I. INTRODUCTION

Einstein's theory of general relativity (GR) is indeed a very successful metric theory of gravity which has seen amazing success in the infrared (IR) [1], including the detection of the first gravitational wave signal [2]. In spite of these successes, classical GR suffers from the ultraviolet (UV) catastrophe at short distances and small time scales, where there are black hole and cosmological singularities [3–5]. It has been recently shown that a ghost-free, quadratic curvature infinite derivative gravity (IDG) can potentially resolve the cosmological [6] and black hole type singularities [7]. Infinite derivatives acting on a point delta-Dirac source smears out the singularity by a Gaussian profile [8–10]. At a quantum level the graviton vertex interactions become nonlocal [11–14], very similar to string field theory [15–18]. Besides strings, nonlocality is also a feature of loop quantum gravity, see [19], spin foam or dynamical triangulation where Wilson loops acts as fundamental operators; for a review, see [20]. The quantum scatterings for such nonlocal interactions in IDG provide a very interesting insight [21,22], where there is a UV-IR connection when a large number of scatterings of particles with nonlocal interactions are taken into account. The scattering amplitude gets exponentially suppressed for external momenta $P^2 > M_s^2$, and the scale of nonlocality gets shifted by $M_s \to M_s/\sqrt{N}$ [23], for N scatterers in the limit when $N \gg 1$. Furthermore, nonlocal thermal field theory provides a resemblance to a Hagedorn phase as shown in [24].

As the gravitational interaction in the UV weakens, both linear [7] and nonlinear equations of motion [25] provide a conformally flat spherically symmetric, static metric solution [8]. A similar scenario also holds in the case of a charged point source [26]. It has also been shown that the singularity and the event horizon does not form in a dynamical context at a linear level [27], as a mass gap can be formed determined by the nonlocal scale [28]. In particular, it has been shown that singular solutions such as Schwarzschild metric [29] and Kasner metric [30] do not satisfy the field equations in the vacuum. Moreover, infinite derivatives acting on the delta-Dirac distribution at the origin are smeared out by a Gaussian profile [7,8], and the region of nonlocality yields a nonvacuum solution as opposed to that in GR. It is also possible to make the gravitational radius as large as the effective scale of nonlocality, $r_{\rm NL}$, which can be larger than the Schwarzschild's radius, $r_{\rm sch} \le r_{\rm NL}$ [8]. At the cosmological front, such nonlocality can potentially replace the cosmological singularity by the big bounce [6] or freezing the Universe in the UV [31]. Outside the region of nonlocality the gravitational interaction becomes that of GR, thus reproducing all the features of gravity being tested in the IR [32]; similar features have been observed for extended objects such as d and p-branes [33]. In Ref. [34], it was shown that in higher curvature gravity with more than four derivatives, the delta source gets smeared out, as for example in the sixth order theory of gravity, and the linearized metric turns out to be singularity-free. However, such local theories always suffer from the presence of ghosts at the tree level.

The aim of this paper will be to understand the rotating metric within IDG and show how to smear out the ring singularity of a Kerr metric [35] in the linear regime by considering a toy model with a delta-Dirac distribution on a rotating ring. We will show that the linear solution approaches conformal flatness in the limit $r \to 0$. We will provide numerical/analytical solutions of the rotating metric and show how it recovers the predictions of GR in the IR. With the help of nonlinear equations of motion we will show that the Kerr metric does not pass through the field equations of ghost-free quadratic curvature IDG.

II. THE INFINITE COVARIANT DERIVATIVE ACTION

The most general quadratic curvature action, which is parity-invariant and torsion-free, is given by [7,25,36]:

$$S = S_{\rm EH} + S_q = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[\mathcal{R} + \alpha_c(\mathcal{R}\mathcal{F}_1(\Box_s)\mathcal{R} + \mathcal{R}^{\mu\nu}\mathcal{F}_2(\Box_s)\mathcal{R}_{\mu\nu} + W^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box_s)W_{\mu\nu\lambda\sigma})], \tag{1}$$

where $S_{\rm EH}$ corresponds to the Einstein-Hilbert and S_q corresponds to the quadratic curvature terms; $G = 1/M_p^2$ is Newton's gravitational constant, and $\alpha_c \sim 1/M_s^2$ is a dimensionful coupling, $\Box_s \equiv \Box/M_s^2$, where M_s represents the scale of nonlocality at which new physics should emerge. In the limit $M_s \to \infty$, the action reduces to the Einstein-Hilbert term, as expected. The d'Alembertian operator is defined as $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$, where μ, $\nu = 0$, 1, 2, 3, and we work with the mostly positive metric convention, $(-,+,+,+)$. The three gravitational form factors, $\mathcal{F}_i$, are the analytic function of $\Box$ and can be expressed in series representation as follows:

$$\mathcal{F}_i(\Box_s) = \sum_{n=0}^{\infty} f_{i,n}\Box_s^n, \tag{2}$$

which are reminiscent of any massless theory possessing *only* derivative interactions. Note that we will always consider analytic operators of $\Box_s$ and not nonanalytic operators such as $1/\Box_s$ [37,38] or $\ln(\Box_s)$ [39]. The ghost-free condition around Minkowski background can be formulated as [7,25,36,40]:

$$6\mathcal{F}_1(\Box_s) + 3\mathcal{F}_2(\Box_s) + 2\mathcal{F}_3(\Box_s) = 0, \qquad a(\Box_s) = 1 + 2\mathcal{F}_2(\Box_s)\Box_s + 4\mathcal{F}_3(\Box_s)\Box_s = e^{-\Box_s}. \tag{3}$$

The field equations for the action in Eq. (1) have been derived in Ref. [25], and they are given by

$$\begin{aligned}
P^{\alpha\beta} = &-\frac{G^{\alpha\beta}}{8\pi G} + \frac{\alpha_c}{8\pi G}(4G^{\alpha\beta}\mathcal{F}_1(\Box_s)\mathcal{R} + g^{\alpha\beta}\mathcal{R}\mathcal{F}_1(\Box_s)\mathcal{R} - 4(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box)\mathcal{F}_1(\Box_s)\mathcal{R}\\
&- 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma} + \bar{\Omega}_1) + 4\mathcal{R}_\mu^\alpha\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\beta}\\
&- g^{\alpha\beta}\mathcal{R}_\nu^\mu\mathcal{F}_2(\Box_s)\mathcal{R}_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\alpha}) + 2\Box(\mathcal{F}_2(\Box_s)\mathcal{R}^{\alpha\beta})\\
&+ 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma} + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta}\\
&- g^{\alpha\beta}W^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box_s)W_{\mu\nu\lambda\sigma} + 4W_{\mu\nu\sigma}^{\alpha}\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\sigma}\\
&- 4(\mathcal{R}_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^{\gamma} + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta})\\
= &-T^{\alpha\beta},
\end{aligned} \tag{4}$$

where $T^{\alpha\beta}$ is the stress-energy tensor of the matter component, and the symmetric tensors are defined as (see Ref. [25])

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1}\nabla^\alpha\mathcal{R}^{(l)}\nabla^\beta\mathcal{R}^{(n-l-1)}, \qquad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1}\mathcal{R}^{(l)}\mathcal{R}^{(n-l)}, \tag{5}$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} \mathcal{R}_{\nu}^{\mu;\alpha(l)} \mathcal{R}_{\mu}^{\nu;\beta(n-l-1)}, \qquad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} \mathcal{R}_{\nu}^{\mu(l)} \mathcal{R}_{\mu}^{\nu(n-l)}, \tag{6}$$

$$\Delta_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n} \sum_{l=0}^{n-1} [\mathcal{R}_{\sigma}^{\nu(l)} \mathcal{R}^{(\beta\sigma;\alpha)(n-l-1)} - \mathcal{R}_{\sigma}^{\nu;\alpha(l)} \mathcal{R}^{\beta\sigma(n-l-1)}]_{;\nu}, \tag{7}$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} W^{\mu;\alpha(l)}_{\ \ \nu\lambda\sigma} W_{\mu}^{\ \ \nu\lambda\sigma;\beta(n-l-1)}, \qquad \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} W^{\mu(l)}_{\ \ \nu\lambda\sigma} W_{\mu}^{\ \ \nu\lambda\sigma(n-l)}, \tag{8}$$

$$\Delta_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n} \sum_{l=0}^{n-1} [W^{\lambda\nu(l)}_{\ \ \sigma\mu} W_{\lambda}^{\ \beta\sigma\mu;\alpha(n-l-1)} - W^{\lambda\nu\ ;\alpha(l)}_{\ \ \sigma\mu} W_{\lambda}^{\ \beta\sigma\mu(n-l-1)}]_{;\nu}. \tag{9}$$

The notation $\mathcal{R}^{(l)} \equiv \Box^l \mathcal{R}$ is only used for the covariant derivatives acting on the curvature tensors. We can also compute the trace of the field equations in Eq. (4) whose expression is a lot simpler and is given by [25]

$$P = \frac{\mathcal{R}}{8\pi G} + \frac{\alpha_c}{8\pi G}(12\Box\mathcal{F}_1(\Box_s)\mathcal{R} + 2\Box(\mathcal{F}_2(\Box_s)\mathcal{R}) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box_s)\mathcal{R}^{\mu\nu}) + 2(\Omega^\sigma_{1\sigma} + 2\bar{\Omega}_1) + 2(\Omega^\sigma_{2\sigma} + 2\bar{\Omega}_2) + 2(\Omega^\sigma_{3\sigma} + 2\bar{\Omega}_3) - 4\Delta^\sigma_{2\sigma} - 8\Delta^\sigma_{3\sigma}) = -T \equiv -g_{\alpha\beta}T^{\alpha\beta}. \tag{10}$$

The static solution for both linerized [7,41,42] and the full nonlinear regime [8,29] has shown that Schwarzschild-like singular solution is not permissible within IDG. In the UV, well inside the region of nonlocality, $r \ll 1/M_s$, the Weyl tensor $W^{\mu\nu\lambda\sigma} \to 0$ as $r \to 0$. In this respect, the system has some similarity to the fuzz ball [43]. The smearing out of the singularity has also been seen in noncommutative geometry, as pointed out first in Ref. [44]. In GR the Schwarzschild metric is derived by imposing the boundary condition at the origin, i.e., by putting a delta-Dirac distribution at $r = 0$ [45,46]; in our case, the IDG smears this singularity at the origin. The entire spacetime metric is regular in the static case, inside the nonlocal region, i.e., $r \ll 2/M_s$, without any singularity. Therefore, perturbation theory can be trusted all the way from $r = 0$ to $r \to \infty$ as long as $mM_s < M_p^2$; see Refs. [7,8]. As the effective scale of nonlocality is given by $M_s \to M_s/\sqrt{N}$, where N is the number of graviton interacting nonlocally, the condition, $mM_s < M_p^2$, can be satisfied for large astrophysical mass m [42].

III. RING SINGULARITY

Let us briefly recall the Kerr metric [35] in rational polynomial coordinates, which is given by (see [47]):

$$ds^2 = -\left(1 - \frac{2mr}{r^2 + a^2\chi^2}\right)dt^2 - \frac{4mar(1-\chi^2)}{r^2 + a^2\chi^2}dtd\varphi + \frac{r^2 + a^2\chi^2}{r^2 - 2mr + a^2}dr^2 + (r^2 + a^2\chi^2)\frac{d\chi^2}{1-\chi^2} + (1-\chi^2)\left(r^2 + a^2 + \frac{2ma^2 r(1-\chi^2)}{r^2 + a^2\chi^2}\right)d\varphi^2, \tag{11}$$

where $\chi = \cos\theta$ is the transformation used to bring the standard Boyer-Lindquist coordinates, whereas m is the mass and $J = am$ is the angular momentum, with a being the rotation parameter. One of the key observations is that the Kerr metric has a ring singularity which is described by the equation (see Ref. [48] for a nice discussion): $r^2 + a^2\cos^2\theta = 0$, where it is clear that a corresponds to the radius of the ring, whereas r is the radial coordinate in Boyer-Lindsquit coordinates, which are defined in terms of the Cartesian ones as follows: $x = \sqrt{r^2 + a^2}\sin\theta\cos\theta$, $y = \sqrt{r^2 + a^2}\sin\theta\sin\varphi$, and $z = r\cos\theta$. The Kretschmann scalar blows up when $r^2 + a^2\cos^2\theta = 0$ is satisfied, i.e., when $r = 0$ and $\theta = \pi/2$, or in Cartesian coordinates, $x^2 + y^2 = a^2, z = 0$, namely the ring singularity lies on a plane which is perpendicular to the rotation axis. Let us first discuss the physics in the linear regime, in analogy with the static case.[1] We consider the source is a Dirac distribution on a ring of radius a, which is rotating with a constant angular velocity, ω, in the plane x–y ($z = 0$). Thus,

[1]There were attempts to understand the Kerr metric in IDG; see [49]. However, we have found an error in our analysis, which we have rectified here. Unfortunately, the rotation was not taken into account correctly in the paper.

the components of the energy momentum tensor of the source are

$$T_{00} = m\delta(z)\frac{\delta(x^2+y^2-a^2)}{\pi}, \qquad T_{0i} = T_{00}v_i. \quad (12)$$

Note that the factor π in the denominator comes from the fact that $\delta(x,y) \equiv \delta(x)\delta(y) = \pi\delta(x^2+y^2)$, and v_i is the tangential velocity whose magnitude can be expressed as $v = \omega a$, and assuming that the rotation happens around the z-axis, we have $v_x = -y\omega$, $v_y = x\omega$, $v_z = 0$. Note that this choice of the stress-energy tensor, in analogy with the static case, is compatible with the fact that in order for the Einstein equations and the Kerr metric to be defined in the entire spacetime we need a nonvanishing stress-energy tensor at the ring. In fact, by using the theory of distribution [46], it was rigorously shown that the stress-energy tensor for a Kerr metric has a structure similar to the one we have written in Eq. (12). For example, the (00) component of the Einstein tensor in the case of the Kerr metric is $G_{00} \sim m\delta(z)\delta(x^2+y^2-a^2)$ [46]. A general linearized metric, which can describe the spacetime in the presence of a rotating source can be written, in isotropic coordinates, as

$$ds^2 = -(1+2\Phi)dt^2 + 2\vec{h}\cdot d\vec{x}dt + (1-2\Psi)d\vec{x}^2, \quad (13)$$

where $h_{00} = -2\Phi < 1$, $h_{ij} = -2\Psi\delta_{ij} < 1$ and $h_{0i} \equiv h_i < 1$ signify the weak-field and the slow rotation regime, and now the metric components depend on the isotropic radius, r, which should not be confused with the Boyer-Lindsquit radial coordinate used above. To find the form of the metric components, we would need to solve the following differential equations:

$$\begin{aligned} &e^{-\nabla^2/M_s^2}\nabla^2\Phi(\vec{r}) = e^{-\nabla^2/M_s^2}\nabla^2\Psi(\vec{r}) = 4Gm\delta(z)\delta(x^2+y^2-a^2),\\ &e^{-\nabla^2/M_s^2}\nabla^2 h_{0x}(\vec{r}) = -16Gm\omega y\delta(z)\delta(x^2+y^2-a^2),\\ &e^{-\nabla^2/M_s^2}\nabla^2 h_{0y}(\vec{r}) = 16Gm\omega x\delta(z)\delta(x^2+y^2-a^2), \end{aligned} \quad (14)$$

where we are assuming the ghost-free condition in Eq. (3). To solve the differential equations in Eq. (14) we can go to the Fourier space and then antitransform back to coordinate space; thus, first of all, we need to compute the Fourier transforms of the stress-energy tensor components, i.e., of T_{00} and T_{0i}.

A. Smearing out the ring singularity at the linearized level

Let us first compute the corresponding gravitational potential, $\Phi = \Psi$; we need the Fourier transform of the ring distribution in Eq. (12):

$$\mathcal{F}[\delta(z)\delta(x^2+y^2-a^2)] = \int dxdydz\delta(z)\delta(x^2+y^2-a^2)e^{ik_xx}e^{ik_yy}e^{ik_zz}. \quad (15)$$

It can be computed by performing the integral in cylindrical coordinates: $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = z$, so that

$$\mathcal{F}[\delta(z)\delta(x^2+y^2-a^2)] = \int_{-\infty}^{\infty} dz\delta(z)e^{ik_zz}\int_0^{\infty} d\rho\rho\delta(\rho^2-a^2)\int_0^{2\pi} d\varphi e^{ik_x\rho\cos\varphi}e^{ik_y\rho\sin\varphi} = \pi I_0\left(ia\sqrt{k_x^2+k_y^2}\right), \quad (16)$$

where I_0 is a modified Bessel function, which is also defined in terms of the Bessel function as $I_0(x) = J_0(ix)$. By antitransforming, we obtain the gravitational potential in coordinate space:

$$\Phi(\vec{r}) = -4\pi Gm\int\frac{d^3k}{(2\pi)^3}\frac{e^{-k^2/M_s^2}}{k^2}I_0\left(ia\sqrt{k_x^2+k_y^2}\right)e^{ik_xx}e^{ik_yy}e^{ik_zz}, \quad (17)$$

where $d^3k = dk_xdk_ydk_z$ and $k^2 = k_x^2+k_y^2+k_z^2$. In order to study whether the ring singularity is still present or not in IDG, we can simplify the integral in Eq. (17), by considering ourselves on the x–y ($z = 0$) plane, where the ring singularity lies in the case of the Kerr metric. Thus, by setting $z = 0$ and going to cylindrical coordinates, $k_x = \zeta\cos\varphi$, $k_y = \zeta\sin\varphi$, $k_z = k_z$, we can rewrite the integral in Eq. (17) as follows:

$$\Phi(\rho) = -Gm\int_0^{\infty} d\zeta I_0(ia\zeta)I_0(i\zeta\rho)\mathrm{Erfc}\left(\frac{\zeta}{M_s}\right), \quad \text{when } M_s\to\infty \Rightarrow \Phi_{GR}(\rho) = -Gm\int_0^{\infty} d\zeta I_0(ia\zeta)I_0(i\zeta\rho), \quad (18)$$

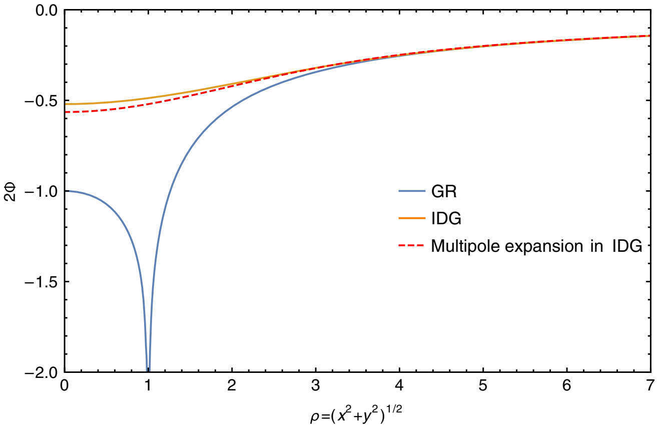

FIG. 1. In this plot we have shown the results of the numerical computation for the integrals in Eq. (18), and the behavior of the metric potential in the case of the multipole expansion in Eq. (31). The blue line corresponds to the behavior of the perturbation, $2\Phi = -h_{00}$, in GR, and the orange line to the behavior of the metric potential in IDG; the dashed red line represents the metric potential in the case of the multipole expansion. We have chosen $G = 1$, $m = 0.5$, $a = 1$ and $M_s = 0.9$. We can notice that the gravitational potential in GR blows up for $\rho = a = 1$, whereas it is finite in IDG; moreover, the metric coming from the multipole expansion is a very good approximation outside the source, i.e., for $\rho > a$.

where the last integral corresponds to the GR case. The two integrals in Eq. (18) cannot be solved analytically, but we can compute them numerically. From the numerical computation one can explicitly see that for $x^2 + y^2 = a^2$, the gravitational potential in GR diverges as expected, whereas in IDG it is singularity-free; see Fig. 1. This is what we expected physically; the IDG smears out a ring distribution very similarly to the case of a point source [6–8,41]. Furthermore, we can trust the linear regime all the way up to $\rho = 0$, as long as $2\Phi(0) < 1$. The integral in Eq. (18) can be evaluated analytically at $\rho = 0$:

$$\Phi(0) = -\frac{Gm}{a}\mathrm{Erf}\left(\frac{M_s a}{2}\right), \tag{19}$$

where the linearized approximation yields $2\Phi(0) < 1$. As $\mathrm{Erf}(M_s a/2) < 1$, the case of $a > 2Gm$ always satisfies the inequality; in the opposite case, $a < 2Gm$, the weak-field inequality is satisfied as long as

$$a < \frac{2}{M_s} \quad \text{(radius of the ring < scale of nonlocality)}. \tag{20}$$

This suggests that ghost-free IDG can indeed avoid the ring-type singularity.

B. Computing h_{0i} components for a rotating ring

So far we have only computed the static gravitational potential generated by a delta-Dirac distribution on the ring. We now wish to study the components h_{0i} which are related to the fact that the ring is also rotating with a constant angular velocity ω. We would need to compute the following Fourier transforms:

$$\mathcal{F}[j\delta(z)\delta(x^2+y^2-a^2)] = \int dxdydz\, j\delta(z)\delta(x^2+y^2-a^2)e^{ik_x x}e^{ik_y y}e^{ik_z z}, \tag{21}$$

where $j = x, y$. The computation can be performed by using cylindrical coordinates as done in Eq. (16):

$$\mathcal{F}[x\delta(z)\delta(x^2+y^2-a^2)] = \int_{-\infty}^{\infty} dz\delta(z)e^{ik_z z}\int_0^{\infty} d\rho\rho^2\delta(\rho^2-a^2)\int_0^{2\pi} d\varphi e^{ik_x\rho\cos\varphi}e^{ik_y\rho\sin\varphi}\cos\varphi = \pi a\frac{k_x}{\sqrt{k_x^2+k_y^2}}I_1\left(ia\sqrt{k_x^2+k_y^2}\right), \tag{22}$$

and by following similar steps we also obtain

$$\mathcal{F}[y\delta(z)\delta(x^2+y^2-a^2)] = \pi a\frac{k_y}{\sqrt{k_x^2+k_y^2}}I_1\left(ia\sqrt{k_x^2+k_y^2}\right), \tag{23}$$

where I_1 is a modified Bessel function. We can express the components h_{0j} in coordinate space as antitransforms:

$$h_{0j}(\vec{r}) = 16Gm\omega a\int\frac{d^3k}{(2\pi)^3}\frac{e^{-k^2/M_s^2}}{k^2}\frac{k_j}{\sqrt{k_x^2+k_y^2}}I_1\left(ia\sqrt{k_x^2+k_y^2}\right)e^{ik_x x}e^{ik_y y}e^{ik_z z}, \tag{24}$$

where $j = x, y$. By using cylindrical coordinates, similar to the integrands in Eq. (18), and setting $z = 0$, we obtain similar expressions for the cross-terms:

$$h_{0x}(x,y) = 4Gm\omega a\frac{y}{\rho}\int_0^\infty d\zeta I_1(ia\zeta)I_1(i\zeta\rho)\mathrm{Erfc}\left(\frac{\zeta}{M_s}\right), \qquad h_{0y}(x,y) = -4Gm\omega a\frac{x}{\rho}\int_0^\infty d\zeta I_1(ia\zeta)I_1(i\zeta\rho)\mathrm{Erfc}\left(\frac{\zeta}{M_s}\right), \quad (25)$$

where $\rho = \sqrt{x^2+y^2}$ is the radial cylindrical coordinate in the plane $z = 0$. As $\theta = \pi/2$, we have $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, and thus all the radial dependence and the singularity structures are taken into account by the integrals:

$$H(\rho) \coloneqq \int_0^\infty d\zeta I_1(ia\zeta)I_1(i\zeta\rho)\mathrm{Erfc}\left(\frac{\zeta}{M_s}\right), \quad \text{when } M_s \to \infty \Rightarrow H_{GR}(\rho) \coloneqq \int_0^\infty d\zeta I_1(ia\zeta)I_1(i\zeta\rho), \qquad (26)$$

where the last integral corresponds to the GR case. The two integrals in Eq. (26) cannot be solved analytically, but we can compute them numerically and check the absence of any singularities. As it also happens for the potentials h_{00} and h_{ij}, the cross-term h_{0i} shows the presence of a ring singularity in GR; indeed, from the numerical analysis one can explicitly see that for $x^2 + y^2 = a^2$ the function H_{GR} diverges in GR. Whereas in IDG the cross-term turns out to be singularity-free, indeed, the function H is finite everywhere. In analogy with the static scenario, also in the rotating case, IDG is responsible for a smearing effect, in this case of the delta-Dirac ring distribution. Note that at the origin, $\rho = 0$, $z = 0$, the cross-term vanishes, which implies that in IDG the spacetime metric approaches conformal flatness; indeed, at $r = 0$ the rotating metric becomes that of the static case [41].

In the IR regime, for $\rho \gg a$, the metric components found above match extremely well with the case of GR. Indeed, for distances larger than the radius of the ring and the scale of nonlocality, i.e., $\rho \gg 2/M_s > a$, we recover the Lense-Thirring metric [50].[2] To exactly recover the Lense-Thirring metric at large distances, we need to identify $J = ma^2\omega$, which is nothing but the relation $J = I\omega$, where $I = ma^2$ is the moment of inertia of the delta-Dirac ring distribution. Note that the relation $J = am$ does not hold, but the angular momentum is related to the parameter a through the momentum of inertia of the source.

C. Rotating metric outside the source: multipole expansion in IDG

We now wish to determine the generic form of the metric in IDG outside the rotating source, without assuming any large distance limit. In this regime, the linear treatment is valid; see Eq. (13). The components h_{00} and h_{ij} will be the same as that already obtained in the static case; to compute the $(0i)$ components we can consider a multipole expansion for $\mathrm{Erf}(M_s|\vec{r}-\vec{r}'|/2)/|\vec{r}-\vec{r}'|$,

$$\frac{1}{|\vec{r}-\vec{r}'|}\mathrm{Erf}\left(\frac{M_s|\vec{r}-\vec{r}'|}{2}\right) = \frac{1}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right) + \left[\frac{1}{r^3}\mathrm{Erf}\left(\frac{M_s r}{2}\right) - \frac{M_s}{\sqrt{\pi}r^2}e^{-\frac{M_s^2r^2}{4}}\right]\sum_{j=1}^{3}x^j x'^j + \cdots, \qquad (28)$$

which recovers the GR case in the large distance regime, $M_s r \gg 2$, as expected. Such a multipole expansion holds true for $r > r' \sim a$, which means outside the source. By using Eq. (28), we can now compute the h_{0i} components:

$$h_{0i}(\vec{r}) = 4G\int d^3r'\frac{T_{0i}(\vec{r}')}{|\vec{r}-\vec{r}'|}\mathrm{Erf}\left(\frac{M_s|\vec{r}-\vec{r}'|}{2}\right) = 2G\left[\frac{1}{r^3}\mathrm{Erf}\left(\frac{M_s r}{2}\right) - \frac{M_s}{\sqrt{\pi}r^2}e^{-\frac{M_s^2r^2}{4}}\right](\vec{r}\times\vec{J})_i, \qquad (29)$$

We can move from Cartesian to isotropic coordinates, so that the $d\varphi dt$ component of the metric will be given by

$$2\vec{h}\cdot d\vec{x}dt = -4GJ\left[\frac{1}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right) - \frac{M_s}{\sqrt{\pi}}e^{-\frac{M_s^2r^2}{4}}\right]\sin^2\theta d\varphi dt, \qquad (30)$$

[2] Recall that the Lense-Thirring metric represents the weak-field and slow-rotation limit of the Kerr metric [50]:

$$ds^2 = -\left(1-\frac{2Gm}{r}\right)dt^2 + \frac{4GJ}{r^3}(ydxdt - xdydt) + \left(1+\frac{2Gm}{r}\right)(dr^2 + r^2d\Omega^2). \qquad (27)$$

which in the regime $M_s r \gg 2$ recovers GR result, as expected. Moreover, by expressing $J = I\omega = ma^2\omega$ and imposing $|h_{0i}| \sim GmM_s^2\omega a^2 < 1$, we can notice that the slow rotation regime means $\omega < 1/a$, when we also require $GmM_s < 1$ and $aM_s < 1$. Note also that by recasting the cross-term in terms of the angular momentum and imposing the linearized regime we obtain $|h_{0i}| \sim GM_s^2 J < 1$, which also means $J < (M_p/M_s)^2$. From the last inequality, as $M_s \le M_p$, the angular momentum J may also exceed one in IDG. The linearized spacetime metric in Eq. (13) outside the source, $r > a$, in the case of IDG reads as

$$ds^2 = -\left(1 - \frac{2Gm}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right)\right)dt^2 + \left(1 + \frac{2Gm}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right)\right)(dr^2 + r^2 d\Omega^2) - 4GJ\left[\frac{1}{r}\mathrm{Erf}\left(\frac{M_s r}{2}\right) - \frac{M_s}{\sqrt{\pi}}e^{-\frac{M_s^2 r^2}{4}}\right]\sin^2\theta d\varphi dt. \tag{31}$$

From Figs. 1 and 2, it is clear that the metric constructed by using the multipole expansion is a very good approximation to describe the spacetime outside the source, $r > a$; whereas in the regime $M_s r \gg 2$, we recover the GR predictions; indeed Eq. (31) reduces to the Lense-Thirring metric [50] in Eq. (27). Thus, in the case of ghost-free IDG, for a rotating source we have found a hierarchy of scales: the radius of the source a, the Schwarzschild radius $r_{\mathrm{sch}} = 2Gm$ and the scale of nonlocality $r_{NL} \sim 2/M_s$, which have to satisfy the following set of inequalities to preserve the linearity:

$$r_{NL} \sim \frac{2}{M_s} > r_{\mathrm{sch}} = \frac{2m}{M_p^2} > a. \tag{32}$$

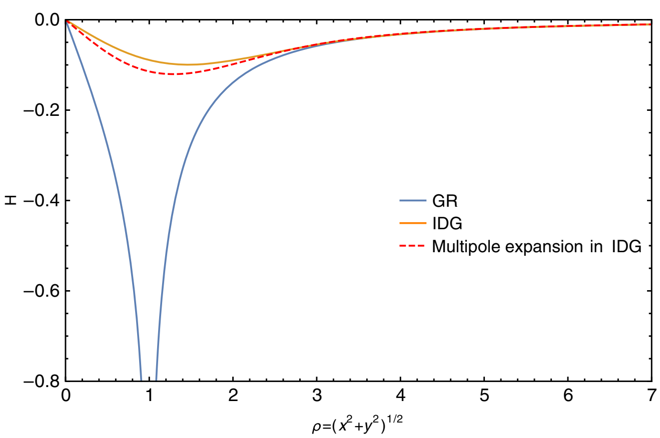

FIG. 2. In this plot we have shown the results of the numerical computation for the integrals in Eq. (26) and the behavior of the same function in the case of the multipole expansion in Eq. (30). The blue line corresponds to the behavior of the function H_{GR}, and so of the cross-term in GR; the orange line to the behavior of the function H_{IDG}, and so of the cross-term in IDG; the dashed red line represents the cross-term in the case of the multipole expansion. For convenience we have chosen $a = 1$ and $M_s = 1.5$. We can notice that the metric components h_{0i} blow up in GR for $\rho = a = 1$, whereas they are finite in IDG; moreover, the metric coming from the multipole expansion is a very good approximation outside the source, i.e., for $\rho > a$.

As long as the inequality in Eq. (32) holds, the spacetime metric is valid all the way from $r = \infty$ up to $r = 0$, and it turns out to be free from any curvature singularity and also devoid of any horizons. Furthermore, because in our case, the h_{00} component is always bounded below unity, there is no ergo region, as first pointed out in [49].

IV. NON-KERR TYPE METRIC IN THE FULL NONLINEAR THEORY

We now wish to move towards the full nonlinear regime and show that the Kerr metric does not solve the full nonlinear field equations in Eq. (4). First of all, note that strictly speaking the Schwarzschild metric in GR is not a vacuum solution everywhere; indeed there is a delta-Dirac distribution at the origin, so that the stress-energy tensor is nonvanishing at $r = 0$. Thus, even in the absence of the Weyl squared term $W_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box_s)W^{\mu\nu\rho\sigma}$, the full nonlinear IDG field equations will not allow the Schwarzschild metric as a solution, due to the presence of infinite order covariant derivatives acting on a delta-Dirac source. We can argue the same also in the case of the Kerr metric. As it was rigorously shown in Ref. [46] by using the theory of distribution, the Kerr metric is not a vacuum solution everywhere, but there is a nonvanishing stress-energy tensor expressed as combinations of delta Dirac and theta Heaviside on the ring [46]. Thus, the infinite order covariant derivatives acting on the theta-Heaviside and delta-Dirac distributions on a ring generically will generate an object which will have a nonpoint support. In this sense, the Kerr metric will not pass as a vacuum solution of the IDG field equations.

We now wish to show that the Kerr metric does not pass as a *pure* vacuum solution (i.e., $T_{\mu\nu} = 0$ everywhere) if the Weyl squared term with a nonconstant form factor (either local or nonlocal), $\mathcal{F}_3(\Box_s) \neq \mathrm{const}$, is taken into account in the action. Let us first *demand* that the Kerr metric, Eq. (11), is a vacuum solution for the full nonlinear equations (4); i.e., let us impose the Ricci flatness, $\mathcal{R} = 0$, $\mathcal{R}_{\mu\nu} = 0$, whereas the Weyl tensor is nonvanishing but coincides with the Riemann tensor. Let us now check whether the Kerr metric is allowed as a vacuum solution with $P_{\alpha\beta}=0$ and $\mathcal{R}=0$ and $\mathcal{R}_{\mu\nu}=0$ (which also means $G^{\alpha\beta}=0$). Thus, by imposing the Ricci flatness, the full field equations in Eq. (4) become

$$P^{\alpha\beta}=0=P_3^{\alpha\beta}=\frac{\alpha_c}{8\pi G}(-g^{\alpha\beta}W^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box_s)W_{\mu\nu\lambda\sigma}+4W^{\alpha}_{\ \mu\nu\sigma}\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\sigma}$$
$$-8\nabla_\mu\nabla_\nu(\mathcal{F}_3(\Box_s)W^{\beta\mu\nu\alpha})-2\Omega_3^{\alpha\beta}+g^{\alpha\beta}(\Omega^{\gamma}_{3\gamma}+\bar{\Omega}_3)-8\Delta_3^{\alpha\beta}). \tag{33}$$

n order to obtain some insight into this problem, let us first consider the right hand side of $P_3^{\alpha\beta}$ up to second order in $\Box_s$, namely

$$\mathcal{F}_3(\Box_s)=(f_{30}+f_{31}\Box_s+f_{32}\Box_s^2), \tag{34}$$

and study the field equations order by order, as we had done for the static case in Ref. [29]. After some computations (see Appendix), we have obtained the following results.

At the zeroth order in $\Box_s$: This is the case of local fourth order gravity of Stelle [51]:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}(\mathcal{R}+\alpha_c[f_{10}\mathcal{R}^2+f_{20}\mathcal{R}^{\mu\nu}\mathcal{R}_{\mu\nu}+f_{30}W^{\mu\nu\lambda\sigma}W_{\mu\nu\lambda\sigma}]). \tag{35}$$

As we are requiring the condition to be Ricci flat, the full field equations in Eq. (4) are explicitly reduced to Eq. (33), where the only relevant terms that remain to be analyzed are those corresponding to the form-factor coefficient f_{30}. However, in this case the local contribution from the Weyl squared term with a constant form factor, f_{30}, vanishes in four dimensions as we can use the Gauss-Bonnet topological invariant to rewrite the Weyl squared in terms of Ricci scalar squared and Ricci tensor squared. Thus, the Kerr metric is still an exact solution for the local fourth order quadratic gravity in Eq. (35) [51].

At the first order in $\Box_s$: Even though the Weyl contribution vanishes at zeroth order, this is not the case for the higher powers of box, i.e., $\Box_s^n$, with $n>0$. Indeed, at the first order in box, we obtain

$$P_3^{(1)\alpha\beta}(\Box_s)=\frac{\alpha_c}{8\pi G}f_{31}\begin{pmatrix}a_{00} & 0 & 0 & a_{03}\\ 0 & a_{11} & 0 & 0\\ 0 & 0 & a_{22} & 0\\ a_{30} & 0 & 0 & a_{33}\end{pmatrix}, \tag{36}$$

with the dimensionless matrix elements given by

$$a_{00}=\frac{144G^2m^2(-8a^4Gm+a^2r(100G^2m^2-8Gmr+5r^2)+5r^4(r-2Gm))}{r^{11}M_s^2(a^2+r(r-2Gm))},\quad a_{03}=-\frac{288aG^3m^3(4a^2+25r(r-2Gm))}{r^{11}M_s^2(a^2+r(r-2Gm))},$$
$$a_{11}=-\frac{1008G^2m^2(4a^4+5a^2r(r-2Gm)+r^2(r-2Gm)^2)}{r^{12}M_s^2},\quad a_{22}=\frac{144G^2m^2(28a^2+r(21r-50Gm))}{r^{12}M_s^2},$$
$$a_{30}=-\frac{288aG^3m^3(4a^2+25r(r-2Gm))}{r^{11}M_s^2(a^2+r(r-2Gm))},\quad a_{33}=\frac{144G^2m^2(r(100G^2m^2-92Gmr+21r^2)-8a^2Gm)}{r^{11}M_s^2(a^2+r(r-2Gm))},$$

where we have fixed the equatorial plane, $\chi=\cos(\pi/2)=0$, without any loss of generality. We can also compute the two-rank symmetric tensor $P_3^{\alpha\beta}(\Box_s)$ at higher order in box; see e.g., Appendix for the computations of the second order in box and for the explicit expression of $P_3^{(2)\alpha\beta}(\Box_s)$.

Generic orders in $\Box_s$: We can now ask what would happen for generic higher orders in $\Box_s$. Note that for the Kerr metric one has $\Box_s\sim\frac{2r}{M_s^2(r^2+a^2\chi^2)}\partial_r$,[3] and by dimensional analysis we can find the behavior of the lowest order in power of $1/r$ at each order in box. We have already seen that the lowest order in $1/r$ at one box goes like $1/r^{10}$, and at two boxes we have $1/r^{12}$; see Appendix. By proceeding in the same way, we can notice that at third order in box, the lowest contribution in powers of $1/r$ is $f_{33}(G^2m^2/r^{14}M_s^6)$, and at fourth order in box $f_{34}(G^2m^2/r^{16}M_s^8)$. Finally, we can hint that at n-th order in box, the lowest contribution in powers of $1/r$ will be always proportional to $f_{3n}(G^2m^2/r^{8+2n}M_s^{2n})$. By just looking at the lowest order contributions at each order in box, we can notice that the tensor $P_3^{\alpha\beta}$ satisfies the following relation:

[3] More precisely, for the Kerr metric the box operator reads $\Box_s=\frac{1}{M_s^2}g^{\nu\mu}\nabla_\nu\nabla_\mu=\frac{1}{M_s^2(a^2\chi^2+r^2)}[(a^2+r(r-2Gm))\partial_r^2+2(r-Gm)\partial_r]$.

$$P_3^{\alpha\beta} \sim f_{31}\mathcal{O}\left(\frac{1}{r^{10}}\right) + f_{32}\mathcal{O}\left(\frac{1}{r^{12}}\right) + \cdots + f_{3n}\mathcal{O}\left(\frac{1}{r^{8+2n}}\right) + \cdots, \quad (37)$$

from which it is clear that in order to vanish we would require an unlikely fine-tuning among all coefficients f_{3n}. In this respect, Kerr-like metric as in Eq. (11) cannot be a vacuum solution of the full nonlinear field equations in Eq. (4); indeed it does not pass through at any order in box, $W_{\mu\nu\rho\sigma}\Box^n W^{\mu\nu\rho\sigma}$ with $n \geq 1$.

V. CONCLUSIONS

Let us briefly conclude our study. In this paper we have studied the rotating metric in the case of ghost-free IDG [7]. First, we have worked in the linear regime and found the spacetime metric in the case of a stress-energy tensor given by a delta-Dirac distribution on a rotating ring. In GR, this kind of source generates a metric solution which suffers from the presence of a ring singularity, where the Kretschmann scalar blows up, and indeed the metric components diverge on the ring, i.e., for $x^2 + y^2 = a^2$ and $z = 0$, which mimics the ring singularity appearing in the Kerr metric [35]. Instead, we have found that in the IDG the spacetime metric turns out to be singularity-free, and for $r \to 0$ the metric becomes *conformally flat*, i.e., the cross-term vanishes at the origin, where the metric coincides with the static one [7,8]. Moreover, the linear approximation can be trusted all the way from the IR to the UV regime, provided we require slow rotations, $mM_s < M_p^2$, and $a < 2/M_s$. The last inequality means that the region of nonlocality has to engulf the ring source of radius a. In IDG the angular momentum has to satisfy the inequality $J < (M_p/M_s)^2$ which implies that its value may also exceed one, unlike in GR. We have shown that outside the source, $r > a$, the spacetime metric can be well described by a multipole expansion which recovers the Lense-Thirring metric in the local regime, $r > 2/M_s$. Finally, we have analyzed the full field equations and shown that the Kerr metric, seen as Ricci flat, will not pass as a vacuum solution if the form factor $\mathcal{F}_3(\Box_s)$ is not constant; indeed, the Weyl contribution does not vanish at each order in box.

Hence, the notion of a rotating black hole that we have in GR would be different in IDG, i.e., without singularity, without event horizons, and without the ergo region. Indeed, our study might have an interesting impact in astrophysical black holes, which should be discussed elsewhere in some detail. Hopefully, our analysis will also shed some light in the presence of LIGO/VIRGO data and understanding the spacetime near a rotating nonsingular compact object.

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APPENDIX: SECOND ORDER CONTRIBUTIONS FROM THE WEYL TERM

We now wish to present the explicit expression of the two-rank symmetric tensor $P_3^{\alpha\beta}$ at second order in $\Box_s$. It is given by

$$P_3^{(2)\alpha\beta}(\Box_s) = \frac{\alpha_c}{8\pi G} f_{32}\begin{pmatrix} a_{00} & 0 & 0 & a_{03} \\ 0 & a_{11} & 0 & 0 \\ 0 & 0 & a_{22} & 0 \\ a_{30} & 0 & 0 & a_{33} \end{pmatrix}, \quad (A1)$$

with the dimensionless matrix elements, defined as

$$a_{00} = \frac{576G^2m^2}{r^{15}M_s^4(a^2 + r(r-2Gm))}[4a^4Gmr(89Gm - 66r) - 72a^6Gm + a^2r^2(-1578G^3m^3 + 927G^2m^2r - 656Gmr^2 + 140r^3) + r^5(939G^2m^2 - 744Gmr + 140r^2)],$$

$$a_{03} = -\frac{1152aG^3m^3}{r^{15}M_s^4(a^2 + r(r-2Gm))}[36a^4 - 2a^2r(89Gm - 52r) + r^2(789G^2m^2 - 696Gmr + 148r^2)],$$

$$a_{11} = \frac{576G^2m^2}{r^{15}M_s^4}[2a^4(193Gm - 50r) + a^2r(-967G^2m^2 + 718Gmr - 120r^2) + r^2(390G^3m^3 - 459G^2m^2r + 172Gmr^2 - 20r^3)],$$

$$a_{22} = \frac{576G^2m^2(a^2(100r - 426Gm) + r(789G^2m^2 - 534Gmr + 80r^2))}{r^{15}M_s^4},$$

$$a_{30} = -\frac{1152aG^3m^3(36a^4 - 2a^2r(89Gm - 52r) + r^2(789G^2m^2 - 696Gmr + 148r^2))}{r^{15}M_s^4(a^2 + r(r - 2Gm))},$$

$$a_{33} = -\frac{576G^2m^2}{r^{15}M_s^4(a^2 + r(r - 2Gm))}[72a^4Gm - 4a^2Gmr(89Gm - 38r) + r^2(1578G^3m^3 - 1857G^2m^2r + 694Gmr^2 - 80r^3)].$$

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