Exact d-dimensional Bardeen-de Sitter black holes and thermodynamics

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The Bardeen metric is the first spherically symmetric regular black hole solution of Einstein's equations coupled to nonlinear electrodynamics, which has an additional parameter (e) due to nonlinear charge apart from mass (M). We find a d-dimensional Bardeen-de Sitter black hole and analyze its horizon structure and thermodynamical properties. Interestingly, in each spacetime dimension d, there exists a critical mass parameter $\mu = \mu_E$, which corresponds to an extremal black hole when Cauchy and event horizons coincide, which for $\mu > \mu_E$ describes a nonextremal black hole with two horizons and no black hole for $\mu < \mu_E$. We also find that the extremal value μ_E is influenced by the spacetime dimension d. Owing to the nonlinear charge corrected metric, the thermodynamic quantities of the black holes also get modified and a Hawking-Page-like phase transition exists. The phase transition is characterized by a divergence of the heat capacity at a critical radius $r_+ = r_+^C$, with the stable (unstable) branch for $C_e > (<)0$. The Hawking evaporation of black holes leads to a thermodynamically stable double-horizon black hole remnant with the vanishing temperature.

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I. INTRODUCTION

The gravitational collapse of a sufficiently massive star ($\sim 3.5\ M_\odot$) necessarily forms a spacetime singularity—this is a fact established by the famous theorem due to Hawking and Penrose [1,2]. The existence of singularity by its very definition means spacetime fails to exist and therefore signaling a breakdown of physics laws. Sakharov [3] and Gliner [4] suggest that singularities could be avoided by matter with a de Sitter core. The first regular black hole solution, based on this idea, was proposed by Bardeen [5] with horizons but no singularity. The Bardeen black hole was reinterpreted as an exact solution to Einstein equations coupled to nonlinear electrodynamics [6]. Recently, the spherically symmetric Bardeen-de Sitter black hole was derived [7] whose metric reads

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where $f(r)$ is a nonlinear metric function given by

$$f(r) = 1 - \frac{2mr^2}{(r^2+e^2)^{3/2}} - \frac{\Lambda r^2}{3}, \qquad r \geq 0.$$

Here m represents black hole mass and e is the nonlinear charge of a self-gravitating magnetic field of a nonlinear electrodynamic source. An analysis of $f(r) = 0$ in the absence of cosmological constant Λ reveals the existence of a critical e^* such that $f(r)$ has a double root if $e = e^*$, two roots if $e < e^*$ and no root if $e > e^*$, with $e^* = 2m/3\sqrt{3}$. These cases, respectively, correspond to an extreme black hole with degenerate horizons, a black hole with Cauchy and event horizons, and no black hole [8]. It can be seen that the metric (1) asymptotically behaves as [7]

$$f(r) \sim 1 - \frac{2m}{r} + \frac{3me^2}{r^3} - \frac{\Lambda r^2}{3} + O\left(\frac{1}{r^5}\right). \tag{2}$$

The Bardeen-de Sitter metric, in the limit $e \to 0$, becomes the Schwarzschild-de Sitter metric, and for small r

$$f(r) \sim 1 - \frac{\Lambda_{\rm eff}}{3}r^2, \quad \text{for } r \sim 0, \tag{3}$$

where $\Lambda_{\rm eff} = \Lambda + 6m/e^3$. The Bardeen-de Sitter solution is regular everywhere which can be realized from the behavior of scalar invariant $R = R^{ab}R_{ab}$ (R_{ab} is the Ricci tensor) and the Kretschmann invariant $K = R^{abcd}R_{abcd}$ (R_{abcd} is the Riemann tensor) which are given by

$$R = \frac{6mg^2(4e^2 - r^2)}{(r^2+e^2)^{7/2}} + 4\Lambda,$$

$$K = \frac{12m^2}{(r^2+e^2)^7}[8e^8 - 4e^6r^2 + 47e^4r^4 - 12e^2r^6 + 4r^8] + 8e^2\Lambda m\left[\frac{4e^2 - r^2}{(e^2+r^2)^{7/2}}\right] + \frac{8}{3}\Lambda^2. \tag{4}$$

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These invariants are well behaved everywhere including at $r = 0$ [2,8–13]. Thus, the black hole does not result in a singularity but develops a de Sitter region, eventually settling with a regular center. Hence, it is a maximal extension of Reissner-Nordström spacetime but with a regular center [14,15].

It turns out that the subsequent analysis of all the regular black hole solutions is based on Bardeen's idea [6,16–18]. Hence, the Bardeen model is the most important regular black hole which triggered a flurry of activities in regular black hole research. The Bardeen solution is extended to noncommutative inspired geometry [19] and the Bardeen-de Sitter black holes is also obtained [7]. The Bardeen black hole has received significant attention in the recent past, e.g., the stability of the Bardeen black hole was performed by Moreno and Sarbach [20]. The Keplerian disks orbiting around the Bardeen black holes were discussed in [21] to obtain profiled spectral lines. The quasinormal modes of the Bardeen black holes have been studied by several authors [22–24]. The antievaporation phenomenon of the Bardeen-de Sitter black holes was investigated in [25]. Thermodynamic quantities of the Bardeen black holes were studied by Man and Cheng [26]. The motion of a test particle in the Bardeen black holes spacetime was studied by Zhou *et al.* [27]. The rotating Bardeen black holes were also discussed [9,12]. Over the past decade there has been an increasing interest in the study of black holes, and related objects, in higher dimensions, motivated to a large extent by developments in string theory as it requires higher dimensions. The first successful statistical counting of black hole entropy was performed for a higher dimensional black hole [28]. Also, the production of higher-dimensional black holes at LHC becomes a possibility in scenarios involving large extra dimensions and TeV-scale gravity [29,30].

Hence, it is pertinent to consider the d-dimensional analog of Bardeen-de Sitter black holes and discuss their thermodynamical properties. To be precise, we analyze an exact d-dimensional solution of the Einstein gravity coupled with nonlinear electrodynamics thereby generalizing those discussions on the Bardeen black holes. There is growing evidence that the physics of higher-dimensional black holes can be markedly different, and much richer than its four-dimensional counterpart. As a consequence, there is a considerable interest towards the understanding of the black holes in higher dimensions, as the growing volume of recent literature indicates. In particular, the Meyers-Perry has found Schwarzschild, Reissner-Nordström and Kerr solutions in asymptotically flat higher dimensional spacetimes [31], which were extended by Dianyan [32] to find charged-dS black holes and later by Liu and Sabra [33] for d-dimensional charged black holes in (A)dS spaces. The Bañados-Teitelboim-Zanelli black holes have been also extended to higher dimensions [34,35] and so are the radiating black holes [36] (see also [37], for a review). Other examples from the higher dimensional spacetime include the gravitational collapse of different fluids [38–41]. It is therefore interesting to find the Bardeen solution in higher dimensional spacetimes.

The paper is organized as follows. In Sec. II, we obtain an exact solution of the d-dimensional Bardeen-de Sitter black holes considering a prototypical form of the Maxwell tensor and the Lagrangian of a nonlinear electromagnetic source of a charge. Section III discusses thermodynamics associated with the black holes. We conclude the paper in Sec. IV.

II. *d*-DIMENSIONAL STATIC SPHERICALLY SYMMETRIC BARDEEN-DE SITTER BLACK HOLE

The spherically symmetric Schwarzschild black holes when generalized to d-dimensional spacetime, it results into the Schwarzschild-Tangherlini black holes [42]. Here, we wish to derive static spherically symmetric d-dimensional Bardeen-de Sitter black holes. The Einstein-Hilbert action coupled to a nonlinear electrodynamics source with a positive cosmological constant term in d-dimensional spacetime [34,35] is given by

$$I = \frac{1}{16\pi}\int d^d x\sqrt{-g}((R - 2\Lambda) - 4\mathcal{L}(\mathcal{F})), \tag{5}$$

where d is the spacetime dimension, Λ is the cosmological constant, R is the Ricci scalar and the Lagrangian $\mathcal{L}(\mathcal{F})$ is a function of $\mathcal{F}$ [6,7], which has a form

$$\mathcal{F} = F_{\mu\nu}F^{\mu\nu}/4, \qquad F_{\mu\nu} = 2\nabla_{[\mu}A_{\nu]},$$

where $F_{\mu\nu}$ is the field strength tensor of the electromagnetic field and A_μ is the gauge potential. To derive Bardeen-de Sitter black holes in arbitrary d-dimensional spacetime, we suitably modify the Lagrangian density $L(\mathcal{F})$ in [6,7] as

$$\mathcal{L}(\mathcal{F}) = \frac{(d-2)}{4se^2}\left[\frac{(\sqrt{2e^2\mathcal{F}})}{1+\sqrt{2e^2\mathcal{F}}}\right]^{\frac{2d-3}{d-2}}, \tag{6}$$

where

$$s = \frac{e^{d-3}}{(d-1)\mu'^{d-3}}$$

On varying of action (5) yields the following field equations [6,7]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = T_{\mu\nu}, \tag{7}$$

$$\nabla_\mu(\mathcal{L}_{\mathcal{F}}F^{\mu\nu}) = 0, \qquad \nabla_\mu(*F^{\mu\nu}) = 0, \tag{8}$$

where $\mathcal{L}_{\mathcal{F}} = \partial\mathcal{L}/\partial\mathcal{F}$ and energy-momentum tensor $T_{\mu\nu}$ reads [7]

$$T_{\mu\nu} = 2(\mathcal{L}_{\mathcal{F}} F^2_{\mu\nu} - g_{\mu\nu}\mathcal{L}). \tag{9}$$

We assume the spherically symmetric metric *ansatz* in d-dimensional spacetime [31,32] as

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-2}, \tag{10}$$

where $f(r)$ is a metric function to be determined by solving the field equations and

$$d\Omega^2_{d-2} = d\theta_1^2 + \sum_{i=2}^{d-2}\left[\prod_{j=2}^{i}\sin^2\theta_{j-1}\right]d\theta_i^2 \tag{11}$$

is the line element of a unit $(d-2)$-dimensional sphere [31,32]. For the consistent field equations with the charge e included, we define the Maxwell field tensor [7,34,35] in d-dimensions as

$$F_{\mu\nu} = 2\delta^{\theta_{d-3}}_{[\mu}\delta^{\theta_{d-2}}_{\nu]}\frac{e^{d-3}}{r^{d-4}}\sin\theta_{d-3}\left[\prod_{j=1}^{d-4}\sin^2\theta_j\right], \tag{12}$$

with $\mathcal{F}$ as

$$\mathcal{F} = \frac{e^{2(d-3)}}{2r^{2(d-2)}}. \tag{13}$$

Substituting Eq. (13) into Eq. (6), one obtains

$$\mathcal{L}(\mathcal{F}) = \frac{(d-1)(d-2)\mu'^{d-3}e^{d-2}}{4(r^{d-2}+e^{d-2})^{\frac{2d-3}{d-2}}}. \tag{14}$$

Next, the (t, t) component of the field equations (7) can be written as

$$-\frac{(d-2)f}{2r^2}[rf' + (d-3)(f-1)] + \Lambda g_{tt} = -2\mathcal{L}(\mathcal{F})g_{tt}. \tag{15}$$

It is useful to introduce the mass function [36] as

$$f(r) = 1 - \frac{m(r)}{r^{d-3}}, \tag{16}$$

which is a measure of mass contained within the radius r. On using Eqs. (14) and (16) in (15), we get

$$m'(r) - \frac{2\Lambda}{d-2}r^{d-2} = \frac{(d-1)\mu'^{d-3}e^{d-2}r^{d-2}}{(r^{d-2}+e^{d-2})^{\frac{2d-3}{d-2}}}, \tag{17}$$

which can be easily integrated to

$$\begin{aligned} &-m(r) + \frac{2\Lambda}{(d-1)(d-2)}r^{d-1} + C \\ &= \mu'^{d-3} - \frac{\mu'^{d-3}r^{d-1}}{(r^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}}, \end{aligned} \tag{18}$$

where integration constant C can be fixed via

$$C = \lim_{r\to\infty}\left[m(r) - \frac{2\Lambda}{(d-1)(d-2)}r^{d-1}\right] = \mu'^{d-3}. \tag{19}$$

Hence, one obtains

$$m(r) = \frac{\mu r^{d-1}}{(r^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}} + \frac{2\Lambda r^{d-1}}{(d-1)(d-2)}, \tag{20}$$

and thus the metric function reads

$$f(r) = 1 - \frac{\mu r^2}{(r^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}} - \frac{2\Lambda r^2}{(d-1)(d-2)}, \tag{21}$$

where the parameter $\mu = \mu'^{d-3}$ is an integration constant related to the black hole Arnowitt-Deser-Misner mass, via [31]

$$\mu = \frac{16\pi M}{(d-2)\Omega_{d-2}}, \qquad \Omega_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma(\frac{d-1}{2})}. \tag{22}$$

In summary, we have shown that the metric of the d-dimensional Bardeen-de Sitter black holes has a form

$$\begin{aligned} ds^2 = &-\left(1 - \frac{\mu r^2}{(r^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}} - \frac{2\Lambda r^2}{(d-1)(d-2)}\right)dt^2 \\ &+ \frac{1}{1 - \frac{\mu r^2}{(r^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}} - \frac{2\Lambda r^2}{(d-1)(d-2)}}dr^2 + r^2 d\Omega^2_{d-2}, \end{aligned} \tag{23}$$

i.e., we have found that Eq. (23) is the solution of field equations (7) and (8) for the energy-momentum tensor (9). Thus, the black hole interior does not terminate on a singularity but crosses the Cauchy horizon and develops in a region that becomes more and more de-Sitter-like eventually ending with a regular origin at $r = 0$ [8]. The metric function, $f(r)$ for large and small r, respectively, reads

$$\begin{aligned} f(r) &\sim 1 - \frac{\mu}{r^{d-3}} - \frac{2\Lambda r^2}{(d-1)(d-2)}, \qquad r \gg 1, \\ f(r) &\sim 1 - \Lambda'_{\text{eff}} r^2, \qquad r \sim 0, \end{aligned} \tag{24}$$

where Λ'_{eff} reads as

$$\Lambda'_{\text{eff}} = \frac{\mu}{e^{d-1}} + \frac{2\Lambda}{(d-1)(d-2)}. \tag{25}$$

When $d = 4$, Eq. (23) goes over to the Bardeen-de Sitter solution (2) and, in the limit, $e \to 0$, reduces to the usual

d-dimensional Schwarzschild-Tangherlini-de Sitter black hole [42]

$$ds^2 = -\left(1 - \frac{\mu}{r^{d-3}} - \frac{2\Lambda r^2}{(d-1)(d-2)}\right)dt^2 + \frac{1}{(1 - \frac{\mu}{r^{d-3}} - \frac{2\Lambda r^2}{(d-1)(d-2)})}dr^2 + r^2 d\Omega_{d-2}^2. \quad (26)$$

As shown by Tangherlini [42], the metric (26) is indeed Ricci flat when $\Lambda = 0$. If the mass parameter $\mu < 0$ then we get a naked singularity, which is not physical. If $\mu > 0$, the black hole horizon radius r_+ is obtained by solving $g^{rr}(r_+) = 0$, which for $\Lambda = 0$ reads

$$r_+ = \left[\frac{16\pi M}{(d-2)\Omega_{d-2}}\right]^{\frac{1}{d-3}}. \quad (27)$$

However, unlike the Schwarzschild-Tangherlini-de Sitter black hole, the Bardeen-de Sitter black hole is regular which can be realized from the invariants, R and K:

$$R = \frac{(d-1)e^{d-2}\mu}{(r^{d-2}+e^{d-2})^{\frac{3d-5}{d-2}}}[e^{d-2}d - (d-3)r^{d-2}] + \frac{8}{d-2}\Lambda,$$
$$K = \frac{(d-1)\mu^2}{(r^{d-2}+e^{d-2})^{\frac{2(3d-5)}{d-2}}}[(d-2)^2(d-3)r^{4(d-2)} - 2(d-1)(d-2)(d-3)e^{d-2}r^{3(d-2)} + (d^3+3d^2-23d+27)e^{2(d-2)}r^{2(d-2)} - 4(d-3)e^{3(d-2)}r^{d-2} + 2de^{4(d-2)}]$$
$$- \frac{1}{(e^{d-2}+r^{d-2})^{\frac{3d-5}{d-2}}}\left[\frac{8(12-9d+d^2)}{(2-3d+d^2)^2}\Lambda^2 - \frac{\Lambda\mu}{(d-1)(d-2)}[2(d-2)(d-3)r^{2(d-2)} + (5d^2-24d+27)e^{d-2}r^{d-2} + (d^2-9d+12)e^{2(d-2)}]\right]. \quad (28)$$

It is easy to see that these invariants are regular everywhere including at $r = 0$. Hence, the charge e removes the curvature singularity which occurs in the Schwarzschild-Tangherlini-de Sitter black hole.

A. Horizons

The horizons of a black hole, if it exists, are zeros of $g^{rr} = f(r) = 0$. Depending upon the choice of the parameters e and μ, we have three distinct horizons, namely, the smaller inner or Cauchy horizon (r_-), the event horizon ($r_+ > r_-$), and the largest cosmological horizon ($r_c > r_+$) such that

$$1 - \frac{\mu r_i^2}{(r_i^{d-2}+e^{d-2})^{\frac{d-1}{d-2}}} - \frac{2\Lambda r_i^2}{(d-1)(d-2)} = 0, \quad (29)$$

where $r_i = \{r_c, r_+, r_-\}$. The metric function $f(r)$ decreases first for $r < r_{\min}$, reaches a minimum at $r = r_{\min}$, and then increases towards zero for $r_{\min} < r < r_+$ (cf. Fig. 2). Further, for $r > r_+$, the function $f(r)$ increases and approaches a maximum at $r = r_{\max}$, and then starts decreasing towards zero for $r_{\max} < r < r_c$. The Schwarzschild-Tangherlini-de Sitter black holes (26) consist of only two horizons r_+ and r_c, which correspond, respectively, to the event and cosmological horizons. The behavior in between is easily found by requiring $f'(r = r_\star) = 0$, where $r_\star$ reads [43]

$$r_\star = \left[\frac{(d-1)(d-2)(d-3)\mu}{4\Lambda}\right]^{\frac{1}{d-1}}. \quad (30)$$

The metric function starts increasing first for $r > r_+$, reaches a maximum at $r = r_\star$, and decreases towards zero for $r_\star < r < r_c$. Therefore, $r_\star$ is easily found to correspond to a global maximum.

A numerical analysis of the zeros of $f(r_+) = 0$ of the Bardeen-de Sitter black holes (23) on varying the values of the parameters μ, e and Λ in different dimensions d reveals a critical value of the mass parameter $\mu = \mu_E$ when the Cauchy and event horizons coincide (cf. Fig. 1). Similarly, the event and cosmological horizons coincide with a critical value of the mass parameter $\mu = \mu_C$ (cf. Fig. 3) such that Eq. (29) admits three horizons when the mass parameter is in the range $\mu_E < \mu < \mu_C$ (cf. Fig. 2) which corresponds to a nondegenerate de Sitter black hole [7]. The value $\mu = \mu_E$ corresponds to the extremal black holes with degenerate horizons $r_- = r_+ = r_H^E$ (cf. Fig. 1) whereas the value $\mu = \mu_C$ describes the degenerate black hole where $r_+ = r_c = r_c^E$ (cf. Fig. 3) as in the Narai solution [7]. Then for $\mu = \mu_C$, we have a regular d-dimensional Narai kind solution and no black hole for $\mu < \mu_E$.

III. THERMODYNAMICS OF BLACK HOLE

In this section, we calculate the thermodynamical quantities associated with the d-dimensional Bardeen-de Sitter black holes (23). The black hole mass can be obtained in

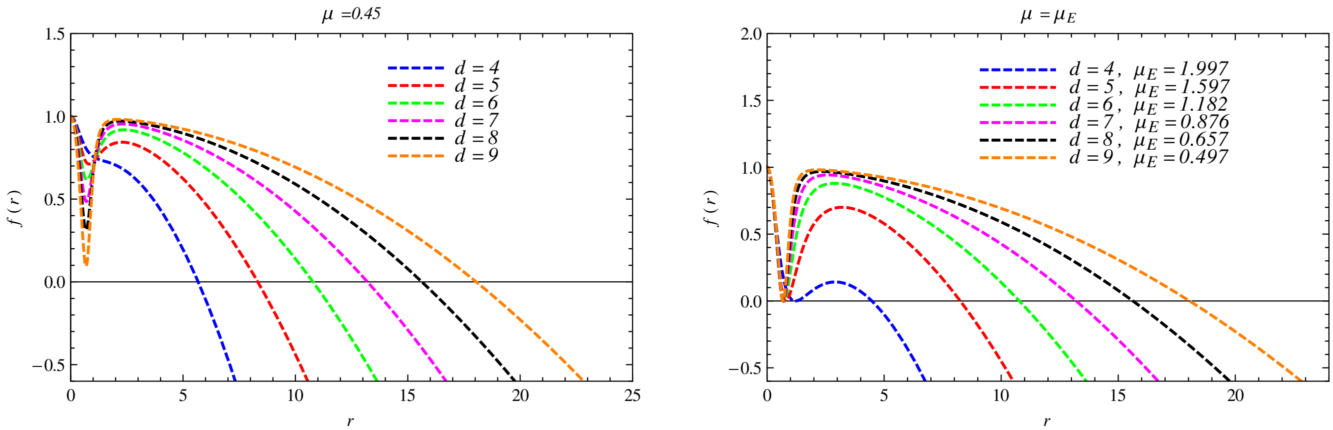

FIG. 1. Plot showing the metric function $f(r)$ vs r for $e = 0.8$ and $\Lambda = 0.086$. Here the mass parameter $\mu = \mu_E$ corresponds to an extremal d-dimensional Bardeen-de Sitter black holes with degenerate horizons when Cauchy and event horizons coincide. For $\mu = 0.45(<\mu_E)$ we have only the cosmological horizon.

terms of horizon radius r_+ by solving $f(r_+) = 0$ which leads to

$$M_+ = \frac{(d-2)\Omega_{d-2}}{16\pi} \frac{[{r_+}^{d-2} + e^{d-2}]^{\frac{d-1}{d-2}} [1 - \frac{2\Lambda}{(d-1)(d-2)} r_+^2]}{r_+^2}. \quad (31)$$

Obviously, when $e = 0$ and $\Lambda = 0$, Eq. (31) reduces to the mass of the d-dimensional Schwarzschild-Tangherlini black hole [44]. The black hole has a Hawking temperature, which can be obtained through the surface gravity given by

$$\kappa = \sqrt{-\frac{1}{2} \nabla_\mu \xi_\nu \nabla^\mu \xi^\nu}, \quad (32)$$

where ξ^μ is the Killing vector. The Killing vector ξ^μ for the static spherically symmetric spacetime has the form $\xi^\mu = \partial_t^\mu$ corresponding to the time-translational symmetry. The surface gravity of the black holes reads

$$\kappa = \frac{1}{2} \frac{\partial \sqrt{-g^{rr} g_{tt}}}{\partial r}\bigg|_{r=r_+} = \frac{1}{2} \frac{df(r)}{dr}\bigg|_{r=r_+}. \quad (33)$$

Then, the Hawking temperature ($T_+ = \kappa/2\pi$) of the Bardeen-de Sitter black holes (23) reads

$$T_+ = \frac{1}{4\pi r_+} \frac{\left[(d-3) - 2\frac{e^{d-2}}{r_+^{d-2}} - (\frac{2}{d-2})\Lambda r_+^2\right]}{\left[1 + \frac{e^{d-2}}{r_+^{d-2}}\right]}. \quad (34)$$

When $\Lambda = 0$, the temperature becomes zero, positive and negative, respectively, when $r_+ = r_0$, $r_+ > r_0$, and $r_+ < r_0$, where r_0 is given by

$$r_0 = e\left[\frac{2}{d-3}\right]^{\frac{1}{d-2}}. \quad (35)$$

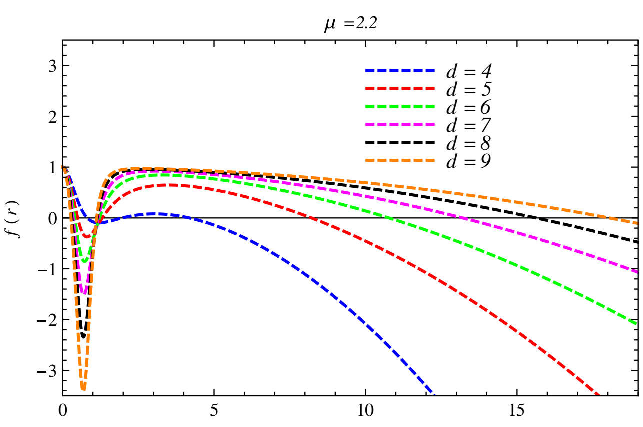

FIG. 2. Plot showing the metric function $f(r)$ vs r for $e = 0.8$ and $\Lambda = 0.086$. For $\mu = 2.2$, we have three distinct horizons in each dimension.

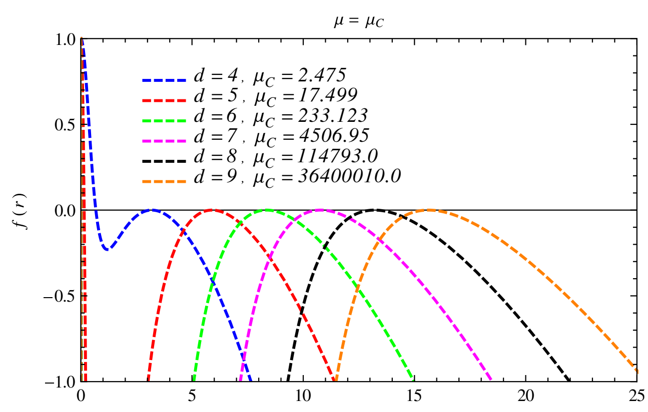

FIG. 3. Plot showing the metric function $f(r)$ vs r for $e = 0.8$ and $\Lambda = 0.086$. Here the mass parameter $\mu = \mu_C$ corresponds to an extremal d-dimensional Narai black holes with the degenerate horizons when the event and cosmological horizons coincide.

For $e = 0$ and $\Lambda = 0$, Eq. (34) reduces to the temperature of the d-dimensional Schwarzschild-Tangherlini black holes [44],

$$T_+ = \frac{(d-3)}{4\pi r_+}. \tag{36}$$

The temperature of the d-dimensional Schwarzschild-Tangherilini black hole diverges when the horizon radius shrinks to zero which puts a limit on the validity of the Hawking evaporation process, whereas the temperature of the d-dimensional Bardeen-de Sitter black holes does not diverge at short distances comparable to the Planck scale [45–47]. When $d = 4$ and $\Lambda = 0$, Eq. (34) reduces to the temperature of the Bardeen black hole [48–50],

$$T_+ = \frac{1}{4\pi r_+}\left[\frac{r_+^2 - 2e^2}{r_+^2 + e^2}\right]. \tag{37}$$

The Hawking temperature of the d-dimensional Bardeen-de Sitter black hole for different values e and Λ is depicted in Fig. 4. The behavior of the temperature profile (cf. Fig. 4) suggests that temperature no more diverges unlike the Schwarzschild-Tangherlini black holes. The maxima of the temperature (34) cannot be obtained analytically for non-zero Λ. However, if $\Lambda = 0$, the maximum of d-dimensional Bardeen black holes occurs at

$$r_+ = e\left[\frac{\beta_d + \sqrt{\beta_d^2 + 8(d-3)}}{2(d-3)}\right]^{\frac{1}{d-2}}. \tag{38}$$

Here and henceforth we use $\beta_d = d^2 - 4d + 7$. Figure 4 shows that the Hawking temperature of d-dimensional Bardeen-de Sitter black holes exhibits a peak that decreases and moves to the right when the charge e increases. In Fig. 4 the peak in the temperature decreases and moves to the left when we increase the dimension d at fixed e.

Wald [51] has shown that the entropy of the black hole in any gravity must be a function of the horizon. In general relativity, the black hole entropy obeys the so-called horizon area formula, $S = A_{d-2}/4$, where A_{d-2} is the horizon area. Since a black hole is a thermodynamical system, the entropy of the black hole can be determined from the first law of thermodynamics [19,52,53]:

$$dM = T_+ dS + \Phi de,$$

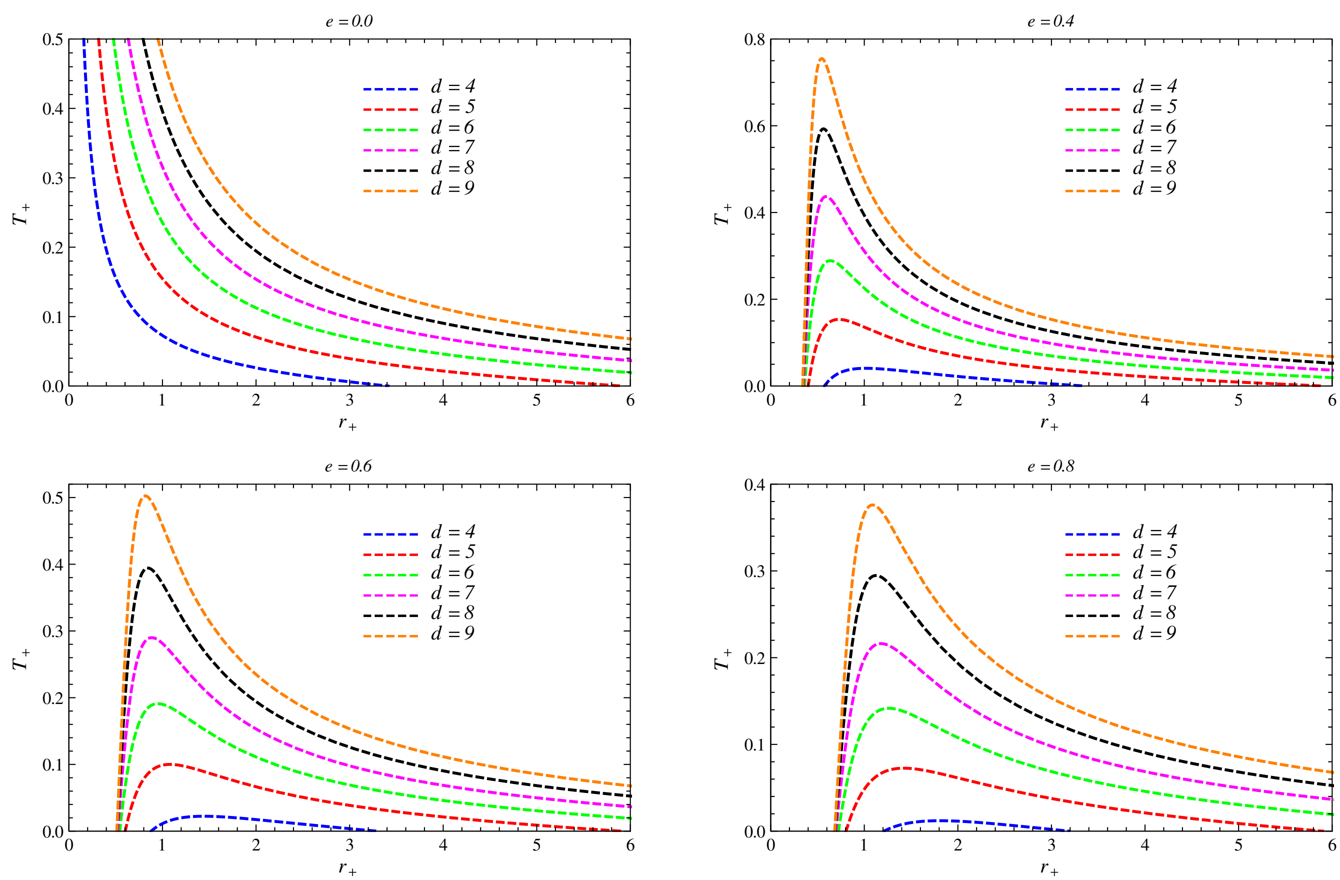

FIG. 4. The Hawking temperature (T_+) vs the horizon radius r_+ in various dimensions for different values of e. The Schwarzschild black hole ($e = 0$) shows a divergent phase in the final stage of black hole evaporation. Here we put $\Lambda = 0.086$.

where Φ is the potential conjugate to e. Thus, the entropy at constant e reads

$$S = \int T_+^{-1}\left(\frac{\partial M}{\partial r_+}\right)_e dr_+. \tag{39}$$

Substituting Eqs. (31) and (34) into Eq. (39) and integrating, the entropy for d-dimensional Bardeen-de Sitter black holes read

$$S = \frac{(d-2)\Omega_{d-2}}{4}\left[-\frac{1}{d-1}\frac{e^{d-1}}{{r_+}^{d-1}}H\right], \quad \text{with}$$
$$H = {}_2F_1\left[-\frac{d-1}{d-2}, -\frac{d-1}{d-2}, -\frac{1}{d-2}, -\frac{{r_+}^{d-2}}{e^{d-2}}\right], \tag{40}$$

where ${}_2F_1$ stands for the hypergeometric function of first kind. The presence of parameter e in the solution affects the entropy significantly and area law does not hold. For $d=4$, Eq. (40) reduces to [52,53]

$$S = \frac{\pi}{r_+}\left[({r_+}^2 - 2e^2)\sqrt{r_+^2 + e^2} + 3e^2\log\left[r_+ + \sqrt{e^2 + {r_+}^2}\right]\right]. \tag{41}$$

Notice that for $e=0$, the entropy formulas (40) and (41) of the Bardeen-de Sitter black holes respectively go over to the entropy of the Schwarzschild-Tangherilini and Schwarzschild black holes and they always obey the area law [43].

A. Phase transition

Next, we focus our attention on the local thermodynamical stability of d-dimensional Bardeen-de Sitter black holes by calculating the heat capacity (C_e) and discuss the effect of the charge e. The condition for a change in the sign of heat capacity determines the possible phase transition of the black hole [46]. When the heat capacity $C_e > 0$, the black hole is locally stable to thermal fluctuations, whereas for $C_e < 0$ it is locally unstable. The heat capacity for constant e can be given by [44]

$$C_e = \left(\frac{\partial M}{\partial r_+}\right)_e \left(\frac{\partial r_+}{\partial T}\right)_e. \tag{42}$$

On using Eqs. (31) and (34) in Eq. (42), the heat capacity of d-dimensional Bardeen-de Sitter black holes turns out to be

$$C_e = \frac{(d-2)\Omega_{d-2}}{4}\frac{r_+^{d-2}\left[(d-3) - 2\frac{e^{d-2}}{r_+^{d-2}} - (\frac{2}{d-2})\Lambda r_+^2\right]\left[1 + \frac{e^{d-2}}{r_+^{d-2}}\right]^{\frac{2d-3}{d-2}}}{\left[2(\frac{e^{d-2}}{{r_+}^{d-2}})^2 + \beta_d\frac{e^{d-2}}{{r_+}^{d-2}} - (d-3) - (1 + (d-1)\frac{e^{d-2}}{r_+^{d-2}})(\frac{2}{d-2})\Lambda r_+^2\right]}. \tag{43}$$

The heat capacity (43) is plotted in Figs. 5 and 6 for different values of e in different dimensions. A numerical analysis shows that the heat capacity changes from negative infinity to positive infinity at the point where the temperature reaches its maximum value, and is identified as the critical radius $r_+ = r_+^C$, where the phase transition of the black hole occurs (cf. Figs. 5 and 6), and is also known as the Davies' point [50,54,55]. The heat capacity becomes positive when r_+ corresponds to $r_+^{(2)} < r_+ < r_+^C$, where $r_+^{(2)}$ is the point where both the temperature and heat

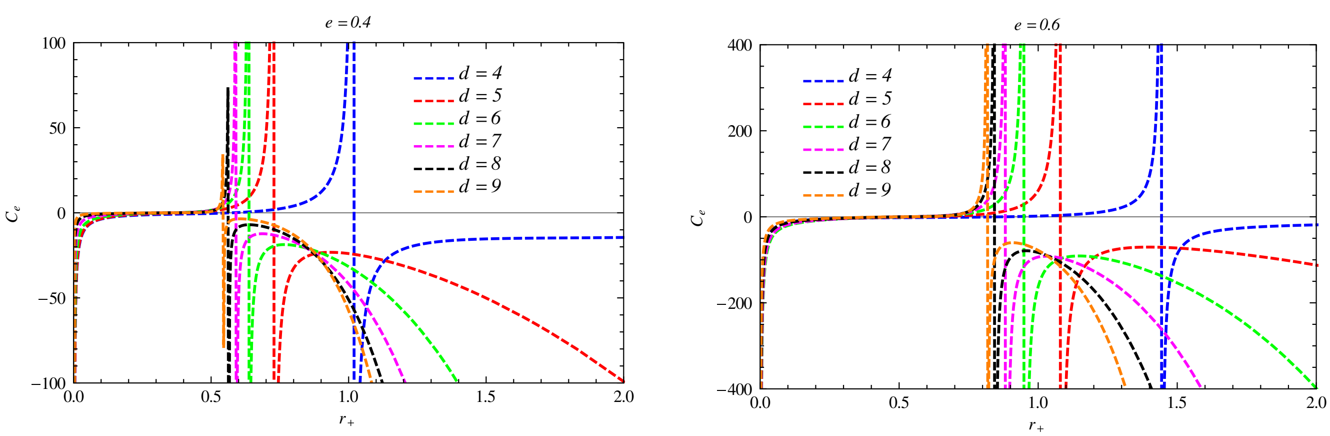

FIG. 5. The heat capacity C_e vs the horizon radius r_+ for the Bardeen black hole in various dimensions for different values of the charge e. Here we put $\Lambda = 0.86$.

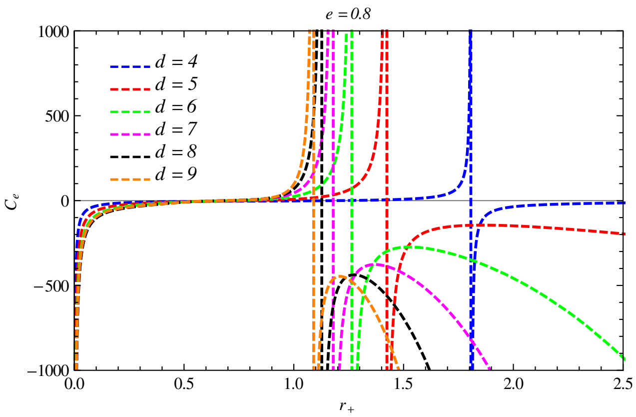

FIG. 6. The heat capacity C_e vs the horizon radius r_+ for the Bardeen black hole in various dimensions for $e = 0.8$ and $\Lambda = 0.86$.

capacity becomes zero. It becomes negative in the regions $0 < r_+ < r_+^{(2)}$ and $r_+ > r_+^C$. For $\Lambda = 0$, Eq. (43) corresponds, respectively, to the negative and positive heat capacities of the black hole when

$$0 < r_+ < r_0, \quad \text{and} \quad r_+ > r_+^{C\star}, \tag{44}$$

and

$$r_0 < r_+ < r_+^{C\star}. \tag{45}$$

Here, $r_+^{C\star}$ is the critical radius for $\Lambda = 0$, which reads

$$r_+^{C\star} = e\left[\frac{\beta_d + \sqrt{\beta_d^2 + 8(d-3)}}{2(d-3)}\right]^{\frac{1}{d-2}}. \tag{46}$$

Therefore, for $\Lambda = 0$, the black hole is thermodynamically unstable and stable, respectively, when r_+ corresponds to Eqs. (44) and (45). The phase transition at $r = r_+^{C\star}$ represents that our black hole goes to an unstable phase from a stable phase. Further, the value of $r_+^{C\star}$ increases with the increase in the charge parameter e in all spacetime dimensions. When $C_e = 0$, we have the corresponding extremal black hole configuration where the temperature also becomes zero. In the limit $e \to 0$ and for $\Lambda = 0$, we get [44]

$$C = -\frac{(d-2)\Omega_{d-2}}{4} r_+^{d-2}, \tag{47}$$

which is the heat capacity of the d-dimensional Schwarzschild-Tangherlini black hole suggesting that the black hole is thermodynamically unstable. The heat capacity of the four-dimensional Bardeen black hole, i.e., when $d = 4$, reads

$$C_e = \frac{2\pi(r_+^2 - 2e^2)(r_+^2 + e^2)^{5/2}}{r_+(2e^4 + 7e^2 r_+^2 - r_+^4)}, \tag{48}$$

which is exactly the same as obtained by [48–50].

The black hole remnant is an important consequence of the Hawking evaporation process to resolve the information loss puzzle [56,57]. The black hole loses its energy continuously through Hawking radiation and, finally, we are left with a stable remnant beyond which no further evaporation is possible. For $\Lambda = 0$, the remnant mass corresponding to the extremal black hole with degenerate horizon radius r_0 reads

$$M_0 = \frac{(d-2)\Omega_{d-2}}{16\pi}\left(\frac{2}{d-3}\right)^{\frac{d-3}{d-2}}\left(\frac{d-1}{2}\right)^{\frac{d-1}{d-2}} e^{d-3}. \tag{49}$$

For $M < M_0$, no horizon exists and for $M > M_0$ there exists two distinct horizons. When $d = 4$, the remnant mass of the Bardeen black hole reduces to $M_0 = (3\sqrt{3}/4)e$ [48]. In the limit $\Lambda \to 0$, both T_+ and C_e approaches to zero exactly at $r_+ = r_0$ and we are left with a stable remnant. Hence, the d-dimensional Bardeen black hole has a stable remnant. In general, for $\Lambda \neq 0$, the black hole remnant is obtained numerically.

IV. CONCLUSION

General relativity with the parameter spacetime dimension d should lead to valuable insights into the nature of the theory, in particular of the black holes, which has been actively investigated for more than a decade. One of the distinct aspects of the higher dimensional black hole is that horizons can have nonspherical topologies, even in asymptotically flat spacetime [37] and the first concrete evidence challenging the weak cosmic censorship conjecture can occur in five dimensions [58]. In this paper, we obtain an exact static spherically symmetric regular Bardeen-de Sitter black hole in an arbitrary d-dimensional spacetimes by solving Einstein equations coupled to nonlinear electrodynamics. We characterized the solution, calculating the possible horizons, which could be at most three. Besides, we confirmed the regular structure of the spacetime everywhere including at the origin by evaluating the quadratic invariants viz. Ricci and Kretschmann scalars. We performed a detailed thermodynamical analysis, focusing mainly on the stability. In general relativity, the black hole entropy satisfies the Bekenstein-Hawking area law, whereas, for the Bardeen black hole, this area law is no longer valid and should be corrected. Further, the thermodynamical quantities such as the Hawking temperature and heat capacity have been derived and plotted. The phase transition is characterized by the divergence of heat capacity at a critical radius r_+^C which is changing with spacetime dimension d. In particular, the black hole is thermodynamically stable, with a positive heat capacity for the range $r_+ < r_+^C$ and unstable for $r > r_+^C$ (cf. Figs. 5 and 6). It would be important to understand how these black holes with positive heat capacity ($C_e > 0$) would emerge from thermal radiation through a phase transition.

In the absence of charge ($e = 0$), our analysis goes over to the Schwarzschild-Tangherlini-de Sitter black holes [42], and for $d = 4$, we recover the results of the Bardeen-de Sitter black hole [7]. The results presented here are the generalization of previous discussions on the Schwarzschild-Tangherlini-de Sitter and Bardeen-de Sitter black holes, and in a more general setting, the stability of Bardeen-de Sitter black holes is an interesting problem for future research. Our results may have importance in the context of the string theory as the dimensions of space has led to several developments in string theory.

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