Hunting for statistical anisotropy in tensor modes with B-mode observations

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(Received 3 September 2018; published 16 October 2018)

We investigate the possibility of constraining statistical anisotropies of the primordial tensor perturbations by using future observations of the cosmic microwave background (CMB) B-mode polarization. By parametrizing a statistically anisotropic tensor power spectrum as $P_h(\boldsymbol{k}) = P_h(k)\sum_n g_n \cos^n\theta_{\boldsymbol{k}}$, where $\theta_{\boldsymbol{k}}$ is an angle of the direction of $\hat{k} = \boldsymbol{k}/k$ from a preferred direction, we find that it is possible for future B-mode observations such as CMB-S4 to detect the tensor statistical anisotropy at the level of $g_n \sim \mathcal{O}(0.1)$.

DOI: 10.1103/PhysRevD.98.083522

I. INTRODUCTION

The detection of the B-mode polarization signal in cosmic microwave background (CMB) is one of the most important challenges in cosmology, because it is sensitive to the primordial gravitational waves (PGWs) generated during inflation. In the standard inflationary scenario, the amplitude of the PGWs generated from vacuum fluctuations during inflation is expected to depend only on the energy density of the inflation, $\rho_{\rm inf}$, and hence its detection was considered as a direct probe for the scale of unknown physics. The current constraint on the amplitude of the PGWs is $r \lesssim 0.07$, where r represents the ratio of the power spectrum of the PGWs to that of the primordial curvature perturbations [1–3]. In the 2020s, next-generation CMB experiments such as LiteBIRD [4] and CMB-S4 [5] are expected to achieve higher sensitivities reaching $r \simeq 10^{-3}$, which corresponds to $\rho_{\rm inf}^{1/4} \simeq 6 \times 10^{15}$ GeV in the conventional vacuum fluctuation case. Recently, other mechanisms of generating PGWs during inflation by introducing some matter fields (e.g., gauge fields) have been proposed, [6,7] and then this means that the vacuum PGWs are no longer the unique target of the B-mode observation. Interestingly, the PGWs generated in these new mechanisms not only have the different relations between r and $\rho_{\rm inf}$ but also observable signatures distinct from the vacuum one, e.g., non-Gaussianity [8,9]. Among them, statistical anisotropies of the PGWs should be useful to distinguish the generation mechanisms and to extract richer information on the early universe from the B-mode observation.

The statistical anisotropy has been pursued mainly in the power spectrum of the curvature perturbation P_ζ. This is because the anisotropic inflation and solid inflation models predict a quadrupole anisotropy in the curvature perturbation, $P_\zeta(\boldsymbol{k}) = P_\zeta(k)(1 + g_* \cos^2\theta_{\boldsymbol{k}})$ [10–17]. Furthermore, recent studies [18,19] argue that higher spin fields generate statistical anisotropies beyond quadrupole in P_ζ during inflation. Indeed, these kinds of anisotropies imprint interesting signatures in the CMB angular power spectrum. While in the standard picture CMB power spectra have only diagonal components in the angular multipole space due to a rotational invariance, statistical anisotropies can create specific nonzero off-diagonal correlations between temperature and polarization in CMB data. Several works have been discussed to test such kind of correlations due to the statistically-anisotropic curvature perturbation [20–24]. So far, however, there is no evidence of the quadrupole anisotropy in P_ζ, and we have an upper bound $|g_*| \lesssim 10^{-2}$ [1,25].

In this paper, we study the statistical anisotropy in the power spectrum of the PGWs. Although little attention has been paid to the statistically-anisotropic PGWs, recent study [26] has proposed a model where large statistical anisotropies in P_h can be generated. In this model, U(1) gauge field is kinematically coupled to a spectator scalar field and gains a large background expectation value which breaks the isotropy of the Universe, and then due to the aniostropy of the Universe the perturbations of the spectator field and the gauge field could source the anisotropic tensor modes. Remarkably, the higher-order statistical anisotropies beyond quadrupole in P_h can be predicted irrespective of the model parameters. Although statistically-anisotropic curvature perturbations are also generated from the spectator and gauge fields in this model, they would be suppressed in compared to those originated from the vacuum fluctuations which are statistically isotopic. This fact promotes us to study the anisotropies of PGWs since they can be better constrained or detected from the observations of the PGWs than from those of the curvature perturbations. A similar prediction is also obtained when the two-form field takes over the role of the U(1) gauge field [27]. Other than this type of model, several works have suggested the generation of testable statistical anisotropies in tensor modes [21,28,29]. Inspired by these predictions, we explore a possibility to test these higher statistical anisotropies of tensor modes through the B-mode angular power spectrum. We model the tensor statistical anisotropies as $P_h(\boldsymbol{k}) = P_h(k)\sum_n g_n (k/k_0)^\gamma \cos^n\theta_{\boldsymbol{k}}$ and evaluate detectabilities of the coefficients g_n in future missions. Compared with the previous study [28], we further investigate the sensitivities of g_n up to $n = 6$. This paper is the first paper studying the statistical anisotropy in the power spectrum of PGWs taking the higher-order statistical anisotropies into account.

This paper is organized as follows. In Sec. II, we describe basic equations for our Fisher analysis. In Sec. III, we obtain 1σ uncertainties of the anisotropic parameters, g_n and q_{LM}. We conclude in Sec. IV.

II. BASIC EQUATIONS

A. Anisotropies

Harmonic coefficients of B-mode anisotropies induced by the primordial tensor perturbations $h_{\pm 2}(\boldsymbol{k})$ can be written in terms of the transfer function $T_\ell^{(B)}(k)$ (see, e.g., Ref. [22]),

$$a_{\ell m}^{(B)} = 4\pi(-i)^\ell \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm 2} h_s(\boldsymbol{k}) T_\ell^{(B)}(k)\,{}_{-s}Y_{\ell m}^*(\hat{k}), \quad (1)$$

with ${}_sY_{\ell m}$ being the spin-s spherical harmonics. The power spectrum of the tensor perturbations is defined as

$$\langle h_{+2}(\boldsymbol{k}_1) h_{-2}(\boldsymbol{k}_2)\rangle = \frac{1}{2}(2\pi)^3 P_h(\boldsymbol{k}_1)\delta^{(3)}(\boldsymbol{k}_1 - \boldsymbol{k}_2), \quad (2)$$

where we have used $h_{-2}(\boldsymbol{k}) = h_{+2}^*(\boldsymbol{k})$. If the rotational invariance is broken, the power spectrum could have the directional dependence, which can be parametrized as [30],

$$P_h(\boldsymbol{k}) = P_h(k) \sum_{LM} Q_{LM}(k) Y_{LM}(\hat{k}), \quad (3)$$

with L running over even numbers, $0, 2, 4, \ldots,$ where $P_h(k)$ is the isotropic (monopole) part, and $\hat{k} \coloneqq \boldsymbol{k}/k$. Taking into account the directional dependence, we obtain the correlation of the harmonic coefficients [28],

$$\begin{aligned} C_{\ell_1 m_1;\ell_2 m_2}^{BB} &\coloneqq \langle a_{\ell_1 m_1}^{(B)} a_{\ell_2 m_2}^{(B)*}\rangle \\ &= \frac{2}{\pi} i^{\ell_2-\ell_1}(-1)^{m_1}\delta_{\ell_1+\ell_2}^{\text{even}} \sum_{LM} \mathcal{G}_{\ell_1\ell_2 L}^{-m_1 m_2 M;-220} \\ &\quad \times \int dk k^2 P_h(k) Q_{LM}(k) T_{\ell_1}^{(B)}(k) T_{\ell_2}^{(B)}(k), \end{aligned} \quad (4)$$

where δ_a^{even} is 1 if a is even, and 0 otherwise, and $\mathcal{G}_{\ell_1\ell_2\ell_3}^{m_1m_2m_3;s_1s_2s_3}$ is the spin-weighted Gaunt integral that is written in terms of the product of Wigner's $3j$-symbols,

$$\begin{aligned} \mathcal{G}_{\ell_1\ell_2\ell_3}^{m_1m_2m_3;s_1s_2s_3} &\coloneqq \int d\Omega\, {}_{s_1}Y_{\ell_1 m_1}(\Omega)\,{}_{s_2}Y_{\ell_2 m_2}(\Omega)\,{}_{s_3}Y_{\ell_3 m_3}(\Omega) \\ &= \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{4\pi}} \\ &\quad \times \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ -s_1 & -s_2 & -s_3 \end{pmatrix}. \end{aligned} \quad (5)$$

In the present study, we assume that the scale dependence of the anisotropic parameter is given as [28]

$$Q_{LM}(k) = q_{LM}\left(\frac{k}{k_0}\right)^\gamma, \quad (6)$$

with constants q_{LM} and γ. Finally, Eq. (4) reads

$$\begin{aligned} C_{\ell_1 m_1;\ell_2 m_2}^{BB}(\gamma) &= \frac{2}{\pi} i^{\ell_2-\ell_1}(-1)^{m_1} \\ &\quad \times \sum_{LM} \delta_{\ell_1+\ell_2+L}^{\text{even}} \mathcal{G}_{\ell_1\ell_2 L}^{-m_1 m_2 M;-220} q_{LM} C_{\ell_1\ell_2}^{BB}(\gamma), \end{aligned} \quad (7)$$

where

$$C_{\ell_1\ell_2}^{BB}(\gamma) \coloneqq \frac{2}{\pi}\int dk k^2 P_h(k) T_{\ell_1}^{(B)}(k) T_{\ell_2}^{(B)}(k)\left(\frac{k}{k_0}\right)^\gamma. \quad (8)$$

Note that for the case with $q_{LM} = \delta_{L0}\delta_{M0}$, that is, statistically-isotropic power spectrum, one can find that the above expression is equivalent to the standard form of the angular power spectrum.

In the theoretical models which predict the statistical anisotropy in primordial tensor modes, the statistical anisotropy is often parametrized in terms of the power series of the cosine function (e.g., [1,25]), as

$$P_h(\boldsymbol{k}) = P_h(k) \sum_{n=\text{even}}^{N} g_n \left(\frac{k}{k_0}\right)^\gamma \cos^n\theta_{\boldsymbol{k}}, \quad (9)$$

where $\theta_{\boldsymbol{k}}$ measures the angle of the direction of $\hat{k}$ from a preferred direction. Thus, it should be useful to give a relation between the parameters g_n and q_{LM}, and according to Eq. (A11) their relations are given by

$$q_{0M} = 2\sqrt{\pi}\left(g_0 + \frac{g_2}{3} + \frac{g_4}{5} + \frac{g_6}{7}\right)\delta_{M0}, \quad (10)$$

$$q_{2M} = 4\sqrt{\frac{\pi}{5}}\left(\frac{g_2}{3} + \frac{2}{7}g_4 + \frac{5}{21}g_6\right)\delta_{M0}, \quad (11)$$

$$q_{4M} = 16\sqrt{\pi}\left(\frac{g_4}{105} + \frac{g_6}{77}\right)\delta_{M0}, \quad (12)$$

$$q_{6M} = \frac{32}{231}\sqrt{\frac{\pi}{13}}g_6\delta_{M0}. \quad (13)$$

B. Fisher information matrix

To quantify the 1σ uncertainties of the anisotropic parameters, $\{q_{LM}\}$ or $\{g_n\}$, we use the Fisher information matrix. The details of the computation of the Fisher information matrix in our study are provided in the Appendix. Here we consider only the B-mode in the full expression in Eq. (A8) or Eq. (A10) with Eq. (A9), and it reads

$$F_L^{BB} = \frac{f_{\rm sky}}{4\pi}\sum_{\ell_1\ell_2}(2\ell_1+1)(2\ell_2+1)\begin{pmatrix}\ell_1 & \ell_2 & L\\ -2 & 2 & 0\end{pmatrix}^2 \times \frac{(C^{BB}_{\ell_1\ell_2})^2}{\tilde{C}^{BB}_{\ell_1}\tilde{C}^{BB}_{\ell_2}}, \quad (14)$$

where $\tilde{C}^{BB}_\ell$ is the total angular power spectrum of B-mode polarization defined in Eq. (A7). Using Eq. (A8) or Eq. (A10), we can estimate the uncertainties of the measurement of the anisotropic parameters,

$$\sigma^2_{q_{LM}} = (F_{LM;LM})^{-1}, \qquad \sigma^2_{g_n} = (F_{nn})^{-1}. \quad (15)$$

A noise model we adopt in the present study is

$$\mathcal{N}^{BB}_\ell = N^{BB}_\ell e^{\ell^2\sigma_b^2}, \quad (16)$$

in which we assume the detector noise N^{BB}_ℓ and the beam effect σ_b are parametrized as [31]

$$N^{BB}_\ell = \left(\frac{\pi}{10800}\frac{w_{BB}{}^{-1/2}}{\mu{\rm K\ arcmin}}\right)^2 \mu{\rm K}^2\ {\rm str}, \quad (17)$$

$$\sigma_b = \frac{\pi}{10800}\frac{\theta_{\rm FWHM}}{\rm arcmin}\frac{1}{\sqrt{8\ln 2}}. \quad (18)$$

with $\theta_{\rm FWHM}$ being the full width at half maximum (FWHM) of the beam in the unit of arcmin. Although we do not take into account neither the lensing effect from the E mode induced by scalar perturbations nor the foreground noises sourced by dust emission, it is possible to emulate the cases including them by increasing the noise parameter $w_{BB}^{-1/2}$.

TABLE I. Cosmological parameters used in the present study. The all parameters except for the tensor-to-scalar ratio are provided by Planck 2015 results (TT, TE, EE + lowP + lensing + ext in Ref. [33]). The amplitude of curvature perturbation and the tensor-to-scalar ratio are evaluated at $k = k_{\rm pivot}$, and we assume $n_{s,0.002} = n_{s,0.05}$ in the notation of Ref. [33].

Parameter		Value
Amplitude of curvature perturbation	$\mathcal{P}_{\mathcal{R}0}$	2.384×10^{-9}
Tensor-to-scalar ratio	r	0.001,0.01,0.05
Pivot scale	$k_{\rm pivot}$	0.002 Mpc^{-1}
Spectral index	n_s	0.9667
Reduced Hubble parameter	h	0.6774
Dark matter fraction	$h^2\Omega_{\rm CDM}$	0.1188
Baryon fraction	$h^2\Omega_{\rm b}$	0.02230
Effective number of neutrinos	$N_{\rm eff}$	3.046
Photon's temperature	$T_{\gamma,0}$	2.7255 K
Optical depth	τ	0.066
Helium abundance	Y_p	0.24667

III. DETECTABILITY OF STATISTICAL ANISOTROPIES OF TENSOR PERTURBATIONS

We use `cmb2nd`[1] to compute the transfer function of the B mode with the cosmological parameters from the Planck 2015 results (TT, TE, EE + lowP + lensing + ext in Ref. [33]), which are tabulated in Table I. Since there is no detection of PGWs, their amplitude is unknown. In this paper, respecting the current constraint on the amplitude of PGWs, $r \coloneqq P_h(k_{\rm pivot})/P_\zeta(k_{\rm pivot}) \lesssim 0.07$, given by the combined analysis of Planck and BICEP2/Keck [1–3], we study the cases with $r = 0.05$, 0.01 and 0.001 to show the dependence of our predictions on the amplitude. We assume a 0.5 degree FWHM beam (designed in LiteBIRD [4]) and the noise level with $w_{\rm BB}^{-1/2} = 1.0$, 5.0, 63.1 μK arcmin which correspond to CMB-S4 [5], LiteBIRD [4], and Planck [34], respectively. In addition to them, we also compute the cosmic-variance-limited (CVL) case with $w_{BB}^{-1/2} = 0$.

The 1σ uncertainties of the measurements of g_n and q_{LM} with even numbers of n and L up to n, $L \leq 6$ are summarized in Tables II–VI which are computed with fixed γ; $\gamma = 0$ (Tables II–IV), $\gamma = -1, -1/2$ (Table V) and $\gamma = 1/2$, 1 (Table VI). Throughout this paper, the isotropic part of angular power spectrum is supposed to be $C^{BB}_\ell \coloneqq C^{BB}_{\ell\ell}(\gamma = 0)$. Hence, the observed signal is given as

$$C^{\rm obs}_{\ell_1m_1;\ell_2m_2}(\gamma) = C^{BB}_{\ell_1}\delta_{\ell_1\ell_2}\delta_{m_1m_2} + C^{BB}_{\ell_1m_1;\ell_2m_2}(\gamma). \quad (19)$$

[1]This Boltzmann code is not public yet, but we have confirmed that the transfer functions obtained from this precisely agree with those from CAMB (https://camb.info/). See also Ref. [32] in which we used the same code.

TABLE II. σ_{g_n} for $w_{BB}^{-1/2} = 63.1,\ 5.0,\ 1.0\ \mu$K arcmin and the CVL case with $\gamma = 0$, $f_{\rm sky} = 1$ and $r = 0.05$.

	CVL	1.0	5.0	63.1
g_2	1.93×10^{-3}	4.61×10^{-2}	9.33×10^{-2}	1.25
g_2	8.23×10^{-3}	2.02×10^{-1}	4.18×10^{-1}	7.16
g_4	1.24×10^{-2}	2.99×10^{-1}	6.08×10^{-1}	1.11×10^{1}
g_2	2.33×10^{-2}	5.02×10^{-1}	9.00×10^{-1}	7.36
g_4	8.27×10^{-2}	1.79	3.17	1.30×10^{1}
g_6	6.46×10^{-2}	1.42	2.55	5.55
q_{2M}	4.98×10^{-3}	1.02×10^{-1}	1.88×10^{-1}	3.73
q_{4M}	4.28×10^{-3}	1.13×10^{-1}	2.47×10^{-1}	4.63
q_{6M}	5.71×10^{-3}	1.21×10^{-1}	2.04×10^{-1}	3.79×10^{-1}

In the case with $\gamma = 0$ (Tables II–IV), we fix $g_0 = 1$ or $q_{00} = 1$ and vary g_n (q_{LM}) for $n \geq 2$ ($L \geq 2$), whereas in the cases with $\gamma \neq 0$ (Tables V and VI), we vary also g_0 or q_{00}. In the Tables, we show the results with $f_{\rm sky} = 1$. One can obtain those with $f_{\rm sky} < 1$ by multiplying the values by $1/\sqrt{f_{\rm sky}}$.

Note that σ_{g_n} with $n \leq N$ has a strong dependence on the number of parameters N due to the nonvanishing off-diagonal components of the Fisher information matrix, whereas $\sigma_{q_{LM}}$ is independent of N since the corresponding Fisher information matrix is diagonal. Hence, as for σ_{g_n}, we compute their uncertainties for $N = 2,\ 4,\ 6$, respectively.

In Table III, where we assumed $r = 0.01$, we find that, when we take into account both g_4 and g_6, their uncertainties are greater than the unity even in the case with $w_{BB}^{-1/2} = 1\ \mu K$ arcmin, which leads to the difficulty of measurement of such higher-order anisotropies. On the other hand, when we take into account up to g_4, it is implied that the hexadecapole anisotropy with $g_4 = \mathcal{O}(1)$ can be detected by an observatory whose specification is similar to CMB-S4.

The 1σ uncertainties mildly depend on r, particularly in CMB-S4 and LiteBIRD, as shown in Table II with $r = 0.05$ and Table IV with $r = 0.001$. From Eq. (14), it is easy to find that the 1σ uncertainty is independent to r in the cosmic-variance limited case, whereas the uncertainty becomes simply proportional to r if the detector noise is

TABLE III. σ_{g_n} for $w_{BB}^{-1/2} = 63.1,\ 5.0,\ 1.0\ \mu$K arcmin and the CVL case with $\gamma = 0$, $f_{\rm sky} = 1$ and $r = 0.01$.

	CVL	1.0	5.0	63.1
g_2	1.93×10^{-3}	5.98×10^{-2}	2.03×10^{-1}	3.25
g_2	8.23×10^{-3}	2.66×10^{-1}	9.25×10^{-1}	1.89×10^{1}
g_4	1.24×10^{-2}	3.90×10^{-1}	1.35	2.94×10^{1}
g_2	2.33×10^{-2}	6.29×10^{-1}	1.47	1.92×10^{1}
g_4	8.27×10^{-2}	2.25	4.67	3.23×10^{1}
g_6	6.46×10^{-2}	1.80	3.67	1.11×10^{1}
q_{2M}	4.98×10^{-3}	1.26×10^{-1}	4.10×10^{-1}	9.52
q_{4M}	4.28×10^{-3}	1.53×10^{-1}	5.59×10^{-1}	1.27×10^{1}
q_{6M}	5.71×10^{-3}	1.50×10^{-1}	2.66×10^{-1}	7.55×10^{-1}

TABLE IV. σ_{g_n} for $w_{BB}^{-1/2} = 63.1,\ 5.0,\ 1.0\ \mu$K arcmin and the CVL case with $\gamma = 0$, $f_{\rm sky} = 1$ and $r = 0.001$.

	CVL	1.0	5.0	63.1
g_2	1.93×10^{-3}	1.25×10^{-1}	7.37×10^{-1}	2.37×10^{1}
g_2	8.23×10^{-3}	5.61×10^{-1}	3.96	1.39×10^{2}
g_4	1.24×10^{-2}	8.13×10^{-1}	6.08	2.15×10^{2}
g_2	2.33×10^{-2}	1.10	4.20	1.41×10^{2}
g_4	8.27×10^{-2}	3.78	8.23	2.33×10^{2}
g_6	6.46×10^{-2}	3.03	4.58	7.31×10^{1}
q_{2M}	4.98×10^{-3}	2.49×10^{-1}	1.95	6.68×10^{1}
q_{4M}	4.28×10^{-3}	3.34×10^{-1}	2.60	9.98×10^{1}
q_{6M}	5.71×10^{-3}	2.33×10^{-1}	3.13×10^{-1}	4.98

TABLE V. σ_{g_n} for $w_{BB}^{-1/2} = 63.1,\ 5.0,\ 1.0\ \mu$K arcmin and the CVL case with $\gamma = -1$ (top), $-1/2$ (bottom) and $k_0 = k_{\rm pivot}$ and $f_{\rm sky} = 1$.

	CVL	1.0	5.0	63.1
g_0	5.28×10^{-2}	9.67×10^{-2}	1.31×10^{-1}	5.93×10^{-1}
g_2	1.43×10^{-1}	2.61×10^{-1}	3.58×10^{-1}	1.64
g_0	7.23×10^{-2}	1.56×10^{-1}	2.23×10^{-1}	1.04
g_2	5.14×10^{-1}	1.25	1.85	8.65
g_4	5.76×10^{-1}	1.42	2.11	9.91
g_0	7.38×10^{-2}	1.56×10^{-1}	2.24×10^{-1}	1.04
g_2	6.02×10^{-1}	1.29	1.88	8.69
g_4	1.10	1.72	2.33	1.02×10^{1}
g_6	6.91×10^{-1}	7.08×10^{-1}	7.17×10^{-1}	1.88
q_{0M}	8.08×10^{-2}	1.49×10^{-1}	1.88×10^{-1}	8.21×10^{-1}
q_{2M}	1.51×10^{-1}	2.76×10^{-1}	3.79×10^{-1}	1.73
q_{4M}	1.56×10^{-1}	3.84×10^{-1}	5.71×10^{-1}	2.68
q_{6M}	4.70×10^{-2}	4.82×10^{-2}	4.89×10^{-2}	1.28×10^{-1}

	CVL	1.0	5.0	63.1
g_0	1.23×10^{-2}	8.26×10^{-2}	1.96×10^{-1}	1.42
g_2	3.36×10^{-2}	2.18×10^{-1}	5.26×10^{-1}	3.96
g_0	1.59×10^{-2}	1.22×10^{-1}	3.23×10^{-1}	2.38
g_2	1.06×10^{-1}	9.23×10^{-1}	2.62	1.94×10^{1}
g_4	1.17×10^{-1}	1.05	2.99	2.22×10^{1}
g_0	2.02×10^{-2}	1.27×10^{-1}	3.25×10^{-1}	2.38
g_2	2.82×10^{-1}	1.19	2.74	1.95×10^{1}
g_4	7.93×10^{-1}	2.49	3.84	2.31×10^{1}
g_6	5.75×10^{-1}	1.66	1.77	4.66
q_{0M}	1.82×10^{-2}	1.39×10^{-1}	3.07×10^{-1}	1.90
q_{2M}	3.55×10^{-2}	2.30×10^{-1}	5.56×10^{-1}	4.18
q_{4M}	3.17×10^{-2}	2.83×10^{-1}	8.09×10^{-1}	5.99
q_{6M}	3.92×10^{-2}	1.13×10^{-1}	1.20×10^{-1}	3.18×10^{-1}

TABLE VI. σ_{g_n} for $w_{BB}^{-1/2} = 63.1$, 5.0, 1.0 μK arcmin and the CVL case with $\gamma = 1/2$ (top), 1 (bottom) and $k_0 = k_{\rm pivot}$ and $f_{\rm sky} = 1$.

	CVL	1.0	5.0	63.1
g_0	2.00×10^{-4}	2.31×10^{-2}	8.37×10^{-2}	5.37
g_2	5.47×10^{-4}	5.94×10^{-2}	2.08×10^{-1}	1.48×10^{1}
g_0	2.55×10^{-4}	3.37×10^{-2}	1.25×10^{-1}	8.29
g_2	1.67×10^{-3}	2.53×10^{-1}	9.56×10^{-1}	6.49×10^{1}
g_4	1.84×10^{-3}	2.86×10^{-1}	1.09	7.37×10^{1}
g_0	3.29×10^{-4}	4.36×10^{-2}	1.61×10^{-1}	8.31
g_2	4.67×10^{-3}	6.34×10^{-1}	2.32	6.59×10^{1}
g_4	1.32×10^{-2}	1.77	6.44	8.12×10^{1}
g_6	9.59×10^{-3}	1.28	4.65	2.48×10^{1}
q_{0M}	2.92×10^{-4}	4.20×10^{-2}	1.67×10^{-1}	7.49
q_{2M}	5.78×10^{-4}	6.28×10^{-2}	2.19×10^{-1}	1.57×10^{1}
q_{4M}	4.97×10^{-4}	7.74×10^{-2}	2.94×10^{-1}	1.99×10^{1}
q_{6M}	6.53×10^{-4}	8.71×10^{-2}	3.17×10^{-1}	1.69

	CVL	1.0	5.0	63.1
g_0	2.11×10^{-5}	1.10×10^{-2}	4.19×10^{-2}	4.01
g_2	5.76×10^{-5}	2.80×10^{-2}	1.02×10^{-1}	1.02×10^{1}
g_0	2.72×10^{-5}	1.61×10^{-2}	6.18×10^{-2}	5.95
g_2	1.80×10^{-4}	1.22×10^{-1}	4.66×10^{-1}	4.51×10^{1}
g_4	1.99×10^{-4}	1.38×10^{-1}	5.30×10^{-1}	5.12×10^{1}
g_0	3.47×10^{-5}	2.10×10^{-2}	8.33×10^{-2}	6.04
g_2	4.88×10^{-4}	3.06×10^{-1}	1.26	5.02×10^{1}
g_4	1.38×10^{-3}	8.53×10^{-1}	3.56	8.40×10^{1}
g_6	9.98×10^{-4}	6.17×10^{-1}	2.58	4.88×10^{1}
q_{0M}	3.13×10^{-5}	2.05×10^{-2}	8.61×10^{-2}	7.59
q_{2M}	6.08×10^{-5}	2.96×10^{-2}	1.08×10^{-1}	1.08×10^{1}
q_{4M}	5.38×10^{-5}	3.73×10^{-2}	1.43×10^{-1}	1.38×10^{1}
q_{6M}	6.80×10^{-5}	4.20×10^{-2}	1.76×10^{-1}	3.32

dominated in the wide range of ℓ, $C_\ell^{BB} \ll \mathcal{N}_\ell$. In fact, CMB-S4 and LiteBIRD are capable of detecting PGWs with $r \sim 0.001$ from the B-mode signal. That is why the uncertainties for these obsevatories behave in between the two limits. As a result, we find that even in the case with $r = 0.001$ CMB-S4-like observations can marginally detect the anisotropies up to g_4.

In Tables V and VI, we estimate the uncertainties with various γ with $r = 0.01$. In the red-tilted cases, our results even with $\gamma = -1$ indicate the possibility to detect g_2 by LiteBIRD and CMB-S4, while it is fairly difficult to get a signal of g_4 even with CMB-S4. On the other hand, in the blue-tilted cases, we can marginally detect g_4, since much power is induced to the angular power spectrum on large ℓ. Note that, in Ref. [28], the authors reported the uncertainties with $\gamma = -2$, $\sigma_{q_{0M}} = 30$ and $\sigma_{q_{2M}} = 58$ in our notations. In the present study, we obtained $\sigma_{g_0} = 20.6$ and $\sigma_{g_2} = 56.4$ with $\gamma = -2$ in the CVL case, which are well consistent to the previous results.

Let us compare our result with theoretical predictions. The model in Ref. [26] predicts $g_0 = 1$, $g_2 = -1$, $g_4 = 1$, $g_6 = -1$ irrespective of the model parameters, while $\gamma \lesssim -1/2$ is required to produce a detectable amplitude of the sourced PGW (i.e., $r_{\rm source} \gtrsim 10^{-3}$). In the case of $\gamma = -1$, $-1/2$ (Table V), these result show that the predicted g_0 and g_2 are marginally detectable, whereas it is challenging to measure g_4 and g_6 even with the CMB-S4 experiment at 1σ level. Let us also consider the discrimination between the models. The prediction in Ref. [27] is $g_0 = 1$, $g_2 = 1$, $g_4 = -2$, $g_6 = 1$, and the sign of g_2 is flipped from that of Ref. [26]. This difference is originated in the distinction between the particle types which generate the PGWs (i.e., U(1) gauge field or two-form field). Therefore, once $g_n (n \geq 2)$ is detected, we may gain an insight what type of particle plays an important role in the primordial universe.

Note that, although we fix $k_0 = k_{\rm pivot}$ in Tables V and VI, the uncertainties with different k_0 can be easily obtained by use of the scaling, $\sigma_{g_n}, \sigma_{q_{LM}} \propto k_0^{\gamma}$, since Eq. (8) is proportional to $k_0^{-\gamma}$ and $\sigma_{g_n}, \sigma_{q_{LM}}$ is thus proportional to the inverse of Eq. (8).

IV. CONCLUSION

We investigated the detectability of the statistical anisotropies of the primordial tensor power spectrum using the Fisher information matrix assuming the observations by CMB-S4, LiteBIRD and Planck. We parametrize the primordial tensor power spectrum in Eq. (3) with Eqs. (6) and (9), and estimate the 1σ uncertainties of q_{LM} and g_n given in Eq. (15) with the fiducial values $q_{LM} = g_n = 0$ for L (or n) ≥ 2 in the case of $\gamma = 0$, and $q_{LM} = g_n = 0$ for L (or n) ≥ 0 in the case of $\gamma \neq 0$.

Our results are tabulated in Tables II–VI. In Table III, we find that a relatively large statistical anisotropy $g_n \sim \mathcal{O}(0.1)$ would possibly be detected by CMB-S4 even with $r = 0.001$ as long as we take into account up to g_4, and by LiteBIRD up to g_2, whereas unfortunately the results imply difficulties to detect anisotropies by Planck since it is contaminated by large noises. In addition, in order to detect a higher multipole coefficient g_6, we need further observatories whose noise level is much more suppressed than CMB-S4.

Note that, in our present study, we do not take the contamination from the CMB-lensing, unwanted B-mode signal converted from the E-mode through the gravitational interaction, into account. In the actual observations, the detectability of the anisotropies depends on how well we can remove the contamination, namely, delensing. Roughly speaking, this effect can be included in our result by increasing $w_{BB}^{-1/2}$ defined in Eq. (17). If the delensing is not perfectly performed, the detectability of the anisotropies by CMB-S4 will be worse.

ACKNOWLEDGMENTS

This work was partially supported by JSPS KAKENHI No. JP16K17695 (T. H.), No. JP15K17659 (S. Y.) and No. JP17J09103 (T. F.) and also MEXT KAKENHI No. 15H05888 (S. Y.) and No. 18H04356 (S. Y.). T. H. was also supported by the MEXT Supported Program for the Strategic Research Foundation at Private Universities (S1411024).

APPENDIX: FISHER INFORMATION MATRIX IN THE ANISOTROPIC CASES

The covariance matrix taking into account the correlations between different ℓ's is given by

$$\boldsymbol{C}_{\ell_1 m_1;\ell_2 m_2} = \begin{pmatrix} C^{TT}_{\ell_1 m_1;\ell_2 m_2} & C^{TE}_{\ell_1 m_1;\ell_2 m_2} & 0 \\ C^{TE}_{\ell_1 m_1;\ell_2 m_2} & C^{EE}_{\ell_1 m_1;\ell_2 m_2} & 0 \\ 0 & 0 & C^{BB}_{\ell_1 m_1;\ell_2 m_2} \end{pmatrix}. \tag{A1}$$

The Fisher information matrix based on this covariance matrix is given as [35,36]

$$F_{ij} = \frac{f_{\rm sky}}{2} \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} {\rm Tr}\left[\boldsymbol{C}^{-1}_{\ell_1} \frac{\partial \boldsymbol{C}_{\ell_1 m_1;\ell_2 m_2}}{\partial \theta_i} \boldsymbol{C}^{-1}_{\ell_2} \frac{\partial \boldsymbol{C}_{\ell_2 m_2;\ell_1 m_1}}{\partial \theta_j}\right] \tag{A2}$$

$$= f_{\rm sky} \sum_{XY} \sum_{\ell_1 m_1} \sum_{\ell_2 m_2} \frac{\partial C^X_{\ell_1 m_1;\ell_2 m_2}}{\partial \theta_i} (\mathcal{C}^{-1}_{\ell_1 \ell_2})^{XY} \frac{\partial C^Y_{\ell_2 m_2;\ell_1 m_1}}{\partial \theta_j}, \tag{A3}$$

where $f_{\rm sky}$ denotes the fraction of the sky covered, $X, Y = TT, TE, EE, BB$ and

$$\mathcal{C}^{-1}_{\ell_1\ell_2} = \frac{1}{2} \begin{pmatrix} C^{EE}_{\ell_1} C^{EE}_{\ell_2}/\Delta_{\ell_1\ell_2} & -\mathcal{C}^{TE,EE}_{\ell_1\ell_2}/\Delta_{\ell_1\ell_2} & C^{TE}_{\ell_1} C^{TE}_{\ell_2}/\Delta_{\ell_1\ell_2} & 0 \\ -\mathcal{C}^{TE,EE}_{\ell_1\ell_2}/\Delta_{\ell_1\ell_2} & (\mathcal{C}^{TT,EE}_{\ell_1\ell_2} + 2C^{TE}_{\ell_1} C^{TE}_{\ell_2})/\Delta_{\ell_1\ell_2} & -\mathcal{C}^{TT,TE}_{\ell_1\ell_2}/\Delta_{\ell_1\ell_2} & 0 \\ C^{TE}_{\ell_1} C^{TE}_{\ell_2}/\Delta_{\ell_1\ell_2} & -\mathcal{C}^{TT,TE}_{\ell_1\ell_2}/\Delta_{\ell_1\ell_2} & C^{TT}_{\ell_1} C^{TT}_{\ell_2}/\Delta_{\ell_1\ell_2} & 0 \\ 0 & 0 & 0 & \frac{1}{C^{BB}_{\ell_1} C^{BB}_{\ell_2}} \end{pmatrix}, \tag{A4}$$

where

$$\Delta_{\ell_1\ell_2} = [C^{TT}_{\ell_1} C^{EE}_{\ell_1} - (C^{TE}_{\ell_1})^2][C^{TT}_{\ell_2} C^{EE}_{\ell_2} - (C^{TE}_{\ell_2})^2], \tag{A5}$$

$$\mathcal{C}^{XY}_{\ell_1\ell_2} = C^X_{\ell_1} C^Y_{\ell_2} + C^Y_{\ell_1} C^X_{\ell_2}, \tag{A6}$$

and

$$\tilde{C}^X_\ell \coloneqq C^X_\ell + \mathcal{N}^X_\ell, \tag{A7}$$

with $\mathcal{N}^X_\ell$ being the noises for the detection of $X = TT, TE, EE, BB$. The angular power spectrum C^X_ℓ with $X = BB$ is given by Eq. (4) with $\ell_1 = \ell_2 = \ell$ and $\gamma = 0$, and those of $X = TT, TE, EE$ can be also calculated by replacing the transfer functions in Eq. (4) with the corresponding ones. If we choose $\{\theta_i\} = \{q_{LM}\}$, the Fisher matrix becomes

$$F_{LM;L'M'} = \delta_{LL'} \delta_{MM'} F_L, \tag{A8}$$

where

$$F_L = \frac{f_{\rm sky}}{4\pi} \sum_{s_1 s_2 s_3 s_4} (-1)^{s_1+s_3} \sum_{\ell_1 \ell_2} (2l_1+1)(2l_2+1) \times \begin{pmatrix} l_1 & l_2 & L \\ -s_1 & s_2 & s_1 - s_2 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & L \\ -s_3 & s_4 & s_3 - s_4 \end{pmatrix} \times \sum_{XY} C^{s_1 s_2 X}_{\ell_1 \ell_2} (\mathcal{C}^{-1}_{\ell_1 \ell_2})^{XY} C^{s_3 s_4 Y}_{\ell_2 \ell_1}, \tag{A9}$$

Alternatively, if we choose $\{\theta_i\} = \{g_n\}$, we have

$$F_{mn} = \sum_{LM} \frac{\partial q_{LM}}{\partial g_m} \frac{\partial q_{LM}}{\partial g_n} F^{XY}_L. \tag{A10}$$

The coefficients in the right-hand side are found to be

$$\frac{\partial q_{LM}}{\partial g_n} = \delta_{M0} \sqrt{2L+1} \sqrt{\pi} \int_{-1}^{1} \mu^n P_L(\mu) d\mu, \tag{A11}$$

where $P_L(\mu)$ is the Legendre polynomials.

[1] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A20 (2016).
[2] P. A. R. Ade *et al.* (BICEP2 and Keck Array Collaborations), Phys. Rev. Lett. **116**, 031302 (2016).
[3] Y. Akrami *et al.* (Planck Collaboration), arXiv:1807.06211.
[4] T. Matsumura *et al.*, J. Low Temp. Phys. **176**, 733 (2014).
[5] K. N. Abazajian *et al.* (CMB-S4 Collaboration), arXiv:1610.02743.
[6] R. Namba, M. Peloso, M. Shiraishi, L. Sorbo, and C. Unal, J. Cosmol. Astropart. Phys. 01 (2016) 041.
[7] E. Dimastrogiovanni, M. Fasiello, and T. Fujita, J. Cosmol. Astropart. Phys. 01 (2017) 019.
[8] M. Shiraishi, C. Hikage, R. Namba, T. Namikawa, and M. Hazumi, Phys. Rev. D **94**, 043506 (2016).
[9] A. Agrawal, T. Fujita, and E. Komatsu, Phys. Rev. D **97**, 103526 (2018).
[10] M. a. Watanabe, S. Kanno, and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009).
[11] M. a. Watanabe, S. Kanno, and J. Soda, Prog. Theor. Phys. **123**, 1041 (2010).
[12] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, Phys. Rev. D **87**, 023504 (2013).
[13] T. Fujita and I. Obata, J. Cosmol. Astropart. Phys. 01 (2018) 049.
[14] A. A. Abolhasani, R. Emami, and H. Firouzjahi, J. Cosmol. Astropart. Phys. 05 (2014) 016.
[15] N. Bartolo, S. Matarrese, M. Peloso, and A. Ricciardone, J. Cosmol. Astropart. Phys. 08 (2013) 022.
[16] M. Akhshik, R. Emami, H. Firouzjahi, and Y. Wang, J. Cosmol. Astropart. Phys. 09 (2014) 012.
[17] N. Bartolo, M. Peloso, A. Ricciardone, and C. Unal, J. Cosmol. Astropart. Phys. 11 (2014) 009.
[18] A. Kehagias and A. Riotto, J. Cosmol. Astropart. Phys. 07 (2017) 046.
[19] G. Franciolini, A. Kehagias, and A. Riotto, J. Cosmol. Astropart. Phys. 02 (2018) 023.
[20] M. a. Watanabe, S. Kanno, and J. Soda, Mon. Not. R. Astron. Soc. **412**, L83 (2011).
[21] X. Chen, R. Emami, H. Firouzjahi, and Y. Wang, J. Cosmol. Astropart. Phys. 08 (2014) 027.
[22] N. Bartolo, S. Matarrese, M. Peloso, and M. Shiraishi, J. Cosmol. Astropart. Phys. 01 (2015) 027.
[23] N. Bartolo, A. Kehagias, M. Liguori, A. Riotto, M. Shiraishi, and V. Tansella, Phys. Rev. D **97**, 023503 (2018).
[24] G. Franciolini, A. Kehagias, A. Riotto, and M. Shiraishi, Phys. Rev. D **98**, 043533 (2018).
[25] J. Kim and E. Komatsu, Phys. Rev. D **88**, 101301 (2013).
[26] T. Fujita, I. Obata, T. Tanaka, and S. Yokoyama, J. Cosmol. Astropart. Phys. 07 (2018) 023.
[27] I. Obata and T. Fujita, arXiv:1808.00548.
[28] M. Shiraishi, D. F. Mota, A. Ricciardone, and F. Arroja, J. Cosmol. Astropart. Phys. 07 (2014) 047.
[29] N. Bartolo, A. Hoseinpour, G. Orlando, S. Matarrese, and M. Zarei, Phys. Rev. D **98**, 023518 (2018).
[30] A. Valentini, arXiv:1510.02523.
[31] N. Katayama and E. Komatsu, Astrophys. J. **737**, 78 (2011).
[32] T. Hiramatsu, E. Komatsu, M. Hazumi, and M. Sasaki, Phys. Rev. D **97**, 123511 (2018).
[33] P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016).
[34] J. Tauber *et al.* (Planck Collaboration), arXiv:astro-ph/0604069.
[35] L. Verde, Lect. Notes Phys. **800**, 147 (2010).
[36] Y. Z. Ma, G. Efstathiou, and A. Challinor, Phys. Rev. D **83**, 083005 (2011); **89**, 129901(E) (2014).