

# Nonminimally coupled pseudoscalar inflaton

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We consider a scenario in which the inflaton  $\phi$  is a pseudoscalar field nonminimally coupled to gravity through a term of the form  $\mathcal{K}R\phi^2$ . The pseudoscalar is also coupled to a  $U(1)$  gauge field (or an ensemble of  $\mathcal{N}$  gauge fields) through an axial coupling of the form  $\phi F\tilde{F}$ . After [M. M. Anber and L. Sorbo, *Phys. Rev. D* **81**, 043534 (2010)], it is well known that this axial coupling leads to a production of gauge particles which acts as a friction term in the dynamics of the inflaton, producing a slow-roll regime even in presence of a steep potential. A remarkable result in this scenario is that the spectrum of the chiral gravitational waves sourced by the scalar-gauge field interplay can be enhanced due to the nonminimal coupling with gravity, leading to measurable signatures, while maintaining agreement with current observational constraints on  $n_s$  and  $r$ . The inclusion of nonminimal coupling could be helpful to alleviate tensions with non-Gaussianity bounds in models including axial couplings.

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## I. INTRODUCTION

In this work we revisit the scenario in which a pseudoscalar field is responsible for driving inflation. We consider a pseudoscalar inflaton coupled to a set of  $\mathcal{N}$   $U(1)$  gauge fields through an axial coupling term of the form  $(\alpha/f)\phi F\tilde{F}$ , where  $\tilde{F}$  is the dual of the field strength  $F$  of the gauge field  $A_\mu$ . This scenario was studied in Refs. [1,2] with a steep natural cosine potential and then in Refs. [3–5] for arbitrary potentials supporting slow-roll evolution. This model presents very appealing possibilities such as the sourcing of chiral gravitational waves due to the axial coupling [2–6], the raising of signatures of parity-violating and anisotropic correlations [7,8], possible connections with magnetogenesis [9,10], generation of the baryon asymmetry of the Universe [6], and production of primordial black holes [11] among others. Despite several of its appealing features, this mechanism is severely constrained given that the generation of any measurable signature is unavoidably accompanied with the production of a large amount of non-Gaussianities in the correlations [3,5,12]. It was also pointed out that the model might suffer from severe perturbativity constraints [13], but recent analyses [14] claim that in the perturbative regime

the model still allows for observable signals at cosmic microwave background (CMB) scales.<sup>1</sup>

A number of studies have focused on different versions of models including axial couplings and have addressed several theoretical and phenomenological aspects of them. An incomplete list of references includes [10–29]. Very recently, a closely related approach, studying the production of fermions through a derivative coupling with a pseudoscalar inflaton, was addressed [30].

Moreover, nonminimal couplings to gravity in the context of inflation have a long history. The main interest in some early references on the subject [31–37] was to avoid (or rephrase) fine-tuning problems in chaotic inflationary models. Later, the interest was boosted by the study of Higgs inflation [38], Higgs-dilaton models [39] and  $\alpha$  attractors [40–44]. The case of multifield inflationary models with nonminimal couplings to gravity [45–47] was also considered, and more recently there have been several works on nonminimally coupled inflaton in non-metric formulations of gravity [48–50].

<sup>1</sup>However, in the case considered here, the backreaction due to the gauge field production is responsible for the slow-roll evolution; the backreaction constraints are therefore broken by construction. We assume perturbativity of the scalar and tensor perturbations but we consider that the classical background is driven by the nonperturbative gauge field production process.

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The aim of this work is to understand how the introduction of a nonminimal coupling to gravity affects the dynamics and the inflationary predictions, in models where an axion plays the role of the inflaton. To this end, we focus specifically in the axion inflation model with a naturally steep potential model presented in Refs. [1,2] in the presence of nonminimal coupling with gravity. In this context, the possibility of generating dark matter from primordial black holes was recently discussed [51] in the case of the so-called  $T$  models [40–43] which lead to the class of  $\alpha$  attractors. Complementary to that discussion, here we consider the perturbations in a steep cosinelike potential. We perform a detailed calculation of the spectrum of scalar and tensor perturbations, and we extract the spectral index of the scalar perturbations  $n_s$ , together with the tensor-to-scalar ratio  $r$  in order to contrast them with current observational bounds. An interesting result in this case is that, instead of a global suppression effect, for certain regions of the parameter space the nonminimal coupling term suppresses vacuum gravitational waves due to the scalar-gravity interaction, but at the same time, it acts as an enhancer for the sourced gravitational waves due to the axial coupling. The interest in this combined effect is twofold: On the one hand, the sourced gravitational waves are chiral because of the preferred helicity in this model, a fact that can be seen as a signature of parity violation in the CMB correlations. Additionally, the predicted tensor-to-scalar ratio could be within the reach of current or planned future experiments such as BICEP3 [52], LiteBIRD [27,53], and the Simons Observatory [54]. On the other hand, we want to explore the constraints coming from interactions between the inflaton, massless gauge fields, and gravity, leading to particular signatures and predictions.

The paper is organized as follows: In Sec. II we briefly revisit the axion dynamics in the minimally coupled inflationary setup. In Sec. II A we discuss scalar and tensor perturbations and we review the calculation of the corresponding correlators. We calculate  $n_s$  and  $r$ , and we also comment on the constraints coming from non-Gaussianities. In Sec. III we introduce nonminimal coupling with gravity for the pseudoscalar field and study its implications in the correlation functions and other observables. Then, Sec. IV presents our main results, where we scan the parameter space and compare it with Planck measurements of  $n_s$  and  $r$ . Finally, in Sec. V we end up with our conclusions.

## II. MINIMALLY COUPLED PSEUDOSCALAR AND GAUGE FIELD DYNAMICS

Here, we briefly summarize the particle production mechanism due to the pseudoscalar-vector coupling. We revisit some details of the amplification of the gauge fields depending on the helicity of the vector mode [1–5,9,12,14–17,20,22,25].

Starting from the Lagrangian for  $\mathcal{N}$   $U(1)$  vector fields coupled to the axion field  $\phi^2$

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{\mathcal{N}}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right], \quad (1)$$

we derive the system of equations of motion for the pseudoscalar and the vector fields

$$\square\phi - V_\phi - \frac{\mathcal{N}\alpha}{4f} F^{\mu\nu} \tilde{F}_{\mu\nu} = 0, \quad (2)$$

$$\nabla_\mu \left( F^{\mu\nu} + \frac{\alpha}{f} \phi \tilde{F}^{\mu\nu} \right) = 0, \quad (3)$$

where  $V_\phi \equiv \partial V / \partial \phi$ . For the vector field we also add the Bianchi identity  $\nabla_\mu \tilde{F}^{\mu\nu} = 0$ . We work with the vector potential  $A_\mu$  corresponding to the field strength  $F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$ , in the Coulomb gauge  $A_0 = \partial_i A_i = 0$ . We consider the de Sitter (or quasi-de Sitter) background metric

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2 = a^2(\tau) [-d\tau^2 + d\vec{x}^2], \quad (4)$$

where  $a \approx -1/(H\tau)$  and  $\tau$  is the conformal time. The gradient of the pseudoscalar field  $\vec{\nabla}\phi = 0$  is neglected, since homogeneous background solutions are considered. All in all, Eqs. (2) and (3) are reduced to

$$\phi'' + 2aH\phi' + a^2 V_\phi = a^2 \frac{\mathcal{N}\alpha}{f} \vec{E} \cdot \vec{B}, \quad (5)$$

$$A_i'' - \nabla^2 A_i - \frac{\alpha}{f} \phi' \epsilon_{ijk} \nabla_j A_k = 0, \quad (6)$$

where primes denote derivatives with respect to conformal time and the electric and magnetic components are defined as

$$E_i = -\frac{1}{a(\tau)^2} F_{0i} = -\frac{1}{a(\tau)^2} \frac{\partial A_i}{\partial \tau} \quad \text{and} \quad B_i = \frac{1}{a(\tau)^2} \tilde{F}_{0i} = \frac{1}{a(\tau)^2} \epsilon_{ijk} \nabla_j A_k. \quad (7)$$

With these definitions it is possible to check that

$$F\tilde{F} = -4\vec{E} \cdot \vec{B} \quad \text{and} \quad F^2 = 2(\vec{B}^2 - \vec{E}^2). \quad (8)$$

The dynamics of the system is completed with Einstein's equations:

<sup>2</sup>Throughout this paper the signature  $(-, +, +, +)$  is used.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_P^2}T_{\mu\nu}, \quad (9)$$

where the energy-momentum tensor is

$$\begin{aligned} T_{\mu\nu} &= \frac{-2}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}} \\ &= \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left( \frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi + V \right) \\ &\quad + \mathcal{N} F_{\mu\alpha} F_\nu{}^\alpha - g_{\mu\nu} \frac{\mathcal{N}}{4} F^2. \end{aligned} \quad (10)$$

The Friedmann equations obtained from the Einstein equations (9) are

$$H^2 = \frac{1}{3M_P^2} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{\mathcal{N}}{2} (\vec{E}^2 + \vec{B}^2) \right), \quad (11)$$

$$\dot{H} = -\frac{1}{2M_P^2} \dot{\phi}^2 - \frac{\mathcal{N}}{3M_P^2} (\vec{E}^2 + \vec{B}^2). \quad (12)$$

To quantize the system, we write the vector potential as an operator and decompose it in terms of creation and annihilation operators as

$$\hat{A}_i(\tau, \vec{x}) = \sum_{\lambda=\pm} \int \frac{d^3k}{(2\pi)^{3/2}} [e_\lambda^i(\vec{k}) A_\lambda(\tau, \vec{k}) \hat{a}_\lambda(\vec{k}) e^{i\vec{k}\cdot\vec{x}} + \text{c.c.}], \quad (13)$$

where the (transverse) polarization vectors are defined as such:

$$\begin{aligned} \vec{e}_\lambda(\vec{k}) \cdot \vec{k} &= 0, \quad \vec{k} \times \vec{e}_\lambda = -i\lambda |\vec{k}| \vec{e}_\lambda, \\ \vec{e}_\lambda \cdot \vec{e}_{\lambda'} &= \delta_{\lambda, -\lambda'}, \quad \vec{e}_\lambda^*(\hat{k}) = \vec{e}_{-\lambda}(\hat{k}) = \vec{e}_\lambda(-\hat{k}). \end{aligned} \quad (14)$$

Then, the vector equation in momentum space is written as

$$A_\pm'' + \left( k^2 \mp \frac{\alpha k}{f} \phi' \right) A_\pm = 0. \quad (15)$$

During the inflationary regime, the classical background solution of the pseudoscalar field is such that  $\frac{d\phi}{dt} = \dot{\phi}_0 \approx \text{constant}$ . Then

$$\phi' = \frac{d\phi}{dt} \frac{dt}{d\tau} = -\frac{1}{H\tau} \frac{d\phi}{dt} \approx -\frac{1}{H\tau} \dot{\phi}_0 \quad (16)$$

and Eq. (15) becomes

$$A_\pm'' + \left( k^2 \pm \frac{2k\xi}{\tau} \right) A_\pm = 0 \quad \text{with} \quad \xi \equiv \frac{\alpha \dot{\phi}_0}{2fH}. \quad (17)$$

Equation (17) is the starting point for the study of the amplification of the gauge fields. Notice that this equation shows manifestly the parity-violating nature of the system, since one of the modes ( $A_+$ ) is amplified,<sup>3</sup> while the other ( $A_-$ ) is exponentially suppressed (since  $\tau < 0$ ). The solution consistent with the Bunch-Davies vacuum initial conditions for the regime  $|k\tau| \ll 2\xi$  is [1]

$$A_+ \approx \frac{1}{\sqrt{2k}} \left( \frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}}. \quad (18)$$

Here we notice that the  $A_+$  mode is exponentially amplified by the factor  $e^{\pi\xi}$ . Hence, the size of the backreaction effects due to the presence of the vector field coupling [1] are

$$\begin{aligned} \langle \vec{E} \cdot \vec{B} \rangle &\approx -\mathcal{I} \frac{H^4}{\xi^4} e^{2\pi\xi}, \\ \frac{1}{2} \langle \vec{E}^2 + \vec{B}^2 \rangle &\approx \frac{4\mathcal{I}}{7} \frac{H^4}{\xi^3} e^{2\pi\xi}, \quad \text{with} \quad \mathcal{I} \approx 2.4 \times 10^{-4}. \end{aligned} \quad (19)$$

### A. Perturbations

Now we can write the equations for the perturbations of the system including gravity, the inflaton, and the gauge fields. They can be obtained varying Eq. (5) and taking the decomposition for the inflaton as a background value plus a perturbation of the field  $\phi(\tau, x) = \phi_0(\tau) + \delta\phi(\tau, x)$ :

$$\phi_0'' + 2aH\phi_0' + a^2 V_\phi = a^2 \frac{\mathcal{N}\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle, \quad (20)$$

$$\delta\phi'' + 2aH\delta\phi' - \nabla^2 \delta\phi + a^2 V_{\phi\phi} \delta\phi = a^2 \frac{\mathcal{N}\alpha}{f} \delta[\vec{E} \cdot \vec{B}]. \quad (21)$$

The original calculation was performed in Ref. [1]; for completeness we revisit all the relevant details in Appendix A. For small inhomogeneities the  $\nabla^2$  term can be neglected. We consider also that  $V \approx 3M_P^2 H^2$  and  $V_\phi \approx V/f$ . Assuming  $\alpha \gg 1$  and  $f \lesssim M_P$ , which means that the dominant contribution comes from the backreaction term  $\delta[\vec{E} \cdot \vec{B}]$ , Eq. (21) becomes

$$\delta\phi'' + \frac{1}{\tau} \frac{\alpha\pi V_\phi(\phi_0)}{fH^2} \delta\phi' + \frac{1}{\tau^2} \frac{V_{\phi\phi}(\phi_0)}{H^2} \delta\phi \approx a^2 \frac{\mathcal{N}\alpha}{f} \delta_{\vec{E}\cdot\vec{B}}, \quad (22)$$

which, in Fourier space, can be formally solved as

$$\delta\phi(\tau, \vec{k}) = \frac{\mathcal{N}\alpha}{f} \int_{-\infty}^{\tau} d\tau_1 a^2(\tau_1) G(\tau, \tau_1) \int d^3x e^{-i\vec{k}\cdot\vec{x}} \delta_{\vec{E}\cdot\vec{B}}(\tau_1, \vec{x}). \quad (23)$$

<sup>3</sup>Taking  $\alpha > 0$  and  $\xi > 0$ .

$G(\tau, \tau_1)$  is the solution of the Green's function associated with Eq. (22), which is

$$\frac{\partial^2 G(\tau, \tau')}{\partial \tau^2} + \frac{1}{\tau} \frac{\alpha \pi V_\phi(\phi_0)}{f H^2} \frac{\partial G(\tau, \tau')}{\partial \tau} + \frac{1}{\tau^2} \frac{V_{\phi\phi}(\phi_0)}{H^2} G(\tau, \tau') = \delta(\tau - \tau'), \quad (24)$$

with the boundary conditions  $G(\tau', \tau') = 0$ ,  $\partial G(\tau', \tau') / \partial \tau = 1$ . The solution of Eq. (24) is

$$G(\tau, \tau') = \frac{\tau'}{\Delta} \left[ \left( \frac{\tau}{\tau'} \right)^{\nu_+} - \left( \frac{\tau}{\tau'} \right)^{\nu_-} \right] \Theta(\tau - \tau'), \quad (25)$$

where

$$\nu_{\pm} \equiv \frac{1}{2} \left( 1 - \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \right) \pm \frac{1}{2} \Delta, \quad \text{and} \quad \Delta \equiv \sqrt{\left( 1 - \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \right)^2 - \frac{4 V_{\phi\phi}(\phi_0)}{H^2}}. \quad (26)$$

Notice that, even if the homogeneous solution is suppressed by a  $1/\Delta$  factor, the nonhomogeneous solution sourced by Eq. (23) is enhanced by the term  $\frac{a}{f} \delta_{\vec{E} \cdot \vec{B}}$ , allowing perturbations to grow large. Using Eq. (25) one can obtain the  $n$ -point correlator as

$$\begin{aligned} \langle \delta\phi(\vec{p}_1) \cdots \delta\phi(\vec{p}_n) \rangle &= \delta(\vec{p}_1 + \cdots + \vec{p}_n) \left( \frac{\mathcal{N} \alpha}{f} \right)^n \int d\tau_1 \cdots d\tau_n a_1^2 \cdots a_n^2 G(\tau, \tau_1) \cdots G(\tau, \tau_n) \\ &\times \int d^3 x_1 \cdots d^3 x_n e^{-i(\vec{p}_1 \cdot \vec{x}_1 + \cdots + \vec{p}_n \cdot \vec{x}_n)} \langle \delta_{\vec{E} \cdot \vec{B}}(\tau_1, \vec{x}_1) \cdots \delta_{\vec{E} \cdot \vec{B}}(\tau_n, \vec{x}_n) \rangle, \end{aligned} \quad (27)$$

where  $a_i \equiv a(\tau_i)$ .

## B. Power spectrum

From Eq. (27) the spectrum of the perturbations is

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &= \delta(\vec{p} + \vec{p}') \left( \frac{\mathcal{N} \alpha}{f} \right)^2 \int d\tau_1 d\tau_2 a_1^2 a_2^2 G(\tau, \tau_1) G(\tau, \tau_2) \int d^3 x e^{i\vec{p} \cdot \vec{x}} \langle \delta_{\vec{E} \cdot \vec{B}}(\tau_1, 0) \delta_{\vec{E} \cdot \vec{B}}(\tau_2, \vec{x}) \rangle \\ &\approx \mathcal{F}(\nu_+) \frac{\delta(\vec{p} + \vec{p}') \mathcal{N}^2 \alpha^2 H^4}{p^3 \Delta^2 f^2 \xi^8} e^{4\pi\xi} (-2^5 \xi p \tau)^{2\nu_+}. \end{aligned} \quad (28)$$

As discussed in Appendix A, the dependence of  $\mathcal{F}$  on  $\nu_+$  is very small. We can thus safely take  $\mathcal{F} \equiv \mathcal{F}(0) \approx 2.13 \times 10^{-6}$ . With the previous results we can calculate the power spectrum of the primordial curvature perturbation  $\zeta = -H\delta\phi/\dot{\phi}_0$

$$\begin{aligned} \delta(\vec{p} + \vec{p}') \mathcal{P}_\zeta(p) &= \frac{p^3}{2\pi^2} \frac{H^2}{\dot{\phi}_0^2} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle \\ &\approx \delta(\vec{p} + \vec{p}') \frac{\mathcal{F}}{2\pi^2} \frac{H^2 \mathcal{N}^2 \alpha^2 H^4}{\dot{\phi}_0^2 \Delta^2 f^2 \xi^8} \\ &\times e^{4\pi\xi} (-2^5 \xi p \tau)^{2\nu_+}. \end{aligned} \quad (29)$$

Now, considering that  $V_\phi \approx \frac{\mathcal{N} \alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle$  and using Eq. (19) one has

$$\frac{V_\phi^2(\phi_0)}{\mathcal{I}^2} \approx \frac{\mathcal{N}^2 \alpha^2 H^8}{f^2 \xi^8} e^{4\pi\xi}. \quad (30)$$

Expressing  $\dot{\phi}_0$  in terms of  $\xi$ , and using Eq. (A14) for  $\Delta$  results in

$$\mathcal{P}_\zeta(p) \approx \frac{\mathcal{F}(\nu_+)}{8\pi^4 \mathcal{I}^2 \mathcal{N} \xi^2} (-2^5 \xi p \tau)^{2\nu_+} \approx \frac{5 \times 10^{-2}}{\mathcal{N} \xi^2} (-2^5 \xi p \tau)^{2\nu_+}, \quad (31)$$

where we used the fact that the contribution of the  $\mathcal{N}$  gauge fields adds incoherently to the spectrum, which results in a suppression by a  $1/\mathcal{N}$  factor. Here we notice that, as pointed out in Ref. [1] for the minimally coupled case, in order to reproduce the observed amplitude of the scalar spectrum it is necessary to have a large number of gauge fields  $\mathcal{N} \sim 10^5$ , for  $\xi \sim 10$ . This is an unnatural feature of the model that persists in the presence of nonminimal coupling, as we will see in Sec. III C. Possible ways out of this problem are string theory inspired models containing a large number  $\mathcal{N}$  of branes, each one with an associated  $U(1)$  gauge field, or large symmetry groups like  $SU(\sqrt{\mathcal{N}})$ . Another possibility is to assume a slow-roll potential instead of the cosinlike, or to use auxiliary scalar fields instead of the inflaton coupled to the  $U(1)$  gauge field, non-Abelian symmetry groups, or massive vector fields [1]. Such scenarios are, however, not considered here.

From the previous expression we extract the spectral index [1]<sup>4</sup>

$$n_s - 1 \approx 2\nu_+ = -\frac{2fV_{\phi\phi}(\phi_0)}{\pi\alpha V_{\phi}(\phi_0)}. \quad (32)$$

### C. Tensor perturbations

The calculation of the spectrum of the tensor perturbations in this model was originally done in Refs. [2,4,20]; here we briefly revisit the main details. The tensor perturbations with polarization  $\lambda$  are obtained by solving the equation

$$h''_{\lambda} - \frac{2}{\tau} h'_{\lambda} + k^2 h_{\lambda} = \frac{2}{M_P^2} \Pi_{\lambda}^{lm} T_{lm}^{\text{EM}},$$

with  $h^{ij}(\vec{k}) = \sqrt{2} \sum_{\lambda=\pm} \epsilon_{\lambda}^i(\vec{k}) \epsilon_{\lambda}^j(\vec{k}) h_{\lambda}(\tau, \vec{k}), \quad (33)$

where the energy-momentum tensor ( $T^{\text{EM}}$ ) and the projector ( $\Pi_{\lambda}$ ) for the polarization plane are defined as

$$T_{ij}^{\text{EM}} = -a^2(E_i E_j + B_i B_j) + C\delta_{ij} = -a^{-2}A'_i A'_j + C\delta_{ij}$$

and  $\Pi_{\lambda}^{lm} = \frac{1}{\sqrt{2}} \epsilon_{-\lambda}^l(\vec{k}) \epsilon_{-\lambda}^m(\vec{k}).$

The  $C$ -term does not contribute to the tensor perturbation because it is diagonal. Equation (33) can be rewritten as

$$h_{\lambda}(\vec{k}) = -\frac{2H^2}{M_P^2} \int d\tau' G_h(\tau, \tau') (-\tau')^2 \times \int \frac{d^3 q}{(2\pi)^{3/2}} \Pi_{\lambda}^{ij} A'_i(\vec{q}, \tau') A'_j(\vec{k} - \vec{q}, \tau'),$$

where  $G_h(\tau, \tau')$  corresponds to the Green's function for the equation of the tensor perturbations (33) and is

$$G_h(\tau, \tau') = \frac{1}{k^3 \tau^2} [(1 + k^2 \tau \tau') \sin[k(\tau - \tau')] + k(\tau - \tau') \cos[k(\tau - \tau')]] \Theta(\tau, \tau'), \quad (34)$$

with  $\Theta$  being the Heaviside's function. Using Eq. (18) we get the tensor perturbation spectrum [4]

$$\mathcal{P}^{\pm} = \frac{H^2}{\pi^2 M_P^2} \left( 1 + \mathcal{A}^{\pm} \frac{\mathcal{N} H^2 e^{4\pi\xi}}{M_P^2 \xi^6} \right), \quad (35)$$

where  $\mathcal{A}^+ \approx 8.6 \times 10^{-7}$  and  $\mathcal{A}^- \approx 1.8 \times 10^{-9}$  are constants for each mode polarization. They are different as a consequence of the parity-violating nature of this model. With them, we obtain the tensor-to-scalar ratio

$$r = \frac{\mathcal{P}^{+} + \mathcal{P}^{-}}{\mathcal{P}_{\zeta}} = \frac{H^2}{\pi^2 M_P^2} \frac{2 + (\mathcal{A}^+ + \mathcal{A}^-) \frac{\mathcal{N} H^2 e^{4\pi\xi}}{M_P^2 \xi^6}}{\mathcal{P}_{\zeta}} \approx \frac{H^2}{\pi^2 M_P^2} \frac{2 + \mathcal{A}^+ \frac{\mathcal{N} H^2 e^{4\pi\xi}}{M_P^2 \xi^6}}{\mathcal{P}_{\zeta}}. \quad (36)$$

Using the approximate Friedmann equation  $H^2 \approx V/(3M_P^2)$  together with Eqs. (30) and (31) we obtain [2]

$$r \approx \frac{2V}{3\pi^2 M_P^4 \mathcal{P}_{\zeta}} + \frac{72\pi^2 \mathcal{A}^+ \xi^4}{\mathcal{F}} \left( \frac{fV_{\phi}}{V} \right)^2 \approx \frac{2V}{3\pi^2 M_P^4 \mathcal{P}_{\zeta}} + 2.9 \times 10^2 \frac{\xi^4}{\alpha^2} \left( \frac{fV_{\phi}}{V} \right)^2, \quad (37)$$

where  $\mathcal{P}_{\zeta} \simeq \frac{0.05}{\mathcal{N} \xi^2} \simeq 2.5 \times 10^{-9}$  is the amplitude of the power spectrum with the so-called COBE normalization. This comes from Eq. (31), neglecting the scale dependent part. In Eq. (37) we can notice the separation of the vacuum gravitational waves spectrum and the gravitational waves sourced by the coupling with the gauge fields. The sourced part is amplified for large values of  $\xi$  and have a definite polarization: they are chiral. Let us emphasize that  $\xi$  does not affect the spectrum of the vacuum gravitational waves, but only the sourced component.

### D. Numerics of the minimally coupled case

In this section we inspect the parameter space of this model. We use the spectral index  $n_s$  in Eq. (32) and the tensor-to-scalar ratio  $r$  in Eq. (37) to compare with current Planck bounds [55]. To this end, we approximate the value of  $\xi$  by solving the equation of motion of the inflaton (5) neglecting time derivatives

$$\xi \simeq \frac{1}{2\pi} \log \left[ \frac{1.3 \times 10^4}{\mathcal{N} \alpha} \xi^4 \frac{M_P^2 f |V_{\phi}|}{H^2 V} \right], \quad (38)$$

and use it to compute the slow-roll parameters

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{2\xi^2 f^2}{\alpha^2 M_P^2} + \frac{8\xi f V_{\phi}}{7\alpha V}, \quad (39)$$

$$\eta \equiv -\frac{\ddot{\phi}}{\dot{\phi} H} = \epsilon - \frac{1}{2H\epsilon} \frac{d\epsilon}{dt} = -\frac{1}{H} \left[ \frac{\dot{H}}{H} + \frac{\dot{\xi}}{\xi} \right] \simeq -\frac{1}{H} \left[ \frac{f\xi}{\sqrt{3}\alpha M_P} \frac{V_{\phi}}{\sqrt{V}} + \frac{-2\frac{\dot{H}}{H} V_{\phi} + \dot{\phi} (V_{\phi\phi} - \frac{V_{\phi}^2}{V})}{2\pi M_P^3 - \frac{4}{\xi} V_{\phi}} \right]. \quad (40)$$

<sup>4</sup>Notice that there is a difference of a global sign with respect to the result in Ref. [1].

The field excursion can be related with the number  $N_e$  of  $e$ -folds that lasts inflation



$$N_e \simeq \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi = \frac{\alpha}{2f} \int_{\phi_i}^{\phi_f} \frac{1}{\xi} d\phi \simeq \frac{\alpha}{2\xi} \frac{\phi_f - \phi_i}{f}, \quad (41)$$

given that the rate  $H/\dot{\phi}$  is approximately constant.

Hereafter we consider the natural inflation potential [56,57]

$$V(\phi) = \Lambda^4 \left[ 1 + \cos\left(\frac{\phi}{f}\right) \right], \quad (42)$$

since, with this choice, it is possible to generate slow roll due to the friction caused by the axial coupling term. This implies that, differently from the case of the original natural inflation, one can have a steep potential, which means that  $f$  can acquire sub-Planckian values.

For the sake of completeness, it is instructive to do a more precise statement about the regime in which the backreaction term dominates the slow-roll evolution. To this end, we compare the derivative of the potential term against the backreaction term in the rhs of Eq. (5). Using the cosine potential and the estimate for the backreaction term in Eq. (19) we find that the friction term dominates when

$$\frac{e^{2\pi\xi}}{\xi^4} > \frac{9M_P^4}{\mathcal{N}I\alpha\Lambda^4}. \quad (43)$$

As an example, even if we take a single gauge field  $\mathcal{N} = 1$ ,  $\alpha = 200$ , and  $\Lambda = 4.5 \times 10^{-3} M_P$  (which correspond to typical values, see, e.g., Figs. 2 and 3), we get that the friction term dominates when  $\xi \gtrsim 5.4$ .

Figure 1 shows the 68% (light blue) and 95% (dark blue) C.L. regions for the tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  from Planck [55]. The predictions in the case of natural inflation with the inflaton coupled with  $\mathcal{N}$  gauge fields are also shown, for 50 (red band) and 60 (black band)  $e$ -folds.

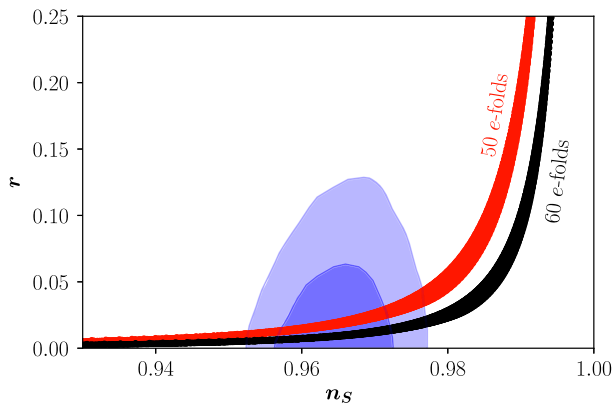


FIG. 1. Tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  for 50 and 60  $e$ -folds, in the case of natural inflation with the inflaton coupled to  $\mathcal{N}$   $U(1)$  gauge fields. The blue bands correspond to the 68% (light) and 95% (dark) C.L. regions from Planck.

60  $e$ -folds (black band). These regions were constructed by varying the parameters in the ranges:  $\xi = [2.5, 10]$  and  $\alpha = [80, 400]$ . For a given number of  $e$ -folds, the number  $\mathcal{N}$  of gauge fields is fixed by the COBE normalization, the scale  $\Lambda$  of the potential by the equation of motion of the inflaton. Let us also note that in the minimally coupled case, both  $r$  and  $n_s$  are independent on the coupling  $f$  of the potential, in the regime where  $\xi$  is almost constant. It is clear from the figure that there are regions compatible with the 95% C.L. Planck limits, for the cases with both 50 and 60  $e$ -folds.<sup>5</sup>

Moreover, Fig. 2 depicts the values of  $\xi$ ,  $\alpha$ ,  $\mathcal{N}$ , and  $\Lambda$  compatibles with Planck limits at 68% C.L. Note that while  $\mathcal{N}$  is independent from the number of  $e$ -folds,  $\Lambda$  is almost insensitive to that parameter. The region  $\xi < 2.5$  is beyond the validity of the current approximations and hence not considered. Let us point out that the number of  $U(1)$  gauge fields required to reproduce the amplitude of the scalar spectrum has to be about  $\mathcal{N} \sim 10^6$ . The inflationary scale  $\Lambda$  is always sub-Planckian, going down to  $\mathcal{O}(10^{-7})M_P$  for  $\xi \simeq 10$ . Additionally, the coupling constant  $\alpha$  is of the order  $10^2$ ; this is, however, not a problem *per se* for perturbativity, because it always appears suppressed by  $f$ , which is at the Planck scale.

### E. Comments on non-Gaussianities

For the sake of completeness, we present the non-Gaussianities produced by this model [2,3,5]. Using Eq. (27) we calculate the three-point function of the scalar perturbations. In the equilateral limit

$$\begin{aligned} & \langle \delta\phi(\vec{p}_1)\delta\phi(\vec{p}_2)\delta\phi(\vec{p}_3) \rangle^{(\text{equil})} \\ & \propto \mathcal{F}_3 \frac{\delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)}{p^6} \frac{\alpha^3 H^6 e^{6\pi\xi}}{f^3 \xi^9}, \end{aligned} \quad (44)$$

where  $\mathcal{F}_3 \approx 5 \times 10^{-11}$  [3,5]. The bispectrum of the primordial curvature perturbation is then

$$\begin{aligned} & \langle \zeta(\vec{p}_1)\zeta(\vec{p}_2)\zeta(\vec{p}_3) \rangle^{(\text{equil})} \\ & \propto -\mathcal{F}_3 \frac{H^3}{\dot{\phi}_0^3} \frac{\delta(\vec{p}_1 + \vec{p}_2 + \vec{p}_3)}{p^6} \frac{\alpha^3 H^6 e^{6\pi\xi}}{f^3 \xi^9}. \end{aligned} \quad (45)$$

For an arbitrary potential able to support slow roll the  $f_{\text{NL}}$  parameter in the equilateral configuration is given by [3,5]

$$f_{\text{NL}}^{\text{equil}} \approx 4.4 \times 10^{10} \mathcal{P}^3 \frac{e^{6\pi\xi}}{\xi^9}, \quad (46)$$

<sup>5</sup>For this to be possible, the minus sign in the expression for the spectral index in Eq. (32) is necessary, otherwise, there is no region in the parameter space compatible with Planck's constraints.

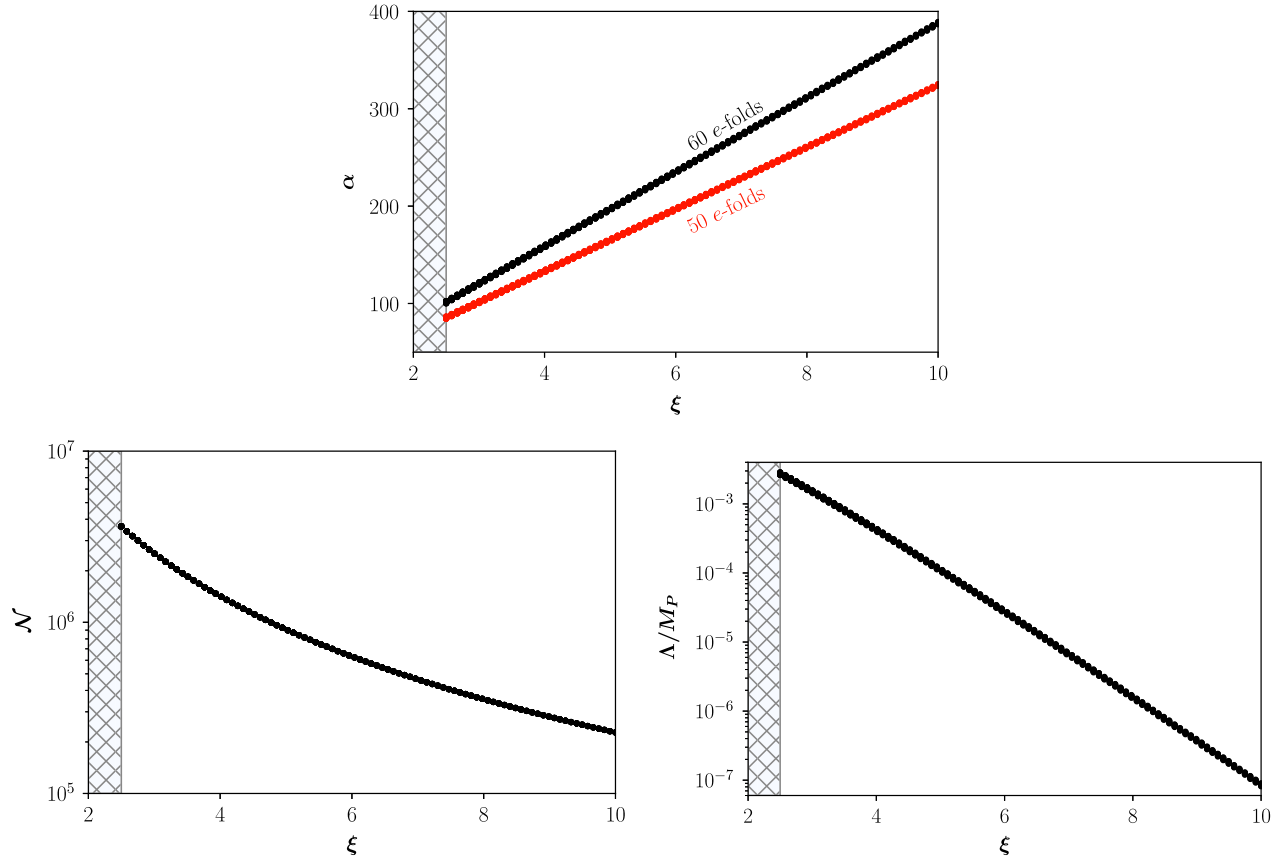


FIG. 2. Values of  $\xi$ ,  $\alpha$ ,  $\mathcal{N}$ , and  $\Lambda$  compatible with Planck limits at 68% C.L. Note that while  $\mathcal{N}$  is independent from the number of  $e$ -folds,  $\Lambda$  is almost insensitive to that parameter. The region  $\xi < 2.5$  is beyond the validity of the current approximations and hence disregarded. Large  $\xi$  are considered since the slow-roll evolution is driven by the gauge field production, obtained precisely for large  $\xi$  values.

where  $\mathcal{P}^{1/2} = H^2/(2\pi\dot{\phi}_0)$ . The current bound on equilateral non-Gaussianity is  $f_{\text{NL}}^{\text{equil}} = -4 \pm 43$  [58]. This is a stringent constraint for this kind of model since it requires  $\xi \lesssim 2.5$ . For such small values of the parameter  $\xi$ , the signatures from the chiral tensor perturbations sourced by the coupling with the axial term  $\phi\tilde{F}F$  are suppressed and indistinguishable from the vacuum perturbations.

However, for models strongly coupled to gauge fields where the slow roll is achieved due to the friction term, the equilateral non-Gaussianity is given by [2]

$$f_{\text{NL}}^{\text{equil}} \approx -1.3\xi, \quad (47)$$

which allows us to consider values above  $\xi \approx 2.5$ , where  $\xi$  reaches a plateau and can be fairly enough approximated by a constant. This is the case considered here. In the next section, the situation in the nonminimal coupling case is discussed.

### III. INCLUDING NONMINIMAL COUPLING WITH GRAVITY

We consider the inclusion of a nonminimal coupling between the pseudoscalar field and gravity. The Lagrangian for the gravity-scalar system is

$$\mathcal{L} = \sqrt{-g} \left[ \frac{M_P^2}{2} \left( 1 + \frac{2h(\phi)}{M_P^2} \right) R - \frac{1}{2} (\partial\phi)^2 - V(\phi) - \frac{\mathcal{N}}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \phi F^{\mu\nu} \tilde{F}_{\mu\nu} \right], \quad (48)$$

where  $h(\phi)$  introduces a particular form of nonminimal coupling with gravity. In the following we restrict to an  $h(\phi)$  of the form

$$h(\phi) = \frac{1}{2} \mathcal{X} \phi^2, \quad (49)$$

where  $\mathcal{X}$  is a dimensionless constant, the nonminimal coupling parameter between gravity and the pseudoscalar

field. This leads to the equation of motion for the pseudoscalar field

$$\square\phi - V_\phi + \mathcal{X}R\phi - \frac{\mathcal{N}\alpha}{4f}F^{\mu\nu}\tilde{F}_{\mu\nu} = 0. \quad (50)$$

Let us note that for a free field (i.e.,  $V_\phi = 0$  and  $\alpha = 0$ ), this equation is invariant under conformal transformations when  $\mathcal{X} = -1/6$ . Moreover, the form of the gauge field equation remains unaltered by the nonminimal coupling to gravity

$$\frac{d^2 A_i}{d\tau^2} - \nabla^2 A_i - \frac{\alpha}{f} \frac{d\phi}{d\tau} (\nabla \times A)_i = 0. \quad (51)$$

However, the effect of the nonminimal coupling is introduced only through the dynamics of the pseudoscalar field, i.e., through the coupling parameter  $\xi$  which depends both on the time and on the nonminimal coupling parameter.

### A. Jordan and Einstein frames

The action (48) describes the system in the so-called Jordan frame in which the nonminimal coupling between the pseudoscalar field and gravity appears explicitly. We can do a conformal scaling of the metric in the following form:

$$\bar{g}_{\mu\nu} = \Omega(\phi)g_{\mu\nu}, \quad \text{where } \Omega(\phi) \equiv 1 + \frac{2h(\phi)}{M_P^2}. \quad (52)$$

For nonminimal couplings of the form (49), the conformal factor is

$$\Omega(\phi) = 1 + \mathcal{X} \left( \frac{\phi}{M_P} \right)^2. \quad (53)$$

Under this transformation the action (48) becomes

$$\mathcal{L} = \sqrt{-\bar{g}} \left[ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} K(\phi) \bar{g}^{\mu\nu} \bar{\nabla}_\mu \bar{\phi} \bar{\nabla}_\nu \bar{\phi} - \bar{V}(\bar{\phi}) - \frac{\mathcal{N}}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \bar{\phi} \bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} \right], \quad (54)$$

where  $\bar{V}(\bar{\phi}) \equiv \frac{V(\phi)}{\Omega^2}$ ,  $\bar{F}^{\mu\nu} \bar{F}_{\mu\nu} \equiv \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} F_{\alpha\beta} F_{\mu\nu}$ , and  $\bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} \equiv \bar{g}^{\mu\alpha} \bar{g}^{\nu\beta} F_{\alpha\beta} \tilde{F}_{\mu\nu}$ . The details of the derivation of Eq. (54) are discussed in Appendix C. For a single pseudoscalar we can write the Lagrangian as a canonical pseudoscalar field coupled minimally to gravity and to the vector field through the axial term

$$\mathcal{L} = \sqrt{-\bar{g}} \left[ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \bar{\phi} \bar{\nabla}_\nu \bar{\phi} - \bar{V}(\bar{\phi}) - \frac{\mathcal{N}}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \bar{\phi}(\bar{\phi}) \bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} \right], \quad (55)$$

where the canonical pseudoscalar field  $\bar{\phi}$  is defined through the transformation  $d\bar{\phi}/d\phi = K^{1/2}$ . The action (55) is now in the Einstein frame [59]: it is written using only fields with canonical kinetic terms and a metric that allows the action to acquire the traditional form of the Einstein-Hilbert gravity. Using the particular form of the nonminimal coupling function (49), we get the transformation function

$$K(\phi) = \frac{1}{\Omega} + \frac{6}{M_P^2 \Omega^2} (\mathcal{X}\phi)^2 = \frac{1 + 6(\mathcal{X} + \frac{1}{6})\mathcal{X}(\frac{\phi}{M_P})^2}{[1 + \mathcal{X}(\frac{\phi}{M_P})^2]^2}. \quad (56)$$

### B. Equations of motion in the Einstein frame

We can now derive the equations of motion for the canonical fields in the Einstein frame from the action (55). It is important to derive the equation of motion for the canonical field  $\bar{\phi}$  since, for this field, the construction of the perturbation correlations are obtained following the usual canonical quantization procedure [33]. Since the action is written in terms of the coordinates  $\bar{x}$  and the metric  $\bar{g}_{\mu\nu}$ , all the derivatives are compatible with this metric. The result for the pseudoscalar field is

$$\square \bar{\phi} - \bar{V}_{\bar{\phi}} - \frac{\mathcal{N}\alpha}{4f K^{1/2}} \bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} = 0, \quad (57)$$

where  $\square \bar{\phi} \equiv \bar{\nabla}^\mu \bar{\nabla}_\mu \bar{\phi} = \frac{1}{\sqrt{-\bar{g}}} \partial_\mu (\sqrt{-\bar{g}} \bar{g}^{\mu\nu} \partial_\nu \bar{\phi})$ . Now, we consider the isotropic solution

$$\begin{aligned} d\bar{s}^2 &= -d\bar{t}^2 + \bar{a}^2 \delta_{ij} d\bar{x}_i d\bar{x}_j \\ &= \bar{a}^2(\bar{\tau}) [-d\bar{\tau}^2 + \delta_{ij} d\bar{x}_i d\bar{x}_j]. \end{aligned} \quad (58)$$

To go from the Jordan to the Einstein frame variables for the isotropic solution (58) we take into account that  $d\bar{s}^2 = \Omega ds^2$ , which implies

$$\begin{aligned} d\bar{t} &= \Omega^{1/2} dt, & \bar{a}(\bar{t}) &= \Omega^{1/2} a(t), \\ d\bar{\tau} &= d\tau \quad \text{and} \quad \bar{H} &= \frac{1}{\bar{a}} \frac{d\bar{a}}{d\bar{t}} = \frac{1}{\Omega^{3/2}} \left[ H\Omega + \frac{1}{2} \dot{\Omega} \right]. \end{aligned} \quad (59)$$

In conformal coordinates, the equation for the canonical pseudoscalar field becomes

$$\frac{\partial^2 \bar{\phi}}{\partial \bar{\tau}^2} + 2\bar{a} \bar{H} \frac{\partial \bar{\phi}}{\partial \bar{\tau}} - \bar{\nabla}^2 \bar{\phi} + \bar{a}^2 \bar{V}_{\bar{\phi}} = \frac{\bar{a}^2}{\Omega^2 K^{1/2}} \frac{\mathcal{N}\alpha}{f} \vec{E} \cdot \vec{B}, \quad (60)$$

where we use the definition for the electric and magnetic field components as in Eqs. (7) and (8) in the Jordan frame.



For gravity, we derive the Einstein equations varying with respect to  $\bar{g}_{\mu\nu}$ :

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{1}{M_P^2}\bar{T}_{\mu\nu}, \quad (61)$$

with the energy-momentum tensor

$$\begin{aligned} \bar{T}_{\mu\nu} &\equiv \frac{-2}{\sqrt{-\bar{g}}}\frac{\delta\mathcal{L}}{\delta\bar{g}^{\mu\nu}} \\ &= \bar{\partial}_\mu\bar{\phi}\bar{\partial}_\nu\bar{\phi} - \bar{g}_{\mu\nu}\left(\frac{1}{2}\bar{g}^{\alpha\beta}\bar{\partial}_\alpha\bar{\phi}\bar{\partial}_\beta\bar{\phi} + \bar{V}\right) \\ &\quad + \bar{g}^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} - \bar{g}_{\mu\nu}\frac{1}{4}\bar{F}^2, \end{aligned} \quad (62)$$

which leads to the Friedmann equations for the metric  $\bar{g}_{\mu\nu}$

$$\bar{H}^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\left(\frac{\partial\bar{\phi}}{\partial\bar{t}}\right)^2 + \bar{V} + \frac{1}{2\Omega^2}\langle\vec{E}^2 + \vec{B}^2\rangle\right], \quad (63)$$

$$\frac{d\bar{H}}{d\bar{t}} = -\frac{1}{2M_P^2}\left(\frac{\partial\bar{\phi}}{\partial\bar{t}}\right)^2 - \frac{1}{3M_P^2\Omega^2}\langle\vec{E}^2 + \vec{B}^2\rangle. \quad (64)$$

Finally, for the equation of motion for the gauge fields it is important to notice that the derivatives with respect to the conformal time and with respect to spacial coordinates in both frames are the same, then the equation obtained is the same Eq. (51). If one goes to Fourier space and projects into the transverse polarizations, one gets the same Eq. (17) obtained in the minimally coupled case

$$A''_{\pm} + \left(k^2 \pm \frac{2k\xi}{\tau}\right)A_{\pm} = 0, \quad \text{with} \quad \xi \equiv \frac{\alpha\dot{\phi}_0}{2fH}. \quad (65)$$

Notice that in this equation appears the velocity of the pseudoscalar field  $\dot{\phi}_0$  and not the velocity of the canonical field. Both quantities can be related through the expression

$$\xi = \frac{\alpha\dot{\phi}_0}{2fH} \approx \frac{\alpha}{2f}\frac{1}{\bar{H}K^{1/2}}\frac{d\bar{\phi}_0}{d\bar{t}} \equiv \frac{1}{K^{1/2}}\bar{\xi}. \quad (66)$$

### C. Perturbations

Starting from Eq. (60), we write the equations for the background canonical field and its perturbation  $\bar{\phi}(\bar{x}, \bar{\tau}) = \bar{\phi}_0(\bar{\tau}) + \delta\bar{\phi}(\bar{x}, \bar{\tau})$

$$\bar{\phi}_0'' + 2\bar{a}\bar{H}\bar{\phi}_0' + \bar{a}^2\bar{V}_{\bar{\phi}} = \frac{\bar{a}^2}{\Omega^2 K^{1/2}}\frac{\mathcal{N}\alpha}{f}\langle\vec{E} \cdot \vec{B}\rangle. \quad (67)$$

$$\begin{aligned} \delta\bar{\phi}'' + 2\bar{a}\bar{H}\delta\bar{\phi}' + \bar{a}^2\bar{V}_{\bar{\phi}\bar{\phi}}\delta\bar{\phi} \\ = \frac{\bar{a}^2\mathcal{N}\alpha}{f}\left[\delta\left[\frac{1}{\Omega^2 K^{1/2}}\right]\langle\vec{E} \cdot \vec{B}\rangle + \frac{1}{\Omega^2 K^{1/2}}\delta[\vec{E} \cdot \vec{B}]\right], \end{aligned} \quad (68)$$

where the primes here represent derivatives with respect to the conformal time in the Einstein frame  $\bar{\tau}$ . We neglected the gradient term since we assume that the perturbations are homogeneous. As we did for the minimally coupled case, we calculate the dependence of the term  $\vec{E} \cdot \vec{B}$  with the velocity of the canonical field

$$\delta[\vec{E} \cdot \vec{B}] \approx \delta_{\vec{E} \cdot \vec{B}} + \frac{\partial\langle\vec{E} \cdot \vec{B}\rangle}{\partial(\partial\bar{\phi}/\partial\bar{t})}\frac{\partial\delta\bar{\phi}}{\partial\bar{t}}. \quad (69)$$

The second term can be approximated as

$$\begin{aligned} \frac{\partial\langle\vec{E} \cdot \vec{B}\rangle}{\partial(\partial\bar{\phi}/\partial\bar{t})}\frac{\partial\delta\bar{\phi}}{\partial\bar{t}} &\approx \frac{\partial\langle\vec{E} \cdot \vec{B}\rangle}{\partial\xi}\frac{\partial\xi}{\partial(\partial\bar{\phi}/\partial\bar{t})}\frac{\delta\bar{\phi}'}{\bar{a}} \\ &\approx 2\pi\langle\vec{E} \cdot \vec{B}\rangle\frac{\partial\xi}{\partial(\partial\bar{\phi}/\partial\bar{t})}\frac{\delta\bar{\phi}'}{\bar{a}} \\ &\approx \frac{\alpha\pi\langle\vec{E} \cdot \vec{B}\rangle}{f}\frac{\delta\bar{\phi}'}{K^{1/2}\bar{H}\bar{a}}, \end{aligned} \quad (70)$$

where we have used Eq. (66) in the last step. We can relate the potential with the value of the term  $\langle\vec{E} \cdot \vec{B}\rangle$  through the equation for the background field  $\bar{\phi}_0$ . Neglecting time derivatives in Eq. (67) and using the approximation  $\bar{V}_{\bar{\phi}} \approx \frac{\mathcal{N}\alpha}{\Omega^2 K^{1/2}f}\langle\vec{E} \cdot \vec{B}\rangle$ , the perturbation of the source term becomes

$$\delta[\vec{E} \cdot \vec{B}] \approx \delta_{\vec{E} \cdot \vec{B}} + \pi\frac{\Omega^2}{\mathcal{N}}\bar{V}_{\bar{\phi}}\frac{\delta\bar{\phi}'}{\bar{H}\bar{a}}. \quad (71)$$

With these results, the equation for the perturbations can be rewritten as

$$\begin{aligned} \delta\bar{\phi}'' + 2\bar{a}\bar{H}\left(1 - \frac{\pi\alpha\bar{V}_{\bar{\phi}}}{2K^{1/2}f\bar{H}^2}\right)\delta\bar{\phi}' \\ + \bar{a}^2\left(\bar{V}_{\bar{\phi}\bar{\phi}} + \frac{d\ln(\Omega^2 K^{1/2})}{d\bar{\phi}}\bar{V}_{\bar{\phi}}\right)\delta\bar{\phi} = \frac{\bar{a}^2\mathcal{N}\alpha}{\Omega^2 K^{1/2}f}\delta_{\vec{E} \cdot \vec{B}}, \end{aligned} \quad (72)$$

where it is understood that all the terms depending on the canonical pseudoscalar field are evaluated at  $\bar{\phi}_0$ . The formal solution to this equation is

$$\delta\bar{\phi}(\bar{\tau}, \vec{k}) = \frac{\mathcal{N}\alpha}{f}\int_{-\infty}^{\bar{\tau}} d\bar{\tau}_1 \frac{\bar{a}^2(\bar{\tau}_1)\bar{G}(\bar{\tau}, \bar{\tau}_1)}{\Omega^2 K^{1/2}} \int d^3x e^{-i\vec{k}\cdot\vec{x}} \delta_{\vec{E} \cdot \vec{B}}(\bar{\tau}_1, \vec{x}), \quad (73)$$

where the Green function is obtained from

$$\begin{aligned} \left[\frac{d^2}{d\bar{\tau}^2} - \frac{2}{\bar{\tau}}\left(1 - \frac{\pi\alpha\bar{V}_{\bar{\phi}}}{2K^{1/2}f\bar{H}^2}\right)\frac{d}{d\bar{\tau}}\right. \\ \left. + \frac{\bar{V}_{\bar{\phi}\bar{\phi}}}{\bar{H}^2\bar{\tau}^2}\left(1 + \frac{d\ln(\Omega^2 K^{1/2})}{d\bar{\phi}}\frac{\bar{V}_{\bar{\phi}}}{\bar{V}_{\bar{\phi}\bar{\phi}}}\right)\right]\bar{G}(\bar{\tau}, \bar{\tau}') = \delta(\bar{\tau} - \bar{\tau}'). \end{aligned} \quad (74)$$

The solution to this expression with the boundary conditions  $\bar{G}(\bar{\tau}', \bar{\tau}') = 0$ ,  $\bar{G}'(\bar{\tau}', \bar{\tau}') = 1$  is

$$\bar{G}(\bar{\tau}, \bar{\tau}') = \frac{\bar{\tau}'}{\bar{\Delta}} \left[ \left( \frac{\bar{\tau}}{\bar{\tau}'} \right)^{\bar{\nu}_+} - \left( \frac{\bar{\tau}}{\bar{\tau}'} \right)^{\bar{\nu}_-} \right] \Theta(\bar{\tau} - \bar{\tau}'), \quad \text{with} \quad (75)$$

$$\nu_{\pm} \equiv \frac{1}{2} \left( 1 - \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right) \pm \frac{1}{2} \bar{\Delta} \approx -\frac{1}{2} \left( \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right) \pm \frac{1}{2} \bar{\Delta}, \quad (76)$$

$$\begin{aligned} \bar{\Delta} &\equiv \sqrt{\left( 1 - \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right)^2 - \frac{4}{\bar{H}^2} \left( \bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0) + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \bar{V}_{\bar{\phi}}(\bar{\phi}_0) \right)} \\ &\approx \sqrt{\left( \frac{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{K^{1/2} f \bar{H}^2} \right)^2 - \frac{4}{\bar{H}^2} \left( \bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0) + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \bar{V}_{\bar{\phi}}(\bar{\phi}_0) \right)}, \end{aligned} \quad (77)$$

where we used the approximation  $\frac{\pi \alpha \bar{V}_{\bar{\phi}}}{2 K^{1/2} f \bar{H}^2} \gg 1$ . The late time evolution of the Green's function is dominated by the  $\nu_+$  term, so we can approximate the Green's function with

$$\bar{G}(\bar{\tau}, \bar{\tau}') \approx \frac{\bar{\tau}'}{\bar{\Delta}} \left( \frac{\bar{\tau}}{\bar{\tau}'} \right)^{\bar{\nu}_+} \quad \text{for } \bar{\tau} > \bar{\tau}'. \quad (78)$$

Following the same steps of the minimally coupled case, we calculate the spectrum of the scalar perturbations

$$\begin{aligned} \langle \delta \bar{\phi}(\vec{p}) \delta \bar{\phi}(\vec{p}') \rangle &\approx \mathcal{F}(\bar{\nu}_+) \frac{\delta(\vec{p} + \vec{p}')}{p^3} \frac{\mathcal{N}^2 \alpha^2 \bar{H}^4}{K \bar{\Delta}^2 f^2 \xi^8} \\ &\times e^{4\pi \xi} (-2^5 \xi p \bar{\tau})^{2\bar{\nu}_+}. \end{aligned} \quad (79)$$

At this point one can calculate the spectrum of the primordial curvature perturbation  $\bar{\zeta} = -\bar{H} \delta \bar{\phi} / \dot{\bar{\phi}}_0$ . However, it has been shown that it is unaltered by a conformal transformation [33], meaning that this variable and its correlators are the same in Einstein and Jordan frames<sup>6</sup>:

$$\bar{\mathcal{P}}_{\zeta}(p) \approx \frac{\mathcal{F}}{2\pi^2} \frac{\bar{H}^2 \mathcal{N}^2 \alpha^2 \bar{H}^4}{\bar{\phi}_0^2 K \bar{\Delta}^2 f^2 \xi^8} e^{4\pi \xi} (-2^5 \xi p \bar{\tau})^{2\bar{\nu}_+}, \quad (80)$$

which using Eqs. (19), (77), and  $\bar{V}_{\bar{\phi}} \approx \frac{\mathcal{N} \alpha}{\Omega^2 K^{1/2} f} \langle \vec{E} \cdot \vec{B} \rangle$  gives

$$\begin{aligned} \bar{\mathcal{P}}_{\zeta}(p) &\approx \frac{\mathcal{F}(\bar{\nu}_+)}{8\pi^4 \mathcal{I}^2 \mathcal{N} \xi^2} (-2^5 \xi p \bar{\tau})^{2\bar{\nu}_+} \\ &\approx \frac{5 \times 10^{-2}}{\mathcal{N} \xi^2} (-2^5 \xi p \bar{\tau})^{2\bar{\nu}_+}. \end{aligned} \quad (81)$$

<sup>6</sup>See, e.g., Ref. [60] for a discussion of quantum equivalence between the Jordan frame and Einstein frame.

The spectral index can then be extracted:

$$\begin{aligned} \bar{n}_s - 1 &\approx 2\bar{\nu}_+ \\ &= -K^{1/2} \frac{2f \bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)}{\pi \alpha \bar{V}_{\bar{\phi}}(\bar{\phi}_0)} \left( 1 + \frac{d \ln(\Omega^2 K^{1/2})}{d\bar{\phi}} \frac{\bar{V}_{\bar{\phi}}(\bar{\phi}_0)}{\bar{V}_{\bar{\phi}\bar{\phi}}(\bar{\phi}_0)} \right). \end{aligned} \quad (82)$$

We can see from the previous results that the amplitude of the scalar perturbation is kept unaltered:

$$\bar{\mathcal{P}}_{\zeta} \approx \mathcal{P}_{\zeta}, \quad (83)$$

but the spectral index is modified by a  $K^{1/2}$  factor. It is important to notice that, even though the expression for the spectrum is equal both for minimal and for nonminimal couplings, the dynamics of the parameter  $\xi$  is different in the two cases. Figure 3 reflects that difference with an example of the evolution of  $\xi$  with  $\mathcal{X} = 0$  (solid blue line) and  $\mathcal{X} = 10$  (dotted red line). These solutions are obtained by solving Eq. (50), with  $H$  obtained from the Friedman equation (63) and using approximations (18) and (19). We also choose  $\Lambda = 4.5 \times 10^{-3} M_P$ ,  $\alpha = 400$ ,  $\mathcal{N} = 10^5$ , and  $f = 0.1 M_P$ . As it can be seen, even for a large value of the nonminimal coupling such as  $\mathcal{X} = 10$ , the difference is not very significant, in the ballpark of a few percent. Note that throughout the paper we used the assumption of a constant  $\xi$ . Figure 3 shows that this assumption for late times is justified; nevertheless, a more precise statement of this assumption can be made if we consider the ratio  $\dot{\xi}/(H\xi)$ . This analysis is shown in the Appendix B.

## D. Tensor perturbations

In this section we discuss how nonminimal couplings can modify the production of sourced gravitational waves. Knowing that the tensor perturbation variable  $h_{\lambda}$  is not

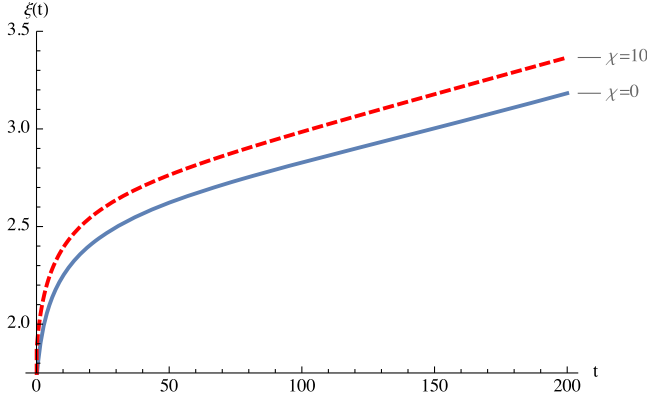


FIG. 3. An example of the evolution of the parameter  $\xi$  for minimal coupling (solid blue) and nonminimal coupling with  $\mathcal{X} = 10$  (dashed red). We use the values  $\Lambda = 4.5 \times 10^{-3} M_P$ ,  $\alpha = 400$ ,  $\mathcal{N} = 10^5$ ,  $f = 0.1 M_P$ . The time is measured in units of  $M_P/\Lambda^2$ .

altered by conformal transformations, we can do the calculation for the metric

$$d\bar{s}^2 = -\bar{a}^2(\bar{\tau})[-d\bar{\tau}^2 + (\delta_{ij} + \bar{h}_{ij})d\bar{x}_i d\bar{x}_j]. \quad (84)$$

As in the minimally coupled case, the spectrum is obtained by solving Eq. (33) with the metric (84) and with the energy-momentum tensor for the sourced part as in Eq. (62):

$$\begin{aligned} \bar{T}_{ij}^{\text{EM}} &= -\bar{a}^2(E_i E_j + B_i B_j) + C\delta_{ij} \\ &= -\frac{1}{\bar{a}^2} A'_i A'_j + C\delta_{ij}. \end{aligned} \quad (85)$$

The result is the same as the one obtained in the minimally coupled case but, in the conformal coordinates  $(\bar{\tau}, \bar{x}_i)$ , it is

$$\bar{\mathcal{P}}^{\pm} = \frac{\bar{H}^2}{\pi^2 \bar{M}_P^2} \left( 1 + \mathcal{A}^{\pm} \frac{\mathcal{N} \bar{H}^2 e^{4\pi\bar{\xi}}}{\bar{M}_P^2 \bar{\xi}^6} \right), \quad (86)$$

then, the total tensor spectrum reads

$$\bar{\mathcal{P}}^t \approx \frac{2\bar{H}^2}{\pi^2 \bar{M}_P^2} \left( 1 + \frac{(\mathcal{A}^+ + \mathcal{A}^-)}{2} \frac{\mathcal{N} \bar{H}^2 e^{4\pi\bar{\xi}}}{\bar{M}_P^2 \bar{\xi}^6} \right), \quad (87)$$

and the tensor-to-scalar ratio

$$\begin{aligned} \bar{r} &= \frac{\bar{\mathcal{P}}^{t+} + \bar{\mathcal{P}}^{t-}}{\bar{\mathcal{P}}_{\zeta}} = \frac{2\bar{H}^2}{\pi^2 \bar{M}_P^2} \frac{1 + \frac{(\mathcal{A}^+ + \mathcal{A}^-)}{2} \frac{\mathcal{N} \bar{H}^2 e^{4\pi\bar{\xi}}}{\bar{M}_P^2 \bar{\xi}^6}}{\bar{\mathcal{P}}_{\zeta}} \\ &\approx \frac{2\bar{H}^2}{\pi^2 \bar{M}_P^2} \frac{1 + \frac{\mathcal{A}^+}{2} \frac{\mathcal{N} \bar{H}^2 e^{4\pi\bar{\xi}}}{\bar{M}_P^2 \bar{\xi}^6}}{\bar{\mathcal{P}}_{\zeta}}. \end{aligned} \quad (88)$$

We can express the previous results in terms of the potential by using the approximate Friedmann equation

$\bar{H}^2 \approx \bar{V}/(3\bar{M}_P^2)$ , the scalar spectrum  $\bar{\mathcal{P}}_{\zeta} \approx \mathcal{P}_{\zeta}$  of Eq. (81), and the derivative of the potential

$$\bar{V}_{\bar{\phi}} \approx \frac{\mathcal{N}\alpha}{f\Omega^2 K^{1/2}} \langle \vec{E} \cdot \vec{B} \rangle \approx -\frac{\mathcal{N}\alpha\mathcal{I}}{f\Omega^2 K^{1/2}} \frac{H^4}{\xi^4} e^{2\pi\xi}. \quad (89)$$

With them, we can write

$$\begin{aligned} \bar{r} &\approx \frac{2\bar{V}}{3\pi^2 \bar{M}_P^4 \mathcal{P}_{\zeta}} + \frac{72\pi^2 \mathcal{A}^+ K \xi^4}{\mathcal{F}} \left( \frac{f\bar{V}_{\bar{\phi}}}{\bar{V}} \right)^2 \\ &\approx \frac{2V}{3\pi^2 \bar{M}_P^4 \Omega^2 \mathcal{P}_{\zeta}} + 2.9 \times 10^2 K \frac{\xi^4}{\alpha^2} \left( \frac{f\bar{V}_{\bar{\phi}}}{\bar{V}} \right)^2. \end{aligned} \quad (90)$$

It is worth emphasizing that the first term of the last equation, the one corresponding to the vacuum fluctuations, is suppressed by the factor  $\Omega^2$  in the denominator. The second term, the one responsible for the “sourced” tensor perturbations, is modified by the factor  $K$ . This factor changes significantly for small field values depending on the value of the nonminimal coupling parameter  $\mathcal{X}$ . This difference in the behavior between the vacuum and sourced terms offers a possibility to generate observable chiral sourced gravitational waves which would acquire an enhancement due to the nonminimal coupling. We can look for a region in the parameters space in which the current observational constraints over  $n_s$  and  $r$  are respected. We explore this possibility in the next section.

## IV. NUMERICAL ANALYSIS OF THE NONMINIMAL COUPLED CASE

### A. Natural inflation

Before going to the full system with the axial coupling, it is interesting to consider a single pseudoscalar field driving inflation. We do not introduce any other auxiliary field as a source of the primordial curvature perturbation or any spectator field. The most widely studied example of small field inflation with a pseudo-Nambu-Goldstone scalar, able to produce an inflationary expansion is natural inflation [56,57], characterized by a potential given in Eq. (42). We also introduce a coupling with gravity of the form  $h(\phi)R = \frac{1}{2}\mathcal{X}\phi^2 R$ .<sup>7</sup>

<sup>7</sup>This coupling is still consistent with parity breaking, but breaks the shift symmetry. A related approach that preserves the tree-level shift symmetry is discussed in Refs. [61,62]. Instead of introducing the nonminimal coupling in the form of a function  $h(\phi)R$ , they use a nonminimal derivative coupling of the type  $K(\phi)\partial\phi\partial\phi$ , keeping the scale of the coupling constant  $f$  far below the Planck scale and achieving in that way an UV protected completion of the theory. Other possibilities also include coupling to Gauss-Bonnet and Chern-Simons terms.

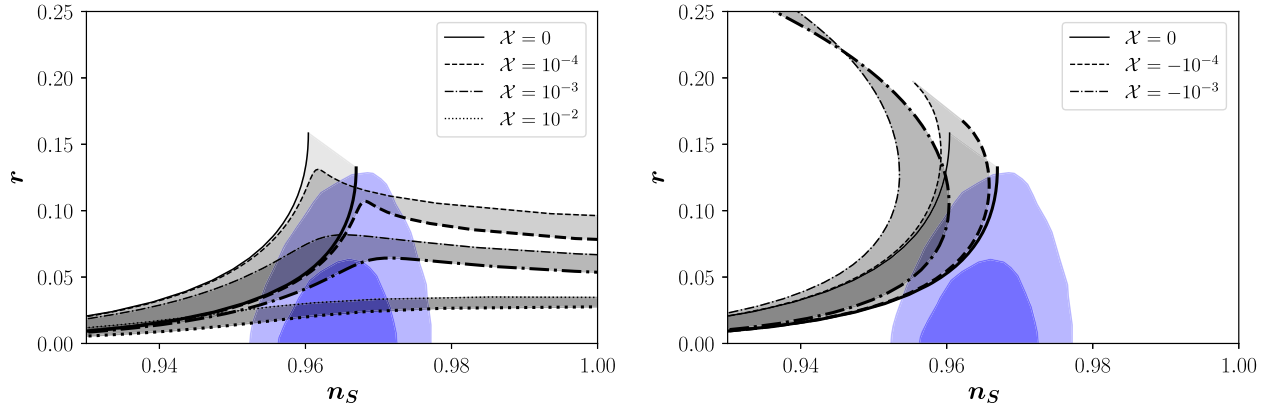


FIG. 4. Tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$ , in the case of natural inflation with nonminimal coupling  $\mathcal{X}$ . Black thin and thick lines correspond to 50 and 60  $e$ -folds, respectively. The blue bands correspond to the 68% (light) and 95% (dark) C.L. regions from Planck.

We are interested in the observables  $n_s$  and  $r$  related with the spectrum of the scalar and the tensor perturbations, respectively. To this end, we need to evaluate the slow-roll parameters for this potential:

$$\bar{\epsilon}_V \equiv \frac{M_P^2}{2K} \left( \frac{\bar{V}_\phi}{\bar{V}} \right)^2 = \frac{[4f\mathcal{X}\phi + (\mathcal{X}\phi^2 + M_P^2) \tan \frac{\phi}{2f}]^2}{2f^2[M_P^2 + \mathcal{X}\phi^2(6\mathcal{X} + 1)]}, \quad (91)$$

$$\begin{aligned} \bar{\eta}_V &\equiv \frac{M_P^2}{K\bar{V}} \left( \bar{V}_{\phi\phi} - \frac{K'}{2K} \bar{V}_\phi \right) \\ &= \frac{1}{2M_P f (M_P^2 + \mathcal{X}\phi^2(6\mathcal{X} + 1))^2} \left[ 2f\mathcal{X}\phi(\mathcal{X}\phi^2 + M_P^2)(7M_P^2 + 6M_P^2\mathcal{X} + 7\mathcal{X}\phi^2(6\mathcal{X} + 1)) \tan \frac{\phi}{2f} \right. \\ &\quad - 8f^2\mathcal{X}(M_P^4 - \mathcal{X}\phi^2(3M_P^2 + 4\mathcal{X}\phi^2(6\mathcal{X} + 1))) + (\mathcal{X}\phi^2 + M_P^2)^2(M_P^2 + \mathcal{X}\phi^2(6\mathcal{X} + 1)) \sec^2 \frac{\phi}{2f} \\ &\quad \left. - 2(\mathcal{X}\phi^2 + M_P^2)^2(M_P^2 + \mathcal{X}\phi^2(6\mathcal{X} + 1)) \right]. \end{aligned} \quad (92)$$

We use the fact that the slow-roll parameters associated with the potential  $\epsilon_V$  and  $\eta_V$  are approximately equal to the Hubble parameter slow-roll  $\epsilon_H \approx \epsilon_V$  and  $\eta_H \approx \eta_V - \epsilon_V$ . Moreover, we rely here on the fact that the primordial curvature perturbation observables are invariant under a conformal transformation [33] and that the slow-roll variables in the canonical variables of the Einstein frame are invariant as well [63]:  $\epsilon_V = \bar{\epsilon}_V$  and  $\eta_V \approx \bar{\eta}_V$ .

We express our results in terms of the number of  $e$ -folds of the inflationary expansion:

$$N_e = -\frac{1}{M_P^2} \int_{\phi_i}^{\phi_f} K \frac{\bar{V}}{\bar{V}_\phi} d\phi, \quad (93)$$

and we let the field roll from the moment at which the perturbations cross the horizon at  $k = aH$ , until the moment in which the slow-roll conditions are broken:  $\bar{\epsilon}_V(\phi) > 1$  or  $\bar{\eta}_V(\phi) > 1$ . The spectral index of the scalar

perturbations  $n_s$  and the tensor-to-scalar ratio  $r$  are obtained from the slow-roll parameters as

$$\bar{n}_s = n_s = 1 - 6\bar{\epsilon}_V + 2\bar{\eta}_V \quad \text{and} \quad \bar{r} = r = 16\bar{\epsilon}_V. \quad (94)$$

Figure 4 shows the 68% (light blue) and 95% (dark blue) C.L. regions for the tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  from Planck. The predictions in the case of natural inflation with nonminimal coupling are also shown, for different values of  $\mathcal{X}$ . The back thin and thick lines correspond to 50 and 60  $e$ -folds, respectively.<sup>8</sup> We use small values for  $\mathcal{X}$  since in this small field case, the dynamics is sensible to small values of the nonminimal coupling. On the one hand, we see that positive values of  $\mathcal{X}$

<sup>8</sup>An analysis of natural inflation with nonminimal coupling was done before in Ref. [64]. Even if we agree with their analytical results, we diverge on the numerics.

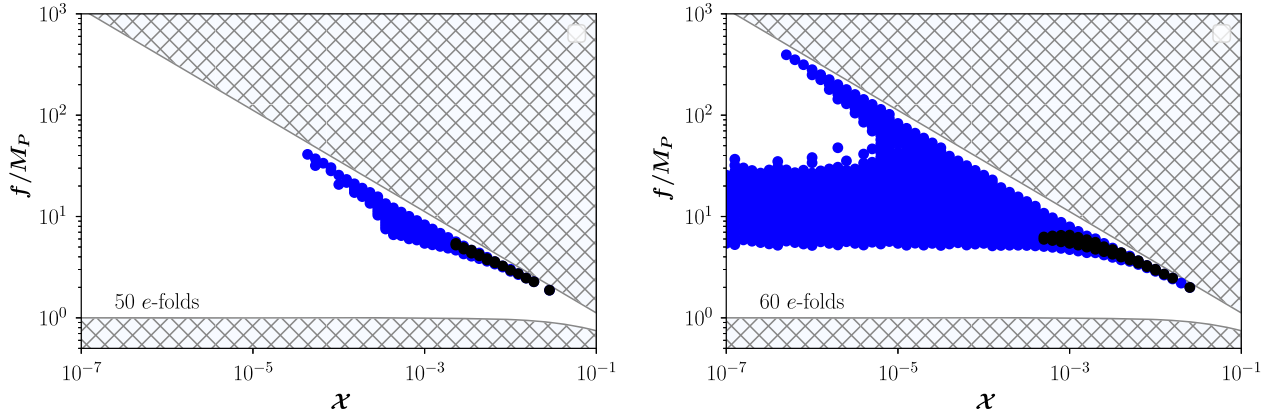


FIG. 5. The blue and black points correspond to the parameter space compatible with the Planck observation at 95% and 68% C.L., respectively. The left (right) panel shows the case of 50 (60)  $e$ -folds.

tend to suppress the amplitude of tensor perturbations and render the scalar perturbations closer to the scale invariance. On the other hand, negative values of  $\mathcal{X}$  tend to enhance the production of tensor mode perturbations and diverge from scale invariance. The case of negative  $\mathcal{X}$  is disfavored by current Planck constraints.

Figure 5 depicts the values of  $f$  and  $\xi$  compatible with Planck limits at 68% (black dots) and 95% C.L. (blue dots), for positive  $\mathcal{X}$ . The left (right) panel shows the case of 50 (60)  $e$ -folds. The hashed regions correspond to the parameter space where  $N_e$  and  $n_s$  tend to be negative at small field values, respectively, and are therefore disregarded. We note that in order to agree with Planck measurements, the best fit favors small but finite nonminimal couplings of the order  $\mathcal{O}(10^{-3})$  to  $\mathcal{O}(10^{-2})$ , and  $f$  at the Planck scale.

### B. Naturally steep potential with nonminimal coupling to gravity

In this case where we have simultaneously a nonminimal coupling to gravity and interactions with gauge fields, the slow-roll parameter  $\epsilon$  can be approximated as

$$\bar{\epsilon} = -\frac{\dot{H}}{\bar{H}^2} \approx \frac{3\Omega K \dot{\phi}^2}{2V} + \frac{\langle \vec{E}^2 + \vec{B}^2 \rangle}{V}. \quad (95)$$

Taking into account that

$$\bar{H}^2 = \frac{H^2}{\Omega} + \frac{\dot{\Omega} H}{\Omega^2} + \frac{1}{4} \frac{\dot{\Omega}^2}{\Omega^3} \approx \frac{H^2}{\Omega}, \quad (96)$$

where we neglected the  $\dot{\Omega}$  terms since they are proportional to  $\dot{\phi}$ , we get

$$\begin{aligned} \bar{\epsilon} &= 3\Omega \frac{K}{2} \frac{4\xi^2 f^2 H^2}{\alpha^2 V} + \frac{\langle \vec{E}^2 + \vec{B}^2 \rangle}{V} \\ &= \frac{2K\xi^2 f^2}{\alpha^2 M_P^2} + \frac{8\xi f K^{1/2} \bar{V}_{\bar{\phi}}}{7\alpha \bar{V}}, \end{aligned} \quad (97)$$

where we have used Eq. (18). We also consider the slow-roll parameter  $\bar{\eta}$

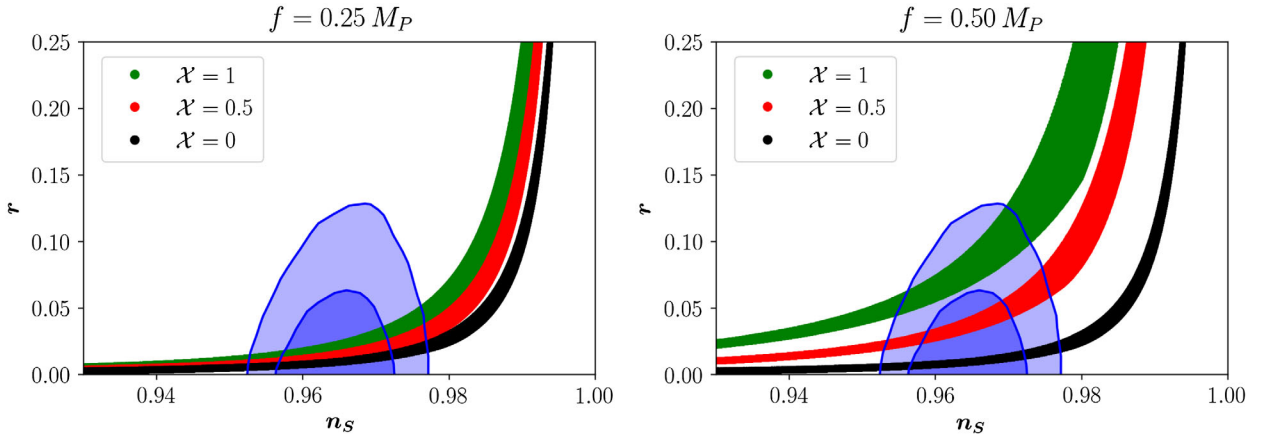


FIG. 6. Tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  for 60  $e$ -folds, in the case where the inflaton is coupled with  $\mathcal{N}$  gauge fields and nonminimally coupled to gravity.



$$\bar{\eta} = \bar{\epsilon} - \frac{1}{2\bar{\epsilon}} \frac{d\bar{\epsilon}}{d\bar{N}} = \bar{\epsilon} - \frac{1}{2\bar{H}\bar{\epsilon}} \frac{d\bar{\epsilon}}{d\bar{t}} = -\frac{\frac{d^2\bar{\phi}}{d\bar{t}^2}}{\frac{d\bar{\phi}}{d\bar{t}}\bar{H}}. \quad (98)$$

Figure 6 is the equivalent of Fig. 1 but now with the inclusion of nonminimal couplings. It again shows the 68% (light blue) and 95% (dark blue) C.L. regions for the tensor-to-scalar ratio  $r$  versus the scalar spectral index  $n_s$  from Planck. The predictions in the case of natural inflation with the inflaton nonminimally coupled to gravity and also coupled with  $\mathcal{N}$  gauge fields are also shown, for  $f = 0.25M_P$  (left panel) and  $f = 0.5M_P$  (right panel), and  $\mathcal{X} = 1$  (green), 0.5 (red), and 0 (black). These regions were constructed by fixing  $N_e = 60$  but varying the parameters in the ranges:  $\xi = [2.5, 10]$  and  $\alpha = [80, 400]$ . The number  $\mathcal{N}$  of gauge fields is fixed by the COBE normalization, the scale  $\Lambda$  of the potential by the equation of motion of the inflaton. In the case where  $\mathcal{X} = 0$  (i.e., inflaton minimally coupled to gravity), the results are independent from  $f$ , and coincide with the ones of Fig. 1. Higher values for  $\mathcal{X}$  tend to increase  $r$ , even above the preferred region by Planck. That is particularly true for large values of  $f$ . The values for  $\xi$ ,  $\alpha$ ,  $\mathcal{N}$ , and  $\Lambda$  compatible with Planck limits are not presented here because they are basically the same as in the minimal case, i.e., Fig. 2.

## V. CONCLUSIONS AND FINAL REMARKS

We studied a scenario in which a pseudoscalar field  $\phi$  is coupled with an ensemble of gauge fields through the axial term  $\phi\tilde{F}F$  in the presence of a nonminimal coupling to gravity. Because of the axial coupling, there is a significant production of chiral gravitational waves which would leave a characteristic signature in the CMB. However, there are strong constraints over this mechanism coming mainly from non-Gaussianity and perturbativity [12–14].<sup>9</sup> In this paper, we added a nonminimal coupling between the pseudoscalar inflaton and gravity, and studied the spectrum of the scalar and tensor perturbations. We tracked the effects of this nonminimal coupling on the spectral index of scalar perturbations  $n_s$  and on the tensor-to-scalar ratio  $r$ , in order to find regions of the parameter space compatible with current observational bounds. The main results of this study are Eqs. (81), (82), (90) and the scan of parameters represented in Fig. 6. We found that, as a result of the gauge fields and gravity interactions with the pseudoscalar inflaton for a steep cosine potential,  $n_s$  and  $r$  are typically mildly modified. However, there are also regions of the parameters space that present a suppression of the vacuum gravitational waves that are accompanied by an enhancement of the gravity waves produced by the axial coupling interaction. This enhancement of the sourced gravitational waves

happens because the dynamics of the gauge fields is affected by the nonminimal coupling through the function  $K$  in Eq. (56) and the parameter  $\xi$  in Eq. (17). The  $K$  function allows for significant modifications in the sourced gravitational wave term, while  $\xi$  is almost unaltered.

An interesting possibility offered by the interaction with gauge fields and a nonminimal coupling is that the amplification of gravitational waves could alleviate some tension with non-Gaussianity bounds. In fact, it was pointed out that non-Gaussianity strongly constrains axionic models with slow-roll potential [12], requiring  $\xi \lesssim 2.5$ . For such small values, the gravitational waves generated by the axial coupling are comparable to the vacuum ones, and hence practically unobservable. However the situation here is different because the non-minimal coupling modifies the evolution of the parameter  $\xi$  and the slow-roll parameter  $\bar{\epsilon}$  for the canonical variables. We will follow the steps of the reasoning of Ref. [12] to track the modifications due to the nonminimal coupling. In the same way that the scalar spectrum is unaltered in the Einstein frame, the bispectrum and hence the non-Gaussianity parameter  $\bar{f}_{\text{NL}}$  do not change either, so

$$\bar{f}_{\text{NL}}^{\text{equil}} \approx f_{\text{NL}}^{\text{equil}} \approx 4.4 \times 10^{10} \bar{\mathcal{P}}_\zeta^3 \frac{e^{6\pi\xi}}{\xi^9}. \quad (99)$$

With this, we get the bound

$$\bar{\mathcal{P}}_\zeta^2 \frac{e^{4\pi\xi}}{\xi^6} < \left( \frac{f_{\text{NL}}^{\text{obs}}}{4.4 \times 10^{10}} \right)^{2/3}. \quad (100)$$

Inserting this expression in Eq. (88) we obtain

$$\bar{r} \approx \frac{\mathcal{A}^+ \bar{H}^4 \frac{e^{4\pi\xi}}{\xi^6}}{\pi^2 M_P^4 \bar{\mathcal{P}}_\zeta} < 35 \frac{\bar{H}^4 e^{4\pi\xi}}{M_P^4 \xi^6} < 1.7 \times 10^{-2} (f_{\text{NL}}^{\text{obs}})^{2/3} \bar{\epsilon}^2. \quad (101)$$

In the last expression,  $\bar{\epsilon}$  is the slow-roll parameter for the canonical field  $\bar{\phi}$ . It can be seen that this parameter is related with the  $\epsilon$  parameter for the noncanonical field  $\phi$  through

$$\epsilon = \frac{\dot{\phi}^2}{2H^2} \approx \frac{1}{K} \frac{\left(\frac{d\bar{\phi}}{d\bar{t}}\right)^2}{2\bar{H}^2} = \frac{1}{K} \bar{\epsilon}, \quad (102)$$

where we used Eq. (59). With that, we rewrite the bound as

$$\bar{r} < 1.7 \times 10^{-2} (f_{\text{NL}}^{\text{obs}})^{2/3} K^2 \epsilon^2. \quad (103)$$

Here we notice that the slow-roll parameter  $\epsilon$  for the noncanonical field is not modified significantly due to the nonminimal coupling function. However, the function  $K$  can act as an amplification factor for certain values of the nonminimal coupling parameter  $\mathcal{X}$ . This mechanism offers then a possibility to leave some measurable signatures of sourced gravitational waves.

<sup>9</sup>Nevertheless, as we mentioned in the Introduction, here the perturbativity of the background evolution is broken by construction. The scalar and tensor perturbations remain perturbative.

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## APPENDIX A: DETAILS OF THE CALCULATION OF THE PERTURBATIONS SPECTRUM

## 1. The Green function

Here we revisit the details of the approximate solution for the scalar perturbations. The rhs of Eq. (21) can be decomposed in two parts, one due to the intrinsic inhomogeneities in  $\vec{E} \cdot \vec{B}$  even with  $\phi = 0$ , and a second component due to the dependence on  $\dot{\phi}$ :

$$\begin{aligned} \delta[\vec{E} \cdot \vec{B}] &\approx [\vec{E} \cdot \vec{B} - \langle \vec{E} \cdot \vec{B} \rangle]_{\delta\phi=0} + \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \dot{\phi}} \delta\dot{\phi} \\ &\equiv \delta_{\vec{E} \cdot \vec{B}} + \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \dot{\phi}} \delta\dot{\phi}. \end{aligned} \quad (\text{A1})$$

Using the definition of  $\xi$  and the approximations in Eq. (19), the  $\dot{\phi}$  dependent term can be written as

$$\begin{aligned} \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \dot{\phi}} \delta\dot{\phi} &\approx \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \xi} \frac{\partial \xi}{\partial \dot{\phi}} \delta\dot{\phi} \approx \frac{\partial \langle \vec{E} \cdot \vec{B} \rangle}{\partial \xi} \frac{\alpha}{2fH} \delta\dot{\phi} \\ &\approx 2\pi \langle \vec{E} \cdot \vec{B} \rangle \frac{\alpha}{2faH} \delta\phi', \end{aligned} \quad (\text{A2})$$

where we have used  $dt = a d\tau$  in the last part. Dots and primes correspond to  $t$  and  $\tau$  derivatives, respectively. If we use the homogeneous equation (20), and assuming that the dynamics is governed by the source term, we can neglect the time derivatives of the pseudoscalar field and hence

$$V_\phi \approx \frac{\mathcal{N}\alpha}{f} \langle \vec{E} \cdot \vec{B} \rangle. \quad (\text{A3})$$

Putting everything together, we write the perturbations equation as

$$\begin{aligned} \delta\phi'' + 2aH\delta\phi' + [-\nabla^2 + a^2 V_{\phi\phi}(\phi_0)]\delta\phi \\ \approx a^2 \frac{\mathcal{N}\alpha}{f} \left( \delta_{\vec{E} \cdot \vec{B}} + \frac{\pi V_\phi(\phi_0)}{\mathcal{N}aH} \delta\phi' \right), \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} \delta\phi'' + 2aH \left( 1 - \frac{\alpha\pi V_\phi(\phi_0)}{2fH^2} \right) \delta\phi' + [-\nabla^2 + a^2 V_{\phi\phi}(\phi_0)]\delta\phi \\ \approx a^2 \frac{\mathcal{N}\alpha}{f} \delta_{\vec{E} \cdot \vec{B}}. \end{aligned} \quad (\text{A5})$$

Using  $a \approx -1/(H\tau)$  during inflation (de Sitter metric), we get

$$\begin{aligned} \delta\phi'' - \frac{2}{\tau} \left( 1 - \frac{\alpha\pi V_\phi(\phi_0)}{2fH^2} \right) \delta\phi' + \left[ -\nabla^2 + \frac{1}{\tau^2} \frac{V_{\phi\phi}(\phi_0)}{H^2} \right] \delta\phi \\ \approx a^2 \frac{\mathcal{N}\alpha}{f} \delta_{\vec{E} \cdot \vec{B}}. \end{aligned} \quad (\text{A6})$$

Assuming that the inhomogeneities are small, and considering also that  $V_\phi \approx V/f$  and  $V \approx 3M_P^2 H^2$  and assuming  $\alpha \gg 1$  and  $f \lesssim M_P$ , the factor accompanying  $\delta\phi'$  can be approximated as

$$-\frac{2}{\tau} \left( 1 - \frac{\alpha\pi V_\phi(\phi_0)}{2fH^2} \right) \approx +\frac{1}{\tau} \frac{\alpha\pi V_\phi(\phi_0)}{fH^2}. \quad (\text{A7})$$

The last approximation is valid in this case where the backreaction term, the one coming from the variation  $\delta[\vec{E} \cdot \vec{B}]$  in the rhs of Eq. (A4) is the dominant. In Eq. (A6), the gradient can also be neglected. In fact, the bulk of the analysis relies on the solution for the vector potential  $A^\mu$ , Eq. (18), which is valid in the regime  $|k\tau| \ll 2\xi$ , with  $\xi > 1$ . Considering the third term in Eq. (A6), one can compare the relevance of the gradient term with the friction term. It can be approximated by

$$k^2 + \frac{1}{\tau^2} \frac{V''}{H^2} \approx k^2 + \frac{1}{\tau^2} \frac{V}{f^2 H^2} \approx k^2 + \frac{3M_P^2}{\tau^2 f^2}. \quad (\text{A8})$$

We can therefore neglect the gradient term whenever  $|k\tau| \lesssim M_P/f$ . Taking into account that  $|k\tau| \ll 2\xi$ , we deduce that this condition is always satisfied as long as  $f \lesssim M_P/\xi$ : sub-Planckian values for the coupling  $f$ , making the potential steep, are required in this mechanism and ensure that the electromagnetic friction term dominates over the gradient of the scalar potential.

Therefore, Eq. (A6) becomes

$$\delta\phi'' + \frac{1}{\tau} \frac{\alpha\pi V_\phi(\phi_0)}{fH^2} \delta\phi' + \frac{1}{\tau^2} \frac{V_{\phi\phi}(\phi_0)}{H^2} \delta\phi \approx a^2 \frac{\mathcal{N}\alpha}{f} \delta_{\vec{E} \cdot \vec{B}}. \quad (\text{A9})$$

We can solve the Green's function for this equation:

$$\frac{\partial^2 G(\tau, \tau')}{\partial \tau^2} + \frac{1}{\tau} \frac{\alpha \pi V_\phi(\phi_0)}{f H^2} \frac{\partial G(\tau, \tau')}{\partial \tau} + \frac{1}{\tau^2} \frac{V_{\phi\phi}(\phi_0)}{H^2} G(\tau, \tau') = \delta(\tau - \tau') \quad (\text{A10})$$

with the boundary conditions  $G(\tau', \tau') = 0$ ,  $\partial G(\tau', \tau') / \partial \tau = 1$ , obtaining

$$G(\tau, \tau') = \frac{\tau'}{\Delta} \left[ \left( \frac{\tau}{\tau'} \right)^{\nu_+} - \left( \frac{\tau}{\tau'} \right)^{\nu_-} \right] \Theta(\tau - \tau'), \quad (\text{A11})$$

where

$$\nu_{\pm} \equiv \frac{1}{2} \left( 1 - \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \right) \pm \frac{1}{2} \Delta, \quad \text{and} \quad \Delta \equiv \sqrt{\left( 1 - \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \right)^2 - \frac{4 V_{\phi\phi}(\phi_0)}{H^2}}. \quad (\text{A12})$$

As said before,  $\frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \gg 1$ ; then we can approximate the previous expressions as

$$\Delta \approx \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \sqrt{1 - \frac{4 f^2 H^2 V_{\phi\phi}(\phi_0)}{\pi^2 \alpha^2 V_\phi(\phi_0)^2}}, \quad \nu_{\pm} \approx -\frac{1}{2} \frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \left( 1 \mp \sqrt{1 - \frac{4 f^2 H^2 V_{\phi\phi}(\phi_0)}{\pi^2 \alpha^2 V_\phi(\phi_0)^2}} \right). \quad (\text{A13})$$

Taking into account that  $[f / (\alpha M_P)]^2 \ll 1$ , previous equations can be rewritten as

$$\Delta \approx \frac{\pi \alpha V_\phi(\phi_0)}{f H^2}, \quad \nu_+ \approx -\frac{f V_{\phi\phi}(\phi_0)}{\pi \alpha V_\phi(\phi_0)} \ll 1, \quad \nu_- \approx -\frac{\pi \alpha V_\phi(\phi_0)}{f H^2} \approx -\Delta \gg 1. \quad (\text{A14})$$

At late times or for scales at which  $|k\tau| \ll 1$ , the Green's function becomes

$$G(\tau, \tau') \approx \frac{\tau'}{\Delta} \left( \frac{\tau}{\tau'} \right)^{\nu_+}, \quad \tau > \tau'. \quad (\text{A15})$$

## 2. Power spectrum

We need to calculate the spectrum of the scalar perturbations:

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &= \delta(\vec{p} + \vec{p}') \left( \frac{\mathcal{N}\alpha}{f} \right)^2 \int d\tau_1 d\tau_2 a_1^2 a_2^2 G(\tau, \tau_1) G(\tau, \tau_2) \\ &\times \int d^3 x e^{i\vec{p}\cdot\vec{x}} \langle \delta_{\vec{E}\cdot\vec{B}}(\tau_1, 0) \delta_{\vec{E}\cdot\vec{B}}(\tau_2, \vec{x}) \rangle. \end{aligned} \quad (\text{A16})$$

Using Eq. (A15) we can approximate the previous expression to

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &\approx \delta(\vec{p} + \vec{p}') \frac{\mathcal{N}^2 \alpha^2 \tau^{2\nu_+}}{H^4 \Delta^2 f^2} \int \frac{d\tau_1 d\tau_2}{(\tau_1 \tau_2)^{1+\nu_+}} \\ &\times \int d^3 x e^{i\vec{k}\cdot\vec{x}} \langle \delta_{\vec{E}\cdot\vec{B}}(\tau_1, 0) \delta_{\vec{E}\cdot\vec{B}}(\tau_2, \vec{x}) \rangle. \end{aligned} \quad (\text{A17})$$

The factor  $\tau^{2\nu_+}$  carries the scale dependence in the power spectrum. Following Refs. [1,2] we evaluate the spacial integral using the approximation (18), and neglecting the  $A_-$  polarization one finds

$$\begin{aligned} &\int d^3 x e^{i\vec{k}\cdot\vec{x}} \langle \delta_{\vec{E}\cdot\vec{B}}(\tau_1, 0) \delta_{\vec{E}\cdot\vec{B}}(\tau_2, \vec{x}) \rangle \\ &= \frac{e^{4\pi\xi}}{4a_1^4 a_2^4} \int \frac{d^3 k}{(2\pi)^3} |\epsilon_+(-\vec{k}) \cdot \epsilon_+(\vec{p} + \vec{k})|^2 \\ &\times [|\vec{k}| |\vec{k} + \vec{p}| + |\vec{k}|^{3/2} |\vec{k} + \vec{p}|^{1/2}] e^{-4\sqrt{2\xi/\bar{a}H}(\sqrt{|\vec{k}|} + \sqrt{|\vec{k} + \vec{p}|})} \end{aligned} \quad (\text{A18})$$

where  $2/\sqrt{\bar{a}} \equiv 1/\sqrt{a_1} + 1/\sqrt{a_2}$ . We can choose the vector  $\vec{p}$  to be along the  $\hat{z}$  axis, then  $\vec{p} = p\hat{z}$ , and we change variables to  $\vec{q} \equiv \vec{k}/p$ . With this, the integral is expressed as

$$\begin{aligned} &\int d^3 x e^{i\vec{k}\cdot\vec{x}} \langle \delta_{\vec{E}\cdot\vec{B}}(\tau_1, 0) \delta_{\vec{E}\cdot\vec{B}}(\tau_2, \vec{x}) \rangle \\ &= \frac{e^{4\pi\xi} p^5}{4a_1^4 a_2^4} \int \frac{d^3 q}{(2\pi)^3} |\epsilon_+(-\vec{q}) \cdot \epsilon_+(\hat{z} + \vec{q})|^2 \\ &\times q |\hat{z} + \vec{q}| \left[ 1 + \frac{q^{1/2}}{|\hat{z} + \vec{q}|^{1/2}} \right] e^{-\sqrt{2^5 \xi p / \bar{a} H}(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})}. \end{aligned} \quad (\text{A19})$$

The time dependence of the exponential factor is separated as

$$\begin{aligned} &e^{-\sqrt{2^5 \xi p / \bar{a} H}(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})} \\ &= e^{-\frac{1}{2}(\sqrt{-2^5 \xi p \tau_1} + \sqrt{-2^5 \xi p \tau_2})(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})} \end{aligned} \quad (\text{A20})$$

which, introducing the change of variables  $x_{1,2} = (-2^5 \xi p \tau_{1,2})^{1/4}$  becomes

$$e^{-\frac{1}{2}(\sqrt{-2^5\xi p\tau_1} + \sqrt{-2^5\xi p\tau_2})(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})} = e^{-\frac{1}{2}(x_1^2 + x_2^2)(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})}, \quad (\text{A21})$$

and the integral (A19) is expressed as

$$\int d^3x e^{i\vec{k}\cdot\vec{x}} \langle \delta_{\vec{E}\cdot\vec{B}}(\tau_1, 0) \delta_{\vec{E}\cdot\vec{B}}(\tau_2, \vec{x}) \rangle = \frac{e^{4\pi\xi} p^5 H^8 x_1^{16} x_2^{16}}{(2^5 \xi p)^8 2^5 \pi^3} \mathcal{M}(x_1, x_2), \quad (\text{A22})$$

where

$$\begin{aligned} \mathcal{M}(x_1, x_2) &\equiv \int d^3q \mathcal{J}(\vec{q}) e^{-\frac{1}{2}(x_1^2 + x_2^2)(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})} \\ &\equiv \int d^3q |\epsilon_+(-\vec{q}) \cdot \epsilon_+(\hat{z} + \vec{q})|^2 q |\hat{z} + \vec{q}| \left[ 1 + \frac{q^{1/2}}{|\hat{z} + \vec{q}|^{1/2}} \right] e^{-\frac{1}{2}(x_1^2 + x_2^2)(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})}. \end{aligned} \quad (\text{A23})$$

Inserting this into Eq. (A17) and doing the time variables change, we get

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &= \delta(\vec{p} + \vec{p}') \frac{\mathcal{N}^2 \alpha^2 H^4}{p^3 \Delta^2 f^2 \xi^8} \frac{e^{4\pi\xi}}{2^{15}} (-2^5 \xi p \tau)^{2\nu_+} \\ &\times \int dx_1 dx_2 (x_1 x_2)^{15-4\nu_+} \frac{16\mathcal{M}(x_1, x_2)}{2^{30} \pi^3}. \end{aligned} \quad (\text{A24})$$

The time integrals in the variables  $x_i$  can be performed as integrals of the form  $x^\beta e^{-\alpha x^2}$

$$\begin{aligned} &\int dx_1 dx_2 (x_1 x_2)^{15-4\nu_+} \frac{16\mathcal{M}(x_1, x_2)}{2^{30} \pi^3} \\ &= \int d^3q \mathcal{J}(\vec{q}) \frac{16}{2^{30} \pi^3} \int dx_1 dx_2 (x_1 x_2)^{15-4\nu_+} \\ &\times e^{-\frac{1}{2}(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})(x_1^2 + x_2^2)} \\ &= \frac{\Gamma[8 - 2\nu_+]^2}{2^{12+4\nu_+} \pi^3} \int d^3q \frac{\mathcal{J}(\vec{q})}{(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})^{16-4\nu_+}}. \end{aligned} \quad (\text{A25})$$

Using

$$|\epsilon_+(-\vec{q}) \cdot \epsilon_+(\hat{z} + \vec{q})|^2 = \frac{1}{4} \left( \frac{q + \cos\theta - |\vec{q} + \hat{z}|}{|\vec{q} + \hat{z}|} \right)^2, \quad (\text{A26})$$

where  $\theta$  is the angle between  $\hat{z}$  and  $\vec{q}$ , the momentum integral becomes

$$\begin{aligned} &\int d^3q \frac{\mathcal{J}(\vec{q})}{(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})^{16-4\nu_+}} \\ &= \int d^3q \frac{(q + \cos\theta - |\vec{q} + \hat{z}|)^2 q}{4|\vec{q} + \hat{z}|(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})^{16-4\nu_+}} \\ &\times \left[ 1 + \frac{q^{1/2}}{|\hat{z} + \vec{q}|^{1/2}} \right]. \end{aligned} \quad (\text{A27})$$

Then the momentum integral can be evaluated numerically. Neglecting  $\nu_+ \ll 1$  we obtain

$$\int d^3q \frac{\mathcal{J}(\vec{q})}{(\sqrt{|\vec{q}|} + \sqrt{|\vec{q} + \hat{z}|})^{16-4\nu_+}} \approx 3.49 \times 10^{-4}, \quad (\text{A28})$$

and the time integral is evaluated as

$$\begin{aligned} \mathcal{T}(\nu_+ = 0) &= \int dx_1 dx_2 (x_1 x_2)^{15-4\nu_+} \frac{16\mathcal{M}(x_1, x_2)}{2^{30} \pi^3} \\ &\approx 3.49 \times 10^{-4} \frac{\Gamma[8]^2}{2^{12} \pi^3} \approx 7 \times 10^{-2}. \end{aligned} \quad (\text{A29})$$

Notice that, if we use  $\nu_+ \approx -0.0175$  to be in agreement with Planck results for the spectral index, the result is modified in a small amount

$$\begin{aligned} \mathcal{T}(\nu_+ \approx -0.0175) &= \int dx_1 dx_2 (x_1 x_2)^{15-4\nu_+} \frac{16\mathcal{M}(x_1, x_2)}{2^{30} \pi^3} \\ &\approx 8.2 \times 10^{-2}. \end{aligned} \quad (\text{A30})$$

Using our previous results, the two point correlator of the perturbations is

$$\begin{aligned} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle &\approx \frac{\mathcal{T}(\nu_+)}{2^{15}} \frac{\delta(\vec{p} + \vec{p}')}{p^3} \frac{\mathcal{N}^2 \alpha^2 H^4}{\Delta^2 f^2 \xi^8} e^{4\pi\xi} (-2^5 \xi p \tau)^{2\nu_+} \\ &\approx \mathcal{F}(\nu_+) \frac{\delta(\vec{p} + \vec{p}')}{p^3} \frac{\mathcal{N}^2 \alpha^2 H^4}{\Delta^2 f^2 \xi^8} \\ &\times e^{4\pi\xi} (-2^5 \xi p \tau)^{2\nu_+}, \end{aligned} \quad (\text{A31})$$

with  $\mathcal{F}(\nu_+ = 0) \approx 2.13 \times 10^{-6}$ . With the previous results we can calculate the power spectrum of the primordial curvature perturbation  $\zeta = -H\delta\phi/\dot{\phi}_0$

$$\begin{aligned} \delta(\vec{p} + \vec{p}') \mathcal{P}_\zeta(p) &= \frac{p^3}{2\pi^2} \frac{H^2}{\dot{\phi}_0^2} \langle \delta\phi(\vec{p}) \delta\phi(\vec{p}') \rangle \\ &\approx \delta(\vec{p} + \vec{p}') \frac{\mathcal{F}}{2\pi^2} \frac{H^2}{\dot{\phi}_0^2} \frac{\mathcal{N}^2 \alpha^2 H^4}{\Delta^2 f^2 \xi^8} \\ &\times e^{4\pi\xi} (-2^5 \xi p \tau)^{2\nu_+}. \end{aligned} \quad (\text{A32})$$

Using Eqs. (17), (19), (20), and (A14), the spectrum can be rewritten as

$$\begin{aligned}\mathcal{P}_\xi(p) &\approx \frac{\mathcal{F}(\nu_+)}{8\pi^4 \mathcal{I}^2 \xi^2} (-2^5 \xi p \tau)^{2\nu_+} \\ &\approx \frac{4.7 \times 10^{-2}}{\mathcal{N} \xi^2} (-2^5 \xi p \tau)^{2\nu_+},\end{aligned}\quad (\text{A33})$$

where the factor  $\mathcal{N}$  comes from the fact that the gauge fields add incoherently in the spectrum. From this expression we extract the spectral index

$$n_s - 1 \approx 2\nu_+ = -\frac{2fV_{\phi\phi}(\phi_0)}{\pi\alpha V_\phi(\phi_0)}. \quad (\text{A34})$$

## APPENDIX B: CONSTANT $\xi$ REGIME

In this section we justify the assumption of constant  $\xi$  used throughout the paper. Figure 7 shows the evolution of  $\xi$  and the ratio  $\dot{\xi}/(H\xi)$ , for the same benchmark point used in Fig. 3 ( $\Lambda = 4.5 \times 10^{-3} M_P$ ,  $\alpha = 400$ ,  $\mathcal{N} = 10^5$ ,  $f = 0.1 M_P$ ) and taking  $\mathcal{X} = 0$  (blue) and  $\mathcal{X} = 10$  (red). Both quantities are small, which shows that the assumption of constant  $\xi$  is well justified for the scales and the regime considered in the calculation of the perturbations.

Additionally, the right panel shows that the ratio  $\dot{\xi}/(H\xi)$  tends to stabilize around  $2.5 \times 10^{-3}$  for the time of the horizon exit. As the dependence of the solution for the vector field in Eq. (18) is roughly exponential ( $A_+ \sim e^{\pi\xi}$ ), a variation of  $\xi$  of order  $2.5 \times 10^{-3}$  has an impact below the percent level in the vector field.

## APPENDIX C: JORDAN AND EINSTEIN FRAMES

Here we briefly discuss the transformation from Jordan to Einstein frames used in Sec. III A applied to a single pseudoscalar field nonminimally coupled with gravity. Further details of the relation between Jordan and Einstein frames can be found, for instance, in Ref. [59]. We start with the action for the system in Eq. (48). By doing

the conformal transformation  $\bar{g}_{\mu\nu} = \Omega(\phi)g_{\mu\nu}$ , the action (48) is transformed into

$$\begin{aligned}\mathcal{L} = \sqrt{-\bar{g}} &\left[ \frac{M_P^2}{2} \left( \bar{R} + \frac{3\Box\Omega}{\Omega^2} - \frac{3}{2\Omega^2} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \Omega \bar{\nabla}_\nu \Omega \right) \right. \\ &- \frac{1}{2\Omega} (\bar{\nabla}\phi)^2 - \bar{V}(\phi) \\ &\left. - \frac{\mathcal{N}}{4\Omega^2} \left( F^{\mu\nu} F_{\mu\nu} + \frac{\alpha}{f} \phi F^{\mu\nu} \bar{F}_{\mu\nu} \right) \right],\end{aligned}\quad (\text{C1})$$

where  $\bar{V}(\phi) \equiv \frac{V(\phi)}{\Omega^2}$  and we have used the fact that the spacetime coordinates are not affected by the conformal transformation:  $\partial_\mu = \bar{\partial}_\mu$ . Moreover, as the derivatives act on pseudoscalar fields, we can write  $\bar{\nabla}_\mu \Omega = \bar{\partial}_\mu \Omega = \nabla_\mu \Omega = \partial_\mu \Omega$ , where the covariant derivative  $\bar{\nabla}_\mu$  is compatible with the metric  $\bar{g}_{\mu\nu}$ . The vector part is not altered since the field strength  $F_{\mu\nu}$  is not affected by the conformal transformation. Now, recalling that  $\Box\Omega = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Omega)$  and that  $\sqrt{-\bar{g}} = \Omega^2 \sqrt{-g}$ , we realize that

$$\sqrt{-\bar{g}} \frac{M_P^2}{2} \frac{3\Box\Omega}{\Omega^2} \quad (\text{C2})$$

is a boundary term. Additionally, the derivatives of the function  $\Omega$  read

$$\bar{g}^{\mu\nu} \bar{\nabla}_\mu \Omega \bar{\nabla}_\nu \Omega = \left( \frac{\partial\Omega}{\partial\phi} \right)^2 \bar{g}^{\mu\nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi, \quad (\text{C3})$$

and then, the kinetic term can be rewritten as

$$\frac{3M_P^2}{4\Omega^2} (\bar{\nabla}\Omega)^2 + \frac{1}{2\Omega} (\bar{\nabla}\phi)^2 = \frac{1}{2} K(\phi) \bar{g}^{\mu\nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi \quad (\text{C4})$$

where

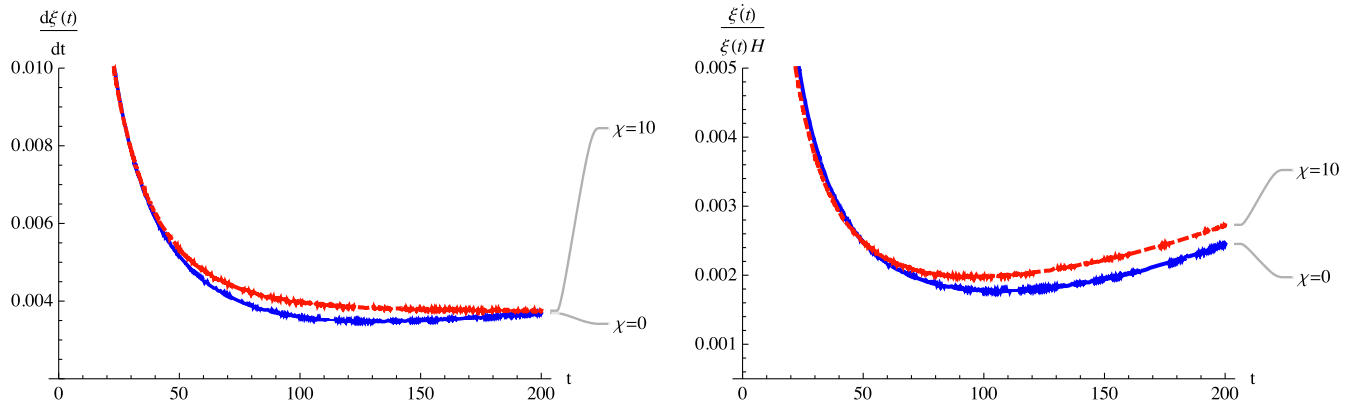


FIG. 7. In this figure we see the evolution of the following quantities:  $\dot{\xi}$  at the left panel and  $\dot{\xi}/(H\xi)$  at the right panel. The values used for those numerical evaluations are the same employed in Fig. 3. This is  $\Lambda = 4.5 \times 10^{-3} M_P$ ,  $\alpha = 400$ ,  $\mathcal{N} = 10^5$ ,  $f = 0.1 M_P$ , and for minimal coupling (blue) and nonminimal coupling with  $\mathcal{X} = 10$  (red). The time is measured in units of  $M_P/\Lambda^2$ .



$$K(\phi) = \frac{1}{\Omega} + \frac{3M_P^2}{2\Omega^2} \left( \frac{\partial\Omega}{\partial\phi} \right)^2. \quad (\text{C5})$$

So, the Lagrangian (C1) in the Einstein frame reduces to

$$\mathcal{L} = \sqrt{-\bar{g}} \left[ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} K(\phi) \bar{g}^{\mu\nu} \bar{\nabla}_\mu \phi \bar{\nabla}_\nu \phi - \bar{V}(\phi) - \frac{\mathcal{N}}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \phi \bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} \right]. \quad (\text{C6})$$

In the case of a single field, we can always define a canonical field  $\bar{\phi}$  through the transformation  $d\bar{\phi}/d\phi = K^{1/2}$ . The action for the system becomes

$$\mathcal{L} = \sqrt{-\bar{g}} \left[ \frac{M_P^2}{2} \bar{R} - \frac{1}{2} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \bar{\phi} \bar{\nabla}_\nu \bar{\phi} - \bar{V}(\bar{\phi}) - \frac{\mathcal{N}}{4} \bar{F}^{\mu\nu} \bar{F}_{\mu\nu} - \frac{\mathcal{N}\alpha}{4f} \phi(\bar{\phi}) \bar{F}^{\mu\nu} \bar{\tilde{F}}_{\mu\nu} \right], \quad (\text{C7})$$

which is the action for a canonical pseudoscalar field minimally coupled with gravity.

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