Tale of two anomalies

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A recent improved determination of the fine structure constant, $\alpha = 1/137.035999046(27)$, leads to a $\sim 2.4\sigma$ negative discrepancy between the measured electron anomalous magnetic moment and the standard model prediction. That situation is to be compared with the muon anomalous magnetic moment where a positive $\sim 3.7\sigma$ discrepancy has existed for some time. A single scalar solution to both anomalies is shown to be possible if the two-loop electron Barr-Zee diagrams dominate the scalar one-loop electron anomaly effect and the scalar couplings to the electron and two photons are relatively large. We also briefly discuss the implications of that scenario.

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So far, neither the LHC experiments nor direct searches for dark matter have uncovered any signs of a "natural" Higgs sector nor weak scale dark matter (DM) states. However, there have been mild deviations from the standard model (SM) predictions over the years. Of these, a long-standing one is the $\sim 3.7\sigma$ discrepancy between experiment [1,2] and theory (see, e.g., Refs. [3,4]) for the muon anomalous magnetic moment $a_\mu \equiv (g_\mu - 2)/2$,

$$\Delta a_\mu \equiv a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = (274 \pm 73) \times 10^{-11}, \qquad (1)$$

which has withstood various theoretical refinements and is being currently remeasured at Fermilab with higher precision. While the final word on $g_\mu - 2$ remains to be decided by the new measurements and ongoing theoretical improvements of the SM prediction, the deviation has been a subject of intense phenomenological interest. As new physics at the TeV scale gets more constrained, the parameter space for weak scale models that could explain $g_\mu - 2$ starts to close.

Meanwhile, the search for new "dark" or "hidden" states at low mass scales $\lesssim 1$ GeV has ben receiving increasing attention [5,6], partially spurred by astrophysical considerations related to DM models [7] and perhaps also by the dearth of indications for new high-energy phenomena. In fact, $g_\mu - 2$ has emerged as an interesting target for dark sector searches, since light states with feeble couplings to the SM can, in principle, explain the anomaly. An early and motivated possibility was offered by the "dark photon" hypothesis, where a new vector boson that kinetically mixes with the photon [8] could have provided a solution [9]. This idea and its simple extensions have now been essentially ruled out. However, other light states from a dark sector, e.g., a light scalar that very weakly couples to muons, could still furnish a potential solution [10].

A recent precise determination of the fine structure constant, α, has introduced a new twist to this story. An improvement in the measured h/M_{Cs} of atomic cesium, where h is Planck's constant, used in conjunction with other precisely known mass ratios and the Rydberg constant leads to the new best value [11],

$$\alpha^{-1}(\mathrm{Cs}) = 137.035999046(27). \qquad (2)$$

(For a detailed explanation of that prescription and its use in determining the SM prediction for the electron anomalous magnetic moment, $a_e = (g_e - 2)/2$, see the articles by G. Gabrielse in Ref. [12].) As a result, comparison of the theoretical prediction of a_e^{SM} [13] with the existing experimental measurement of a_e^{exp} [14,15] now leads to a discrepancy,

$$\Delta a_e \equiv a_e^{\mathrm{exp}} - a_e^{\mathrm{SM}} = [-87 \pm 28(\mathrm{exp}) \pm 23(\alpha) \pm 2(\mathrm{theory})] \times 10^{-14}, \qquad (3)$$

or, when the uncertainties are added in quadrature,

$$\Delta a_e = (-87 \pm 36) \times 10^{-14}. \qquad (4)$$

The above result represents a 2.4σ discrepancy that is opposite in sign from the long-standing muon discrepancy

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previously mentioned and larger in magnitude than lepton-mass-scaling m_e^2/m_μ^2 might suggest.

Note that the discrepancy in Eqs. (3) and (4) results from an improvement in α^{-1} from 137.035998995(85) which previously [13] gave $\Delta a_e = -130(77) \times 10^{-14}$ and represented a 1.7σ effect. The central value has decreased in magnitude, but its significance has increased. The errors from the experimental determinations of a_e and α are now the dominant sources of uncertainty and they are expected to further improve in the near future. An alternative perspective is that $\alpha^{-1}(a_e) = 137.035999149(33)$, derived from a comparison of a_e theory [13] and experiment [14,15], differs by 2.4σ from Eq. (2), and follow-up experimental improvements may resolve the current discrepancy or significantly diminish its magnitude.

Interestingly, dark photon models [9] and their simple extensions predict a one-loop positive deviation for both $g_\mu - 2$ and $g_e - 2$. Therefore, the negative $\sim 2.4\sigma$ deviation in $g_e - 2$ cannot be simultaneously explained together with the $\sim 3.7\sigma$ anomaly in $g_\mu - 2$ in the simplest versions of those models, even if one could circumvent existing experimental constraints.

In this paper, we would like to point out that a minimal model based on a single light real scalar ϕ, can in principle explain the deviations of both $g_\mu - 2$ and $g_e - 2$, in a relatively economical fashion. We will show that a two-loop Barr-Zee diagram [16,17] might explain Δa_e while a one-loop contribution could be the primary origin of Δa_μ [10,18], with both corrections mediated by the same scalar ϕ. For more detailed discussions of these loop processes and their contributions to the electron and muon anomalous magnetic moments, see Refs. [19,20], where the authors discuss the relative contributions of one- and two-loop diagrams, but focus primarily on the case of a pseudoscalar boson.

Before going further, we note that somewhat less minimal solutions, e.g., with a scalar coupled to the muon and a pseudo-scalar coupled to the electron, can potentially yield the right size and sign for the deviations in $g_\mu - 2$ and $g_e - 2$, respectively, and satisfy experimental constraints. However, here, we focus on the effect of a single light scalar where inclusion of the Barr-Zee contribution represents an extension of earlier work in Ref. [10]. Studies of the contribution of Barr-Zee type diagrams to $g_\mu - 2$ in the context of two Higgs doublet models and supersymmetry can also be found in Ref. [21].

Let us consider the following effective Lagrangian for the real scalar ϕ of mass m_ϕ

$$\mathcal{L}_\phi = -\frac{1}{2} m_\phi^2 \phi^2 - \sum_f \lambda_f \phi \bar{f} f - \frac{\kappa_\gamma}{4} \phi F_{\mu\nu} F^{\mu\nu}, \qquad (5)$$

where we only include explicit couplings with strengths λ_f to a set of fermions f and have omitted various kinetic terms and fermion masses. In this work, we allow f to correspond to SM fermions, as well as other potential more massive charged fermions. The λ_f are constrained by phenomenology, as will be discussed later. We assume that the ϕ coupling to photons, through the field strength tensor $F_{\mu\nu}$, is governed by the constant κ_γ which has mass dimension -1. The sum over $\phi\gamma\gamma$ triangle diagrams mediated by f will induce a contribution to κ_γ, but we do not specify the properties of all charged states that couple to ϕ.

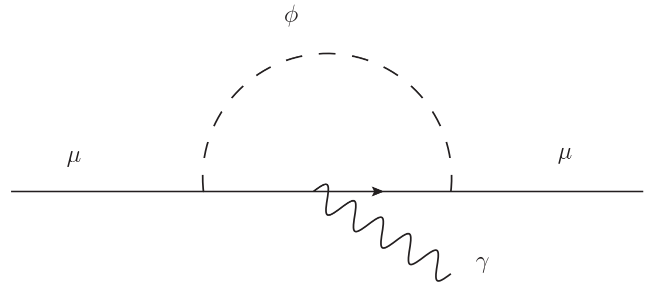

FIG. 1. One-loop ϕ contribution to $g_\mu - 2$.

We will start with the $g_\mu - 2$ discrepancy, assumed to be dominated by the one-loop diagram in Fig. 1, which is given by [10,22,23]

$$\Delta a_\ell = \frac{\lambda_\ell^2}{8\pi^2} x^2 \int_0^1 dz \frac{(1+z)(1-z)^2}{x^2(1-z)^2 + z} \qquad (6)$$

for a lepton ℓ of mass m_ℓ and $x \equiv m_\ell / m_\phi$.

Current experimental constraints, as illustrated in Ref. [24]—under the assumption that ϕ only couples to muons—allow $2m_\mu \lesssim m_\phi \lesssim 100$ GeV and $\lambda_\mu \sim 5 \times 10^{-4} - 0.1$, roughly corresponding to a range of parameters that can explain the 3.7σ deviation in $g_\mu - 2$, given by Eq. (1), which we will approximate as $\Delta a_\mu \approx 3 \times 10^{-9}$. The above lower bound on m_ϕ corresponds to demanding that ϕ decay promptly into muon pairs. In our scenario, couplings to the electron lead to prompt decays $\phi \to e^+e^-$ below the muon pair threshold, allowing $m_\phi \lesssim 200$ MeV. However, for such values of m_ϕ, the one-loop positive contribution to $g_e - 2$ starts to become significant and cancel out the desired two-loop effect that we will discuss below. For m_ϕ well above the GeV scale, we also find it difficult to accommodate the suggested $g_e - 2$ anomaly in Eq. (4) with reasonable values of λ_e and κ_γ. In addition, for $m_\phi \gg 1$ GeV, typical low energy probes of ϕ at intense beam facilities become less efficient, adversely affecting experimental prospects for testing the scenario. For the above reasons, we mostly focus on the ϕ mass range $2m_\mu \lesssim m_\phi \lesssim$ few GeV, in what follows.

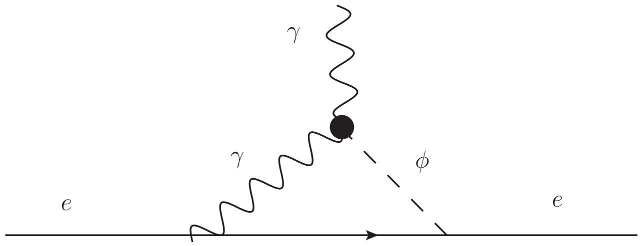

FIG. 2. Effective two-loop Barr-Zee diagram contribution to $g_e - 2$, with fermion loops integrated out. The dot represents light and heavy fermion loops that contribute to κ_γ.

Let us choose, for concreteness,

$$m_\phi = 250\ \text{MeV} \quad \text{and} \quad \lambda_\mu = 10^{-3}, \tag{7}$$

which according to Eq. (6) yields $\Delta a_\mu \approx 3 \times 10^{-9}$.

We now address the deviation in Eq. (4). Here, we will concentrate on the "Barr-Zee" diagram contribution to a_ℓ in Fig. 2, for a heavy fermion f loop that is represented by the dot in the figure, given by [16,19]

$$\Delta a_\ell^{\text{BZ}}(f) = -\frac{\alpha}{6\pi}\frac{m_\ell}{m_f}\frac{\lambda_\ell \lambda_f}{\pi^2} Q_f^2 N_c^f I(y), \tag{8}$$

where

$$I(y) = \frac{3}{4} y^2 \int_0^1 dz \frac{1 - 2z(1-z)}{z(1-z) - y^2} \ln \frac{z(1-z)}{y^2}, \tag{9}$$

with $y \equiv m_f/m_\phi$; Q_f and N_c^f are the electric charge and the number of colors of f, respectively, with $N_c^f = 1(3)$ for ordinary leptons (quarks). For multiple heavy fermions, one simply sums over f.

For $y^2 \gg 1$, the above expression for $\Delta a_\ell^{\text{BZ}}$ will reduce to [19,25]

$$\Delta a_\ell^{\text{BZ}} \approx \frac{\lambda_\ell \kappa_\gamma m_\ell}{4\pi^2}(13/12 + \ln y), \tag{10}$$

after integrating out heavy charged fermions of mass m_f in the two-loop Barr-Zee diagram. Here, κ_γ is given by (see, e.g., Ref. [26])

$$\kappa_\gamma \approx -\frac{2\alpha}{3\pi}\sum_f \frac{\lambda_f Q_f^2 N_c^f}{m_f}, \tag{11}$$

where it is assumed that the sum is over fermions with similar values of $\ln(m_f/m_\phi)$. Otherwise, the terms in Eq. (11) should be weighted by the different values of $I(m_f/m_\phi)$. The above formula for κ_γ is obtained in the limit that $y^2 \gg 1$. For heavy fermions to contribute significantly to κ_γ they need to couple to ϕ with sizable strength.

It is instructive to consider various potential contributions to Δa_e^{BZ} provided by κ_γ values and their associated $I(m_f/m_\phi)$ for $m_\phi = 250$ MeV. We consider the relative muon, tau and generic TeV particle BZ contributions, assuming $\lambda_e = 4 \times 10^{-4}$, roughly the maximum value allowed by "dark photon" searches [27]. In terms of their possible λ_f values, one finds the following relative BZ contributions in units of 10^{-14}: $-5\lambda_\mu/10^{-3}, -14\lambda_\tau/10^{-2}$, and $-7.5\lambda_{\text{TeV}}$. The muon value $\lambda_\mu = 10^{-3}$ is fixed by Δa_μ and is only capable of accounting for a small, ~6%, of the central discrepancy in Eq. (4). The tau has more λ_τ freedom. A rather large, but not ruled out, value of $\lambda_\tau = 0.06$ could accommodate the entire Δa_e discrepancy. Of course, such a large λ_τ would have many other phenomenological consequences for tau physics. In the case of new TeV charged states, about ten new states with $\mathcal{O}(1)$ Yukawa couplings are required to account for the discrepancy. The TeV scenario would seem most viable in a theory with dynamical symmetry breaking. Some combination of the above contributions with smaller couplings could also achieve the desired value of Δa_e. For larger m_ϕ values, in the $\mathcal{O}(\text{GeV})$ regime, similar conclusions can be reached.

Let us focus on the contributions of the tau[1] or TeV scale fermions, and for concreteness take

$$\lambda_e = 3 \times 10^{-4}, \tag{12}$$

which is somewhat below the aforementioned experimental upper bound. To account for the $g_e - 2$ discrepancy in Eq. (4), we then require ϕ-photon coupling to be

$$\kappa_\gamma = -7(2) \times 10^{-5}\ \text{GeV}^{-1} \quad \text{for } m_f = m_\tau\ (\sim\text{TeV}). \tag{13}$$

Here, we have assumed that the sign of the κ_γ is negative, which is a choice corresponding to positive fermion Yukawa couplings to ϕ. Assuming a coupling strength $\lambda_\tau \sim 8 \times 10^{-2}$ of ϕ to τ, Eq. (11) would yield $\kappa_\gamma \sim -7 \times 10^{-5}$. We note that a more careful phenomenological study is perhaps required to determine whether a coupling of $\mathcal{O}(8 \times 10^{-2})$ between ϕ and τ is consistent with experimental constraints. Thus, we generally expect that contributions from electrically charged states of mass $\gtrsim$few $\times$ 100 GeV (the new charged states cannot be much lighter given the fair agreement of the TeV scale LHC data with the SM predictions) are needed to generate the requisite strength of κ_γ. As noted before, to account for the entire $g_e - 2$ discrepancy would require constructive contribution of $\mathcal{O}(10)$ fermions at the TeV scale with $\lambda_f \sim 1$. This

[1]We will not consider couplings of ϕ to quarks, in order to avoid potentially severe constraints from flavor-changing neutral currents. Given the current mild B physics anomalies pointing to lepton flavor universality violations such an extension may be worth further examination. However, that analysis is outside the scope of this paper whose focus is on the leptonic interactions of the scalar ϕ.

requirement can be moderated if the tau contribution is significant, say with $\lambda_\tau \sim \text{few} \times 10^{-2}$, assuming constructive interference from same sign Yukawa couplings.

Note that the chosen value of λ_e in Eq. (12) does not follow naive scaling with the lepton mass, i.e., $\lambda_e/\lambda_\mu = m_e/m_\mu$, in reference to that of λ_μ in Eq. (7). However, since ϕ is not assumed to control the masses of the leptons, this is not an inconsistent choice and can be easily obtained from a simple effective theory that does not have hierarchic charged lepton interactions. We would also like to mention that for values of $\ln(m_f/m_\phi)$ larger than those assumed in the preceding discussion, one could choose smaller values of $|\lambda_{e,\mu}|$ due to the enhanced contributions of the Barr-Zee diagrams to both a_e and a_μ, for $\lambda_e\kappa_\gamma < 0$ and $\lambda_\mu\kappa_\gamma > 0$, respectively. This would presumably originate from the coupling of ϕ to the aforementioned charged heavy states. We will see later that such new particles may be motivated in some ultraviolet completions of the effective theory in Eq. (5).

For the above reference values, the one-loop contribution from Eq. (6) to Δa_e is $\sim 5 \times 10^{-14}$, which is small compared to that required by the apparent anomaly in Eq. (4). Similarly, the Barr-Zee diagram contribution to a_μ from Eq. (10) is $\sim -5 \times 10^{-10}$, a factor of ~ 5 too small and of the wrong sign to account for the anomaly in a_μ from Eq. (1). To compensate for this $\sim 20\%$ effect we could change our reference parameters in Eq. (7) slightly, but the values chosen here suffice to illustrate that, in principle, a simultaneous resolution of both current discrepancies in a_e and a_μ can be obtained in our framework. We also note that at larger m_ϕ alternative possibilities arise. For example, for $m_\phi = 3.0$ GeV, using a value of κ_γ that gives a Barr-Zee two-loop solution for Δa_e with $\lambda_e = 3.0 \times 10^{-4}$, we find that there are two values of $\lambda_\mu \sim -3 \times 10^{-3}$ and $\sim 1 \times 10^{-2}$ that yield the $g_\mu - 2$ discrepancy, from the sum of one- and two-loop diagrams. Here, the two-loop contribution for the negative λ_μ is dominant over that of the one-loop diagram, and vice versa for the positive λ_μ.

Aspects of phenomenology related to the coupling of ϕ to muons, including extensions to CP violating couplings, have been discussed before [10,18]. The coupling of light ϕ to leptons could lead to signals for "bump hunt" searches in decay or scattering processes, if kinematically allowed.

Assuming significant ϕ coupling to τ of $\lambda_\tau \gtrsim \text{few} \times 10^{-2}$, leptonic ($l = e,\ \mu$) resonances in τ decay, e.g., $\tau \to e\nu\bar{\nu}\phi(\to l^+l^-)$, or in scattering $e^+e^- \to \tau^+\tau^-\phi(\to l^+l^-)$, as suggested in Ref. [18], could provide potentially promising signals.

Regardless of the production process for ϕ, its dominant decay modes play an important role in its phenomenology and experimental implications. Let us first consider the decay of ϕ into an on-shell fermion pair $\bar{f}f$ that interact with it through the Yukawa coupling in Eq. (5). The partial width for this decay is given by

$$\Gamma(\phi \to \bar{f}f) = \frac{\lambda_f^2}{8\pi} m_\phi \left(1 - \frac{4m_f^2}{m_\phi^2}\right)^{3/2}. \tag{14}$$

The decay of ϕ into photons, assuming the coupling κ_γ in Eq. (5), is given by

$$\Gamma(\phi \to \gamma\gamma) = \frac{\kappa_\gamma^2}{64\pi} m_\phi^3. \tag{15}$$

(See, e.g., Ref. [24].) The coupling κ_γ is generated by the interactions of ϕ with charged fermions as previously discussed. Here, rather than specify all such charged states, we parametrize the $\phi\gamma\gamma$ overall interaction in terms of the effective coupling κ_γ. We find that for the above chosen reference values (7), (12), and (13) we get $\Gamma(\phi \to \mu^+\mu^-) \approx 1.5$ eV, $\Gamma(\phi \to e^+e^-) \approx 0.9$ eV, $\Gamma(\phi \to \gamma\gamma) \approx (3\text{–}40) \times 10^{-5}$ eV, hence ϕ decays into muons and electrons with branching fractions of $\sim 60\%$ and $\sim 40\%$, respectively. However, as m_ϕ gets larger than 250 MeV, the phase space suppression for the $\mu^+\mu^-$ final state becomes less important and the ratio of those branching fractions approaches ~ 11, for our chosen values of λ_μ, λ_e, and $m_\phi \lesssim 1$ GeV.

Let us briefly discuss potential models that could give rise to the types of interactions we have assumed. The couplings of ϕ to μ and e can be obtained from an effective operator of the form

$$c_\ell \frac{\phi H \bar{L} \ell_R}{M}, \tag{16}$$

where M is the typical scale of new physics leading to the effective interaction above, L is a lepton doublet of the SM, and ℓ_R is a right-handed charged lepton. To avoid constraints from flavor-changing neutral current data, we generally assume that the structure of these interactions are flavor-diagonal. Also, for typical parameters similar to our reference values assumed before, the underlying interactions generating the above operators are roughly flavor universal, that is $c_e \sim c_\mu$. We then have $\lambda_\ell \equiv c_\ell \langle H \rangle / M$. Assuming $M \sim 1$ TeV, we find that $c_\ell \lesssim 10^{-2}$.

The scale M in Eq. (16) could be identified with the mass of a vectorlike fermion F, with quantum numbers of ℓ_R. In Ref. [10] a similar setup was assumed, where lepton flavor violation constraints and model building issues were discussed in more detail. Here, we simply take ϕ to be a singlet and not responsible for "dark" gauge symmetry breaking as was done in Ref. [10]. Thus, couplings of the form $y_H H \bar{L} F_R$ and $y_\phi \phi \bar{F}_L \ell_R$ can generate the operator in Eq. (16), with $c_\ell = y_H y_\phi$. We note that this choice of ultraviolet theory is also consistent with the assumption of charged TeV scale particle contributions to the $\phi\gamma\gamma$ coupling κ_γ, discussed earlier.

In summary, we have shown that a simple model, comprising a singlet scalar ϕ of mass $\gtrsim 250$ MeV and couplings $\sim 10^{-3}$, and few $\times 10^{-4}$ to the muon and the electron, respectively, can potentially account for a $\sim 3.7\sigma$ discrepancy in the muon $g-2$ and the $\sim 2.4\sigma$ discrepancy (or a significant part thereof), in the electron $g-2$ of opposite sign. For $m_\phi \sim 250$ MeV, the former anomaly is mediated through a one-loop digram, whereas the latter originates mostly from a two-loop Barr-Zee diagram, using a phenomenologically allowed coupling of the scalar to photons as small as few $\times 10^{-5}$ GeV^{-1}. Variations on this scenario, where two-loop contributions to the muon $g-2$ are important or dominant, can arise for $m_\phi \gtrsim 1$ GeV. The model could give rise to lepton pair signals in rare meson or tau decays, as well as those in electron and muon scattering processes. A simple effective theory that does not lead to naive scaling of the scalar-lepton couplings with the lepton mass can realize our scenario. The effective theory, in turn, could arise from TeV-scale charged vectorlike fermions coupled to the SM Higgs and ϕ. In that case, the LHC could potentially discover those fermions, which would shed further light on the underlying physics manifested in the possible deviations of the electron and muon $g-2$.

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[1] G. W. Bennett *et al.* (Muon G-2 Collaboration), Phys. Rev. D **73**, 072003 (2006).
[2] C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).
[3] T. Blum, P. A. Boyle, V. Gülpers, T. Izubuchi, L. Jin, C. Jung, A. Jüttner, C. Lehner, A. Portelli, and J. T. Tsang (RBC and UKQCD Collaborations), Phys. Rev. Lett. **121**, 022003 (2018).
[4] A. Keshavarzi, D. Nomura, and T. Teubner, Phys. Rev. D **97**, 114025 (2018).
[5] J. D. Bjorken, R. Essig, P. Schuster, and N. Toro, Phys. Rev. D **80**, 075018 (2009).
[6] R. Essig, J. A. Jaros, W. Wester, P. H. Adrian, S. Andreas, T. Averett, O. Baker, B. Batell *et al.*, arXiv:1311.0029.
[7] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D **79**, 015014 (2009).
[8] B. Holdom, Phys. Lett. **166B**, 196 (1986).
[9] M. Pospelov, Phys. Rev. D **80**, 095002 (2009).
[10] C. Y. Chen, H. Davoudiasl, W. J. Marciano, and C. Zhang, Phys. Rev. D **93**, 035006 (2016).
[11] R. H. Parker, C. Yu, W. Zhong, B. Estey, and H. Mueller, Science **360**, 191 (2018).
[12] G. Gabrielse, in *Lepton Dipole Moments*, edited by B. L. Roberts and W. J. Marciano (World Scientific, Singapore, 2009).
[13] T. Aoyama, T. Kinoshita, and M. Nio, Phys. Rev. D **97**, 036001 (2018).
[14] D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008).
[15] D. Hanneke, S. F. Hoogerheide, and G. Gabrielse, Phys. Rev. A **83**, 052122 (2011).
[16] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990); **65**, 2920(E) (1990).
[17] J. D. Bjorken and S. Weinberg, Phys. Rev. Lett. **38**, 622 (1977).
[18] B. Batell, N. Lange, D. McKeen, M. Pospelov, and A. Ritz, Phys. Rev. D **95**, 075003 (2017).
[19] G. F. Giudice, P. Paradisi, and M. Passera, J. High Energy Phys. 11 (2012) 113.
[20] W. J. Marciano, A. Masiero, P. Paradisi, and M. Passera, Phys. Rev. D **94**, 115033 (2016).
[21] K. m. Cheung, C. H. Chou, and O. C. W. Kong, Phys. Rev. D **64**, 111301 (2001).
[22] J. P. Leveille, Nucl. Phys. **B137**, 63 (1978).
[23] D. Tucker-Smith and I. Yavin, Phys. Rev. D **83**, 101702 (2011).
[24] B. Batell, A. Freitas, A. Ismail, and D. Mckeen, Phys. Rev. D **98**, 055026 (2018).
[25] A. Czarnecki and W. J. Marciano, Phys. Rev. D **96**, 113001 (2017); **97**, 019901(E) (2018).
[26] M. Carena, I. Low, and C. E. M. Wagner, J. High Energy Phys. 08 (2012) 060.
[27] J. P. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **113**, 201801 (2014).