Probing gravitational wave polarizations with Advanced LIGO, Advanced Virgo, and KAGRA

Yuki Hagihara, Naoya Era, Daisuke Iikawa, and Hideki Asada

Graduate School of Science and Technology, Hirosaki University, Aomori 036-8561, Japan

Assuming that, for a given source of gravitational waves (GWs), we know its sky position, as is the case for GW events with an electromagnetic counterpart such as GW170817, we discuss a null stream method to probe GW polarizations including spin-0 (scalar) GW modes and spin-1 (vector) modes, especially with an expected network of Advanced LIGO, Advanced Virgo, and KAGRA. For two independent null streams for four noncoaligned GW detectors, we study a location on the sky, exactly at which the spin-0 modes of GWs vanish in any null stream for the GW detector network, though the strain output at a detector may contain the spin-0 modes. Our numerical calculations show that there exist 70 sky positions that satisfy this condition of killing the spin-0 modes in the null streams. If a GW source with an electromagnetic counterpart is found in one of the 70 sky positions, the spin-1 modes will be testable separately from the spin-0 modes by the null stream method. In addition, we study a superposition of the two null streams to show that any one of the three modes (one combined spin-0 and two spin-1 modes) can be eliminated by suitably adjusting a weighted superposition of the null streams and thereby a set of the remaining polarization modes can be experimentally tested.

I. INTRODUCTION

In Einstein's theory of general relativity (GR), gravitational waves (GWs) are spacetime perturbations that propagate at the speed of light and can be expressed in terms of the so-called "tensor" modes which are spin-2 transverse and traceless (TT) [1,2]. The propagation speed of GWs has been confirmed at the remarkable level $\sim O(10^{-15})$ by the celebrated GW170817 event with an electromagnetic counterpart that is the first observation of GWs from a a neutron star and a neutron star merger [3]. In the rest of this paper, we assume that the propagation speed of GWs is the same as the speed of light. In alternative theories to GR, specifically in general metric theories of gravity, GWs can contain extra degrees of freedom with spin 0 and spin 1, which are often called scalar and vector modes, respectively [4,5]. Future detection of scalar or vector polarization would provide serious evidence against GR, or future GW observations would put a stringent constraint on scalar and vector modes of GWs, which will lead to a new test of modified gravity theories, some of which might be excluded. Therefore, several attempts for polarization tests of GWs, not only from bursts, compact binary coalescences, and pulsars, but also from stochastic sources, have been discussed by many authors (e.g., [6–11]).

So far, aLIGO-Hanford (H), aLIGO-Livingston (L), and Virgo (V) have detected GW signals. However, three detectors are not enough for distinguishing polarizations of GWs. Very recently, Allen [12] has examined whether

the observation of the merging binary black hole event GW170814 by the aLIGO-Virgo network [13] can be explained by a mimic model in which pure vector GWs mimic pure tensor ones for the detector network of HLV. He shows that, if the vector polarizations in a hypothetical pure vector theory are allowed to rotate as the GWs propagate, then for a certain source location on the sky, the strain outputs of three GW detectors can reproduce the strain outputs that GR predicts. A kilometer-scale interferometer called KAGRA (K) is expected to join the GW detector network within a few years and to contribute as a fourth detector to break partly a degeneracy in polarization tests. See Ref. [14] for a comprehensive review on the expanding network of Advanced LIGO, Advanced Virgo, and KAGRA (denoted as HLVK). Therefore, it is interesting to discuss how to probe GW polarizations using HLVK.

The main purpose of this paper is to study a possible method for probing GW polarizations, including scalar and vector modes especially, by taking into account the near-future network of HLVK, where we focus on GW events with an electromagnetic counterpart such as GW170817. For such a GW source, we know its location on the sky. In the rest of the paper, we use the information only on the direction of the GW source but not on the distance to the GW source. As a useful tool for our study, we consider null streams. A general method used in data analysis of GWs to separate signals from noise is that of null streams. The null stream approach was first introduced by Gürsel and Tinto [15] and was extended later by Wen and Schutz [16] and

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Chatterji et al. [17]. The idea behind this method is that there exists a linear combination of the data from a network of detectors, which contains no tensor modes but only noise in GR cases. Chatziioannou et al. have argued that null streams should be used for six (or more) GW detectors to probe GW polarizations [18].

This paper is organized as follows. In Sec. II, we describe null streams for four noncoaligned detectors and it is shown, by explicit calculations, that there are two independent null streams. By using two independent null streams, in Sec. III, we study a certain source location on the sky, exactly at which the null streams contain no spin-0 GW modes. We examine also superpositions of null streams for probing GW polarizations. Section IV is devoted to the Conclusion. Throughout this paper, latin indices $a, b, \cdots$ run from 1 to 4, corresponding to four detectors.

II. NULL STREAMS FOR FOUR NONCOALIGNED DETECTORS

Let us assume that there exist four detectors with uncorrelated noise and that, for a given source, we know its sky position, as is the case of GW events with an electromagnetic counterpart such as GW170817. Then, one knows exactly how to shift the arrival time of the GW from detector to detector.

For a detector labeled by a ($a = 1, 2, 3,$ and 4), the noise-weighted signal from a GW source at the location denoted as (θ, ϕ) on the sky is

$$S_a = F_a^+ h^+ + F_a^\times h^\times + F_a^S h^S + F_a^L h^L$$
$$+ F_a^V h^V + F_a^W h^W + n_a, \tag{1}$$

where n_a denotes noise; h^+ and $h^\times$ denote the spin-2 modes called the plus and cross mode, respectively; h^S and h^L denote the spin-0 modes called the breathing and longitudinal mode, respectively; and h^V and h^W denote the spin-1 modes often called the vector-x and vector-y mode, respectively [19]; and F_a^+, $F_a^\times$, F_a^S, F_a^L, F_a^V, and F_a^W are the antenna patterns for polarizations of GWs [5,10,20]. The antenna patterns are functions of a GW source location θ and ϕ [21]. In our numerical calculations, θ and ϕ denote the latitude and longitude, respectively.

By noting $F_a^S = -F_a^L$ [22], which was shown by Nishizawa et al. in [14], Eq. (1) can be simplified as

$$S_a = F_a^+ h^+ + F_a^\times h^\times + F_a^S (h^S - h^L)$$
$$+ F_a^V h^V + F_a^W h^W + n_a. \tag{2}$$

Note that the effects of h^S on the detector are exactly the same with the opposite sign as those of h^L. Hence, only the difference as $h^S - h^L$ can be tested as one combined spin-0 mode.

In this and the next paragraphs of the present paper, we follow Gürsel and Tinto [15] to consider only the purely tensorial modes $h^\times$ and h^+. In addition, let us imagine, for its simplicity, an ideal case such that noise is negligible in Eq. (2). By eliminating the two TT modes in signals at three detectors in the ideal case, we obtain a null stream [15] as, for $a = 1, 2,$ and 3 for instance,

$$\delta_{23} S_1 + \delta_{31} S_2 + \delta_{12} S_3 = 0, \tag{3}$$

where we define

$$\delta_{ab} \equiv F_a^+ F_b^\times - F_b^+ F_a^\times. \tag{4}$$

Similarly, the other null streams are obtained as

$$\delta_{34} S_2 + \delta_{42} S_3 + \delta_{23} S_4 = 0, \tag{5}$$

$$\delta_{41} S_3 + \delta_{13} S_4 + \delta_{34} S_1 = 0, \tag{6}$$

$$\delta_{12} S_4 + \delta_{24} S_1 + \delta_{41} S_2 = 0. \tag{7}$$

By explicit calculations, we shall show that any two out of the null streams Eqs. (3) and (5)–(7) can construct the remaining two null streams. Note that it is suggested in Ref. [17] that two null streams are likely to be enough for four noncoaligned detectors. Without loss of generality, we take Eqs. (3) and (5) as two null streams and multiply them by δ_{34} and δ_{31}, respectively. The difference between them is rearranged as

$$\delta_{23}(\delta_{41} S_3 + \delta_{13} S_4 + \delta_{34} S_1) = 0, \tag{8}$$

where we use an identity as

$$\delta_{12}\delta_{34} - \delta_{42}\delta_{31} = \delta_{23}\delta_{41}. \tag{9}$$

We thus obtain Eq. (6). Note that δ_{23} is nonvanishing almost everywhere in the sky for the noncoaligned $a = 2$ and 3 detectors. See Fig. 1 for the zero points of δ_{23} in the
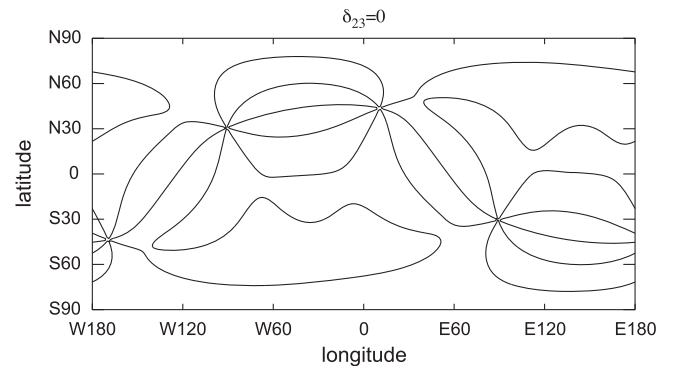

FIG. 1. Curves for $\delta_{23} = 0$ in the sky, where $L = 2$ and $V = 3$ are assumed.

sky, where we assume a case of $L = 2$ and $V = 3$. In our numerical calculations, the reference for polarization angles is chosen as aLIGO-Livingston ($L = 2$) and we assume that the source is located sufficiently far from the detectors, so that the plane wave approximation of GWs can be used. In the same way, we can obtain Eq. (7) from Eqs. (3) and (5) almost everywhere in the sky.

Actually, a real GW detector has noise. Equations (3) and (5)–(7) are thus modified with noise as

$$\delta_{23}S_1 + \delta_{31}S_2 + \delta_{12}S_3 = \delta_{23}n_1 + \delta_{31}n_2 + \delta_{12}n_3, \quad (10)$$

$$\delta_{34}S_2 + \delta_{42}S_3 + \delta_{23}S_4 = \delta_{34}n_2 + \delta_{42}n_3 + \delta_{23}n_4, \quad (11)$$

$$\delta_{41}S_3 + \delta_{13}S_4 + \delta_{34}S_1 = \delta_{41}n_3 + \delta_{13}n_4 + \delta_{34}n_1, \quad (12)$$

$$\delta_{12}S_4 + \delta_{24}S_1 + \delta_{41}S_2 = \delta_{12}n_4 + \delta_{24}n_1 + \delta_{41}n_2. \quad (13)$$

By using identities such as Eq. (9) again, it is clear that any pair among Eqs. (10)–(13) can derive the remaining two equations.

Next, we incorporate scalar and vector polarization modes. Let us denote two null streams including spin-0 and spin-1 polarizations as

$$P_a S_a = (P_b F_b^S)(h^S - h^L) + (P_c F_c^V)h_V$$
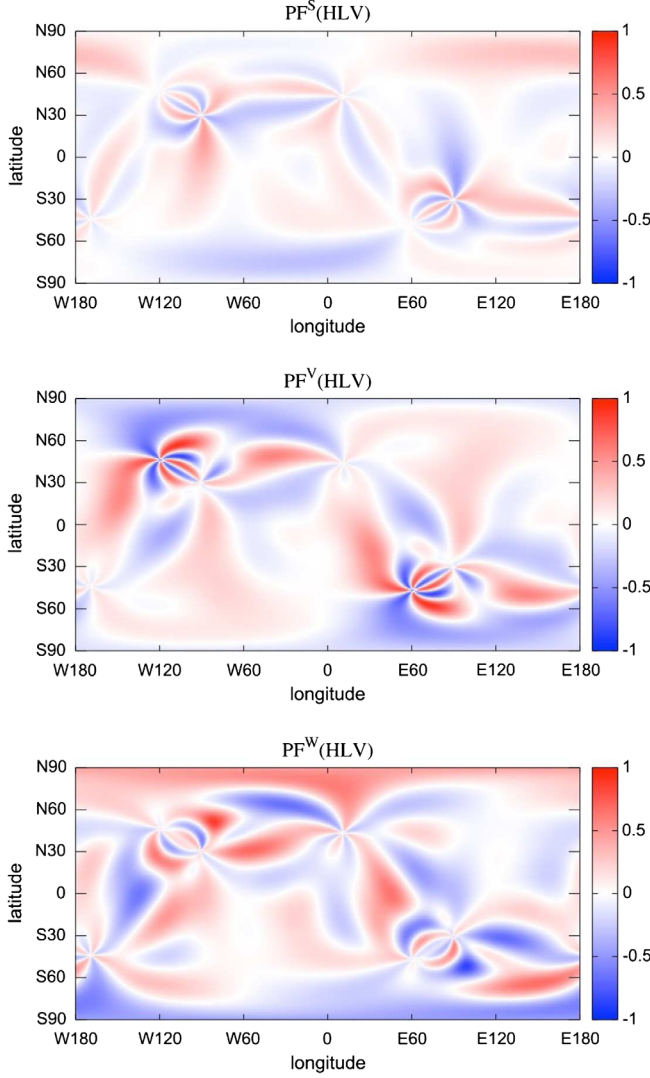$$+ (P_d F_d^W)h_W + P_e n_e, \quad (14)$$

FIG. 2. Contour map of the coefficients in the null stream as Eq. (14), where θ and ϕ denote the latitude and longitude, respectively. The detectors are labeled as $H = 1$, $L = 2$, $V = 3$, and $K = 4$. In this figure, white means zero, red is positive, and blue denotes negative. From top to bottom, the panels show $P_a F_a^S$, $P_b F_b^V$, and $P_c F_c^W$.
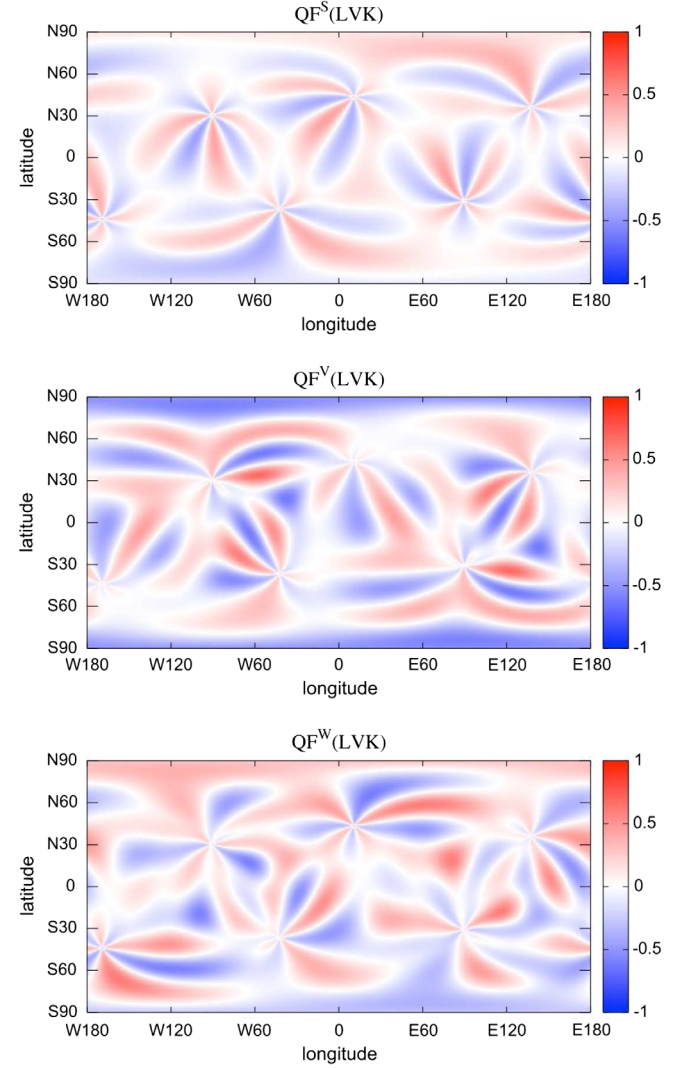

FIG. 3. Contour map of the coefficients in the null stream as Eq. (15). This corresponds to Fig. 2. From top to bottom, the panels show $Q_a F_a^S$, $Q_b F_b^V$, and $Q_c F_c^W$.

$$Q_f S_f = (Q_g F_g^S)(h^S - h^L) + (Q_h F_h^V)h_V$$
$$+ (Q_i F_i^W)h_W + Q_j n_j, \qquad (15)$$

where we use Eq. (2) and the summation is taken over $a, \ldots, j = 1,$ 2, 3 and 4. Note that the tensor null stream is built in and hence h^+ and $h^\times$ do not appear in the above equations. Without loss of generality, we can choose P_a and Q_a as $(P_a) = (\delta_{23}, \delta_{31}, \delta_{12}, 0)$ and $(Q_a) = (0, \delta_{34}, \delta_{42}, \delta_{23})$ for its simplicity, which correspond to Eqs. (10) and (11) in the previous paragraph. In the next section, we shall examine Eqs. (14) and (15) in more detail. In numerical calculations for the HLVK network, we choose $H = 1$, $L = 2$, $V = 3$, and $K = 4$ for $a = 1,$ 2, 3, and 4 for its simplicity. See Figs. 2 and 3 for the network of HLVK.

III. SPIN-0 SILENT LOCATION ON THE SKY AND CANCELING METHOD

A. Source locations on the sky at which null streams contain no spin-0 modes

Let us examine spin-0 modes in the null streams by Eqs. (14) and (15), in which the coefficients in front of $h^S - h^L$ are $P_a F_a^S$ and $Q_a F_a^S$. Note that the tensor null stream is built in and hence h^+ and $h^\times$ do not appear in the equations. If $P_a F_a^S$ and $Q_a F_a^S$ vanish simultaneously, spin-0 GW components do not contribute to the null streams, even if these waves have a non-negligible amplitude. For a source location satisfying simultaneously $P_a F_a^S = 0$ and $Q_a F_a^S = 0$, therefore, the null streams can be used for testing only the vector modes h^V and h^W.

As Sec. II suggests, $h^S - h^L$ for this very particular case will disappear in any null stream that is made of Eqs. (3) and (5)–(7), though the strain output at a detector may contain spin-0 modes. This can be explicitly shown as follows. Without loss of generality, we can consider Eq. (12) as another null stream, for which we define $(R_a) = (\delta_{34}, 0, \delta_{41}, \delta_{13})$. The coefficient of $h^S - h^L$ in this null stream is written as $R_a F_a^S$. By using the same method that we use for obtaining Eq. (8), one can find an identity as

$$\delta_{34}(P_a F_a^S) - \delta_{31}(Q_b F_b^S) = \delta_{23}(R_c F_c^S), \qquad (16)$$

where we use Eq. (9). Here, δ_{23} is nonvanishing almost everywhere for noncoaligned detectors. Therefore, if $P_a F_a^S = 0$ and $Q_b F_b^S = 0$ are satisfied simultaneously, $R_c F_c^S$ vanishes. See Fig. 4 for numerical plots of the sky locations that satisfy both $P_a F_a^S = 0$ and $Q_b F_b^S = 0$ for the HLVK network. There are 70 sky positions that kill the spin-0 modes in the null streams for HLVK.

Why is the number of the specific sky positions $70 \sim O(10^2)$ but not $O(1)$ or $O(10^4)$? Here, let us discuss it from a mathematical viewpoint. Both $P_a F_a^S$ and $Q_a F_a^S$ are expressed in terms of trigonometric functions of θ and ϕ
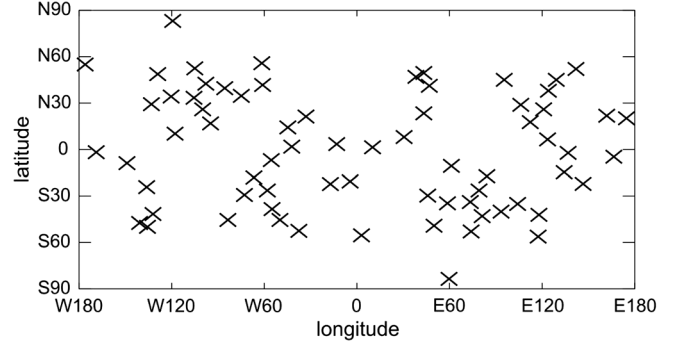

FIG. 4. The 70 sky positions that satisfy simultaneously $P_a F_a^S = 0$ and $Q_a F_a^S = 0$, where we assume $H = 1$, $L = 2$, $V = 3$ and $K = 4$.

of degree 12. Roughly speaking, they can be approximated by polynomials with $O(10)$ degrees. Bézout's theorem in algebraic geometry [23] states that two algebraic curves of degrees m and n on a plane intersect in $m \times n$ points if each point is counted with its intersection multiplicity. In the current paper, we do not count the intersection multiplicity. In our case, therefore, Bézout's theorem gives the upper bound on the number of the intersection points. The number of the specific sky positions is thus below $O(10) \times O(10) = O(10^2)$. Therefore, 70 points seems to be reasonable.

If we are extremely lucky to observe such a GW event with an electromagnetic counterpart at the location at which spin-0 modes fade out from the null streams, Eqs. (14) and (15) will enable us to constrain (or perhaps detect) h^V and h^W, separately. They are solved for h^V and h^W as

$$\begin{pmatrix} h^V \\ h^W \end{pmatrix} = \begin{pmatrix} P_a F_a^V & P_b F_b^W \\ Q_c F_c^V & Q_d F_d^W \end{pmatrix}^{-1} \begin{pmatrix} P_e(S_e - n_e) \\ Q_f(S_f - n_f) \end{pmatrix}, \qquad (17)$$

where $a, \ldots, f$ run from 1 to 4. By observations using the four detectors, Eq. (17) can determine, in principle, h^V and h^W separately. In practice, we have noise in the measurements, so that we would be able to put a constraint on h^V and h^W within the observational error bars.

B. How to adjust a weighted superposition of two null streams

We consider a superposition of the two null streams by Eqs. (14) and (15) as

$$AP_a S_a + BQ_b S_b = (AP_c F_c^S + BQ_d F_d^S)(h^S - h^L)$$
$$+ (AP_e F_e^V + BQ_f F_f^V)h^V$$
$$+ (AP_g F_g^W + BQ_h F_h^W)h^W$$
$$+ (AP_i + BQ_i)n_i, \qquad (18)$$

where A and B are real constants. Note that the tensor null stream is built in and hence h^+ and $h^\times$ do not appear in the

above equations. For the later convenience, let us denote a ratio between A and B as

$$r \equiv \frac{B}{A}. \tag{19}$$

First, we consider a case in which $AP_c F_c^S + BQ_d F_d^S$ in Eq. (18) vanishes, though the strain at a detector may contain spin-0 modes. This case is achieved by choosing a ratio between A and B as

$$r^{S-L} \equiv -\frac{P_a F_a^S}{Q_b F_b^S}, \tag{20}$$

where we assume $Q_b F_b^S \neq 0$. If $Q_b F_b^S = 0$, Eq. (15) has no spin-0 parts, so that Eq. (15) can be used to probe the spin-1 modes as it is. For example, we consider two simple cases as GWs from the direction of the north pole (N) and GWs from the direction of the equator and prime meridian (EPM). For these cases, the ratio for the HLVK network is $r_N^{S-L} = -0.33669$ and $r_{\text{EPM}}^{S-L} = 0.31524$, respectively.

Next, we examine a case in which the coefficient in front of h^V in Eq. (18) vanishes. This case can be achieved by choosing the ratio r as

$$r^V \equiv -\frac{P_a F_a^V}{Q_b F_b^V}, \tag{21}$$

where we assume $Q_b F_b^V \neq 0$. If $Q_b F_b^V$ vanishes, Eq. (15) does not contain h^V, such that Eq. (15) can immediately constrain some combination of $h^S - h^L$ and h^W. The ratio for HLVK is $r_N^V = -0.31112$ for GWs from the direction of the north pole and $r_{\text{EPM}}^V = 0.49763$ for GWs from the EPM direction.

Thirdly, we study a case in which the coefficient of h^W vanishes in Eq. (18). This case can be achieved by choosing the ratio as

$$r^W \equiv -\frac{P_a F_a^W}{Q_b F_b^W}, \tag{22}$$

where we assume $Q_b F_b^W \neq 0$. If $Q_b F_b^W = 0$, h^W disappears in Eq. (15), so that Eq. (15) can immediately constrain some combination of $h^S - h^L$ and h^W. For HLVK, the ratio is $r_N^W = -1.61958$ and $r_{\text{EPM}}^W = 0.54243$ for GWs from the N direction and from the EPM direction, respectively.

IV. CONCLUSION

In expectation of the near-future network of Advanced LIGO, Advanced Virgo, and KAGRA, we discussed a null stream method to probe GW polarizations including spin-0 (scalar) GW modes and spin-1 (vector) modes, where we assumed that, for a given source of GWs, we know its sky position, as is the case for GW events with an electromagnetic counterpart such as GW170817. We studied a location on the sky, exactly at which the spin-0 modes of GWs vanish in null streams for the GW detector network, though the strain output at a detector may contain the spin-0 modes. By numerical calculations, we showed that there are 70 sky positions that kill the spin-0 modes in the null streams. If a GW source with an electromagnetic counterpart is found in one of the 70 sky positions, the spin-1 modes will be testable separately from the spin-0 modes by the null stream method. We examined also a superposition of the two null streams to show that any one of the three modes (one combined spin-0 and two spin-1 modes) can be eliminated by suitably adjusting a weighted superposition of the null streams and thereby a set of the remaining polarization modes can be experimentally tested.

It is left for future work to perform a more comprehensive study, for instance, by taking account of expected noise at each detector of the near-future network HLVK.

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