Stars creating a gravitational repulsion

Igor D. Novikov
Astro-Space Center of P.N. Lebedev Physical Institute, Profsoyusnaya 84/32, Moscow, Russia 117997; The Neil's Bohr International Academy, Neil's Bohr Institute, Blegdamsvej 17, DK-2100, Copenhagen, Denmark; and National Research Center Kurchatov Institute, 123182, Moscow, Russia

G. S. Bisnovatyi-Kogan
Space Research Institute, Profsoyusnaya 84/32, Moscow, Russia 117997; National Research Nuclear University MEPhI, Kashira Highway, 31, Moscow, 115409; and Moscow Institute of Physics and Technology MIPT, Institutskiy Pereulok, 9, Dolgoprudny, Moscow Region, 141701

Dmitry I. Novikov
Astro-Space Center of P.N. Lebedev Physical Institute, Profsoyusnaya 84/32, Moscow, Russia 117997

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In the framework of the theory of general relativity, models of stars with an unusual equation of state $\rho c^2 < 0$, $P > 0$, where ρ is the mass density and P is the pressure, are constructed. These objects create outside themselves the forces of gravitational repulsion. The equilibrium of such stars is ensured by a nonstandard balance of forces. Negative mass density, acting gravitationally on itself, creates an acceleration of the negative mass, directed from the center. Therefore, in the absence of pressure, such an object tends to expand. At the same time, the positive pressure, which falls just like in ordinary stars from the center to the surface, creates a force directed from the center. This force acts on the negative mass density, which causes acceleration directed opposite of the acting force, that is, to the center of the star. This acceleration balances the gravitational repulsion produced by the negative mass. Thus, in our models, gravity and pressure change roles: the negative mass tends to create a gravitational repulsion, while the gradient of the pressure acting on the negative mass tends to compress the star. In this paper, we construct several models of such a star with various equations of state.

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I. INTRODUCTION

In the last century, many amazing and unusual objects and phenomena were discovered, quite unlike those that were known before: white dwarfs, neutron stars, black holes, dark matter, and dark energy. These discoveries were preceded by exotic theoretical predictions. In many cases, the scientific community was very skeptical about such predictions. A striking example of this is the discovery of black holes and the accelerated expansion of the Universe due to dark energy. Sometimes predictions of this kind are not justified for a long time, but there are hopes of finding such objects in the future. An example of this kind is the hypothesis of wormholes [1–3].

In this article, we consider models of objects consisting of a substance with an unusual equation of state. In particular, the negative energy density is under consideration. The appeal of this possibility is connected, of course, to the discovery of gravitational repulsion forces that make the Universe expand faster than it was during inflation and in the modern era. The source of gravitational repulsion in cosmology is the negative pressure P. However, in our case, antigravity is due to negative energy density, rather than negative pressure:

$$P > 0, \qquad \rho c^2 < 0. \tag{1}$$

The possibility of such a condition has been considered in theory for a long time. This consideration is relevant to the cosmological constant Λ, which can be positive or negative and is interpreted as the components of the stress-energy tensor of the vacuum [4–8]. The negative value of Λ corresponds to inequality (1).

Another example of exotic matter with an equation of state that corresponds to this inequality is the scalar field with negative energy density [9,10]. Such an equation of state is widely used in the theory of wormholes [11].

Models of stars with an unusual equation of state were repeatedly considered earlier [12–14]. As indicated above, we refer here to the equation of state satisfying (1), specifically in connection with the consideration of the problem of antigravity in the Universe [15]. We emphasize that a ball of finite radius filled with matter with the equation of state corresponding to dark energy in today's Universe ($P \approx -\rho c^2 < 0$) creates ordinary gravity outside itself in the vacuum, not antigravitation. The question arises: can an object exist that creates antigravity in the space beyond its border? A positive answer to this question has long been known: such objects are the entrances to wormholes in many models [16,17]. Our article is devoted to the study of the question of whether there can be an object with the usual spherical topology, which creates a gravitational repulsion outside itself.

We will consider models with $\rho < 0$ and $P > 0$. The motive for considering such models is a very simple example of the mechanical interaction of two bodies with positive and negative masses. In this example, the ball of mass $m_1 > 0$ moves with the speed $\vec{v}_1$ in the direction of the resting ball with the mass $m_2 < 0$ [18]. If $|m_2| > m_1$, then after the collision both balls will move in the direction opposite to $\vec{v}_1$. If we denote the velocities of two balls after the collision as v_1', v_2', then we always have $|v_1'| > |v_2'|$. A similar example was also considered in [19] for the relativistic linear motion of two particles with masses of opposite signs and a small difference between their absolute values.

In case of a star with $\rho < 0$ and $P > 0$, the negative matter density creates a gravitational force directed toward the center. This force, acting on a negative mass, creates an acceleration directed from the center of the star. Positive pressure falling down from the center towards the surface creates a force directed from the center. This force, acting on the negative mass, creates an acceleration directed against the acting force (as in the example with the balls) that is to the center. Thus, gravity and pressure act in opposite directions, balancing each other. Note that they act in directions opposite to those in which they act in an ordinary star with a positive matter density.

The paper is organized as follows. Section II gives the equation of equilibrium of a star, which we transform using dimensionless quantities. In Sec. III we consider the models of a star with a given equation of state and models with a given density profile. Finally in Sec. IV we make our conclusive remarks.

II. THE EQUILIBRIUM OF THE STAR

The equation of equilibrium for the spherical star in general relativity can be written in the following form [7]:

$$\frac{dP}{dr} = -G\frac{(\rho c^2 + P)(M_r c^2 + 4\pi P r^3)}{r^2 c^4 - 2GM_r r c^2}, \tag{2}$$

where G is the gravitational constant, c is the speed of light, r is the radial coordinate ($r^2 = A/4\pi$, where A is the total area of the 2-sphere), P is the pressure, ρ is the density, and M_r is

$$M_r = 4\pi \int_0^r \rho(\tau)\tau^2 d\tau. \tag{3}$$

The boundary of the star (its surface) is the coordinate $r = r_s$ at which the pressure vanishes: $P(r_s) = 0$. The pressure must be a continuous function everywhere including $r = r_s$. This is always true for our models with finite r_s and, therefore, the interior star solution matches with the exterior vacuum solution, where the pressure is zero. For further consideration, we denote the absolute value of the density at the center of the star as ρ_c and use the dimensionless quantities θ, w, x:

$$\rho = \rho_c\theta, \qquad P = \rho_c c^2 w, \qquad r = Rx, \qquad R^2 = \frac{c^2}{4\pi G\rho_c}. \tag{4}$$

In our models we always have $\theta < 0$ and $w > 0$, so that the equation of state satisfies (1).

Equation (2) in dimensionless units (4) looks like this:

$$\frac{dw}{dx} = -\frac{(\theta + w)(I_x + x^3 w)}{x^2 - 2xI_x}, \tag{5}$$

and

$$I_x = \int_0^x \theta(\tau)\tau^2 d\tau < 0 \tag{6}$$

is the dimensionless mass, which corresponds to the expression (3).

III. MODELS OF A STAR CREATING ANTIGRAVITATION

In this section we consider two types of models of a star with negative mass density: models with a given equation of state and models with a given density profile. In our paper, when specifying the equation of state, we confine ourselves to two models: the linear and quadratic dependence of pressure on density. As for models with a given density profile, we consider here the model of a star filled with matter of constant density and a model with a parabolic dependence of the density on the coordinate.

A. The models with the given equation of state

1. The model with the equation of state $w = -\varepsilon\theta$, $\varepsilon > 0$

The motive for applying this equation of state is the fact that the linear relation between pressure and density has been considered in classical works devoted to the study of the star equilibrium for a positive mass density.

Equation (5) for the object with the negative matter density and under condition $w = -\varepsilon\theta$, where ε is the positive constant, takes the following form:

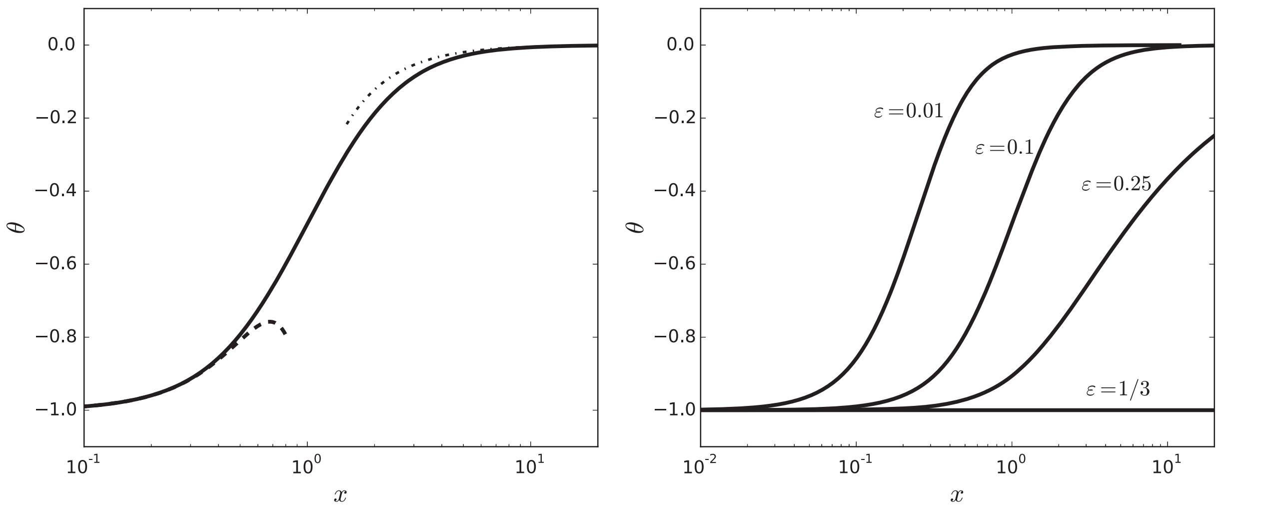

FIG. 1. The density profile as a function of the coordinate x for the model with the equation of state $w = -\varepsilon\theta$. Left panel: The density profile for $\varepsilon = 0.1$ (solid line), together with two asymptotics: the dashed line for $x \ll \frac{1}{\sqrt{d_2}}$ and the dotted line for $x \gg \frac{1}{\sqrt{d_2}}$. Right panel: The density profile for different values of ε.

$$\theta' = \theta \cdot \frac{1-\varepsilon}{\varepsilon} \cdot \frac{I_x - \varepsilon x^3 \theta}{x^2 - 2xI_x}, \qquad \theta' = \frac{d\theta}{dx}. \tag{7}$$

We should mention, first, that this equation has a simple analytical solution (a similar analytical solution was found in [20,21] for a positive mass density and positive pressure):

$$\theta = -\frac{2\varepsilon}{(\varepsilon-3)^2 - 8} \cdot \frac{1}{x^2}, \quad \text{for } \varepsilon < 3 - 2\sqrt{2} \approx 0.17. \tag{8}$$

Such a solution corresponds to infinite density in the center of the star. In order to satisfy the finite density in the center, we should integrate Eq. (7) with the boundary condition $\theta(0) = -1$, $\theta'(0) = 0$. The solution in the vicinity of the center can be found analytically by representing θ in the form of a Taylor series with even terms only:

$$\theta = -1 + d_2 x^2 + d_4 x^4 + \cdots, \qquad d_n = \frac{1}{n!} \frac{d^n\theta}{dx^n}\bigg|_0. \tag{9}$$

We use here even terms to make sure that there is no weird feature in the density or pressure profile around the center of the star. Thus, θ and w are continuous functions, all their derivatives have no discontinuities and our solution is spherically symmetrical. Substituting expression (9) into Eq. (7), we have expressions for d_n:

$$d_2 = \frac{(1-\varepsilon)}{2\epsilon}\left(\frac{1}{3} - \varepsilon\right),$$
$$d_4 = \left[\frac{1-\varepsilon}{2\epsilon}\left(\varepsilon - \frac{4}{15}\right) - \frac{1}{3}\right] d_2. \tag{10}$$

In order to make sure that density is changing monotonically from -1 to 0 in the direction from the center, the equation of state should satisfy the condition $\varepsilon < 1/3$. The asymptotic solution for large x can either be the expression (8) if $\varepsilon < 3 - 2\sqrt{2}$ or, if $\varepsilon > 3 - 2\sqrt{2}$, then $\theta \sim x^\gamma$, $\gamma > -2$. Finally we have the following asymptotics:

$$\theta \approx -1 + d_2 x^2 + d_4 x^4, \qquad x \ll \frac{1}{\sqrt{d_2}}, \qquad \varepsilon < \frac{1}{3};$$
$$\theta \approx -\frac{2\varepsilon}{(\varepsilon-3)^2 - 8} \cdot \frac{1}{x^2}, \qquad x \gg \frac{1}{\sqrt{d_2}}, \qquad \varepsilon < 3 - 2\sqrt{2};$$
$$\theta \approx Cx^\gamma, \qquad \gamma = \frac{(3\varepsilon - 1)(1-\varepsilon)}{\varepsilon(1+\varepsilon)}, \qquad x \gg \frac{1}{\sqrt{d_2}},$$
$$3 - 2\sqrt{2} \qquad < \varepsilon < \frac{1}{3}, \tag{11}$$

where $C < 0$ is some negative constant. Results of our numerical integration of Eq. (7) for different values of ε are shown in Fig. 1.

It should be mentioned that for the equation of state under consideration, when the pressure is proportional to the density, the mass of the star turns out to be infinite, since the integral $I_\infty = \int_0^\infty \tau^2 \theta d\tau$ does not converge. Similar models for the case of positive matter density were considered in [20,21]. It is better to call them "models", not "stars", since they have infinite size. In order to avoid mass infinity, below we consider a model of a star with a different equation of state, which gives us the finite size of the star and hence its finite mass.

2. The model with the equation of state $w = \varepsilon\theta^2$, $\varepsilon > 0$, $\theta < 0$

In the case of the equation of state $w = \varepsilon\theta^2$, the expression (5) becomes

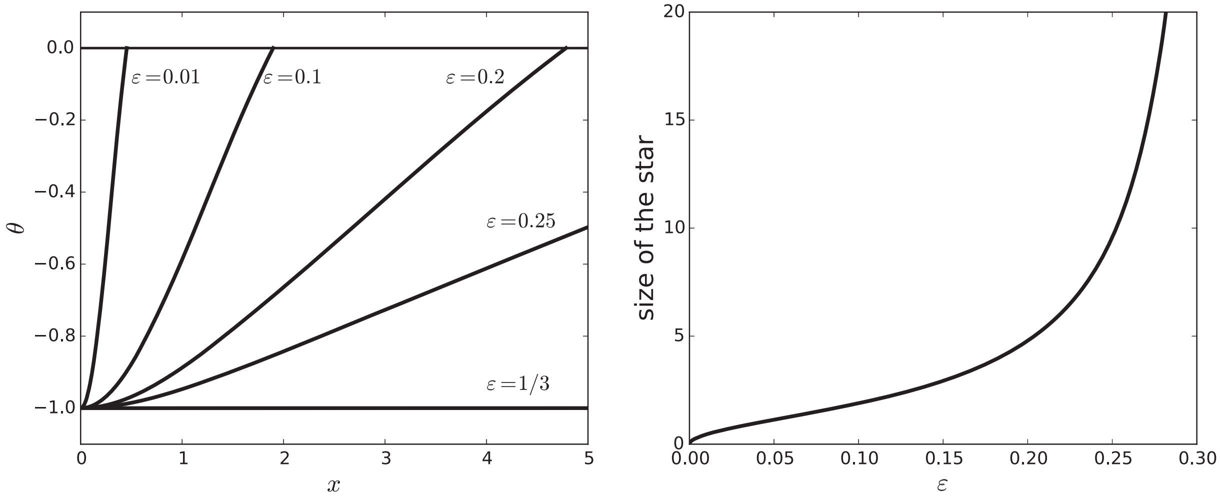

FIG. 2. The model with the equation of state $w = \varepsilon\theta^2$. Left panel: Density as a function of the coordinate x for different values of ε. Right panel: Size of the star as a function of ε.

$$\frac{d\theta}{dx} = -\frac{1+\varepsilon\theta}{2\varepsilon} \cdot \frac{I_x + \varepsilon x^3\theta^2}{x^2 - 2xI_x}. \tag{12}$$

As can easily be seen, if $\theta = 0$, then the density derivative with respect to the coordinate when $\theta = 0$ is a finite positive constant. This means that the model of the star with finite size and mass can be constructed:

$$\left.\frac{d\theta}{dx}\right|_{x=a} = -\frac{1}{2\varepsilon}\frac{I_a}{a^2 - 2aI_a}, \qquad \theta(a) = 0, \qquad I_a < 0. \tag{13}$$

Here and below by a, we denote the size of the star. We define a as a coordinate at which the pressure becomes zero: $w(a) = 0$. In this particular case since $w = \varepsilon\theta^2$, both pressure and its derivative at a are zero: $w(a) = \frac{dw}{dx}|_{x=a} = 0$.

Analogously to the previous case, one can find the asymptotic solution of (12) for small x:

$$\theta \approx -1 + \frac{(1-\varepsilon)}{4\varepsilon}\left(\frac{1}{3} - \varepsilon\right) \cdot x^2. \tag{14}$$

The numerical solution of (12) is shown in Fig. 2. $\theta(x)$ is growing monotonically starting from the negative value $\theta(0) = -1$ and, unlike the previous case, eventually becomes zero at some point $x = a$, $\theta(a) = 0$, where a is the size of the star. In Fig. 2 we also show the dependence of the size of the star on ε. As we can see, this size grows as ε increases and becomes infinite for $\varepsilon = 1/3$ because in this case Eq. (12) has the constant solution $\theta = -1$.

B. The models with a given density profile

Our motivation to consider models with a given density profile is the fact that the equation of state can vary depending on the distance from the center of the star. In this subsection, we consider the models with a given monotonic function θ, $-1 < \theta < 0$, at the range $0 < x < \infty$. In this case, the pressure w satisfies the equilibrium equation (5) and eventually becomes zero at $x = a$, where a is the size of the star.

1. The constant density model

We start our analysis with the model of the star with constant density as a special case of the models with a given density profile. In such a model the density $\theta = -1$ and the equation of equilibrium (5) takes a particularly simple form:

$$\frac{dw}{dx} = -x \cdot \frac{(w-1)(w-\frac{1}{3})}{1+\frac{2}{3}x^2}. \tag{15}$$

This equation can be easily integrated and has the following analytical solution:

$$w = \frac{(3w_c - 1)\sqrt{1+\frac{2}{3}x^2} - (w_c - 1)}{(3w_c - 1)\sqrt{1+\frac{2}{3}x^2} - 3(w_c - 1)},$$
$$a^2 = -\frac{3w_c(4w_c - 2)}{(3w_c - 1)^2},$$
$$\varepsilon < \frac{1}{3}, \tag{16}$$

where w_c is the central pressure and it is easy to see that w becomes zero at $x = a$. In this model the ratio w/θ is changing monotonically:

$$-\frac{1}{3} < \frac{w}{\theta} = -w < 0, \quad \text{as } 0 < x < a, \qquad 0 < a < \infty. \tag{17}$$

The equilibrium equation (5) for the constant density in the case of positive mass density was integrated and the solution was analyzed in [22,23]. For this case, there is a restriction:

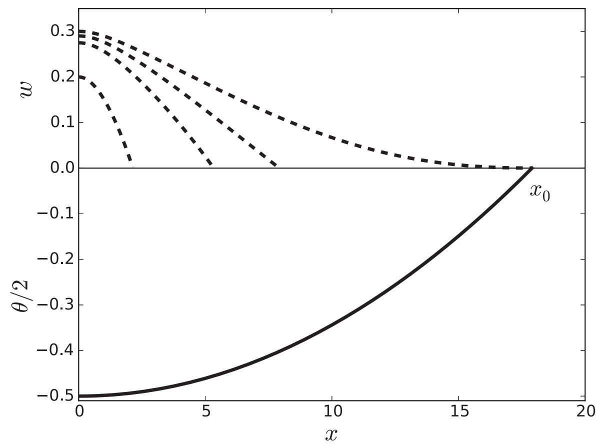

FIG. 3. Pressure as a function of the coordinate for a given density profile. The solid line denotes density and shaded lines denote pressure for different values of the central pressure $w_c = w(0)$. For $w_c = 0.3$, the parameter $x_0 = 17.9$ gives the example in which the pressure and density simultaneously turn to zero.

$$I_a < \frac{4}{9}a. \tag{18}$$

It is important to note that there is no such a restriction for the negative mass density.

2. Models with the parabolic density profile

Here we consider a more general but still extremely simple case of the density profile, namely, the parabolic shape for θ:

$$\theta = -1 + \left(\frac{x}{x_0}\right)^2. \tag{19}$$

Note that in case of $x_0 \to \infty$ this model reduces to a constant density model (see above).

In order to find the solution, we should substitute (19) into (5) and define the boundary condition at $x = 0$ as $w(0) = w_c$ and $\frac{dw}{dx}|_{x=0} = 0$. After the substitution, we get the differential equation:

$$\frac{dw}{dx} = -x \cdot \frac{[w - 1 + (\frac{x}{x_0})^2][w - \frac{1}{3} + \frac{1}{5}(\frac{x}{x_0})^2]}{1 + \frac{2}{3}x^2 - \frac{2}{5}x_0^2(\frac{x}{x_0})^4}. \tag{20}$$

Analogously to Sec. III A, we represent the pressure in the vicinity of the star center in the form of the Taylor series with even terms:

$$w = w_c + p_2 x^2 + p_4 x^4 + \cdots, \qquad p_n = \frac{1}{n!}\frac{d^n w}{dx^n}\bigg|_0 \tag{21}$$

and for coefficients p_n we have

$$\begin{aligned} p_2 &= -\frac{1}{2}(w_c - 1)\left(w_c - \frac{1}{3}\right), \\ p_4 &= -\frac{1}{4}\left[(w_c - 1)\left(p_2 + \frac{1}{5x_0^2}\right)\right. \\ &\quad \left. + \left(w_c - \frac{1}{3}\right)\left(p_2 + \frac{1}{x_0^2}\right) + \frac{4}{3}p_2\right]. \end{aligned} \tag{22}$$

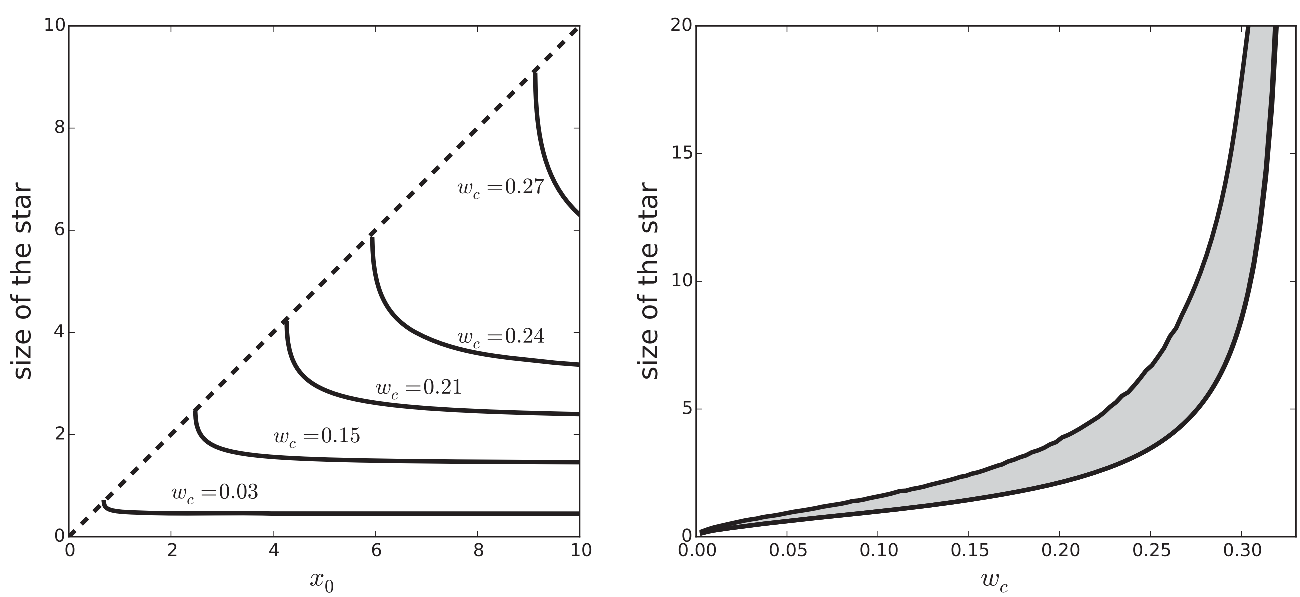

FIG. 4. The model with the given density profile. Left panel: Dependence of the size of the star on the parameter x_0 for different values of the central pressure. The dashed line shows the values of x_0 at which the density and pressure simultaneously turn to zero $[w(x_0) = \theta(x_0) = 0]$. For $x_0 \to \infty$, the size of the star asymptotically tends to a size determined by the model with constant density. Right panel: The size of a star as a function of the central pressure. The lower curve determines the size of the star for the constant density model. The upper curve corresponds to a solution in which the density and pressure become zero at the same coordinate x_0. The shaded zone corresponds to the range of admissible values of the star size.

Therefore, expressions (21) and (22) give us the analytical solution of Eq. (20) for $x \ll \frac{1}{\sqrt{p_2}}$. Results of our numerical integration of this equation are given in Fig. 3. If $x_0 \gg \frac{1}{\sqrt{p_2}}$, then the solution becomes the same as for the model with constant density profile; see Eq. (15). In this case the size of the star (the coordinate $x = a$, at which the pressure becomes zero) is determined by the central pressure only and this size is much less then x_0. The size of the star increases with increasing central pressure and, for any given x_0, there exists a maximum central pressure w_c at which the density and pressure go to zero for the same $x = x_0$: $w(x_0) = \theta(x_0) = 0$.

In Fig. 4 we demonstrate the dependence of the star size a on x_0 and on the central pressure w_c. For a fixed value of w_c, there are two asymptotics for the star size as a function of x_0. For large x_0, the size of the star is determined by Eq. (15). As x_0 decreases, the size of the star increases and eventually reaches the maximum possible value when $w(a) = \theta(a) = 0$, $a = x_0$.

IV. CONCLUSIVE REMARKS

As stated in the Introduction, antigravity in cosmology, which causes the accelerated expansion of the Universe, has been widely discussed during the last quarter century. However, the source of antigravitation in the cosmological models was negative pressure [15]. The energy density was assumed to be positive and such an energy was named in cosmology as dark energy. As stressed in [15,17], an isolated body consisting of dark energy creates outside an attraction and not antigravity. At the same time, a model of the body was indicated in [17], which creates outside itself the gravitational repulsion. Such a body is one of the entrances to a wormhole with a massless scalar field with negative energy density analyzed in [17]. In this paper, we constructed several models of isolated objects of the star type, creating an antigravity. Any test bodies with positive or negative mass outside such an object will be accelerated away from it.

We did not consider here the issues of the stability of the solutions obtained and did not touch upon the problem of whether such objects can have anything to do with the real Universe. Other physical limitations were not considered as well. We would like to mention only that the standard conditions for the homological stability of a star with a power equation of state $P = \varepsilon\rho^\gamma$ and negative mass density $\rho < 0$ will be inverse to those that exist for an ordinary star. Namely, it looks as $\gamma < 4/3$. It is necessary to mention also that in the case $\rho < 0$ there is no gravitational radius in the spherical solution.

We also emphasize that the general problem of the positivity of the energy and the positive nature of its radiation is analyzed in [[23], pp. 285–295].

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