Topological pseudodefects of a supersymmetric $SO(10)$ model and cosmology

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(Received 15 February 2018; published 20 September 2018)

Obtaining realistic supersymmetry preserving vacua in the minimal renormalizable supersymmetric $Spin(10)$ GUT model introduces considerations of the nontrivial topology of the vacuum manifold. The D-parity of low energy unification schemes gets lifted to a one-parameter subgroup $U(1)_D$ of $Spin(10)$. Yet, the choice of the fields signaling spontaneous symmetry breaking leads to disconnected subsets in the vacuum manifold related by the D-parity. The resulting domain walls, existing due to topological reasons but not stable, are identified as topological pseudodefects. We obtain a class of one-parameter paths connecting D-parity flipped vacua and compute the energy barrier height along the same. We consider the various patterns of symmetry breaking which can result in either intermediate scale gauge groups or a supersymmetric extension of the standard model. If the onset of inflation is subsequent to grand unified theory (GUT) breaking, as could happen also if inflation is naturally explained by the same GUT, the existence of such pseudo-defects can leave signatures in the CMB. Specifically, this could have an impact on the scale invariance of the CMB fluctuations and Large Scale Structure data at the largest scale.

DOI: 10.1103/PhysRevD.98.063523

I. INTRODUCTION

There are several indications for physics beyond the standard model (SM) which demands the need to connect with the high energy scales unlikely to be accessible to accelerators. One of them is the minuscule masses of neutrinos [1,2] which through see-saw mechanism [3–6] suggest the existence of a high mass scale. Further the precarious hierarchy of the Higgs mass with respect to the Planck scale is conceptually unnatural and can be easily ameliorated by new physics beyond the SM. Finally, unification of couplings remains ever a desirable feature and can be accomplished by grand unification in supersymmetic or non supersymmetric $SO(10)$, or larger gauge groups. The scale of such models is beyond the reach of accelerators but the early cosmology and its imprints on the CMB data and Large Scale Structure data can be important checks on this model. Other than the CMB, consistent big bang nucleosynthesis and successful inflation remain important requirements on any model with new physics at high energies. Indeed for the class of models that unifies, the high scale physics natural to them is constrained by inflation, by the need to generate baryon asymmetry, and by exotic relics such as cosmic strings and domain walls (DW) that survive the homogenizing effects of the high temperature.

An early study [7] considered the consequence of unstable domain wall formation in $Spin(10)$, which can decay due to the formation of cosmic strings as punctures or boundaries. Several other works have considered these issues, notably [8–12] for the context of topological defects and others [13–19] which have utilized the group theoretic constraints arising from such considerations for unification proposals. The present investigation is concerned with studying the interplay of symmetry breaking patterns with cosmology in the context of supersymmetric $Spin(10)$. There are two broad directions that have been pursued along these ones. One, that is motivated by superstring unification, as in [20–23], and the other class of models relies on the minimal representations of the Higgs and has been explored in [24–29]. It has been advanced as a renormalizable minimal supersymmetric $SO(10)$ grand unified theory (MSGUT). In the present work we shall restrict to the latter class of models due to their rich topological structure. The model utilizes Higgs supermultiplets **10**, **210**, **126** ($\overline{\mathbf{126}}$) required to break the symmetry and provide fermion masses. Among these the **210** and **126** ($\overline{\mathbf{126}}$) are responsible for breaking of $SO(10)$ symmetry down to MSSM, and **10** and $\overline{\mathbf{126}}$ give masses to the fermions. The **16**-dim Spinor representation contains one generation of SM fermions and a right handed neutrino.

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The GUT model considered has many phenomenological as well as cosmological virtues. It can provide inflaton candidates [30,31], mechanism for baryogenesis through leptogenesis and the lightest supersymmetric particle (LSP) of the model can serve as weakly interacting cold dark matter candidate.

A hallmark of this class of models is the occurrence of D-parity, a discrete symmetry that can exchange the chiral matter fields with their charge conjugates and also appropriately the corresponding Higgs bosons which gives them masses. While this is a discrete symmetry of the partial unification model, viz., the left-right symmetric model, when lifted to $SO(10)$ it gets embedded in a one parameter $U(1)_D$ subgroup of the covering group $Spin(10)$. Since the parent group is simply connected there are no stable domain walls. However we argue here that the topology of the vacuum manifold can be nontrivial, and can give rise to defects that are best dubbed topological pseudodefects (TPD). In this paper we want to study the implications to cosmology of TPD's arising in a SUSY $SO(10)$ GUT model. In [26–29] the implicit assumption is that there is a one step breaking from SUSY $SO(10)$ to MSSM. However the physics of big bang implies the existence of many causally disconnected regions in space and the nontrivial vacuum structure would give rise to domain walls [7].

Generically, the presence of domain walls (DW) in a model has important interplay with inflation. One of the successes of inflationary proposal is the removal unpleasant relics of GUTs such as monopoles. The same applies to cosmic strings, whose density can be easily diluted by cosmological inflation. The same is however not true of DW, as these may form a mutually locked structure which may not be blown apart by inflation easily. This is a relevant possibility if inflation has a preceding hot period allowing DW forming phase transition. The walls if stable would conflict with standard cosmology due to the inhomogeneities they would introduce in the CMB. On the other hand, for a network of walls which is unstable, the question shifts to the time scale of the decay and disappearance of the walls. The unified model in this case can be constrained by requiring that the inhomogeneities generated by their early presence should not affect the successful outcomes of inflation, specifically the nearly scale invariant CMB spectrum and the observed Gaussian nature of the density perturbations.

The plan of the paper is as follows. In Sec. II we review the topological role of D-parity as first considered in [7]. In Sec. III, we treat a warm up example of the energy barrier of DW in the minimal supersymmetric left-right where D parity occurs as a discrete symmetry, avoiding the subtleties of the large group $SO(10)$. In Sec. IV, we give a brief introduction to the GUT model of [28] and calculate the energy barrier associated with the D-parity breaking. The height of such a barrier estimates the energy per unit area of the TPD walls that may form. In Sec. V we discuss the implications to cosmological inflation which in turn may imply constraints on the scales the possible symmetry breaking schemes. The conclusions are in Sec. VI.

II. METASTABLE DOMAIN WALLS

Here we briefly review the topological issues relevant to the domain walls, paraphrased from [7]. $Spin(10)$ can be broken to its subgroup $H_0 = Spin(6) \otimes Spin(4)$, where the first factor contains the color $SU(3)_c$ and the second factor contains the SM $SU(2)_L$ and a potential $SU(2)_R$. This breaking can be achieved by using the 54 dimensional scalar χ which takes on a vacuum expectation value (VEV)

$$\langle\chi\rangle = \chi_0 \mathrm{diag}(2,2,2,2,2,2,-3,-3,-3,-3). \tag{1}$$

However, the stability group of this VEV contains a discrete set of additional elements. Consider the one-parameter curve in $Spin(10)$ of the form $U_{J_{67}}(\theta) = \exp(i\theta J_{67})$. The 6–7 submatrix of the VEV of χ transforms under this as

$$U_{J_{67}}(\theta)\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} U^{-1}_{J_{67}}(\theta) = \begin{pmatrix} -\frac{1}{2}+\frac{5}{2}\cos 2\theta & -\frac{5}{2}\sin 2\theta \\ -\frac{5}{2}\sin 2\theta & -\frac{1}{2}-\frac{5}{2}\cos 2\theta \end{pmatrix}. \tag{2}$$

Thus $\langle\chi\rangle$ is left invariant by $U_{J_{67}}(\theta = n\pi)$ with $n \in \mathbb{Z}$. It can be seen that all such choices derived from mixed a and α indices, with $a \in 1,2,\ldots 6$ and $\alpha \in 7,8,9,10$, have the same property, and are indeed equivalent to each other under a suitable transformation by an element from H_0. Thus the full stability group of $\langle\chi\rangle$ is $H_0 \oplus H_0'$ consisting of two disconnected continuous subsets, where H_0' are all the elements of the form $h(iJ_{67})$ with $h \in H_0$. It may be noted that iJ_{67} also enters the D-parity defined as $D \equiv (iJ_{67})(iJ_{23})$, the effective charge conjugation operator due to the charges of the fermions assigned to **16**.

In the model of [7] the sequence of breaking is

$$Spin(10) \underset{\mathbf{54}}{\overset{M_X}{\rightarrow}} Spin(6) \otimes Spin(4) \underset{\mathbf{126}}{\overset{M_R}{\rightarrow}} SU(3) \otimes SU(2) \otimes U(1) \underset{\mathbf{10}}{\overset{M_W}{\rightarrow}} SU(2) \otimes U(1). \tag{3}$$

Stable cosmic strings arise at the first phase transition, due to $\Pi_0(H_0 \oplus H_0') = \mathbb{Z}_2$. At the next stage of breaking when the **126** acquires a VEV, domain walls appear due to the breaking of this Z_2. But the Z_2 comes embedded in a continuous loop $U(1)_D$ the one-parameter subgroup generated by the D-parity generator. Such loops continuously connect a VEV to its D-parity conjugate. Specifically such walls separate vacua with $\langle\mathbf{126}\rangle = (\overline{10},1,3)$ (written in its components with Pati-Salam quantum numbers) from its charge conjugate VEV (10,3,1). Thus the walls are not stable, and decay due to tension of the string boundaries which are liable to shrink. The walls can also disintegrate due to creation of holes formed in them due to quantum tunneling assisted by thermal fluctuations. If the second phase transition is first order, there is a phase of wall domination and the possibility of wall decay only through large black hole formation. This is certainly ruled out by the CMB inputs into primordial fluctuations. On the other hand, a second order phase transition at second stage of breaking creates a short period of wall persistence though the walls do not come to dominate over radiation.

These considerations are a warm up for the study of topology of the vaccum manifold arising in [26–29] which study the breaking of SUSY $SO(10)$ to MSSM. We argue that D-parity TPD walls are a necessary consequence in such a breaking and expect that an epoch similar to the second order phase transition at second stage as reviewed in this section may unfold. A short period of substantial wall presence can have definite consequences to CMB data. We shall discuss this in Secs. IV and V.

III. D-PARITY SYMMETRIC VACUA IN LEFT-RIGHT SYMMETRIC MODEL

Before proceeding to the SUSY $SO(10)$ case, to illustrate the procedure we start with a warm up exercise for a related system, the minimal supersymmetric left-right symmetric model (MSLRM) considered in [32], based on the group $G_{LR} = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. While unlikely to have implications for inflation, the model is interesting in its own right as an intermediate scale group. The walls were studied from the point of view of cosmology and leptogenesis earlier in [33,34].

The Higgs superfields proposed for breaking the G_{LR} symmetry to SM are

$$\begin{aligned} \Delta &= (1,3,1,2); \quad \bar{\Delta} = (1,3,1,-2); \\ \Delta_c &= (1,1,3,-2); \quad \bar{\Delta}_c = (1,1,3,2); \\ \Omega &= (1,3,1,0); \quad \Omega_c = (1,1,3,0). \end{aligned} \tag{4}$$

These fields transform under D-parity as

$$\Delta \to \Delta_c^*; \quad \bar{\Delta} \to \bar{\Delta}_c^*; \quad \Omega \to \Omega_c^*. \tag{5}$$

The renormalizable superpotential corresponding to these Higgs superfields is given as,

$$\begin{aligned} W_{LR} &= m_\Delta (Tr\Delta\bar{\Delta} + Tr\Delta_c\bar{\Delta}_c) + m_\Omega (Tr\Omega^2 + Tr\Omega_c^2) \\ &\quad + a(Tr\Delta\Omega\bar{\Delta} + Tr\Delta_c\Omega_c\bar{\Delta}_c). \end{aligned} \tag{6}$$

The vacua are sought assuming the supersymmetry to be unbroken and remaining so till the electroweak scale [~O(TeV)]. These can be obtained by imposing F-flatness and D-flatness conditions given in [32]. The set of vacuum expectation values (VEV's) for the Higgs fields required to obtain the MSSM is,

$$\begin{aligned} &\langle\Omega_c\rangle = \begin{pmatrix} w_c & 0 \\ 0 & -w_c \end{pmatrix}; \; \langle\Delta_c\rangle = \begin{pmatrix} 0 & 0 \\ d_c & 0 \end{pmatrix}; \; \langle\bar{\Delta}_c\rangle = \begin{pmatrix} 0 & \bar{d}_c \\ 0 & 0 \end{pmatrix}; \\ &\langle\Omega\rangle = 0; \; \langle\Delta\rangle = 0; \; \langle\bar{\Delta}\rangle = 0. \end{aligned} \tag{7}$$

The required minimum is obtained at $w = \frac{-m_\Delta}{a}$ and $d = (\frac{2m_\Delta m_\Omega}{a^2})^{\frac{1}{2}}$ [32]. Here, w and d set two mass scales in the problem. At first step, Ω_c acquires VEV at scale M_R and $SU(2)_R$ is broken to $U(1)_R$, and then the VEV's of the Δ_c, $\bar{\Delta}_c$ break $U(1)_R \times U(1)_{B-L}$ to $U(1)_Y$ at a lower scale M_{B-L}. Thus, at this scale we get the minimal supersymmetric standard model (MSSM). However the D and F-flatness conditions [32] also give another set of possibility of vacuum which is degenerate to the one given by Eq. (7) which preserves the $SU(2)_R \times U(1)_L \times U(1)_{B-L}$ symmetry. The alternative set of VEV's is given by

$$\begin{aligned} &\langle\Omega\rangle = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix}; \quad \langle\Delta\rangle = \begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix}; \quad \langle\bar{\Delta}\rangle = \begin{pmatrix} 0 & \bar{d} \\ 0 & 0 \end{pmatrix}; \\ &\langle\Omega_c\rangle = 0, \quad \langle\Delta_c\rangle = 0, \quad \langle\bar{\Delta}_c\rangle = 0. \end{aligned} \tag{8}$$

Due to the left-right symmetry, numerically $d = \bar{d}$, $d_c = \bar{d}_c$ and $w = w_c$. It is the breaking of this symmetry that leads to the formations of domain walls. Now we have two degenerate vacua separated by a domain wall. Since in this case the D-parity is a discrete symmetry, the walls are topologically stable. Here we consider an ansatz for a trajectory in the group space which connects the two vacua. We parametrize the VEV's as follows with a parameter θ,

$$\begin{aligned} &\langle\Omega_c\rangle = \cos\frac{\theta}{2}\begin{pmatrix} w_c & 0 \\ 0 & -w_c \end{pmatrix}; \quad \langle\Delta_c\rangle = \cos\frac{\theta}{2}\begin{pmatrix} 0 & 0 \\ d_c & 0 \end{pmatrix}; \\ &\langle\bar{\Delta}_c\rangle = \cos\frac{\theta}{2}\begin{pmatrix} 0 & \bar{d}_c \\ 0 & 0 \end{pmatrix}; \quad \langle\Omega\rangle = \sin\frac{\theta}{2}\begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix}; \\ &\langle\Delta\rangle = \sin\frac{\theta}{2}\begin{pmatrix} 0 & 0 \\ d & 0 \end{pmatrix}; \quad \langle\bar{\Delta}\rangle = \sin\frac{\theta}{2}\begin{pmatrix} 0 & \bar{d} \\ 0 & 0 \end{pmatrix}. \end{aligned} \tag{9}$$

When $\theta = 0$, we have left like vacuum and for $\theta = \pi$, right like. On substituting these parametrized VEV's in the superpotential we obtain,

$$\begin{aligned} W_L &= m_\Delta \cos^2\frac{\theta}{2} d_c^2 + 2m_\Omega \cos^2\frac{\theta}{2} w_c^2 + a\cos^3\frac{\theta}{2} d_c^2 w_c \\ W_R &= m_\Delta \sin^2\frac{\theta}{2} d^2 + 2m_\Omega \sin^2\frac{\theta}{2} w^2 + a\sin^3\frac{\theta}{2} d^2 w. \end{aligned} \tag{10}$$

We can then compute θ derivative of the scalar potential as

$$\frac{\partial V}{\partial\theta} = 2Re\sum_i \frac{\delta W}{\delta\phi_i}\frac{\partial}{\partial\theta}\left(\frac{\delta W}{\delta\phi_i}\right). \tag{11}$$

This gives, using the numerical equality of the VEV's noted below Eq. (8),

$$\frac{\delta V_{\text{total}}}{\delta\theta} = -\sin\theta\Big[\cos^2\frac{\theta}{2}\Big\{\Big(m_\Delta d_c + a\cos\frac{\theta}{2}d_c w_c\Big)\Big(m_\Delta d_c + \frac{3a}{2}\cos\frac{\theta}{2}d_c w_c\Big) + \Big(2m_\Omega w_c + a\cos\frac{\theta}{2}d_c^2\Big)\Big(2m_\Omega w_c + \frac{3a}{2}\cos\frac{\theta}{2}d_c^2\Big)\Big\} - \sin^2\frac{\theta}{2}\Big\{\Big(m_\Delta d_c + a\sin\frac{\theta}{2}d_c w_c\Big)\Big(m_\Delta d_c + \frac{3a}{2}\sin\frac{\theta}{2}d_c w_c\Big) + \Big(2m_\Omega w_c + a\sin\frac{\theta}{2}d_c^2\Big)\Big(2m_\Omega w_c + \frac{3a}{2}\sin\frac{\theta}{2}d_c^2\Big)\Big\}\Big]. \tag{12}$$

It is easy to see that the two expressions with the braces mutually cancel at the symmetric point $\theta = \frac{\pi}{2}$. The value of the energy at this point is given by,

$$V_{DW} = (2-\sqrt{2})^2\frac{m_\Delta m_\Omega}{a^2}(m_\Delta^2 + m_\Omega^2). \tag{13}$$

The two set of vacua considered above, Eqs. (7) and (8) are degenerate and related by D-parity, which is a discrete symmetry of the group G_{LR}. The domain walls are therefore topologically stable. The solitonic domain walls thus arising were obtained as solutions of this theory in [35]. The motivation here is different. The considerations of this section illustrate how one rotates from one vacuum to another, not necessarily along energetically optimal path, but in order to estimate the height of the barrier. The same strategy will be utilized even for the more general case when the parent group is simply connected. The main point is that degeneracies that might occur in single field minimization are lifted due to the presence of several mutually coupled fields providing a general quartic polynomial. The two minima of Eqs. (7) and (8) are two of the solutions of the extremization condition. Such extrema are necessarily isolated points, being the zeros of a generic polynomial. Further, supersymmetry ensures that the supersymmetry preserving minima are absolute minima. Thus the third extremum, the intermediate point, is a local maximum determined along the parametrized curve. While this is not guaranteed to be the lowest energy peak separating the two minima, it provides an upper bound on the height of the saddle point lying on the barrier.

IV. DOMAIN WALLS IN MINIMAL SUPERSYMMETRIC *SO*(10) GUT

We now turn to the main problem of the minimal SUSY GUT model (MSGUT) [24–26,28]. The wall ansatz for MSGUT turns out to be complicated due to the presence of several Higgs fields in different representations. At first we proceed to establish the presence of a variety of walls that may appear in this MSGUT model, which may also involve intermediate phases of smaller gauge groups, if we allow some variation in the parameters. In each case we focus on D-parity walls which are unstable yet have non-trivial consequences.

The MSGUT can be broken down to MSSM directly or through intermediate symmetries depending upon the choice of Higgs multiplet getting VEV [27]. During these symmetry breakings, the D-parity is also broken and leads to the formation of TPD domain walls. The Higgs content of this model is **210** (Φ_{ijlk}, four index totally antisymmetric), $\mathbf{126}(\overline{\mathbf{126}})$ (Σ_{ijklm} ($\overline{\Sigma}_{ijklm}$), five index totally antisymmetric self-dual (antiself-dual) representation) and the vector representation **10** (H_i). Here $i, j, k, l, m = 1, 2...10$ run over the vector representation of $SO(10)$. The $\mathbf{126}(\overline{\mathbf{126}})$ and **210** break the SO(10) gauge symmetry to MSSM; the **10** breaks the electroweak symmetry, while the **10** and $\overline{\mathbf{126}}$ give masses to the fermions.

The renormalizable superpotential for the above mentioned Higgs superfields is given by,

$$W = \frac{m_\Phi}{4!}\Phi^2 + \frac{\lambda}{4!}\Phi^3 + \frac{m_\Sigma}{5!}\Sigma\bar{\Sigma} + \frac{\eta}{4!}\Phi\Sigma\bar{\Sigma} + m_H H^2 + \frac{1}{4!}\Phi H(\gamma\Sigma + \bar{\gamma}\,\bar{\Sigma}). \tag{14}$$

To recognize the SM singlets, the decomposition of Higgs supermultiplets required for SO(10) symmetry breaking to MSSM in terms of Pati-Salam gauge group ($SU(4)_C \times SU(2)_L \times SU(2)_R$) is given as [27],

$$210 = (15,1,1) + (1,1,1) + (15,1,3) + (15,3,1) + (6,2,2) + (10,2,2) + (\bar{1}0,2,2)$$
$$126 = (\bar{1}0,1,3) + (10,3,1) + (6,1,1) + (15,2,2)$$
$$\overline{126} = (\bar{1}0,3,1) + (10,1,3) + (6,1,1) + (15,2,2)$$

So, we call the SM singlet fields as $P(1,1,0)$, A (irreducible singlet of (15,1,1)) and Ω_R^0 ((113^0) of (15,1,3)) from 210. Similarly we identify Σ_R^- (($\bar{1}13^-$) of $(\bar{1}0,1,3)$) from **126** and $\bar{\Sigma}_R^+$ (($\bar{1}13^+$) of (10,1,3)) from $\overline{\mathbf{126}}$. The details of how these fields are defined in terms of components having SO(10) indices breaking them in $SO(6) \otimes SO(4)$ indices is elaborated in the Appendix. The VEV of H is not relevant to our considerations. The D-parity is defined as

$$D = \exp(i\pi J_{23})\exp(i\pi J_{67}) \tag{15}$$

Under the action of D-parity these fields transform as

$$P \to -P; \quad A \to A; \quad W_R^0 \to W_L^0$$
$$\Sigma_R^- \to -\Sigma_L^+; \quad \bar{\Sigma}_R^+ \to -\bar{\Sigma}_L^- \tag{16}$$

as further explained in the Appendix. Specific components of these fields are assigned the following VEV's.

$$\begin{aligned}
&\langle\Phi_{78910}\rangle = p\\
&\langle\Phi_{1234}\rangle = \langle\Phi_{1256}\rangle = \langle\Phi_{3456}\rangle = a\\
&\langle\Phi_{1278}\rangle = \langle\Phi_{3478}\rangle = \langle\Phi_{5678}\rangle\\
&\quad = \langle\Phi_{12910}\rangle = \langle\Phi_{34910}\rangle = \langle\Phi_{56910}\rangle = w\\
&\langle\Sigma_{a+1,b+3,c+5,d+7,e+9}\rangle = \frac{1}{2^{5/2}}(i)^{a+b+c-d-e}\sigma\\
&\langle\bar{\Sigma}_{a+1,b+3,c+5,d+7,e+9}\rangle = \frac{1}{2^{5/2}}(-i)^{a+b+c-d-e}\bar{\sigma}
\end{aligned} \tag{17}$$

so that, $\langle\Omega_L^0\rangle = \langle\bar{\Sigma}_L^-\rangle = \langle\Sigma_L^+\rangle = 0$. Here a, b, c, d, e take values 0 or 1.

The superpotential in terms of these VEVs is given by

$$W = m_\Phi(p^2 + a^2 + w^2) + 2\lambda(a^3 + 3pw^2 + 6aw^2) + m_\Sigma\sigma\bar{\sigma} + \eta\sigma\bar{\sigma}(p + 3a + 6w). \tag{18}$$

The SUSY preserving minima using the F-term and D-terms vanishing conditions are given by [26],

$$a = \frac{m_\Phi}{\lambda}\frac{x^2+2x-1}{1-x}; \quad p = \frac{m_\Phi}{\lambda}\frac{x(5x^2-1)}{(1-x)^2};$$
$$\sigma\bar{\sigma} = \frac{2m_\Phi^2}{\eta\lambda}\frac{x(1-3x)(1+x^2)}{\eta(1-x)^2}; \quad w = -\frac{m_\Phi}{\lambda}x. \tag{19}$$

where x is the solution of following cubic equation

$$8x^3 - 15x^2 + 14x - 3 = -\frac{\lambda m_\Sigma}{\eta m_\Phi}(1-x)^2. \tag{20}$$

However we have a list of possible intermediate symmetries depending on the value of x [27].

(1) For $x = 1/2$ and if $\lambda m_\Sigma/\eta m_\Phi = -5$, it gives SU(5) minimum.
(2) For $x = 0$ and if $\lambda m_\Sigma/\eta m_\Phi = 3$, this results in $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ minimum.
(3) For $x = \pm i$ and if $\lambda m_\Sigma/\eta m_\Phi = -3\,(1 \pm 2i)$, it gives $SU(3)_C \times SU(2)_L \times U(1)_R \times U(1)_{B-L}$ symmetry.
(4) For $x = 1/3$ and if $\lambda m_\Sigma/\eta m_\Phi = -2/3$, it results in the flipped $SU(5) \times U(1)$ minimum.
(5) For $x = 1/4$ and if $\lambda m_\Sigma/\eta m_\Phi = 5/9$, it results in MSSM minimum.

Now, consider an arbitrary D-rotation

$$U(\theta)_D = \exp\{i\theta(J_{23} + J_{67})\}. \tag{21}$$

Individual components of the fields transform differently under this generalized U_D-rotation, as follows, (with s_θ, c_θ standing for $\sin\theta$ and $\cos\theta$ respectively)

$$\begin{aligned}
\hat{\Phi}_{78910} &= c_\theta\Phi_{78910} + s_\theta\Phi_{68910}\\
\hat{\Phi}_{1234} &= \Phi_{1234}\\
\hat{\Phi}_{1256} &= c_\theta^2\Phi_{1256} - c_\theta s_\theta(\Phi_{1356} + \Phi_{1257}) + s_\theta^2\Phi_{1357}\\
\hat{\Phi}_{3456} &= c_\theta^2\Phi_{3456} + c_\theta s_\theta(\Phi_{2456} - \Phi_{3457}) - s_\theta^2\Phi_{2457}\\
\hat{\Phi}_{1278} &= c_\theta^2\Phi_{1278} + c_\theta s_\theta(\Phi_{1378} - \Phi_{1268}) - s_\theta^2\Phi_{1368}\\
\hat{\Phi}_{3478} &= c_\theta^2\Phi_{3478} + c_\theta s_\theta(\Phi_{2478} + \Phi_{3468}) + s_\theta^2\Phi_{2468}\\
\hat{\Phi}_{5678} &= W_{5678}\\
\hat{\Phi}_{12910} &= c_\theta\Phi_{12910} - s_\theta\Phi_{13910}\\
\hat{\Phi}_{34910} &= c_\theta\Phi_{34910} + s_\theta\Phi_{24910}\\
\hat{\Phi}_{56910} &= c_\theta\Phi_{56910} - s_\theta\Phi_{57910}\\
\hat{\Sigma}_{13579} &= c_\theta^2\Sigma_{13579} + c_\theta s_\theta(\Sigma_{12579} + \Sigma_{13569}) + s_\theta^2\Sigma_{12569}
\end{aligned} \tag{22}$$

Similarly one can write out for the other field components of Σ_R^- and $\overline{\Sigma}_R^+$ given in (A1).

Now we calculate the θ dependent potential from the corresponding superpotential as,

$$V = \sum_{i=1}^{74}\left|\frac{\partial W}{\partial\phi_i}\right|^2. \tag{23}$$

Here i runs over number of field components given in Eq. (A1). The form of potential for different values of x assuming $|\eta| = |\lambda|$ is

$$\begin{aligned}
V_{x=0}^{DW} &= \frac{|m_\Phi|^4}{|\lambda|^2}(8(\cos 2\theta + \sin 2\theta - 1)^2 + (-2\cos 2\theta + \sin 4\theta + 2)^2)\\
V_{x=1/3}^{DW} &= \frac{16|m_\Phi|^4(26\sin^4\theta + 12\sin^2\theta)}{81|\lambda|^2}\\
V_{x=\pm i}^{DW} &= \frac{|m_\Phi|^4}{|\lambda|^2}(272\sin^4\theta + 160\sin^2\theta + 48\sin^2\theta(4\sin 2\theta + 11\cos 2\theta + 25))\\
V_{x=1/2}^{DW} &= \frac{|m_\Phi|^4\sin^2\theta}{8|\lambda|^2}(-159\cos 2\theta - 5(14\cos 4\theta + \cos 6\theta - 218))\\
V_{x=1/4}^{DW} &= \frac{|m_\Phi|^4\sin^2\theta}{93312|\lambda|^2}(-490680\sin 2\theta + 111780\sin 4\theta - 324597\cos 2\theta + 41142\cos 4\theta + 17613\cos 6\theta + 1127498)
\end{aligned} \tag{24}$$

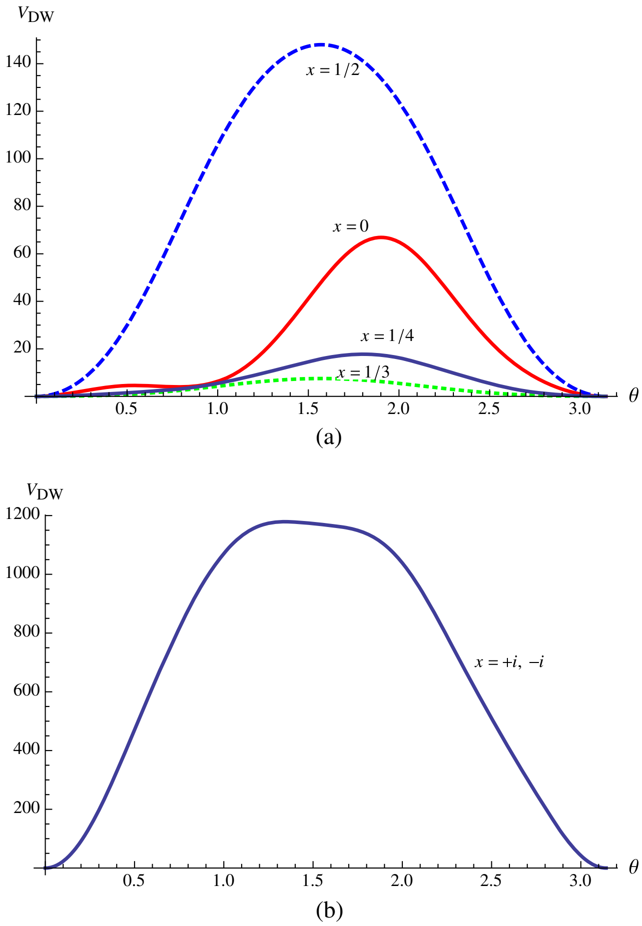

FIG. 1. The two subfigures (a) and (b) show scalar potential in arbitrary units as a function of θ which generates a one parameter subgroup. The curves correspond to different patters of symmetry breaking labeled by x, as listed below Eq. (20). The one step breaking to MSSM corresponds to $x = 1/4$.

The variation of the potential (in units of $\frac{|m_\Phi|^4}{|\lambda|^2}$) as a function of the $U(1)_D$-rotation angle θ is shown in Fig. 1. One can see from Fig. 1 that at $\theta = 0$ and π, the potential energy is zero and the VEVs satisfy the relations in Eq. (19) which are x dependent. For intermediate values of θ the potential is not symmetric under $\theta \to \pi - \theta$ and its overall magnitude is also strongly x dependent.

Our main motivation in performing this detailed calculation is to establish that TPD domain walls indeed form even if the group contains no discrete symmetry. In the last para of Sec. III, it was pointed out that our strategy at least yields an upper bound on the energy barrier separating vacua which are two distinct points. Unlike in that example, the group is simply connected here. However, it may be observed that there is an inadvertent (not accidental) discrete symmetry of the D and F flatness conditions. The flatness conditions cannot single out a unique vacuum but signal two for a given set of parameters, related by the D-parity. We now need to argue that this vacuum pair related by the flip symmetry are necessarily separated by an energy barrier. It is sufficient to focus on the 210 whose three independent sets of MSSM singlet components are assigned different VEV's p, a and w. The parameters in the superpotential are tuned according to Eqs. (19), (20) and the five possibilities listed below them. Then the value $x = 1/4$ is one zero of a cubic polynomial, which is necessarily isolated. Further, any values of the component fields P, A and Ω accessed by small variations of p, a and w are necessarily of higher energy. Thus the preferred vacua with unbroken MSSM are also isolated points at best connected by discrete transformations. This ends the existence proof of isolated vacua. While it is convenient to build low energy phenomenology based on the preferred vacuum, the conditions in the early universe allow domains of both types to form. Eventually the unstable TPD walls must disintegrate or have unfavorable consequences as discussed in II, based on [7]. In the next section we turn to cosmological consequences for MSGUT.

V. TOPOLOGICAL PSEUDODEFECT WALLS AND INFLATION

It is interesting to inquire what kind of signatures the walls can leave. A high scale theory will necessarily have to contend with inflation scale physics. Broadly, we may consider three possibilities, (A) $M_{GUT} < M_{inf}$, (B) $M_{GUT} > M_{inf}$, (C) $M_{GUT} \simeq M_{inf}$, where M_{GUT} is the $SO(10)$ symmetry breaking scale and M_{inf} is the scale of inflation.

Case A is generic to chaotic inflation[36] where inflation originates close to the Planck scale. In this case after reheating, the temperature could be less than or more than M_{GUT}. In the former case the thermal state should be directly in the required MSSM phase. In the latter case however, after the symmetry breaking phase transition, TPD walls would emerge. Due to their unstable nature they eventually disintegrate. The resulting epoch of wall domination would end with entropy dumping with return to pure radiation dominated universe. It has been pointed out [37–39] that there are models of inflation in which the duration of the reheating phase and the effective equation of state during that phase can be correlated with other inflation observables and is being pursued in [40]. For such cases the presence of TPD walls during reheating could have important consequences.

In Case B, the TPD walls would be copiously present when inflation commences. This would produce signatures similar in nature to but more pronounced than in the case C to be discussed below. The epoch over which the power law inflation caused by the walls would compete with the scale invariant inflation would be determined by the ratio M_{GUT}/M_{inf}. In late time observables, this would reflect in deviations from scale invariance at the largest scales. Since there are no strong indications to this effect we do not analyse this further, however the framework would be similar to that we pursue for the case C.

Case C could be accidental, but more interestingly, also occurs if inflationary physics emerges from the same grand unified theory. In this case the formation of TPD walls could occur before the inflaton potential energy dominates, giving rise to the signatures in the primordial fluctuations as encoded in the CMB data [41,42]. Recently this question has been addressed in [43] and it is shown that the presence of frustrated domain walls can alleviate the quadrupole anomaly of the CMB fit occurring in the Lambda-CDM model.

Consider the presence of domain walls which are conformally stretched, [7] and the wall complex as a whole obeys the coarse grained equation of state [44], and corresponding dependence on the Friedmann scale factor,

$$p = -\frac{2}{3}\rho; \quad \rho_{DW}(t) = \frac{\rho_1 a_1}{a(t)}. \tag{25}$$

where $\rho_1 \equiv M_{GUT}^4$, and in the latter equation the numerator sets the initial conditions on its value. The inflaton has a comparable energy density, $V_0 \equiv M_{inf}^4$ so that $H_0^2 = (8\pi/3)GV_0$ would be the Hubble parameter if only the inflaton were present. The combined Friedmann equation,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\left(V_0 + \frac{\rho_1 a_1}{a(t)}\right), \tag{26}$$

has the solution

$$a(t) = \frac{\rho_1 a_1}{2V_0}[\cosh\{H_0(t-t_1) + u_1\} - 1], \tag{27}$$

with

$$\cosh u_1 = 1 + \frac{2V_0}{\rho_1}. \tag{28}$$

In the regime where $\rho_1 > V_0$, and for $H_0(t-t_1) < 1$ one gets the behaviour

$$a^{(1)}(t) \approx \frac{4\pi}{3}G\rho_1 a_1\left(1 + \frac{2V_0}{\rho_1}\right)(t-t_1)^2, \tag{29}$$

characteristic of the $p = -2\rho/3$ equation of state. At late times of course the vacuum energy dominates. But a brief period of wall domination would still have the behavior similar to inflation, in which physical scales like those of the scalar field perturbations would be growing faster than the Hubble horizon. The amplitude of the perturbations would be imprinted on the earliest of the scales to leave the horizon. The ϵ parameter of inflation calculated in the present case gives,

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2}\mathrm{sech}^2\left[\frac{1}{2}\{H_0(t-t_1) + u_1\}\right]. \tag{30}$$

At $t = t_1$ this gives $\epsilon = 1/2$ as expected for a pure power law expansion with domain walls. But it soon turns over to

$$\epsilon \approx \frac{1}{2}e^{-H_0(t-t_1)}, \tag{31}$$

approaching the value 0 of the vacuum energy dominated phase. Thus the early modes to leave the horizon would be far from scale invariant, whereas within a few e-foldings of the time scale H_0^{-1} the slow roll condition is satisfied [40]. The departure from approximate scale invariance could therefore be detected. Further, the presence of domain walls would introduce non-Gaussianities. Since these would affect inflation only in its earliest stages of slow roll, they may not have entered our horizon yet. But in principle these could be detected. While cosmic strings have been studied for their effect on CMB data extensively [45–48], the presence of such primordial domain walls is also warranted, as a countercheck on models of unification as well as inflation.

VI. CONCLUSIONS

We have studied the example of a unification group wherein domain walls can form although the group is simply connected, with no discrete symmetries that break spontaneously. But inadvertent symmetries of the minimization conditions imply the possibility of a discrete set of vacua. Such vacua turn out to be related by discrete symmetries in the parent group. The case in point is the well known D-parity of $Spin(10)$ and its descendants. We have explicitly computed value of the energy along one-parameter paths connecting two possible subgroups to which the symmetry breaking of $Spin(10)$ could have occurred. We thus show that the vacua are indeed separated by an energy barrier. Then the causal structure of the early Universe creates the interesting possibility of topological pseudodefects, dubbed TPD walls, separating regions of such vacua. Even though manifestly unstable, the walls may live long enough to leave imprints on the observables. Such signatures in the CMB signals create the exciting possibility of accessing grand unification in current observations.

We have shown that in the context of inflation with a preceding epoch of radiation domination, (cases B and C), the formation of domain walls would leave scale dependent imprints on the very long wavelengths which leave the horizon at the onset of inflation. At current state of knowledge we do not know if these are indeed the scales being seen in the lowest multipoles. Likewise it is important to study non-Gaussianities resulting from such objects in the phase at the onset of inflation.

APPENDIX: D-PARITY PROPERTIES OF PATI-SALAM IRREDUCIBLE REPRESENTATION OF $SO(10)$ ACQUIRING VACUUM EXPECTATION VALUES

The MSSM singlets components from **210** are (15,1,1), (1,1,1), (15,1,3), each of which is assigned a different VEV.

Further, we have $(\bar{1}0, 1, 3)$ from **126** and (10,3,1) from $\overline{\mathbf{126}}$, all of which acquire VEVs to break $SO(10)$ down to SM. These can be written in terms of $SO(10)$ vector indices. We follow the procedure given in [27] but our conventions are different. We choose $a, b = 1, 2, \ldots 6$ for $SO(6)$ and $\alpha, \beta = 7, 8, 9, 10$ for $SO(4)$. Now the PS group $SU(4)_C \times SU(2)_L \times SU(2)_R$ is isomorphic to $SO(6) \times SO(4) \subset SO(10)$. In [27], the full table of Higgs representations in terms of $SO(10)$ indices is given. Below are the fields given in terms of $SO(10)$ indices in our conventions which are important to us in terms of breaking of $SO(10)$ gauge group.

$$
\begin{aligned}
P &= [7,8,9,10] \\
A &= [1,2,3,4] + [1,2,5,6] + [3,4,5,6] \\
\Omega_R^0 &= [1,2,7,8] + [3,4,7,8] + [5,6,7,8] + [1,2,9,10] + [3,4,9,10] + [5,6,9,10] \\
\Sigma_R^- &= -i([1,3,5,7,9] - [2,4,5,7,9] - [2,3,6,7,9] - [1,4,6,7,9] - i[2,3,5,7,9] - i[1,4,5,7,9] - i[1,3,6,7,9] \\
&\quad + i[2,4,6,7,9]) - (7,9 \to 8,10) + i\{7,9 \to 7,10\} + i\{7,9 \to 8,9\} \\
\overline{\Sigma}_R^+ &= i([1,3,5,7,9] - [2,4,5,7,9] - [2,3,6,7,9] - [1,4,6,7,9] + i[2,3,5,7,9] + i[1,4,5,7,9] + i[1,3,6,7,9] \\
&\quad - i[2,4,6,7,9]) - (7,9 \to 8,10) - i\{7,9 \to 7,10\} - i\{7,9 \to 8,9\}
\end{aligned}
\tag{A1}
$$

The sign $+(-)$ in the superscript represents the T_{3R} value. The D-parity is defined as

$$
D = \exp(i\pi J_{23}) \exp(i\pi J_{67}) \tag{A2}
$$

Using the definition of MSSM singlet fields in Eq. (A1), we find that under the action of D-parity these fields transform as

$$
\begin{gathered}
P \to -P; \quad A \to A; \quad W_R^0 \to W_L^0 \\
\Sigma_R^- \to -\Sigma_L^+; \quad \bar{\Sigma}_R^+ \to -\bar{\Sigma}_L^-
\end{gathered}
\tag{A3}
$$

where,

$$
\begin{aligned}
\Omega_L^0 &= [7,8,1,2] + [7,8,3,4] + [7,8,5,6] - [9,10,1,2] - [9,10,3,4] - [9,10,5,6] \\
\bar{\Sigma}_L^- &= -i([7,9,1,3,5] - [7,9,2,4,5] - [7,9,2,3,6] - [7,9,1,4,6] - i[7,9,2,3,5] - i[7,9,1,4,5] - i[7,9,1,3,6] \\
&\quad + i[7,9,2,4,6]) + (7,9 \to 8,10) - i\{7,9 \to 7,10\} + i\{7,9 \to 8,9\} \\
\Sigma_L^+ &= i([7,9,1,3,5] - [7,9,2,4,5] - [7,9,2,3,6] - [7,9,1,4,6] + i[7,9,2,3,5] + i[7,9,1,4,5] + i[7,9,1,3,6] \\
&\quad - i[7,9,2,4,6]) + (7,9 \to 8,10) + i\{7,9 \to 7,10\} - i\{7,9 \to 8,9\}
\end{aligned}
\tag{A4}
$$

The sign $+(-)$ in the superscript represents the T_{3L} value. These are used to choose the VEV's used in Eq. (17)

Next, the choice of the directions of the VEV's for the D-rotated field components used in calculating the potential in Eq. (23) is made as follows,

$$
\begin{aligned}
&\langle \Phi_{78910} \rangle = \langle \Phi_{68910} \rangle = p \\
&\langle \Phi_{1234} \rangle = \langle \Phi_{1256} \rangle = \langle \Phi_{3456} \rangle = \langle \Phi_{1356} \rangle = \langle \Phi_{1257} \rangle = \langle \Phi_{1357} \rangle = \langle \Phi_{2456} \rangle = \langle \Phi_{3457} \rangle = \langle \Phi_{2457} \rangle = a \\
&\langle \Phi_{1278} \rangle = \langle \Phi_{3478} \rangle = \langle \Phi_{5678} \rangle = \langle \Phi_{12910} \rangle = \langle \Phi_{34910} \rangle = \langle \Phi_{56910} \rangle = \langle \Phi_{1378} \rangle = \langle \Phi_{2478} \rangle = \langle \Phi_{13910} \rangle = \langle \Phi_{24910} \rangle = \langle \Phi_{57910} \rangle \\
&\quad = \langle \Phi_{1268} \rangle = \langle \Phi_{1368} \rangle = \langle \Phi_{3468} \rangle = \langle \Phi_{2468} \rangle = w \\
&\langle \Sigma_{13579} \rangle = \langle \Sigma_{12579} \rangle = \langle \Sigma_{13569} \rangle = \langle \Sigma_{13569} \rangle = \langle \Sigma_{12569} \rangle = \sigma
\end{aligned}
\tag{A5}
$$

Similarly we can write for all the field components which will appear after D-rotation of Σ_R^- and $\overline{\Sigma}_R^+$ (taking care of the i for each component of Σ_R^- and $\overline{\Sigma}_R^+$ appearing in Eq. (17).

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