Mass in cosmological perspective

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We consider the total nonlocal energy associated with a particle at rest in the Hubble flow, i.e., the relational energy between this particle and all connected particles within the causal horizon. The particle, even while at rest, partakes in relative recessional and peculiar motion of connected particles in three dimensions. A geometrical argument due to Berkeley suggests that the nonlocal mass of recessional energy associated with the particle is 3 times its Newtonian mass. It follows that nonlocal recessional and peculiar energy of the Universe are equal, and match Misner-Sharp energy within the apparent horizon. Contributions of recessional and peculiar nonlocal energy are thus shown to generate a 6 times higher level of matter energy than expected from the Newtonian mass. Accordingly, the nonlocal energy density of baryons is expected to be 6 times the standard local energy density of baryons, i.e., $\Omega_{\mathrm{b,eff}} = 6\Omega_{\mathrm{b}}$. At $\Omega_{\mathrm{b}} \sim 0.0484 \pm 0.0017$ [P. A. R. Ade *et al.* (Planck Collaboration), Astron. Astrophys. **594**, A13 (2016)] this predicts a nonlocal baryon energy density $\Omega_{\mathrm{b,eff}} \sim 0.290 \pm 0.010$, in agreement with observed matter density $\Omega_{\mathrm{m}} \sim 0.308 \pm 0.012$. The effect of nonlocal mass on solar system and galactic scales is considered.

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I. INTRODUCTION

Newtonian physics has a concept of both local and nonlocal energy. Kinetic energy is attributed to a particle, so it is localizable at the particle's position. Gravitational potential energy, on the other hand, is mutual and shared between particles; hence, it cannot be localized at a point. This, however, means that the conserved total energy, the sum of both, is necessarily nonlocal too. One of the main criticisms of Newton's theory from the start has been that kinetic energy of an object is only physically meaningful if considered in relation to other matter, ultimately the background of the "fixed stars." Thus, one can argue that in Newtonian physics *all* energy is essentially nonlocal, not just gravitational energy. It is legitimate to ask why this would be any different in general relativity, which actually was intended to satisfy this Machian principle. The rather artificial distinction between local and nonlocal energy becomes even less pertinent in the homogeneous, isotropic universe, where both can only appear as spatially constant energy densities. The question then is whether we can recognize nonlocal components of the density parameter ρ. Considering that in terms of local energy the density of baryonic matter can only explain about 5% of the required total energy density ρ, the remainder (or all) can perhaps be attributed to nonlocal energy contributions.

Theoretical approaches to represent nonlocal energy typically involve the use of pseudotensors (e.g., Einstein, Landau-Lifshitz, Bergmann, Møller) or prescriptions of quasilocal energy (e.g., Misner-Sharp, Hawking, Brown-York, Epp). Although the literature is not conclusive, studies mostly agree on zero, or constant, total energy of the Universe, at least in the flat case [1–11]. Unfortunately, these notions of nonlocal energy (e.g., a zero-energy universe) provide no direct information about the evolution of the density parameter ρ in a way consistent with observation. There are various indications, however, apart from the mass deficit itself, that mass in cosmological context is not necessarily the same as mass in local context. An intrinsic reason comes from a conjecture due to Berkeley [12], which gives rise to a different particle mass in peculiar and recessional motion, as shown hereafter. Notice that mass associated with nonlocal energy is nonlocal too, as it depends on distribution and motion of interacting particles (mass of binding energy being a familiar example). Within the context of general relativity, the notion of (Misner-Sharp) quasilocal energy suggests that the nonlocal energy density of cosmic matter differs from the standard local energy density, as we shall point out first. We use $c = 1$ throughout.

II. MISNER-SHARP ENERGY

Misner-Sharp energy represents internal energy (kinetic and potential) of a perfect fluid contained in a sphere of arbitrary radius [13]. Within the apparent horizon of FLRW universes, it equals (the Schwarzschild mass) [14,15]

$$E_{\mathrm{MS}} = \frac{R_{\mathrm{a}}}{2G}, \tag{1}$$

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while the apparent horizon radius R_a satisfies [16]

$$H^2R_a^2 = 1 + \frac{8}{3}\pi G\rho_k R_a^2, \quad (2)$$

where H is the Hubble parameter and ρ_k is curvature energy density. Energy $E_{MS}(= M_{MS}c^2)$ exerts a potential $\frac{3}{2}GM_{MS}R_a^{-1} = \frac{3}{4}$ at the center of the sphere. Introducing the density of Misner-Sharp energy, $\rho_E \equiv E_{MS}/\frac{4}{3}\pi R_a^3$, and using Eqs. (1) and (2), one obtains the energy equation (per unit mass)

$$T_a = \frac{3}{4}H^2R_a^2 = 2\pi G(\rho_E + \rho_k)R_a^2 = E_a - V_a, \quad (3)$$

where, in classical terms, $T_a \equiv \frac{3}{4}H^2R_a^2$ is kinetic energy, $E_a \equiv 2\pi G\rho_E R_a^2 = \frac{3}{4}$ is conserved total energy and $V_a \equiv -2\pi G\rho_k R_a^2$ is curvature energy, i.e., gravitational potential energy. Since the energies are expressed per unit mass, they can be regarded potentials.

There are some observations to make: (a) conservation of energy seems to hold if defined in terms of nonlocal energy. (b) Equation (3) is actually the Friedmann equation, multiplied on both sides by the common factor R_a^2. Hence, if the Misner-Sharp formalism is correct, then the total density in the Friedmann equation equals $\rho = \rho_E + \rho_k$, and nothing seems to be missing. For as far assumed local matter density ρ_m is represented, it must take the nonlocal form of ρ. We shall investigate this in the next section. (c) The Misner-Sharp formalism employs comoving coordinates [13]. Therefore, recessional speed in these coordinates is zero, so that Misner-Sharp energy only represents peculiar energy of the fluid. (d) Kinetic energy $T_a = \frac{3}{4}H^2R_a^2$ is, for the appearance of H, naturally associated with recessional motion of matter, while Misner-Sharp energy only expresses peculiar energy, which one may not immediately relate to the Hubble parameter. That is, unless the two, peculiar and recessional energy, maintain a fixed ratio. This indeed follows from both equipartition and the relational derivation hereafter.

III. RELATIONAL ENERGY

In the relational (Machian) view [17], energy is exclusively a mutual property *between* causally connected particles, therefore not an intrinsic property of a particle, meaning that local energy in fact does not exist in the relational universe. This may be understood realizing that the potential energy of a particle of mass m equals $m\varphi$, where φ is the cosmic potential. Without the cosmic mass-energy present, the potential energy of the particle would vanish. A similar argument applies to photon energy $h\nu$, where ν is the photon frequency. A vanishing potential would redshift the photon frequency to zero. Note that the Misner-Sharp mass within the apparent horizon indeed equals the Schwarzschild mass. Hence the idea that particle energy disappears in absence of other mass is not uncommon. This dependency can be largely disregarded though in the local frame, where spacetime is just an "empty" flat Minkowski background to local physics. Accordingly, our notions and unit of inertial mass relate to peculiar motion of an object in some particular direction, while in the relational view this object, even when at rest in the Hubble flow, partakes in energy exchange of recessional and peculiar motion of cosmic mass in *all* directions. In other words, the Newtonian mass and energy of an object in peculiar motion express only part of the total energy associated with the object. This follows directly from Berkeley's ontological conjectures [12].

George Berkeley, an early critic of Newton, noted that one can not meaningfully attribute a position or velocity to a single (point) particle in empty space. Consequentially, this applies to kinetic and potential energy too, hence to both inertial and gravitational mass. These properties can only emerge from the interaction with other particles, and are, therefore, necessarily shared, mutual properties between particles, so not localizable in a point and not intrinsic to a particle. Berkeley continues noting that of two particles in otherwise empty space, only their radial distance is observable. Motion in any perpendicular direction, like with these two particles in circular orbit of each other, is unobservable in an empty background. Therefore (and this is crucial), motion in nonradial direction, does *not* represent energy between two point particles. This means that both the kinetic energy T_{ij} and potential energy V_{ij} between point particles i and j depend *only* on their separation R_{ij}, or time derivative thereof, as pointed out by Poincaré and others [18–20]. Note that Newtonian potential energy

$$V_{ij} = -Gm_i m_j R_{ij}^{-1} \quad (4)$$

is perfectly Machian [18]. It is indeed a mutual, frame independent property between two connected particles and depends geometrically only on their separation. Newtonian kinetic energy, on the contrary, is defined relative to a frame of reference, so is not relational. Schrödinger [19,20] reproduced Einstein's expression of the anomalous perihelion precession from the following definition of Machian kinetic energy,

$$T_{ij} = \frac{1}{2}\mu_{ij}\dot{R}_{ij}^2, \quad (5)$$

where μ_{ij} represents the mutual mass between particles i and j,

$$\mu_{ij} \equiv \frac{V_{ij}}{\varphi_{px}}. \quad (6)$$

The effective potential φ_{px}, defined hereafter, normalizes μ_{ij} in order to match Newtonian mass and kinetic energy in peculiar motion [21]. Definition (5) meets the Machian requirements: kinetic energy T_{ij} is mutual between two particles, is frame independent, and depends only on the radial component of motion. The total kinetic and potential energy associated with particle i follows from summation over all particles within the causal radius R_g of particle i, i.e., $T_i = \sum_j T_{ij}$ and $V_i = \sum_j V_{ij} = m_i \varphi_N$, where

$$\varphi_{\mathrm{N}} = -2\pi G\rho R_g^2 \tag{7}$$

is the Newtonian potential at the center of the causal sphere.

A. Nonlocal mass

Like kinetic and potential energy, the total mass μ_i associated with particle i is a nonlocal, distributed property. However, the value of μ_i does not follow from simple addition, i.e., $\mu_i \neq \sum_j \mu_{ij}$, as pointed out next.

Due to the exclusively radial relationship in Eq. (5), particle j only contributes to kinetic energy T_i and mass μ_i if $\dot{R}_{ij} \neq 0$. Hence $\dot{R}_{ij} = 0$ effectively nullifies the contribution of particle j to both μ_i and the Newtonian mass m_i. This implies that only a part of the total connected mass, and therefore only a part $\varphi_{\mathrm{p}x}$ of the total Newtonian potential φ_{N}, contributes to the Newtonian mass m_i of a particle i in peculiar motion in some direction x. In a homogeneous, isotropic sphere, where all particles are in random peculiar motion, this fraction is

$$\xi_{\mathrm{p}x} \equiv \langle \dot{R}_{ij}^2 \rangle / \langle v_{ij}^2 \rangle_{\mathrm{p}x} = \frac{1}{3}, \tag{8}$$

where v_{ij} is the relative speed, and $\dot{R}_{ij}$ the radial component of v_{ij}, so that the effective potential in peculiar motion in an arbitrary direction x is (cf. [19,21])

$$\varphi_{\mathrm{p}x} = \xi_{\mathrm{p}x}\varphi_{\mathrm{N}} = \frac{1}{3}\varphi_{\mathrm{N}}. \tag{9}$$

Likewise the mass of peculiar motion in the x-direction between particle i and all connected particles equals

$$\mu_i^{(\mathrm{p}x)} = \xi_{\mathrm{p}x}\sum_j \mu_{ij} = \frac{1}{3}\frac{\sum_j V_{ij}}{\frac{1}{3}\varphi_{\mathrm{N}}} = m_i, \tag{10}$$

as expected. Thus Newtonian mass agrees with nonlocal mass in peculiar motion in arbitrary direction.

Different from peculiar motion, recession is purely radial motion between all particles, i.e., $\dot{R}_{ij} = v_{ij}$, and hence

$$\xi_{\mathrm{r}} = \langle \dot{R}_{ij}^2 \rangle / \langle v_{ij}^2 \rangle_{\mathrm{r}} = 1. \tag{11}$$

The kinetic energy of recession, therefore, balances with the full potential; all connected particles contribute fully. Thus the effective potential in recessional motion is

$$\varphi_{\mathrm{r}} = \xi_{\mathrm{r}}\varphi_{\mathrm{N}} = \varphi_{\mathrm{N}}. \tag{12}$$

This, however, means that a particle in recessional motion effectively has an effective mass 3 times larger than the Newtonian mass in peculiar motion. It interacts with 3 times as much mass. Indeed the total mass between particle i and all connected receding particles equals

$$\mu_i^{(r)} = \xi_{\mathrm{r}}\sum_j \mu_{ij} = \frac{\sum_j V_{ij}}{\frac{1}{3}\varphi_{\mathrm{N}}} = 3m_i. \tag{13}$$

This is an intriguing consequence of Berkeley's conjectures, evidently hinting at a possible interpretation of unidentified dark matter in the form of existing, but unrecognized, nonlocal energy components associated with each baryonic particle. Referencing Eqs. (13) and (10), the total nonlocal mass associated with particle i in the homogeneous, isotropic universe follows from adding up the components of μ_i

$$\mu_i = \mu_i^{(r)} + \mu_i^{(px)} + \mu_i^{(py)} + \mu_i^{(pz)} = 6m_i. \tag{14}$$

Note that this follows from geometrical considerations only. Equation (14) reflects that nonlocal energy density associated with baryonic matter is 6 times the local energy density. A perhaps conceptually more satisfactory way to derive this result is through actual calculation of the recessional and peculiar energies, as follows.

B. Total recessional and peculiar energy

We consider a unit mass test particle at rest in the Hubble flow at the position of the comoving observer. Adopting Eq. (5), integration over the causal sphere $\mathcal{V}_g$ yields the recessional Machian kinetic energy T_{r} between the test particle and all receding mass within the causal horizon at radius $R_g \equiv a r_g$ [21],

$$T_{\mathrm{r}} = \int_{\mathcal{V}_g} \frac{1}{2}\frac{\mathrm{d}\varphi_{\mathrm{r}}(r,\theta,\phi)}{\frac{1}{3}\varphi_{\mathrm{r}}} r^2\dot{a}^2 = \frac{3}{4}r_g^2\dot{a}^2 = \frac{3}{4}H^2R_g^2. \tag{15}$$

According to Eq. (12), the potential in recessional motion is the total Newtonian potential $\varphi_{\mathrm{r}} = \varphi_{\mathrm{N}} = -2\pi G\rho R_g^2$, where ρ is total density. Hence, the equation of total recessional energy is

$$T_{\mathrm{r}} = \frac{3}{4}H^2R_g^2 = 2\pi G\rho R_g^2 = -\varphi_N. \tag{16}$$

This again is the Friedmann equation, but derived from Machian principle [21].

Recalling that the effective potential in peculiar motion in arbitrary direction x is $\varphi_{\mathrm{p}x} = \frac{1}{3}\varphi_{\mathrm{r}} = \frac{1}{3}\varphi_{\mathrm{N}}$, we expect the balancing kinetic energies to maintain the same ratio, i.e., $T_{\mathrm{p}x} = \frac{1}{3}T_{\mathrm{r}}$. Thus

$$T_{px} = \frac{1}{4} H^2 R_g^2 = -\frac{1}{3} \varphi_N. \qquad (17)$$

Like with recessional motion, this holds for a test particle at rest in the Hubble flow; i.e., Eq. (17) expresses the kinetic energy due to the x-component of peculiar motion (on all physical scales) of all connected particles. The total peculiar energy associated with the test particle, summed over three orthogonal directions, is

$$T_p \equiv T_{px} + T_{py} + T_{pz} = \frac{3}{4} H^2 R_g^2 = 2\pi G \rho R_g^2 = -\varphi_N. \qquad (18)$$

Hence $T_p = T_r$, in agreement with both the equipartion theorem ($T_{px} = T_{py} = T_{pz} = T_{rx} = T_{ry} = T_{rz}$) and Misner-Sharp energy (at $R_g = R_a$). The nonlocal recessional and peculiar energy combined thus add to

$$T \equiv T_r + T_p = \frac{3}{2} H^2 R_g^2 = 4\pi G \rho R_g^2 = -\varphi = -2\varphi_N. \qquad (19)$$

Therefore,

$$T = 6 T_{px}, \qquad (20)$$

consistent with Eq. (14).

Notice that the equivalence of inertial and gravitational mass is implicitly satisfied by all energy equations above. The equations, expressed per unit mass, have the common form $T_\star = -\varphi_\star$. By definition, the kinetic energy $T_\star$ involves inertial mass, and the Newtonian gravitational potential $\varphi_\star$ involves gravitational mass. For an arbitrary test particle with inertial mass m_I and gravitational mass m_G, the equation becomes $m_I T_\star = -m_G \varphi_\star$. Hence $m_I = m_G$.

IV. COSMOLOGICAL OBSERVATION OF NONLOCAL MASS

Contributions of both recessional and peculiar nonlocal energy in three spatial dimensions have been shown to generate a 6 times higher level of matter energy than expected from the Newtonian mass of cosmic matter. This suggests an effective nonlocal baryon energy density of the Universe of 6 times the local energy density of baryons, i.e., $\Omega_{b,\mathrm{eff}} = 6\Omega_b$. According to Planck 2015 data [22], the baryon density is $\Omega_b h^2 \sim 0.0222 \pm 0.0002$. At $h \sim 0.678 \pm 0.009$ this gives $\Omega_b \sim 0.0484 \pm 0.0017$, while estimated matter density is $\Omega_m \sim 0.308 \pm 0.012$. The factor of six then predicts a total nonlocal baryon energy density $\Omega_{b,\mathrm{eff}} = 6\Omega_b \sim 0.290 \pm 0.010$, which matches Ω_m within the 68% confidence limits given. The nonlocal mass associated with baryonic matter thus provides interpretation to dark matter on the cosmological scale.

V. OBSERVATION OF NONLOCAL MASS ON LOCAL SCALES

The above model of nonlocal energy regards the causally connected mass of a homogeneous isotropic Universe. By Mach's principle the only true scale of any gravitational system is the cosmological scale, meaning that cosmic nonlocal energy acts on any scale, even while not necessarily recognized as such. On the other hand, a specifically local aspect of gravitational systems is inhomogeneity and the interaction between the system's constituents. A question then is how nonlocal mass relates to Newtonian mass and general relativistic effects in a gravitational system, discussed as follows.

According to the above, the total nonlocal mass associated with a body in the homogeneous Universe equals $m_{\mathrm{eff}} = 6m$, where m is the Newtonian mass of the body, i.e., the part of m_{eff} that is observed in peculiar motion. Hence, in the relational view Newtonian mass arises from the interaction with cosmic mass. The reason that $\frac{5}{6}$ of m_{eff} is not locally observable from the motion of the body itself is that the body's peculiar motion evidently is in one direction at the time, while recessional motion of bodies in a gravitationally bound local system is negligible or zero. A system, like the solar system or a galaxy, may be seen as a distribution of interacting local bodies superposed on a homogeneous cosmic background of relatively very low density. While the huge total amount of cosmic mass *outside* the system induces the Newtonian mass of all bodies, the interaction of bodies i and j *inside* the system gives rise to additional nonlocal mass μ_{ij}, which, considering Eq. (6), is typically extremely small compared with the Newtonian masses involved. That is, $\mu_{ij} \lll m_i + m_j$. Yet, the tiny effect of the corresponding relational kinetic energy $T_{ij} = \frac{1}{2}\mu_{ij}\dot{R}_{ij}^2$ between the bodies is observable on the solar system scale, for instance as the anomalous perihelion precession, or as Lense-Thirring frame dragging. Schrödinger showed that the effect of relational kinetic energy between two orbiting bodies precisely matches the general relativistic expression of the anomalous precession [19]. Thus Schrödinger's model is meaningful on both the small (solar system) and the large (cosmic) scale.

What this suggests is that the internal, non-Newtonian, part of the system mass arises from the local interaction of bodies within the system, and that the kinetic energy of these local interactions appears to be accountable for general relativistic deviations. Moreover, at an increasing number N of particles within the system, the number of internal interactions grows as $N(N-1)$. One, therefore, expects the total internal part of the system mass [i.e., $\sum_{i\neq j}\sum_j \mu_{ij} \sim N(N-1), i, j = 1, ..N$], to grow exponentially faster than the Newtonian mass of the system, which grows like $\sim N$, thus giving rise to much stronger deviations from Newtonian behavior in more massive larger systems. This may be of interest in the study of galaxy rotation and clusters.

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