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Sensitivity of holographic $\mathcal{N}=4$ SYM plasma hydrodynamics to finite coupling corrections

Alex Buchel

Department of Applied Mathematics, Department of Physics and Astronomy, University of Western Ontario, London, Ontario N6A 5B7, Canada and Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2J 2W9, Canada

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Gauge theory/string theory holographic correspondence for $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory is well under control in the planar limit, and for large (infinitely large) 't Hooft coupling, $\lambda\to\infty$. Certain aspects of the correspondence can be extended including $\mathcal{O}(\lambda^{-3/2})$ corrections. There are no reliable first principle computations of the $\mathcal{N}=4$ plasma nonequilibrium properties beyond the stated order. We show extreme sensitivity of the nonhydrodynamic spectra of holographic $\mathcal{N}=4$ SYM plasma to $\mathcal{O}(\lambda^{-3})$ corrections, challenging any conclusions reached from "resummation" of $\mathcal{O}(\lambda^{-3/2})$ corrections.

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I. INTRODUCTION AND SUMMARY

The most studied example of the holographic correspondence relating gauge theories and string theory is for the maximally supersymmetric $SU(N)$ $\mathcal{N}=4$ Yang-Mills theory (SYM) and type IIB string theory in $AdS_5\times S^5$ [1]. The number of colors N of the SYM is related to the 5-form flux on the string theory side. Furthermore, the asymptotic AdS_5 (or S^5) radius L in units of the string length $\alpha'=\ell_s^2$ along with the asymptotic value of the string coupling g_s establishes a correspondence to the 't Hooft coupling λ on the SYM side:

$$\frac{L^4}{\alpha'^2}=4\pi g_s N=g_{\rm YM}^2 N\equiv\lambda. \tag{1.1}$$

While there has been tremendous progress over the years in developing the correspondence (e.g., see [2]), understanding the full parameter space $\{N,\lambda\}$ is elusive. How much is exactly known depends on what questions one asks. Thermal or nonequilibrium states of SYM plasma at strong coupling are under control in the planar limit, $g_{\rm YM}\to 0$ $N\to\infty$ with λ kept fixed, and (in addition) for large 't Hooft coupling $\lambda\gg 1$. Only first subleading corrections $\propto\mathcal{O}(\lambda^{-3/2})$ are computationally accessible [3]. Here is a sample of SYM plasma results including first subleading corrections in the limit $\lambda\to\infty$:

(i) The thermal equilibrium free energy density of the SYM plasma is [4,5]

$$\mathcal{F}=-\frac{\pi^2}{8}N^2T^4(1+15\gamma+\cdots). \tag{1.2}$$

(ii) The shear viscosity to the entropy density ratio is [6–8]

$$\frac{\eta}{s}=\frac{1}{4\pi}(1+120\gamma+\cdots). \tag{1.3}$$

(iii) The speed of the sound waves and the bulk viscosity is [9]

$$c_s^2=\frac{1}{3}+0\cdot\gamma+\cdots,\qquad \frac{\zeta}{s}=0\cdot\gamma+\cdots. \tag{1.4}$$

(iv) A sample of the second-order transport coefficients (see [10,11] for further details) is [12,13]

$$\begin{aligned}\tau_\Pi T&=\frac{2-\ln 2}{2\pi}+\frac{375}{4\pi}\gamma+\cdots,\\ \kappa&=\frac{\eta}{\pi T}(1-145\gamma+\cdots),\\ \frac{\lambda_1 T}{\eta}&=\frac{1}{2\pi}(1+215\gamma+\cdots).\end{aligned} \tag{1.5}$$

(v) The plasma conductivity is [14][1]

$$\sigma=\sigma_\infty(1+125\gamma+\cdots), \tag{1.6}$$

where σ_∞ is the plasma conductivity at infinite 't Hooft coupling.

[1]Reference [14] corrects the earlier computation [15].

In expressions (1.2)–(1.6) we introduced

$$\gamma = \frac{1}{8}\zeta(3)(\alpha')^3. \tag{1.7}$$

Notice that as one proceeds from the corrections to the equilibrium quantities (1.2) to the first-order (1.3), the second-order (1.5) transport, the conductivity (1.6), the relative "strength" of the corrections grow. The correction strength is even more dramatic, $\propto (10^4\text{–}10^5)\cdot\gamma$ to the spectra of the nonhydrodynamic plasma excitations [the quasinormal modes (QNMs) of the dual gravitational background] [16,17]. This observation led the authors of [18] to propose the idea of an effective resummation of γ-corrections. In a nutshell, on α'-corrected gravity side of the holographic correspondence one typically gets higher-derivative bulk equations of motion. One can use the smallness of γ to eliminated the higher derivatives, reducing the equations to the second-order ones, where γ corrections affect the first-order derivatives at the most—this is precisely what was done for example in computation of the shear viscosity in [6]. The next (new) step is to "forget" that γ must be small in transformed equations and instead treat the equations nonperturbatively in γ. There are two effects of such a resummation at finite γ:

(i) It is possible to compute finite-γ corrections to SYM observables at infinitely large 't Hooft coupling;
(ii) one can discover new phenomena, which are absent in an infinite 't Hooft coupling limit.

It is the latter aspect of the resummation that should be subject to additional scrutiny in drawing physical conclusions. In particular, following the resummation approach of [18], in [19] a new branch of the QNMs was found—these are (purported) SYM plasma excitations with $\Re(\mathfrak{w}) = 0$. The physics of these new excitations was crucial to draw conclusions regarding properties of $\mathcal{N} = 4$ spectral function at intermediate 't Hooft coupling [20].

To our knowledge, there is no discussion in the literature, even at a phenomenological level, on how robust is the resummation approach of [18]. In this note we address this question focusing on $\Re(\mathfrak{w}) = 0$ branch of the QNMs identified in [19]. In the absence of the reliable corrections to type IIB supergravity we proceed as follows. Recall the tree level type IIB low-energy effective action in ten dimensions taking into account the leading-order string corrections [21,22]

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4\cdot 5!}(F_5)^2 + \cdots + \gamma e^{-\frac{3}{2}\phi}W + \cdots\right], \tag{1.8}$$

where W in a certain scheme is proportional to the fourth power of the Weyl tensor

$$W = C^{hmnk}C_{pmnq}C_h{}^{rsp}C^q{}_{rsk} + \frac{1}{2}C^{hkmn}C_{pqmn}C_h{}^{rsp}C^q{}_{rsk}. \tag{1.9}$$

A consistent (for the purpose of QNM spectra computation) Kaluza-Klein reduction of (1.8) on S^5 results in

$$S_5 = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\left(R + \frac{12}{L^2} + \gamma W\right), \tag{1.10}$$

where W is a five-dimensional equivalent of (1.9). We would like to stress that an effective action (1.10) includes all the terms at order γ arising from string theory that are relevant for physics of homogeneous and isotropic thermal equilibrium states of $\mathcal{N} = 4$ SYM plasma and (non)hydrodynamic fluctuations about them. As it stands, results extracted from this action are valid only up to $\mathcal{O}(\gamma)$, i.e., for infinitesimal γ, and thus do not provide information about finite-γ (finite 't Hooft coupling) corrections to $\mathcal{N} = 4$ SYM observables. The resummation procedure advocated in [18] follows the steps:

(a) Derive relevant equations of motion from (1.10) to order $\mathcal{O}(\gamma)$ inclusive.
(b) These equations contain higher (than the second order) space-time derivatives. Using equations of motion at order $\mathcal{O}(\gamma^0)$, all the space-time derivatives (higher than the first order) at order $\mathcal{O}(\gamma)$ can be eliminated; e.g., see [6]. The resulting equations contain at most second space-time derivatives and the space of perturbative in γ solutions of these equations agrees [up to $\mathcal{O}(\gamma)$] with the space of solutions of perturbative equations in (a).
(c) The proposal of [18] is to treat equations in (b) as *exact* in γ.

Clearly, there is no physical justification of step (c) where one extends, without any modifications, equations of motion (EOMs) valid at $\mathcal{O}(\gamma)$ only. On can easily invent infinitely many resummation schemes in the spirit of [18]. Here is one of them:

(A) Derive relevant equations of motion from (1.10) to order $\mathcal{O}(\gamma^k)$ inclusive, where $k \geq 1$ is an arbitrary integer.
(B) These equations contain higher (than the second order) space-time derivatives. Using equations of motion at orders $\mathcal{O}(\gamma^m)$, $m < k$, all the space-time derivatives (higher than the first order) at orders $\mathcal{O}(\gamma^m)$, $1 \leq m \leq k$ can be eliminated; e.g., see Sec. II. The resulting equations contain at most second space-time derivatives and the space of perturbative in γ solutions of these equations agrees [up to $\mathcal{O}(\gamma^k)$] with the space of solutions of perturbative equations in (A).
(C) The *new* resummation is to treat equations in (B) as *exact* in γ.

The *new* truncation and resummation procedure of γ-corrections is as good (or as bad) as the one proposed in [18]. The purpose of our paper is precisely to test the robustness of the different k resummation schemes. Specifically, we consider the simplest extension of the five-dimensional effective action (1.10):

$$\tilde{S}_5=\frac{1}{2\kappa_5^2}\int d^5x\sqrt{g}\left(R+\frac{12}{L^2}+\gamma W+\alpha\gamma^2W^2+\mathcal{O}(\gamma^3)\right), \tag{1.11}$$

where we study a family of a constant α such that $|\alpha\gamma|\lesssim 1$. Notice that the (phenomenological) action (1.11) is assumed to be *exact* up to order γ^2. At $\alpha=0$ the effective action (1.11) is just $k=2$ representative of the new resummation scheme explained above. The order $\mathcal{O}(\alpha)$ term is one of the potential terms that could arise from real string theory computations—we do not claim that it is a dominant one (there could be other terms at this order); neither do we know the precise value of α. The purpose of introducing this α term is to illustrate that physical observables does not necessarily have to be monotonic in γ. Given (1.11), the corrections at order γ^2 arise from the second-order perturbation due to γW term, and directly due to the first-order term in α. In the next section we present results of the computations. In both cases,

(i) setting $\alpha=0$ but treating (1.10) as (1.11),

(ii) fixing $\gamma=10^{-3}$ and exploring $|\alpha|\lesssim 100$,

we find a dramatic variation in the spectrum of QNMs on the branch with $\Re(\mathfrak{w})=0$. Thus, we conclude that physics extracted from (1.10) beyond the leading order in γ [in the absence of explicit and reliable computations of $\mathcal{O}(\gamma^2)$ string theory corrections] have to be treated with caution. We explicitly demonstrated this fact for some branches of the spectra of QNMs; however, this is also true for the relation between the black brane temperature T and the location of its horizon r_0 in the holographic dual to $\mathcal{N}=4$ SYM plasma: From (2.3) the $\mathcal{O}(\gamma^2)$ term (at $\alpha=0$) enters with coefficient over 1400 larger than the $\mathcal{O}(\gamma)$ term. While we believe that a similar fate awaits other observables, the η/s ratio in particular, this remains to be corroborated with explicit computations. On a positive note, it is conceivable that some quantities in $\mathcal{N}=4$ plasma exhibit $\mathcal{O}(\gamma)$ features that remain qualitatively robust upon inclusion of higher-order corrections.

II. TECHNICAL DETAILS

To facilitate comparison and readability, we follow notations of [19].

To order $\mathcal{O}(\gamma^2)$, the black brane solution to the equations of motion following from (1.11) is given by

$$ds^2=\frac{r_0^2}{u}(-f(u)Z_tdt^2+dx^2+dy^2+dz^2)+Z_u\frac{du^2}{4u^2f}, \tag{2.1}$$

where $f(u)=1-u^2$, r_0 is the parameter of nonextremality of the black brane geometry, and

$$\begin{aligned}Z_t&=1-15\gamma(5u^2+5u^4-3u^6)+\gamma^2\left(\frac{161100}{7}u^{14}\alpha+\frac{30}{7}(-6630\alpha+69720)u^{12}\right.\\&\quad+\frac{36}{7}(-5525\alpha-119560)u^{10}+\frac{45}{7}(-4420\alpha-11872)u^8+\frac{60}{7}(-3315\alpha-7329)u^6\\&\quad\left.+\frac{90}{7}(-2210\alpha-6986)u^4+\frac{180}{7}(-1105\alpha-3493)u^2\right),\\Z_u&=1+15\gamma(5u^2+5u^4-19u^6)+\gamma^2\left(\left(\frac{198900}{7}\alpha+89820\right)u^2+\left(\frac{198900}{7}\alpha+95445\right)u^4\right.\\&\quad+\left(\frac{198900}{7}\alpha+20070\right)u^6+\left(\frac{198900}{7}\alpha+57195\right)u^8+\left(\frac{198900}{7}\alpha+2744370\right)u^{10}\\&\quad\left.+\left(\frac{198900}{7}\alpha-3680775\right)u^{12}-\frac{2321100}{7}u^{14}\alpha\right).\end{aligned} \tag{2.2}$$

The γ-corrected Hawking temperature corresponding to the solution (2.1) is

$$T=\frac{r_0}{\pi}\left(1+15\gamma+\gamma^2\left(21420+\frac{47700}{7}\alpha\right)\right). \tag{2.3}$$

Scalar channel.—The QNM equation takes the form

$$\partial_u^2Z_1-\frac{1+u^2}{u(1-u^2)}\partial_uZ_1+\frac{\mathfrak{w}^2-\mathfrak{q}^2(1-u^2)}{u(1-u^2)^2}Z_1=\gamma\mathcal{G}_1[Z_1]+\gamma^2\mathcal{G}_{1,2}[Z_1], \tag{2.4}$$

where Z_1 is a radial profile of the $h_x{}^y$ metric fluctuations. The explicit expression for $\mathcal{G}_1[Z_1,\partial_uZ_1]$ can be found in [19], and we compute

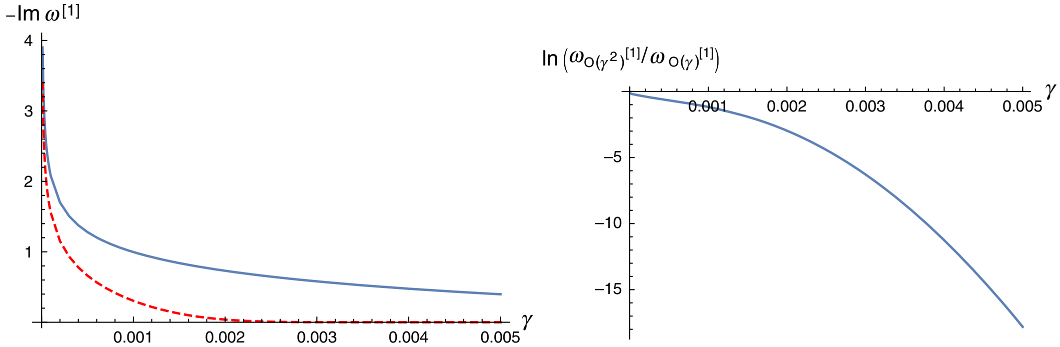

FIG. 1. Left panel: The lowest QNM frequencies $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma)}$ computed with (1.10) (solid blue curve) and the lowest QNM frequencies $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma^2)}$ computed with (1.11) with $\alpha = 0$ (dashed red curve). Right panel: Log-comparison of the lowest QNM frequencies for different orders of the approximation of the gravitational effective action.

$$
\begin{aligned}
\mathcal{G}_{1,2} = & -\frac{2}{7}(3144960\alpha\mathfrak{q}^2u^{11} + 8052660\alpha u^{12} - 1075200\mathfrak{q}^4u^8 + 7878600\alpha u^{10} \\
& + 40025216\mathfrak{q}^2u^9 + 75735891u^{10} - 994500\alpha u^8 - 29659392\mathfrak{q}^2u^7 + 1741824u^7\mathfrak{w}^2 \\
& + 15490125u^8 - 795600\alpha u^6 - 40675194u^6 - 596700\alpha u^4 + 604800\mathfrak{q}^2u^3 - 1040445u^4 \\
& - 397800\alpha u^2 - 843255u^2 - 198900\alpha - 628740)u\partial_u Z_1 + \frac{1}{7u(u^2-1)}(483840\alpha\mathfrak{q}^4u^{13} \\
& - 17476020\alpha\mathfrak{q}^2u^{14} - 258048\mathfrak{q}^6u^{10} + 17945100\alpha\mathfrak{q}^2u^{12} - 15661800\alpha u^{12}\mathfrak{w}^2 \\
& + 14363328\mathfrak{q}^4u^{11} - 135086623\mathfrak{q}^2u^{12} - 198900\alpha\mathfrak{q}^2u^{10} + 2084400\alpha u^{10}\mathfrak{w}^2 - 12425280\mathfrak{q}^4u^9 \\
& + 5246976\mathfrak{q}^2u^9\mathfrak{w}^2 + 213413970\mathfrak{q}^2u^{10} - 104522733u^{10}\mathfrak{w}^2 - 198900\alpha\mathfrak{q}^2u^8 \\
& + 1686600\alpha u^8\mathfrak{w}^2 + 100800\mathfrak{q}^4u^7 - 77651133\mathfrak{q}^2u^8 + 81113193u^8\mathfrak{w}^2 - 198900\alpha\mathfrak{q}^2u^6 \\
& + 1288800\alpha u^6\mathfrak{w}^2 + 282240\mathfrak{q}^4u^5 - 1654800\mathfrak{q}^2u^6 + 3212370u^6\mathfrak{w}^2 - 198900\alpha\mathfrak{q}^2u^4 \\
& + 891000\alpha u^4\mathfrak{w}^2 + 404775\mathfrak{q}^2u^4 + 1908900u^4\mathfrak{w}^2 - 198900\alpha\mathfrak{q}^2u^2 + 493200\alpha u^2\mathfrak{w}^2 \\
& - 644490\mathfrak{q}^2u^2 + 1590435u^2\mathfrak{w}^2 - 95400\alpha\mathfrak{q}^2 + 95400\alpha\mathfrak{w}^2 - 301455\mathfrak{q}^2 + 301455\mathfrak{w}^2)Z_1. \qquad (2.5)
\end{aligned}
$$

Note that the EOM for Z_1 directly obtained from (1.11) involves terms $\propto\gamma$ or $\propto\gamma^2$ with (up to) fourth-order derivatives in u. Following [6], higher-derivative "source" terms with γ dependence can be eliminated using EOM at lower order. We implemented two different schemes:

(i) All the higher derivatives in γ-dependent source terms are eliminated using the $\mathcal{O}(\gamma^0)$ EOM from (2.4):

$$
\partial_u^2 Z_1 = \frac{1+u^2}{u(1-u^2)}\partial_u Z_1 - \frac{\mathfrak{w}^2 - \mathfrak{q}^2(1-u^2)}{u(1-u^2)^2}Z_1;
$$

(ii) the functionals $\mathcal{G}_1$ and $\mathcal{G}_{1,2}$ (dependent on Z_1 and $\partial_u Z_1$ only) are adjusted in such a way that the perturbative solutions to (2.4) agree with the perturbative solutions of the higher-derivative order direct EOM for Z_1 to order $\mathcal{O}(\gamma^2)$ inclusive.

The two reduction procedures are not equivalent: Specifically, $\mathcal{G}_{1,2}$ differs.[2] Expression (2.5) represents the result of the latter of the two reduction schemes.[3]

As in [19],

$$
\mathfrak{w} = \frac{\omega}{2\pi T}, \qquad \mathfrak{q} = \frac{q}{2\pi T}, \qquad (2.6)
$$

with the temperature given by (2.3), and $\{\omega, q\}$ begin the frequency and the momentum of the nonhydro SYM plasma excitation.

We focus on QNMs with $\Re(\mathfrak{w}) = 0$ at $\mathfrak{q} = 0$. Thus, we need to solve (numerically) (2.4) for z_1, defined as

$$
Z_1 = (1-u)^{-i\mathfrak{w}/2}u^2 z_1(u), \qquad (2.7)
$$

[2] Nonetheless, we find that the QNM spectra computed within these two schemes over the parameter range reported in Figs. 1 and 2 differ by less than 5%.

[3] I would like to thank the authors of [20] for independent confirmation of the technical details reported.

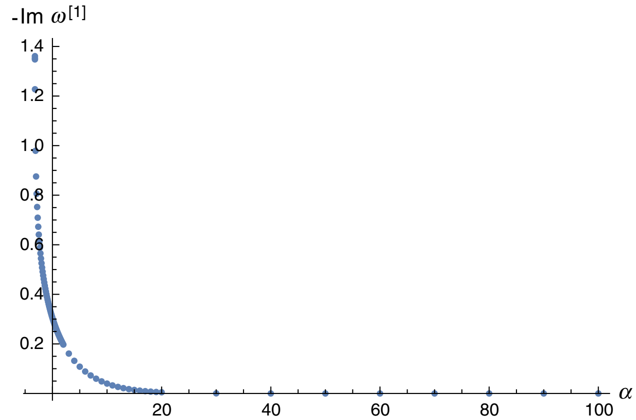

FIG. 2. The lowest QNM frequencies $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma^2)}$ computed with (1.11) at $\gamma = 10^{-3}$ as a function of the phenomenological parameter α.

subject to a regular boundary conditions both as $u \to 0_+$ (the asymptotic AdS_5 boundary) and $u \to 1_-$ (the black brane horizon):

$$\lim_{u\to 1_-} z_1 = 1, \qquad \lim_{u\to 0_+} z_1 = \text{const} \neq 0. \qquad (2.8)$$

Notice that (2.7) automatically accounts for an incoming-wave boundary conditions for Z_1 at the black brane horizon. Results of the numerical computations are presented in Figs. 1 and 2.

(i) We confirm the computations of the QNM frequencies determined in [19] and presented in Fig. 5 there.

(ii) The left panel of Fig. 1 presents the lowest QNM frequencies $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma)}$ computed with (1.10) (solid blue curve) and the lowest QNM frequencies $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma^2)}$ computed with (1.11) with $\alpha = 0$ (dashed red curve) for a range of $\gamma \in [10^{-5}, 5\times 10^{-3}]$. At $\gamma = 10^{-5}$, the two approximations produce frequencies that differ by $\sim 13\%$. As γ increases, the difference becomes dramatic: At $\gamma = 0.005$ the two frequencies differ by a factor of $\sim 5\times 10^7$.

(iii) Figure 2 presents results for $\mathfrak{w}^{[1]}_{\mathcal{O}(\gamma^2)}$ at $\gamma = 10^{-3}$ as parameter α varies within $[-3.21, 100]$. The value of the frequencies varies by a factor of $\sim 10^{11}$. Notice that the presented QNM spectrum has a linear sensitivity to α about $\alpha = 0$. This implies that, lacking the precise knowledge of higher-derivative γ-corrections, observables in $\mathcal{N}=4$ SYM plasma do not have to be monotonic in γ.

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