

Closing the gap on R_{D^*} by including longitudinal effects

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Measurements of the $R_{D^*} \equiv \text{Br}(B \rightarrow \tau \nu D^*)/\text{Br}(B \rightarrow e \nu D^*)$ parameter remain in tension with the standard model prediction, despite recent results helping to close the gap. The standard model prediction it is compared with considers the D^* as an external particle, even though what is detected in experiments is a $D\pi$ pair it decays into, from which it is reconstructed. We argue that the experimental result must be compared with the theoretical prediction considering the full four-body decay ($B \rightarrow l \nu D^* \rightarrow l \nu D \pi$). We show that the longitudinal degree of freedom of the off shell D^* helps to further close the disagreement gap with experimental data. We find values for the ratio $R_{D\pi}^l \equiv \text{Br}(B \rightarrow \tau \nu_\tau D \pi)/\text{Br}(B \rightarrow l \nu_l D \pi)$ of $R_{D\pi}^e = 0.274 \pm 0.003$ and $R_{D\pi}^\mu = 0.275 \pm 0.003$, where the uncertainty comes from the uncertainty of the form factors parameters. Comparing against $R_{D\pi}$ reduces the gap with the latest LHCb result from 1.1σ to 0.48σ , while the gap with the latest Belle result is reduced from 0.42σ to just 0.10σ and with the world average results from 3.7σ to 2.1σ .

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I. INTRODUCTION

The formulation of the standard model (SM) incorporates the three families of leptons with universal couplings to gauge bosons, such that the differences in similar processes for different leptons are only of kinematical origin. This lepton flavor universality has been tested in the weak sector, e.g., by studying the semileptonic decays of heavy mesons. The parameter that is expected to reflect the universality property is defined as the ratio of similar processes into two different leptons, namely,

$$R_X \equiv \text{Br}(P \rightarrow \tau \nu_\tau X)/\text{Br}(P \rightarrow l \nu_l X), \quad (1)$$

where P is the decaying particle, typically a pseudoscalar meson, $l = e, \mu$, and X is the hadronic product.

Early measurements of R_D [1,2] from B meson decays prompted the question of possible lepton flavor violation after finding significant discrepancies with the SM predictions, although a more recent measurement has obtained $R_D = 0.375 \pm 0.064 \pm 0.026$ [3], which is in better agreement with the SM prediction of $R_D^{\text{SM}} = 0.300 \pm 0.008$ [4].

For R_{D^*} , several B factories conducted experiments that have also consistently measured values of R_{D^*} that are higher than the SM prediction [1–3,5,6]. The most

recent results from the LHCb [7,8] and Belle [9,10] Collaborations, which study the process through hadronic channels of the τ decay, have reduced this disagreement gap to a statistical significance of just 1.1σ and 0.42σ , respectively. However, a discrepancy with combined statistical significance of 3.7σ [11] is still found when considering the previous experiments, which use other τ reconstruction methods [1–3,5,6]. Since the deviation is still large in the R_{D^*} case, we can inquire at which extent the nature of the hadronic particle in the final state affects the result. The $B \rightarrow l \nu D^*$ decay is represented by a single tree-level diagram, shown in Fig. 1. Since the D^* decay width is relatively small, its decay process is usually considered not to play any role in the estimation of R_{D^*} (since it is common to both leptonic decay modes, it should cancel in the ratio). Thus, it is common to consider simply this three-body

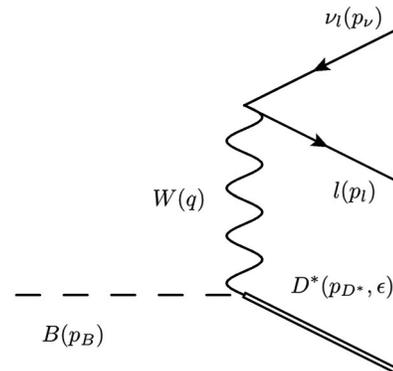


FIG. 1. Feynman diagram for $B \rightarrow l \nu D^*$ decay.

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decay, from which a value of $R_{D^*}^{\text{SM}} = 0.252 \pm 0.003$ is obtained [12]. The error bar accounts for the uncertainties on the form factors of the vertex connecting the B , W , and D^* particles as measured by Belle [13].

We argue that the tension of the theoretical prediction with experimental measurements is due in part to the fact that R_{D^*} is not the proper quantity that the results should be compared with, since in all the experiments the D^* is never measured directly but through its decay into daughter particles, namely, a $D\pi$ pair for the charged D^* (with a branching ratio of 98.4%) and either a $D\pi$ pair (branching ratio of 64.7%) or $D\gamma$ (branching ratio of 35.3%) for the neutral D^* . LHCb and early Belle results on R_{D^*} rely on purely charged D^* [5,7,8] (neutral B 's), while other Belle results [3,9,10] and *BABAR* [1,2] consider both neutral and charged ones. Therefore, it is adequate to consider the ratio

$$R_{D\pi} \equiv \text{Br}(B \rightarrow \tau \nu_\tau D\pi) / \text{Br}(B \rightarrow l \nu_l D\pi), \quad (2)$$

obtained from the full four-body diagram shown in Fig. 2 as a better value to compare to. Upcoming experiments will help to settle down the experimental value, and a solid SM prediction to compare to is mandatory.

In this paper, we show that the corrections that arise from the full process, corresponding to adding the longitudinal degree of freedom of the off shell D^* , are not negligible and help to get the experimental and theoretical results in better agreement. The corrections that we focus on in this paper apply only to the $D\pi$ channel, and the $D\gamma$ channel is ignored throughout. An earlier work has considered the full process to explore the possible effect of other resonances and found it to be negligible [14]. The purely longitudinal contribution ratio has also been computed [12]. Here we exhibit the role of each contribution, transverse, longitudinal, and interference. In particular, the interference turns out to be very different for light and heavy leptons, from which $R_{D\pi}$ is found to deviate from R_{D^*} .

In Sec. II, we elaborate on the origin of these corrections and contrast with the shortened three-body decay. Then, we

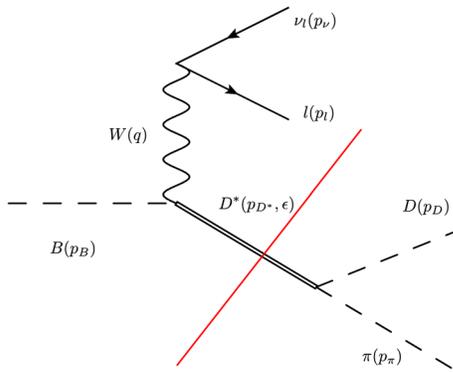


FIG. 2. Feynman diagram for the full $B \rightarrow l \nu_l D\pi$ decay, often thought of as independent $B \rightarrow l \nu_l D^*$ and $D^* \rightarrow D\pi$ decays, as indicated by the red cutting line.

present our result for $R_{D\pi}$, including the uncertainties estimate. The discussion and concluding remarks are presented in Sec. III.

II. LONGITUDINAL CORRECTIONS: FROM THREE- TO FOUR-BODY DECAY

Let us describe the four-body decay process, taking the three-body one as a baseline to identify the role of the longitudinal contribution of the D^* vector meson.

The $B \rightarrow l \nu D^* \rightarrow l \nu D\pi$ decay can be considered as separate subsequent processes by “cutting” the diagram as shown in Fig. 2. This procedure kills the longitudinal contribution by considering the D^* as on shell. In this approach, the total amplitude can be written as a product of three- and two-body decay amplitudes, connected by the outgoing and ingoing D^* polarizations, and a sum over them to account for all the different forms they can match,

$$M = M_{3\mu} \left(\sum \epsilon^{*\mu} \epsilon^\nu \right) M_{2\nu} = M_{3\mu} \left(-g^{\mu\nu} + \frac{p_{D^*}^\mu p_{D^*}^\nu}{m_{D^*}^2} \right) M_{2\nu},$$

where ϵ^ν , p_{D^*} , and m_{D^*} are the polarization tensor, momentum, and mass of the D^* , respectively. $M_{3\mu}$ and $M_{2\nu}$ are the three-body D^* production and two-body D^* decay amplitudes, respectively, with the polarization tensor factored out. The squared amplitude is then

$$|M|^2 = M_{3\mu} M_{3\alpha}^* \left(-g^{\mu\alpha} + \frac{p_{D^*}^\mu p_{D^*}^\alpha}{m_{D^*}^2} \right) \times \left(-g^{\alpha\beta} + \frac{p_{D^*}^\alpha p_{D^*}^\beta}{m_{D^*}^2} \right) M_{2\nu} M_{2\beta}^*. \quad (3)$$

The complete four-body amplitude, on the other hand, is given by

$$M = M_{3\mu} D^{\mu\nu} M_{2\nu},$$

where $D^{\mu\nu}$ is the D^* propagator that, upon considering the absorptive correction (dominated by the $D\pi$ mode as discussed before), can be set in terms of the transverse and longitudinal part as follows [15–17]:

$$D^{\mu\nu} = \frac{-iT^{\mu\nu}}{p_{D^*}^2 - m_{D^*}^2 + i\text{Im}\Pi_T} + \frac{iL^{\mu\nu}}{m_{D^*}^2 - i\text{Im}\Pi_L}, \quad (4)$$

with the corresponding projectors

$$T^{\mu\nu} \equiv g^{\mu\nu} - \frac{p_{D^*}^\mu p_{D^*}^\nu}{p_{D^*}^2} \quad \text{and} \quad L^{\mu\nu} \equiv \frac{p_{D^*}^\mu p_{D^*}^\nu}{p_{D^*}^2}.$$

Here, the transversal correction is proportional to the full decay width, $\text{Im}\Pi_T = \sqrt{p_{D^*}^2} \Gamma_{D^*}(p_{D^*}^2)$, while the longitudinal function $\text{Im}\Pi_L$ is proportional to the $D - \pi$ mass difference, as will be discussed below.

Since $\Gamma_{D^*} \equiv \Gamma_{D^*}(m_{D^*}^2)$ is relatively small, the relevant contribution to the transversal term is just around $p_{D^*}^2 = m_{D^*}^2$, and a narrow width approximation can be used. This allows us to rewrite the transversal part of the squared amplitude as

$$|M_T|^2 = M_{3\mu} M_{3\alpha}^* T^{\mu\nu} T^{\alpha\beta} M_{2\nu} M_{2\beta}^* \frac{\pi\delta(p_{D^*}^2 - m_{D^*}^2)}{m_{D^*}\Gamma_{D^*}}. \quad (5)$$

We note that the delta function forces the transversal part of the D^* to remain on shell, thus recovering the same structure from (3), with global factors in (5) compensated by the splitting of the phase space in (3). Therefore, taking only the transversal part of the D^* propagator is equivalent to working with the on-shell scheme that is often used.

On the other hand, the longitudinal part of the propagator is not restricted to be around $p_{D^*}^2 = m_{D^*}^2$ and gives place to two new terms in the squared amplitude (one purely longitudinal and one of interference) that cannot be accounted for in the $B \rightarrow l\nu_l D^*$ process.

The two-body decay amplitude is $M_{2\nu} = -ig(p_D - p_\pi)_\nu$, where p_D and p_π are the momenta of the D and the π , respectively, and g is the $D^* - D - \pi$ coupling. Thus, the longitudinal part of the amplitude can be written as

$$M_L = igM_{3\mu} \frac{p_{D^*}^\mu}{p_{D^*}^2} \frac{m_D^2 - m_\pi^2}{m_{D^*}^2 - i\text{Im}\Pi_L}. \quad (6)$$

Hence, the longitudinal corrections are modulated by a dimensionless mass-difference parameter $\Delta^2 \equiv (m_D^2 - m_\pi^2)/m_{D^*}^2$. This is a known result for any vector meson that decays into two pseudoscalars, and Δ^2 is usually invoked as a suppression factor. However, due to the large mass difference between the D and π mesons, here it happens to be a relatively large value of $\Delta^2 = 0.86$. Thus, the longitudinal corrections that are missing in the $B \rightarrow l\nu_l D^*$ case may carry an important weight.

The three-body decay amplitude can be written as

$$M_3^\mu = \frac{G_F}{\sqrt{2}} V_{cb} J^{\lambda\mu} l_\lambda, \quad (7)$$

where $l_\lambda \equiv \bar{u}_l \gamma_\lambda (1 - \gamma^5) v_\nu$ is the leptonic current, V_{cb} is the Cabibbo-Kobayashi-Maskawa matrix element, and $J^{\lambda\mu}$ is the hadronic matrix element with the polarization vector of the D^* factored out. The hadronic matrix element can be parametrized in terms of four form factors [18,19] (see Appendix). The parameters of such form factors have been obtained from a heavy quark analysis of B^0 decays with electron and muon products measured by the Belle Collaboration [13].

In calculating the new decay widths, both the transversal and interference parts of the squared amplitudes have been integrated using the narrow width approximation (as the transversal part forces the main contribution from the interference to come from its corresponding dominant

part), while the longitudinal part can be integrated without any restriction in the full four-body phase space. In order to compare with the experimental determination, we restricted D and π momenta to fulfil $(p_D + p_\pi)^2 = (m_{D^*} \pm \delta)^2$, where δ is in the range $\Gamma_{D^*}/2$ to 1 MeV. Our results for $R_{D\pi}$ are the same in this range, as the small pure longitudinal correction is well below the current precision. The kinematics is taken as given in [20] and implemented with the Vegas subroutine. We have found that the interference term makes a slight distinction between the $l = e$ and $l = \mu$ cases. Thus, we quote our final result separately as

$$R_{D\pi}^e = 0.274 \pm 0.003$$

and

$$R_{D\pi}^\mu = 0.275 \pm 0.003,$$

where the uncertainty comes from the uncertainties on the measurement of the form factors (V , A_0 , A_1 , and A_2) that characterize the hadronic vertex between the B , D^* , and W , for which we have used results published by Belle [13] in agreement with world average measurements [11]. Notice that A_0 is not independent, but derived from A_2 as discussed in the Appendix. Thus, an anticorrelation on the form factors is present, which brings the uncertainties lower.

In Table I, we show the contribution to the branching ratio from the transversal, longitudinal ($\delta = \Gamma_{D^*}$), and interference parts of the amplitude for all three lepton flavor products, which are consistent with Belle measurements for the electron and muon [13]. Within parentheses we quote the errors coming from the uncertainties in the form factor parameters and V_{cb} [13]. We note that the pure longitudinal contribution to the branching ratio is the same for the light leptons. An early estimation of the contribution of the D^* longitudinal polarization to the $B \rightarrow l\nu D^*$ process [12] quotes a value for the pure longitudinal rate of 0.115(2), which is in agreement with our result of 0.111(3) after integration in the full four-body phase space. In the last two rows, we show $R_{D\pi}$ for the electron and the

TABLE I. Contribution to the branching ratio of the transversal, longitudinal ($\delta = \Gamma_{D^*}$), and interference parts of the amplitude for all three lepton flavors. Quantities are given in percentage. The last two rows show the value of $R_{D\pi}$ as each contribution is added subsequently from left to right for e and μ .

	Transversal	Longitudinal	Interference
Electron	4.6(3)	$5.0(3) \times 10^{-6}$	$7.6(6) \times 10^{-8}$
Muon	4.6(3)	$5.0(3) \times 10^{-6}$	$1.6(1) \times 10^{-3}$
Tau	1.16(8)	$1.1(6) \times 10^{-6}$	$1.02(7) \times 10^{-1}$
$R_{D\pi}^e$	0.252	0.252	0.274
$R_{D\pi}^\mu$	0.252	0.252	0.275

muon as each part is added, namely, transversal, longitudinal, and interference parts. Notice that, due to the cancellation of global factors in the ratio, $R_{D\pi}$ has a much higher precision than the individual branching ratios.

On the other hand, the relative size of the interferences turns out to increase with the lepton mass, being negligible for the electron and muon at the current precision, while for the tau it becomes on the order of 10% compared to the transversal contribution. An earlier estimate [14] got a negligible effect from the interference that can be traced back to the propagator they used, which differs from ours (in the limit of $\text{Im}\Pi_L = 0$) by a term proportional to $im_{D^*}\Gamma_{D^*}$ in the longitudinal part. The interference, upon integration in the narrow width approximation, leaves this term as the only not null contribution. Notice that this imaginary term makes a real contribution as both the leptonic tensor ($l^{\lambda\theta} = \sum l^\lambda l^{\dagger\theta}$) and the $B - D^* - W$ vertex (see Appendix) carry also an imaginary term. This is the case, e.g., when considering the contraction with the Levi-Civita from the leptonic tensor $\sum_{\text{pol}} l^\mu l^{\nu\dagger}$ and the term proportional to A_1 in the hadronic matrix element. We may interpret the interference as connecting the leptonic and hadronic parts mostly through the chirality of the lepton, which knows about its mass. Thus, the corrections in the hadronic part are modulated by the corresponding leptonic flavor, making in this case, the heavier the lepton, the larger the contribution.

III. DISCUSSION

We have argued that the experimental information for R_{D^*} must be compared with the theoretical $R_{D\pi}$, since the experimental information relies on the reconstruction of the full four-body decay process. We have shown that the longitudinal correction from the D^* propagator introduces a correction to the branching ratios, which produces a value of $R_{D\pi}^e = 0.274 \pm 0.003$ and $R_{D\pi}^\mu = 0.275 \pm 0.003$, where the uncertainty comes from the experimental measurement of the form factor parameters. Within the error bars, $R_{D\pi}$ can be considered as flavor independent. This contrasts with the value of $R_{D^*} = 0.252 \pm 0.003$ [12] often used to compare with. The two quantities are distinguishable from each other at the current precision, and therefore, the correction introduced is meaningful when comparing with the experimental result. Namely, by comparing with $R_{D\pi}$ instead, we find that the difference with the latest results from LHCb [7,8] goes down from 1.1σ to 0.48σ , while the difference with the latest Belle results [9,10] goes down from 0.42σ to just 0.10σ , and the difference with the world average results [11] goes down from 3.7σ to 2.1σ . In all cases, the agreement with the experiments is improved, but there still remains some tension with the world average results that cannot be explained by the longitudinal corrections alone.

In order to exhibit the role of the form factors, in Fig. 3 we show the contribution of each of them to the three-body differential decay width for the case of the tau (upper panel) and electron (lower panel). Interferences are not shown.

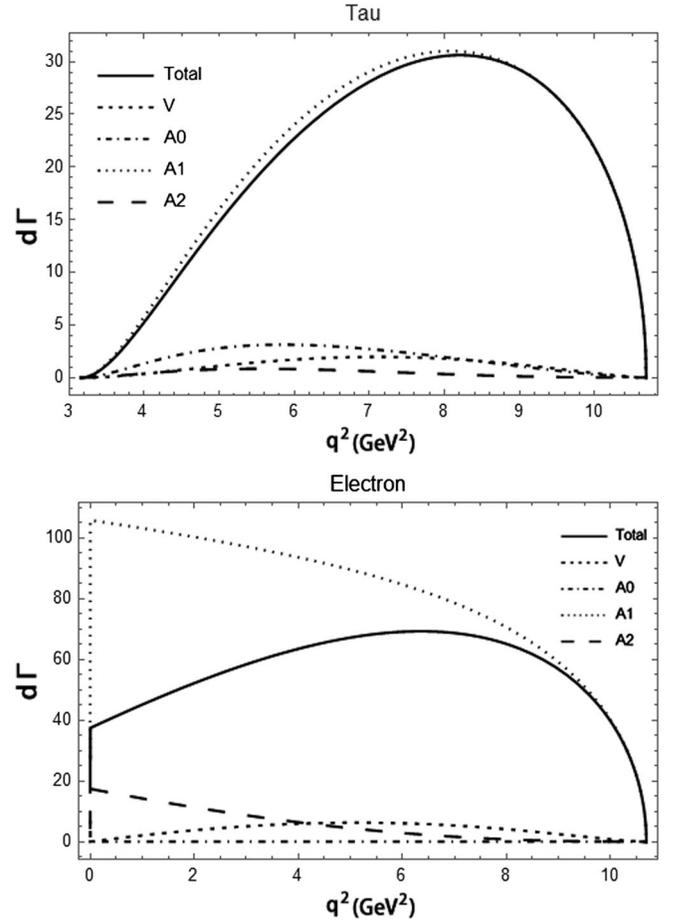


FIG. 3. Contribution to the three-body differential decay width of each form factor as a function of q^2 , for the tau and the electron. For the electron, the A_0 term is nearly zero. Interference terms between form factors are not shown.

The vectorlike contribution A_1 is the dominant in each case, while the other contributions compete among themselves. Lattice calculations have provided information on this form factor at zero recoil, which is consistent with the current experimental information [21]. The A_0 form factor accounts for the longitudinal projector for the transferred momentum q , corresponding to states with helicity zero for the lepton-neutrino system. Because such a state is forbidden in the zero-mass limit for both the lepton and the neutrino, the term is heavily suppressed for both the electron and the muon, but not the tau. Measurements of the vertex have only been done for electric and muonic flavors [11,13], where it is undetectable. Instead, A_0 is obtained from A_2 by invoking an approximate relation derived from heavy quark effective theory [12,18,22] and used as such in the tau system. The drastically different behavior that A_0 has between flavors might be important for understanding the experimental results. Thus, it is important to measure the form factors and, in particular, A_0 using tau decay channels, to shed light on the nature of possible deviations from the SM.

Additional scalar resonances like the $D_0^*(2400)^0$ and longitudinal effects of the vector resonance together can be seen as background contributions. They might be removed from experimental data by subtracting a Monte Carlo generated sample of the full scalar contribution [17]. In that case, the comparison of the experimental result is returned to be done with the three-body transversal part, originally considered. Additional tests of the lepton universality that are independent of the form factors will be useful to decide whether there is or not a deviation from the standard model [23].

It has also been suggested that other observables defined in terms of the longitudinal contribution may be useful to search for new physics signals [24]. Early calculations on the second-order corrections on the form factors [25] and radiative corrections in the pseudoscalar and neutral vector meson [26–28] systems have shown that they are expected to be important at the few percent level in the branching ratios. Thus, upcoming improvements on the experimental side will require one to account for them in the theoretical prediction.

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APPENDIX: HADRONIC VERTEX

For the three-body decay, the hadronic vertex connecting the B , D^* , and W is characterized by four form factors, as follows [12]:

$$\begin{aligned} & \langle D^*(p_{D^*}, \epsilon_\mu) | J^\lambda | B(p_B) \rangle \\ &= \frac{2iV(q^2)}{m_B + m_{D^*}} \epsilon^{\lambda\mu\alpha\beta} \epsilon_\mu^*(p_B)_\alpha (p_{D^*})_\beta - 2m_{D^*} A_0(q^2) \frac{q^\lambda q \cdot \epsilon^*}{q^2} \\ & - (m_B + m_{D^*}) A_1(q^2) \left(\epsilon^{*\lambda} - \frac{q^\lambda q \cdot \epsilon^*}{q^2} \right) \\ & + \frac{A_2(q^2) q \cdot \epsilon^*}{m_B + m_{D^*}} \left((p_B + p_{D^*})^\lambda - \frac{m_B^2 - m_{D^*}^2}{q^2} q^\lambda \right), \end{aligned}$$

where J^λ is the weak current, ϵ_μ^* is the polarization vector of the D^* , and $q = p_B - p_{D^*}$ is the transferred momentum. These terms constitute the most general double-indexed Lorentz structure that can be constructed with the available variables, for an on shell D^* . For the four-body decay, as an approximation, we have used the same hadronic vertex and replaced the polarization vector by a free index to contract with the hadronic part associated with the D^* decay. In this approximation, terms proportional to $p_{D^*}^\mu$ that can coupled to the longitudinal part of the D^* propagator are missing. Since the form factors have been derived from Belle data [13] without including these terms, a new analysis should be necessary. Additional longitudinal terms in the $D^* - D - \pi$ vertex have not been considered either. A full four-body description would require one to consider the most general structure for the four-body decay, similar to K_{14} as discussed in [29], where the form factors were computed in the chiral approach. It would be useful to have them computed at the heavy quark limit and isolate the D^* contribution from other possible resonances that may be part of the internal process.

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