Local scale-invariance breaking in the standard model by two-measure theory

Eduardo I. Guendelman*
Department of Physics, Ben Gurion University of the Negev, Beer Sheva 84105, Israel

Hitoshi Nishino† and Subhash Rajpoot‡
Department of Physics & Astronomy, California State University, 1250 Bellflower Boulevard, Long Beach, California 90840, USA

(Received 2 May 2018; published 18 September 2018)

We introduce Weyl's scale invariance as additional local symmetry in the standard model of electroweak interactions. Under this, the gauge symmetry of the standard model now is $SU(3)\times SU(2)\times U(1)\times \tilde{U}(1)$, where $\tilde{U}(1)$ is for *local* scale invariance, and its gauge boson is called the Weylon. Also introduced are two new scalars σ_1 and σ_2 with the common scaling weight -1. The mechanism for spontaneous breaking of scale invariance is invoked by coupling σ_2 to a metric-independent measure defined in terms of an additional four scalars $\phi^I (I = 1,2,3,4)$. Weyl's scale invariance is now implemented by combining it with internal diffeomorphisms of the four scalars ϕ^i. We show that once local scale invariance is broken, the phenomenon (a) generates Newton's gravitational constant G_N and (b) triggers spontaneous symmetry breaking in the conventional manner resulting in masses for the conventional fermions and bosons. The scale at which Weyl's scale symmetry breaks is of order Planck mass. If right-handed neutrinos are also introduced, their absence at present energy scales is attributed to their mass being tied to the scale at which scale invariance breaks. New *C*- and *CP*-violating effects can also be induced by mixing the Weylon with the hypercharge gauge boson of the standard model.

DOI: 10.1103/PhysRevD.98.055022

I. INTRODUCTION

The notion that the standard model [1] is the underlying theory of elementary particle interactions is without a doubt the prevailing consensus supported by all experiments of the present time. Unfortunately, gravitational interactions elude us. Unlike the standard-model interactions [1], gravitational interactions simply refuse to partake in the successes of quantum field theory and the gauge principle. Despite this, interesting and useful models incorporating gravity can be constructed that may serve, it is hoped, as forerunners to the future correct theory of quantum gravity. One such model emerges when one addresses the issue of particle masses in the standard model. Although the wide disparity in the particle masses provides no clue to any underlying symmetry, scale-invariance symmetry, albeit badly broken, can serve as a guiding principle.

We consider extending the standard model with local scale invariance *à la* Weyl [2,3], the doomed symmetry that gave birth to the gauge principle and ultimately paved the way for implementing gauge invariance as is known and practiced today. A glance at the elementary particle mass spectrum attests to the fact that scale invariance is a badly broken symmetry of nature. As will be shown, in the absence of fine-tuning, the scale at which the scale-invariance symmetry breaks turns out to be of order Planck mass $M_P \approx 1.3\times 10^{19}$ GeV. The extended model predicts the existence of an additional vector particle we will call the Weylon [4–6]. Its mass is tied to the scale at which Weyl's symmetry breaks and in the absence of fine-tuning, is also of order M_P.

Under scale invariance, the parallel transport of a vector around a closed loop in four-dimensional space-time not only changes its direction but also its length, while the angle between two parallel transported vectors around a closed loop remains the same. The fundamental metric tensor $g_{\mu\nu}$ transforms as

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = e^{2\Lambda(x)} g_{\mu\nu}, \tag{1.1}$$

where $\Lambda(x)$ is the parameter of scale transformations. The four-dimensional volume element transforms as

$$d^4x\sqrt{-g} \to e^{4\Lambda(x)}\, d^4x\sqrt{-g}. \tag{1.2}$$

*guendel@bgu.ac.il
†h.nishino@csulb.edu
‡subhash.rajpoot@csulb.edu

Since

$$e_\mu{}^m e_{\nu m} = g_{\mu\nu}, \qquad e_m{}^\mu e_{\mu n} = \eta_{mn},$$
$$(\eta_{mn}) = \mathrm{diag}(1,-1,-1,-1), \tag{1.3}$$

it follows that the transformation properties of $e_\mu{}^m$ and its inverse $e_m{}^\mu$ under Weyl's symmetry are

$$e_\mu{}^m \to e^{\Lambda(x)} e_\mu{}^m, \qquad e_m{}^\mu \to e^{-\Lambda(x)} e_m{}^\mu. \tag{1.4}$$

II. THE MODEL

We extend the standard model of particle interactions to include Weyl's scale invariance as a local symmetry. The electroweak symmetry $SU(2)\times U(1)$ is extended to

$$G^* = SU(2)\times U(1)\times \tilde{U}(1), \tag{2.1}$$

where $\tilde{U}(1)$ represents the local noncompact Abelian symmetry associated with Weyl's scale invariance, and the asterisk on G signifies that the full symmetry respects local scale invariance. The additional particles introduced are the vector boson S_μ associated with $\tilde{U}(1)$ and two real scalar fields σ_1 and σ_2 [7–10] that transform as singlets under G^*. The distinct feature of the new symmetry is that under it, fields transform with a real phase, whereas under the $SU(2)\times U(1)$, symmetries, fields transform with complex phases.

Under $\tilde{U}(1)$, a generic matter field $\Psi^g (g=1,2,3)$ in the action is taken to transform as $e^{w\Lambda(x)}$ with a scaling weight w. Thus, under $G = SU(2)\times U(1)\times \tilde{U}(1)$, the transformation properties of the entire particle content of the extended model are the following.

The e family $(g=1)$,

$$\begin{aligned}
&\Psi_L^{1q} = \begin{pmatrix} u \\ d \end{pmatrix} \sim \left(2, \frac{1}{3}, -\frac{3}{2}\right), \qquad \Psi_L^{1l} = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \sim \left(2, -1, -\frac{3}{2}\right),\\
&\Psi_{1R}^{1q} = u_R \sim \left(1, \frac{4}{3}, -\frac{3}{2}\right), \qquad \Psi_{2R}^{1q} = d_R \sim \left(1, -\frac{2}{3}, -\frac{3}{2}\right),\\
&\Psi_{2R}^{1l} = e_R \sim \left(1, -2, -\frac{3}{2}\right).
\end{aligned} \tag{2.2}$$

The μ family $(g=2)$,

$$\begin{aligned}
&\Psi_L^{2q} = \begin{pmatrix} c \\ s \end{pmatrix} \sim \left(2, \frac{1}{3}, -\frac{3}{2}\right), \qquad \Psi_L^{2l} = \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \sim \left(2, -1, -\frac{3}{2}\right),\\
&\Psi_{1R}^{2q} = c_R \sim \left(1, \frac{4}{3}, -\frac{3}{2}\right), \qquad \Psi_{2R}^{2q} = s_R \sim \left(1, -\frac{2}{3}, -\frac{3}{2}\right),\\
&\Psi_{2R}^{2l} = \mu_R \sim \left(1, -2, -\frac{3}{2}\right).
\end{aligned} \tag{2.3}$$

The τ family $(g=3)$,

$$\begin{aligned}
&\Psi_L^{3q} = \begin{pmatrix} t \\ b \end{pmatrix} \sim \left(2, \frac{1}{3}, -\frac{3}{2}\right), \qquad \Psi_L^{3l} = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \sim \left(2, -1, -\frac{3}{2}\right),\\
&\Psi_{1R}^{3q} = t_R \sim \left(1, \frac{4}{3}, -\frac{3}{2}\right), \qquad \Psi_{2R}^{3q} = b_R \sim \left(1, -\frac{2}{3}, -\frac{3}{2}\right),\\
&\Psi_{2R}^{3l} = \tau_R \sim \left(1, -2, -\frac{3}{2}\right).
\end{aligned} \tag{2.4}$$

All of these fermions have the same scaling weight $w = -3/2$. The scalar bosons sector comprises the usual Higgs doublet φ and the two new real scalars σ_1 and σ_2,

$$\varphi \sim (2,-1,-1), \qquad \sigma_1 \sim (1,0,-1), \qquad \sigma_2 \sim (1,0,-1), \tag{2.5}$$

all transforming with the common scaling weight $w = -1$. The gauge potential fields W_μ, B_μ, and S_μ are, respectively, for the $SU(2)$, $U(1)$, and $\tilde{U}(1)$ symmetries. The gauge fields W_μ and B_μ have zero scaling weight: $W_\mu \to W_\mu$ and $B_\mu \to B_\mu$. The field strengths associated with the gauge potentials W_μ, B_μ, and S_μ are $W_{\mu\nu}$, $B_{\mu\nu}$, and $S_{\mu\nu}$, and carry zero scaling weights. We have suppressed the $SU(3)$ of strong interactions, as neglecting it will not affect our results and conclusions. The electroweak symmetry action I_0 of the model is

$$\begin{aligned} I_0 = \int d^4x\sqrt{-g}\Bigg[&-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}(W_{\mu\nu}W_{\rho\sigma}+B_{\mu\nu}B_{\rho\sigma}+U_{\mu\nu}U_{\rho\sigma}) \\ &+\sum_{\substack{f=q,l\\ g=1,2,3\\ i=1,2}}(\bar{\Psi}^{gf}_L e_m{}^{\mu}\gamma^m D_\mu \Psi^{gf}_L + \bar{\Psi}^{gf}_{iR} e_m{}^{\mu}\gamma^m D_\mu \Psi^{gf}_{iR}) \\ &+g^{\mu\nu}(D_\mu\varphi)^\dagger(D_\nu\varphi)+\frac{1}{2}g^{\mu\nu}(D_\mu\sigma_1)(D_\nu\sigma_1) \\ &+\frac{1}{2}g^{\mu\nu}(D_\mu\sigma_2)(D_\nu\sigma_2) \\ &+\sum_{\substack{f=q,l\\ g,g'=1,2,3\\ i=1,2}}(\mathbf{Y}^{\mathrm{f}}_{gg'}\bar{\Psi}^{gf}_L\varphi\Psi^{gf}_{iR}+\mathbf{Y}'^{f}_{gg'}\bar{\Psi}^{gf}_L\tilde{\varphi}\Psi^{g'f}_{iR})+\mathrm{H.c.} \\ &-\frac{1}{2}(\beta\varphi^\dagger\varphi+\zeta_1\sigma_1^2+\zeta_2\sigma_2^2)\tilde{R}+V(\varphi,\sigma_1,\sigma_2)\Bigg], \end{aligned} \tag{2.6}$$

where $\tilde{\varphi}\equiv i\sigma_2\varphi^*$ [here, σ_2 is one of the $SU(2)$ Pauli matrices, not to be confused with the $\sigma_{1,2}$ scalar fields], the indices (g,g') are for generations, the indices $f=(q,l)$ refer to (quark, lepton) fields, $\mathbf{Y}^f_{gg'}$ or $\mathbf{Y}'^f_{gg'}$ are quark, lepton Yukawa couplings that define the mass matrices after symmetry breaking, the index $i=1$, 2, 3 is needed for right-handed fermions, while β, ζ_1, and ζ_2 are dimensionless couplings, and $\tilde{R}$ is the scalar curvature in the 4D space respecting scale invariance. The various D's acting on the fields represent the covariant derivatives constructed in the usual manner using the principle of minimal substitution. Explicitly,

$$\begin{aligned} D_\mu\Psi^{gf}_L &= \left(\partial_\mu + ig\tau\cdot W_\mu + \frac{i}{2}g'Y^{gf}_L B_\mu \right. \\ &\quad \left. -\frac{3}{2}fS_\mu - \frac{1}{2}\tilde{\omega}_\mu{}^{mn}\sigma_{mn}\right)\Psi^{gf}_L, \\ D_\mu\Psi^{gf}_{iR} &= \left(\partial_\mu + \frac{i}{2}g'Y^{gf}_{iR}B_\mu - \frac{3}{2}fS_\mu - \frac{1}{2}\tilde{\omega}_\mu{}^{mn}\sigma_{mn}\right)\Psi^{gf}_{iR}, \\ D_\mu\varphi &= \left(\partial_\mu + ig\tau\cdot W_\mu - \frac{i}{2}g'B_\mu - fS_\mu\right)\varphi, \\ D_\mu\sigma_1 &= (\partial_\mu - fS_\mu)\sigma_1, \qquad D_\mu\sigma_2 = (\partial_\mu - fS_\mu)\sigma_2. \end{aligned} \tag{2.7}$$

The Y^{gf}_L's, Y^{gf}_{iR}'s represent the hypercharge quantum numbers (e.g., $f=q$, $g=1$, $i=1$, $Y^{1q}_L=1/3$, $Y^{1q}_{1R}=4/3$, etc.), g, g', f are the respective gauge couplings of $SU(2)$, $U(1)$, and $\tilde{U}(1)$, while

$$U_{\mu\nu}\equiv\partial_\mu S_\nu - \partial_\nu S_\mu \tag{2.8}$$

is the field strength associated with Weyl's $\tilde{U}(1)$. It is gauge invariant, since S_μ transforms as

$$S_\mu \to S_\mu - \frac{1}{f}\partial_\mu\Lambda. \tag{2.9}$$

The spin connection $\tilde{\omega}_\mu{}^{mn}$ [11] is defined in terms of the vierbein $e_\mu{}^m$,

$$\begin{aligned} \tilde{\omega}_{mrs} &\equiv \frac{1}{2}(\tilde{C}_{mrs}-\tilde{C}_{msr}+\tilde{C}_{srm}), \\ \tilde{C}_{\mu\nu}{}^r &\equiv (\partial_\mu e_\nu{}^r + fS_\mu e_\nu{}^r) - (\partial_\nu e_\mu{}^r + fS_\nu e_\mu{}^r), \end{aligned} \tag{2.10}$$

while the affine connection $\tilde{\Gamma}^\alpha{}_{\mu\nu}$ is defined by

$$\begin{aligned} \tilde{\Gamma}^\rho{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}[&(\partial_\mu+2fS_\mu)g_{\nu\sigma}+(\partial_\nu+2fS_\nu)g_{\mu\sigma} \\ &-(\partial_\sigma+2fS_\sigma)g_{\mu\nu}]. \end{aligned} \tag{2.11}$$

The Riemann curvature tensor $\tilde{R}^\rho{}_{\sigma\mu\nu}$ is

$$\tilde{R}^\rho{}_{\sigma\mu\nu} = \partial_\mu\tilde{\Gamma}^\rho{}_{\nu\sigma} - \partial_\nu\tilde{\Gamma}^\rho{}_{\mu\sigma} - \tilde{\Gamma}^\lambda{}_{\mu\sigma}\tilde{\Gamma}^\rho{}_{\nu\lambda} + \tilde{\Gamma}^\lambda{}_{\nu\sigma}\tilde{\Gamma}^\rho{}_{\mu\lambda}, \tag{2.12}$$

where $\tilde{\Gamma}^\rho{}_{\mu\nu}$, $\tilde{R}^\rho{}_{\sigma\mu\nu}$, and the Ricci tensor $\tilde{R}^\rho{}_{\mu\rho\nu}=\tilde{R}_{\mu\nu}$ have scaling weight $w=0$, while the scalar curvature $\tilde{R}=g^{\mu\nu}\tilde{R}_{\mu\nu}$ has the form

$$\begin{aligned} \tilde{R} &= R - 6fD_\mu S^\mu + 6f^2 S_\mu S^\mu, \\ D_\kappa S^\mu &= \partial_\kappa S^\mu + \tilde{\Gamma}^\mu{}_{\kappa\nu}S^\nu, \end{aligned} \tag{2.13}$$

and transforms with scaling weight $w=-2$. The potential is given by

$$\begin{aligned} V(\phi,\sigma_1,\sigma_2) = &+\lambda(\phi^\dagger\phi)^2 + \frac{\lambda_1}{4}\sigma_1^4 + \lambda_2\sigma_2^4 \\ &-(\phi^\dagger\phi)(\mu_1\sigma_1^2+\mu_2\sigma_2^2)+\frac{\lambda_3}{2}\sigma_1^2\sigma_2^2 \\ &+a\sigma_1\sigma_2^3 + b\sigma_2\sigma_1^3 + 2c\phi^\dagger\phi\sigma_1\sigma_2, \end{aligned} \tag{2.14}$$

where λ, μ_i, ξ_i, a, b, and c are dimensionless couplings. It is interesting to note that the scalar potential in this model consists of quartic terms only as required by Weyl's scale invariance.

In order to keep things simple, and without loss of generality, we can eliminate the terms with constants a, b, and c either by hand [method (i)] or by imposing discrete symmetry [method (ii)] as follows:

(i) The constants a, b, and c as taken to be small and, hence, are neglected in the analysis.
(ii) Under a discrete symmetry $\sigma_2 \leftrightarrow -\sigma_2$, the last three terms in (2.14) are excluded.

Henceforth, the last three terms in (2.14) will be neglected in the foregoing.

III. SYMMETRY BREAKING

In the primary stage of symmetry breaking, scale-invariance symmetry will be broken spontaneously. This can be achieved in various ways. One way is to break it through quantum corrections via the Coleman-Weinberg mechanism. Another method is to break it explicitly by hand [12] by taking the vacuum expectation value (VEV) $\langle\sigma_2\rangle = \Delta$. This method does not need the second singlet scalar and has already been entertained previously by two of us in the literature [4].[1] Yet another method is to implement the concept of two-measure theory (TMT). In TMT, in addition to the usual metric-dependent measure $\sqrt{-g}$ where $g = \det(g_{\mu\nu})$, one introduces a second measure Φ which is taken to be independent of the metric. The second metric can be inserted in the total action in a variety of ways that depend on the physics being addressed. TMTs can also accommodate both global scale invariance and local scale invariance. This is because under either a global scale-invariance transformation or a local scale-invariance transformation, a measure independent of the metric can have scaling different from that of $\sqrt{-g}$. In one example of a TMT, a globally scale-invariance theory is formulated such that ΦR is globally scale invariant, where R is the invariant scalar curvature [13]. In this case, a dilaton field ϕ with suitable exponential potentials is coupled in scale-invariant ways to $\sqrt{-g}$ and Φ, and provides the possibility of a very small vacuum energy. Additionally, such a TMT scenario coupled with a seesawlike mechanism provides a cosmological seesaw mechanism [14], thus, allowing for both inflation and a small vacuum energy in the late Universe [15]. The TMT approach has also been used for providing the breaking of global scale invariance in a standard-model extension [12], in string and brane models with spontaneously and/or dynamically induced tension [16], and in supersymmetric extended objects on curved backgrounds [17]. In yet another approach, the Gauss-Bonnet surface term is incorporated in TMT to provide a model for small vacuum energy density [18].

In the following, we will follow the TMT approach and break the local scale symmetry by considering the standard model as a TMT extension as follows. Implementing spontaneous symmetry breaking (SSB) of scale invariance is going to be achieved by introducing an additional term to the action, that although scale invariant, can induce the SSB of scale invariance as we will see. The new term is a coupling of the σ_2 field to a metric-independent measure Φ defined in terms of four scalar fields $\phi^I (I = 1, 2, 3, 4)$ as

$$\Phi = \varepsilon_{IJKL}\varepsilon^{\mu\nu\rho\sigma}(\partial_\mu\phi^I)(\partial_\nu\phi^J)(\partial_\rho\phi^K)(\partial_\sigma\phi^L). \qquad (3.1)$$

[1] In [4], the σ scalar serves as the σ_1 scalar, and there is no σ_2 scalar.

In the above, ε_{IJKL} and $\varepsilon^{\mu\nu\rho\sigma}$ represent permutation symbols in internal space and coordinate space, and the former has the same values in any coordinate frame.

Modified measure theories have at their disposal many types of measures of integration in the action. One way that we adapt here is to use, e.g., the standard Riemannian integration measure $\sqrt{-g}$ and the above metric-independent measure Φ:

$$S = \int d^4x\,\Phi\,L_1 + \int d^4x\sqrt{-g}\,L_0. \qquad (3.2)$$

The second part has been defined by (2.6), while L_1 is taken to be $L_1 \equiv K\sigma_2^n$. Here, K is an arbitrary real nonzero constant, whose value is *not* important, since it can be absorbed by a rescaling of the measure fields ϕ^I. The n is a nonzero real number but otherwise arbitrary. We include only σ_2, but *not* σ_1, because the latter is to be absorbed as an additional degree of freedom for the massive Weylon S_μ after the SSB of local scale invariance. For this reason, and to keep matters simple, we will treat the σ_2 field as auxiliary and nonpropagating. Hence, the kinetic term for the σ_2 field is dropped in the Lagrangian. We now discuss the scaling symmetry properties of the fields ϕ^I and how these transformations must be correlated to the transformations of the other fields, in particular, how it correlates to the transformation of the σ_2 field.

Following the way in which conformal invariance is implemented in string theory formulated with a modified measure [16], we consider an internal diffeomorphism in the space of the scalar fields $\phi^I (I = 1, 2, 3, 4)$,

$$\tilde{\phi}^I = \tilde{\phi}^I(\phi^J). \qquad (3.3)$$

Under this internal diffeomorphism, the measure Φ scales according to the Jacobian $\mathcal{J}$ of this transformation:

$$\tilde{\Phi} = \mathcal{J}\Phi, \qquad \mathcal{J} \equiv \det\left(\frac{\partial\tilde{\phi}^I}{\partial\phi^J}\right), \qquad (3.4)$$

so that in order to have that $\Phi L_1 = K\Phi\sigma^n$ be invariant, we require that

$$\mathcal{J} \equiv e^{n\Lambda(x)}. \qquad (3.5)$$

Since the general diffeomorphism (3.4) is x dependent, so is the parameter $\Lambda(x)$ in (3.5).

We can take the viewpoint that the internal diffeomorphism defining $\mathcal{J}$ is our starting point. Accordingly, we define the transformation of the other fields by defining $e^{\Lambda(x)}$ through the above equation.

Let us now turn to the field equations that are obtained from the variation of the measure fields $\phi^I (I = 1, 2, 3, 4)$. These are determined to be

$$A^{\mu I}\partial_\mu\sigma_2^n = 0, \qquad (3.6)$$

where

$$A^{\mu I} = \varepsilon_{IJKL}\varepsilon^{\mu\nu\rho\sigma}(\partial_\nu\phi^J)(\partial_\rho\phi^K)(\partial_\sigma\phi^L). \tag{3.7}$$

It is easy to see that the determinant of ${A_\mu}^I$ is proportional to Φ^3, so if this measure is nonzero, we get that

$$\langle\sigma_2(x)\rangle = \Delta, \tag{3.8}$$

where Δ is a constant of integration and serves as the scale for the SSB of the scale symmetry associated with Weyl's $\tilde{U}(1)$. Notice that this result has been obtained from the integration of the field equations and is conceptually different from the conventional SSB mechanism. The σ_2-field equation determines only $\chi \equiv \Phi/\sqrt{-g} = \text{const}$ because

$$\frac{\delta S}{\delta\sigma_2} = nK\sigma_2^{n-1}\Phi + \sqrt{-g}\left[\left(\frac{\delta V}{\delta\sigma_2}\right) - \zeta_2\sigma_2\tilde{R}\right] = 0$$

$$\Rightarrow \chi \equiv \frac{\Phi}{\sqrt{-g}} = -\frac{1}{nK\sigma_2^{n-1}}\left[\left(\frac{\delta V}{\delta\sigma_2}\right) - \zeta_2\sigma_2\tilde{R}\right] \tag{3.9a}$$

$$= -\frac{1}{nK\sigma_2^{n-1}}\left(\frac{\delta V}{\delta\sigma_2}\right) = \text{const}, \tag{3.9b}$$

where we used the trace of the gravitational field equation

$$\tilde{R} = -2Y^{-1}(D_\mu\varphi)^\dagger(D^\mu\varphi) - Y^{-1}(D_\mu\sigma_1)^2 + 3Y^{-1}D_\mu^2Y + (\text{fermionic and } S_\mu \text{ terms}) = 0, \tag{3.10a}$$

$$Y \equiv +\beta\varphi^\dagger\varphi + \zeta_1\sigma_1^2 + \zeta_2\sigma_2^2. \tag{3.10b}$$

In (3.10a), we skipped the terms where the S_μ field is *explicitly* involved other than minimal couplings, together with fermionic terms. In (3.10a) and (3.9b), we ignored all terms with scalars with derivatives, such as $D_\mu\sigma_1$, etc., and terms with explicit S_μ. The vanishing of the VEV of these terms is justified under the reasonable assumptions $\langle\varphi\rangle = \langle\sigma_1\rangle = \langle\sigma_2\rangle = \text{const}$ and $\langle S_\mu\rangle = 0$.

It is also interesting to notice that the field equations and the action, up to a total derivative, are invariant under a shift of the measure fields by an arbitrary function of σ, $\phi^I \to \phi^I + f^I(\sigma)$. In the primary stage of symmetry breaking, only scale invariance is spontaneously broken. With $\langle\sigma_2\rangle = \Delta$, the Lagrangian is no longer invariant under scale transformations, but still local gauge invariance is respected. This intermediate symmetry without scale invariance is represented by G. Thus, in the primary stage of symmetry breaking,

$$G^* \xrightarrow{\Delta} G. \tag{3.11}$$

After σ_2 gets frozen by (3.8), the potential takes the following form:

$$V(\varphi,\sigma_1,\sigma_2) \to V(\phi,\sigma_1,\Delta) = \lambda(\phi^\dagger\phi)^2 + \frac{\lambda_1}{4}\sigma_1^4 + \lambda_2\Delta^4 - (\phi^\dagger\phi)(\mu_1\sigma_1^2 + \mu_2\Delta^2) + \frac{\mu_3}{2}\sigma_1^2\Delta^2. \tag{3.12}$$

All the conventional particles are still massless at this stage. Note that the gauge symmetry still is $G = SU(2)\times U(1)\times \tilde{U}(1)$ and signifies the symmetry of the model in which scale invariance is no longer intact.

In the intermediate stage of symmetry breaking, local scale invariance is broken. This is achieved by the VEV of the σ_1 field, $\langle\sigma_1\rangle = \tilde{\Delta}$. The σ_1 field becomes the Goldstone boson and is absorbed by the Weylon to become massive.

After the intermediate stage of symmetry breaking, the potential takes the form

$$V(\phi,\sigma_1,\Delta) = \lambda(\phi^\dagger\phi)^2 + \frac{\lambda_1}{4}\tilde{\Delta}^4 + \lambda_2\Delta^4 - (\phi^\dagger\phi)(\mu_1\tilde{\Delta}^2 + \mu_2\Delta^2) + \frac{\lambda_3}{2}\tilde{\Delta}^2\Delta^2. \tag{3.13}$$

It is to be noted that this form of the potential, apart from the vacuum energy density term contributing to the cosmological constant, is of the same form as the standard Higgs potential if we identify μ of the standard model with the effective $\mu = \sqrt{(\mu_1\tilde{\Delta}^2 + \mu_2\Delta^2)}$ of our model. While the μ of the standard model has dimension of mass, the μ_1 and μ_2 of the present model are dimensionless, acquiring mass dimensions from $\tilde{\Delta}^2$ and Δ^2, respectively.

In the final stage of symmetry breaking, the electroweak gauge symmetry is broken to $U(1)$ of electromagnetism by the VEV of the doublet η in the conventional way. Thus, from the minimization of the potential, the relevant VEVs in terms of the parameters of the potential are $\tilde{\Delta}$ and $\langle\phi\rangle = \frac{1}{\sqrt{2}}\binom{\eta}{0}$, where

$$\langle\phi^\dagger\phi\rangle = \eta^2 = \frac{(\lambda_1\mu_2 - \mu_1\lambda_3)\Delta^2}{\lambda\lambda_1 - \mu_1^2},$$
$$\langle\sigma_1^2\rangle = \tilde{\Delta}^2 = \frac{(\lambda\lambda_3 - \mu_1\mu_2)\Delta^2}{\lambda\lambda_1 - \mu_1^2}, \tag{3.14}$$

where η is the electroweak symmetry breaking scale. From weak interactions phenomenology, $\eta = 246$ GeV. We take $\tilde{\Delta} \gg \eta$. In this case, the descent of G to $U(1)_{\text{em}}$ follows the hierarchy

$$G \xrightarrow{\tilde{\Delta}} SU(2)\times U(1) \xrightarrow{\eta} U(1)_{\text{em}}. \tag{3.15}$$

IV. DISCUSSION AND CONCLUDING REMARKS

The breaking of scale symmetry determines Newton's gravitational constant G_N:

$$\zeta_2\Delta^2 + \zeta_1\tilde{\Delta}^2 = M_P^2 = \frac{1}{8\pi G_N}. \qquad (4.1)$$

As one representative scenario, we take $\xi_2\Delta^2 = \zeta_1\tilde{\Delta}^2 = \zeta\Delta^2$. Thus, $\Delta \approx 0.2 \times M_P/\sqrt{\zeta}$, and barring any fine-tuning $\Delta \approx \mathcal{O}(M_P)$,[2] if we take $\zeta \approx \mathcal{O}(1)$.[3] With G_N defined, it is appropriate to work in the weak-field approximation. Henceforth, we set $\sqrt{g}g_{\mu\nu} \approx \eta_{\mu\nu} + \mathcal{O}(\kappa)$ where $\kappa^2 = 8\pi G_N$. In the intermediate stage of the SSB as depicted above, the Weylon absorbs the Goldstone field, which is primarily σ_1, and becomes massive. The Weylon mass is given by

$$M_S = \sqrt{\frac{3f^2}{4\pi G_N}}. \qquad (4.2)$$

The Weylon mass receives an additional small contribution from the intermediate stage of symmetry breaking and its mass gets shifted. Explicitly,

$$M_S \to \sqrt{\frac{3f^2}{4\pi G_N}}\sqrt{1 + \frac{\beta\eta^2}{\zeta_1\tilde{\Delta}^2}}. \qquad (4.3)$$

However, the additional contribution is negligibly small as $\eta^2/\tilde{\Delta}^2 \approx 10^{-33}$. Apart from being superheavy, another distinct property of the Weylon is that it completely decouples from the fermions and the bosons of the standard model. In deriving this result, we are assuming that $\tilde{\Delta} \approx \Delta \approx \mathcal{O}(M_P)$, a reasonable assumption if fine-tuning is to be avoided. With this assumption, the constraint on the parameters defining η and $\tilde{\Delta}$ are

$$\frac{(\lambda_1\mu_2 - \mu_1\lambda_3)}{\lambda\lambda_1 - \mu_1^2} \approx 10^{-32}, \quad \frac{(\lambda\lambda_3 - \mu_1\mu_2)}{\lambda\lambda_1 - \mu_1^2} \approx 1. \qquad (4.4)$$

After symmetry breaking, the constant term in the potential is vacuum energy density and serves as the cosmological constant Λ_0,

$$V_{\text{vacuum}} = \frac{\lambda_2}{\zeta_2^2}M_P^4\left(1 + \frac{\lambda_3}{2\lambda_2}\frac{\tilde{\Delta}^2}{\Delta^2} + \frac{\lambda_1}{4\lambda_2}\frac{\tilde{\Delta}^4}{\Delta^4}\right). \qquad (4.5)$$

From the present-day constraints on the vacuum energy density of the Universe 10^{-3} eV^4, the various parameters entering $V_{\text{vacuum}} = \Lambda_0^4$ are required to satisfy

$$\frac{\lambda_2}{\zeta_2^2}\left(1 + \frac{\lambda_3}{2\lambda_2}\frac{\tilde{\Delta}^2}{\Delta^2} + \frac{\lambda_1}{4\lambda_2}\frac{\tilde{\Delta}^4}{\Delta^4}\right) \leq 10^{-122}. \qquad (4.6)$$

After SSB, the conventional particles acquire masses as in the standard model,

$$M_W = \frac{1}{2}g\eta, \qquad M_Z = \frac{M_W}{\cos\theta_W}, \quad \mathbf{M}^f_{gg'} = \frac{1}{\sqrt{2}}\mathbf{Y}^f_{gg'}\eta, \qquad \mathbf{M}'^f_{gg'} = \frac{1}{\sqrt{2}}\mathbf{Y}'^f_{gg'}\eta, \qquad (4.7)$$

where θ_W is the weak angle and $\mathbf{M}^f_{gg'}$, $\mathbf{M}'^f_{gg'}$ are the quark $(f = q)$ and the charged lepton $(f = l)$ mass matrices. At this stage, neutrinos are still massless. In this model, there is still left over the conventional Higgs particle h_0. The mass of the Higgs particle is given by

$$m_{\text{Higgs}} = \sqrt{2(\mu_1\tilde{\Delta}^2 + \mu_2\Delta^2)}. \qquad (4.8)$$

Since $m_{\text{Higgs}} = 125$ GeV and $\eta = 246$ GeV, the value of the quartic coupling λ is determined to be $\lambda = 0.125$, which is consistent with the bounds derived in [19].

However, although the standard model is a renormalizable theory [20,21], the present model is not. This puts into doubt the validity of the unitarity constraint derived in the renormalizable standard model and extrapolated to the nonrenormalizable extended model considered here.

At the present time, one fundamental issue is that of neutrino masses and their lightness as compared to the masses of other particles. In the standard model and the model under consideration, neutrinos are strictly massless as no right-handed neutral lepton fields were introduced. A popular extension of the standard model that addresses this issue in an aesthetically appealing way introduces right-handed neutrinos $\Psi^{1l}_{1R} = \nu_{eR}$, $\Psi^{2l}_{1R} = \nu_{\mu R}$, $\Psi^{3l}_{1R} = \nu_{\tau R}$ that lead to seesaw masses [22] for the conventional neutrinos. This scenario is usually entertained in the $SO(10)$ grand unified theory, where the right-handed neutrinos acquire superheavy masses. The superheavy scale is determined by the stage at which the internal symmetry $SO(10)$ breaks and has nothing to do with gravitational interactions. If right-handed neutrino fields are also introduced in the present model, the seesaw mechanism can naturally be accommodated due to the presence of the singlet field σ_1. Because of the imposed discrete symmetry, the other singlet field σ_2 does not couple to fermions. The relevant interaction Lagrangian is

[2]We are using the *reduced* Planck mass $M_P \equiv 1/\sqrt{8\pi G_N} = 2.45 \times 10^{18}$ GeV. The original Planck mass as introduced by Planck is $M_P^{(\text{o})} \equiv 1/\sqrt{G_N} = 1.22 \times 10^{19}$ GeV.

[3]Note that Δ and $\boldsymbol{\Delta}$ represent distinct quantities.

$$L_\nu = \sum_{\substack{g,g'=1,2,3 \\ i=1}} \left(\mathbf{Y}^l_{gg'} \bar{\Psi}^{gl}_L \varphi \Psi^{g'l}_{iR} + \text{H.c.} + \frac{1}{\sqrt{2}} \mathbf{Y}^{RR}_{gg'} \sigma^{gl}_{1R}{}^T C \sigma_1 \Psi^{g'l}_{1R} \right). \tag{4.9}$$

The lepton number is explicitly broken by the last term. Scale breaking gives superheavy Majorana masses to the right-handed neutrinos, and SSB subsequently gives Dirac masses that connect the left- and right-handed neutrinos leading to the following familiar 6×6 mass matrix

$$\mathbf{M}_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{0} & \mathbf{Y}^l_{gg'} \eta \\ \mathbf{Y}^l_{g'g} \eta & \mathbf{Y}^{RR}_{gg'} \tilde{\Delta} \end{pmatrix}, \tag{4.10}$$

the eigenvalues of which are three seesaw masses for the light neutrinos and three heavy neutrinos with enough parameters to fit the observed solar and atmospheric neutrino oscillation phenomena. In the present model, the scale of right-handed neutrino masses is tied to the scale Δ associated with Weyl's $\tilde{U}(1)$ breaking, which, in turn, is tied to Newton's constant G_N. This is unlike the GUT scenario where right-handed neutrino masses are tied to the GUT scale at which the grand unification internal symmetry breaks. Thus, the absence of right-handed neutrinos from the low energy scales is attributed to their superheavy masses of $\mathcal{O}(M_P)$ and may be interpreted as an indication that right-handed neutrinos (and also gauge-mediated right-handed currents) and gravitational interactions may ultimately be related.

In the standard model, physical fields and the couplings like electric charge $e = 1/\sqrt{g^{-2} + (g')^{-2}}$ and Fermi constant $G_F = g^2/(8M_W^2)$ get defined *after* SSB. Similarly, in the present model, not only e and G_F, but also G_N get defined *after* symmetry breaking, thus, conforming to the main theme in physics that all phenomena observed in nature are symmetry breaking effects.

There are three kinds of potential anomalies to consider in our model. These are gauge anomalies [23,24], gravitational anomalies [25,26], and Weyl (scale) anomalies [27–29]. Our model is seen to be free of all these anomalies, as follows. The gauge (triangle) anomalies are absent because the anomalies due to the $U(1)$ hypercharge cancel between the quarks and the leptons as *no* new (exotic) fermions are introduced in our model. The Weylon (gauge boson of scale invariance) does not couple to fermions. This follows from the terms in the covariant derivative acting on the fermion fields. The contribution due to the Weylon-dependent spin connection $\tilde{\omega}_\mu{}^{rs}$ exactly cancels the contribution due to the scale-invariance gauge symmetry $(-3f/2)S_\mu$ in the covariant derivative. In fact, from (2.7) and (2.10), we get

$$\gamma^\mu \left(-\frac{3}{2} f S_\mu + \frac{1}{4} \tilde{\omega}_\mu{}^{rs} \Big|_S \gamma_{rs} \right) = \gamma^\mu \left(-\frac{3}{2} f S_\mu + \frac{1}{2} f S_\nu \gamma_\mu{}^\nu \right) = 0. \tag{4.11}$$

Hence, there are *no* triangle anomalies to consider in our model. The Weyon also does *not* couple to the conventional gauge bosons of the standard model. The gravitational anomalies also cancel since the trace of the $U(1)$ hypercharge over the quarks and leptons of our model vanishes. Our model is also free of Weyl anomaly. According to the work of Coriano *et al.* [30], there are three conditions under which the Weyl anomaly appears. One, if the Higgs boson is a composite, second, if there is an interaction term $\phi^2 R$ with a coefficient equal to $-1/6$, and third, if the term responsible for the Weyl anomaly is added by hand. We note that in our model, first, the scalar particles are not composite. Second, we add a term $\xi\phi^2 R$ with an arbitrary coefficient ξ (not equal to $-1/6$), and third, if dimensional regularization is used, then the Weyl anomaly term added by hand is rendered harmless. Thus, in our model we circumvent *all* the three conditions responsible for the Weyl anomaly. It is also to be noted that while the work of Coriano *et al.* [30] pertains to *global* scale invariance, our work is fundamentally different from theirs as our work relates to *local* scale invariance in the standard-model extension. There are no Goldstone bosons in our model.

As for renormalizability of our model, we note the following points. First, in our original scale-invariant extension [4–6] of the standard model, there are limited numbers of possible counterterms [6]. This is seen from the possible scale-invariant counterterms [6]. So, the issue of renormalizability in our model [4–6] is much more softened than in general relativity, in which *infinitely many* counterterms arise. Second, TMT is an additional feature of our scale-invariant standard-model extension because of the new scalars φ^I. The particular feature is that it exhibits some unusual infinite-dimensional symmetries. This symmetry coupled with scale invariance may restrict further the number of counterterms. Since the renormalizability issue is softened for our scale-invariant standard model [4–6], it is reasonable to expect even softer quantum behavior of our model with TMT compared with general relativity.

We have been so far using the Jordan frame for TMT formalism. Some readers may wonder why we need to use the Jordan frame instead of the Einstein frame. It is because the difference might well result in a difference in cosmology. Our standpoint is as follows: When one moves from the Jordan frame to the Einstein frame, physics must *not* change. It is, however, sometimes advantageous to choose a particular frame in order to elucidate the relevant physics. A classic example is using different gauges in the conventional standard model. The masses of the particles are usually discussed in the unitary gauge (physical gauge). In this gauge, the particle masses represent the true (physical) masses of the particles. On the other hand, the renormalizability of the model is more transparent in the renormalizable gauge. In the renormalizable gauge, particle masses are gauge dependent and, hence, unphysical. Similarly, symmetry breaking is more transparent in the Jordan frame, while cosmology is more transparent in the Einstein frame.

As for the possible origin of our model in (super)string models or M theory [31], we have *no* concrete example to offer as yet. But we expect that string theories with gauge symmetries as large as $E_8 \times E_8$ or $SO(32)$ [32] or their originations come from M theory [31], and other candidates based on gauge symmetries as large as $SO(44)$ [33] with an unconventional Lorentzian metric are rich in their scalar and fermion content and are likely to contain all the necessary ingredients of the standard-model extension we have presented here. Their dimensional compactification to our model in light of broken scale invariance and TMT will undoubtedly require further restrictions on the modes of compactification. Also, the possible connection, if any, of the dilaton of string theory to the scalars of our model needs to be understood. It is possible that the dilaton of string theory may eventually turn out to be a linear combination of the scalars of our model. But it seems this is highly unlikely due to the following remarks.

First, the so-called "dilatons" in string theory are with *global* scale symmetry represented typically by the *constant* shift in the dilaton field. Second, in our formalism, our scale symmetry is realized as *local* symmetry with the Weylon, which does *not* necessarily have origins in string theory. Third, at the present time, even before talking about string theory, we do *not* have a supergravity theory consistent with *local* scale invariance with the Weylon.[4] Therefore, any possible relationship between our formalism and string theory seems obscure. As we have no concrete example to offer, we relegate this task to a future study of the issue and publication.

Finally, we note that additional terms involving the Weylon can be added to the action. The first one is

$$\int d^4x \left(-\frac{1}{4} \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} B_{\mu\nu} S_{\rho\sigma} \right). \tag{4.12}$$

This term mixes the weak hypercharge with the Weylon, and it is constrained by neutral current phenomenology. The neutral current couplings of the conventional fermions are well established, and the data can only tolerate deviations of less than 1%. Thus, this mixing is small. The second term that can be added is

$$\int d^4x\, \epsilon^{\mu\nu\rho\sigma} S_{\mu\nu} S_{\rho\sigma}. \tag{4.13}$$

This term is C and CP violating. However, it is topological in nature and does not affect the tree-level field equations.

To conclude, we have accommodated Weyl's scale invariance as local symmetry in the standard electroweak model. This inevitably leads to the introduction of general relativity. The additional particles are a vector particle we call the Weylon and a real scalar singlet that couples to the scalar curvature $\tilde{R}$ *à la* Dirac. The scale at which Weyl's scale invariance breaks defines Newton's gravitational constant G_N. Weyl's vector particle, i.e., the Weylon, absorbs the scalar singlet σ and acquires mass $\mathcal{O}(M_P)$ in the absence of fine-tuning. The scalar potential is unique in the sense that it consists of terms only quartic in the scalar fields and dimensionless couplings. Yet, as we have demonstrated, symmetry breaking is possible such that the leftover symmetry is $U(1)_{\rm em}$, and all particle masses are consistent with present-day phenomenology. If right-handed neutrinos are also introduced, the light neutrinos acquire seesaw masses, and the suppression factor in the neutrino masses is of $\mathcal{O}(M_P)$.

ACKNOWLEDGMENTS

We gratefully acknowledge the referee of this paper for constructive suggestions. S. R. was partially supported by a sabbatical leave from CSULB and acknowledges hospitality at Ben Gurion University, Beer Sheva, Israel, where a part of the work was carried out. E. G. gratefully acknowledges the hospitality at CSULB.

[4]Even though we presented in [34] the gauging of the dilaton-shift symmetry coupled to supergravity, such a symmetry is *not* the same as the scale symmetry in our present paper. For example, our fermions transform under local scale symmetry, while those in [34] do *not*.

[1] S. L. Glashow, Nucl. Phys. **B22**, 579 (1961); S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967); A. Salam, in *Elementary Particle Physics*, Nobel Symposium No. 8, edited by N. Svartholm (Almqvist & Wiksell, Stockholm, 1968), p. 367.

[2] H. Weyl, Sitzungsber. Preuss Akad. Wiss. Berlin 465 (1918); Mathematische Zeitschrift **2**, 384 (1918); Ann. Phys. (N.Y.) **59**, 101 (1919); H. Weyl, *Raum, Zeit, Materie, Vierte Erweiterte Auflage* (Springer, New York, 1921).

[3] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, England, 1922).

[4] H. Nishino and S. Rajpoot, arXiv:hep-th/0403039.

[5] H. Nishino and S. Rajpoot, Phys. Rev. D **79**, 125025 (2009).

[6] H. Nishino and S. Rajpoot, Classical Quantum Gravity **28**, 145014 (2011).

[7] P. A. M. Dirac, Proc. R. Soc. A **333**, 403 (1973).

[8] R. Utayama, Prog. Theor. Phys. **50**, 2080 (1973).

[9] D. K. Sen and K. A. Dunn, J. Math. Phys. (N.Y.) **12**, 578 (1971).
[10] Y. M. Cho, Phys. Rev. Lett. **68**, 3133 (1992).
[11] V. Fock, Zeitschrift für Physik **57**, 261 (1929).
[12] E. Guendelman, H. Nishino, and S. Rajpoot, Phys. Lett. B **765**, 251 (2017).
[13] E. I. Guendelman and A. B. Kaganovich, Phys. Rev. D **60**, 065004 (1999).
[14] E. I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999).
[15] E. I. Guendelman and O. Katz, Classical Quantum Gravity **20**, 1715 (2003).
[16] E. I. Guendelman, A. Kaganovich, E. Nissimov, and S. Pacheva, Phys. Rev. D **66**, 046003 (2002).
[17] H. Nishino and S. Rajpoot, Phys. Lett. B **736**, 350 (2014).
[18] E. Guendelman, H. Nishino, and S. Rajpoot, Eur. Phys. J. C **77**, 240 (2017).
[19] See, e.g., K. G. Chetyrkin and M. F. Zollerb, arXiv: 1604.00853.
[20] G. 't Hooft and M. Veltman, Nucl. Phys. **B44**, 189 (1972); **B50**, 318 (1972).
[21] B. W. Lee and J. Zinn-Justin, Phys. Rev. D **5**, 3121 (1972); **5**, 3121 (1972); **5**, 3137 (1972); **5**, 3155 (1972).
[22] M. Gell-Mann, P. Ramond, and R. Slansky, ; T. Yanagida, KEK Report No. 79-18, 1979, p. 95.
[23] S. L. Adler, Phys. Rev. **177**, 2426 (1969).
[24] J. S. Bell and R. Jackiw, Nuovo Cimento **60**, 47 (1969).
[25] R. Delbourgo and A. Salam, Phys. Lett. **40B**, 381 (1972).
[26] T. Eguchi and P. G. O. Freund, Phys. Rev. Lett. **37**, 1251 (1976).
[27] D. M. Capper and M. J. Duff, Nuovo Cimento A **23A**, 173 (1974); Nucl. Phys. **B82**, 147 (1974).
[28] M. J. Duff, Nucl. Phys. **B125**, 334 (1977).
[29] M. J. Duff, Classical Quantum Gravity **11**, 1387 (1994).
[30] C. Coriano, L. Delle Rose, A. Quintavalle, and M. Serino, J. High Energy Phys. 06 (2013) 077.
[31] C. Hull and P. K. Townsend, Nucl. Phys. **B438**, 109 (1995); E. Witten, Nucl. Phys. **B443**, 85 (1995); P. K. Townsend, arXiv:hep-th/9612121; arXiv:hep-th/9712004; T. Banks, W. Fischler, S. H. Shenker, and L. Susskind, Phys. Rev. D **55**, 5112 (1997).
[32] B. Green and J. H. Schwarz, Phys. Lett. **151B**, 21 (1985); D. J. Gross, J. A. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985).
[33] K. S. Narain, Phys. Lett. **169B**, 41 (1986).
[34] H. Nishino and S. Rajpoot, Phys. Rev. D **78**, 125006 (2008).