

## Coincident general relativity


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The metric-affine variational principle is applied to generate teleparallel and symmetric teleparallel theories of gravity. From the latter we discover an exceptional class which is consistent with a vanishing affine connection. Based on this remarkable property, this work proposes a simpler geometrical formulation of general relativity that is oblivious to the affine spacetime structure, thus fundamentally depriving gravity of any inertial character. The resulting theory is described by the Hilbert action purged from the boundary term and is more robustly underpinned by the spin-2 field theory, where an extra symmetry is now manifest, possibly related to the double-copy structure of the gravity amplitudes. This construction also provides a novel starting point for modified gravity theories, and the paper presents new and simple generalizations where analytical self-accelerating cosmological solutions arise naturally in the early- and late-time Universe.

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### I. INTRODUCTION

In the conventional geometrical interpretation of general relativity (GR), gravitation is described by the curvature

$$R^\alpha{}_{\beta\mu\nu} \equiv 2\partial_{[\mu}\Gamma^\alpha{}_{\nu]\beta} + 2\Gamma^\alpha{}_{[\mu|\lambda}\Gamma^\lambda{}_{\nu]\beta}, \quad (1)$$

where the affine connection  $\Gamma^\alpha{}_{\mu\nu} = \{\alpha{}_{\beta\gamma}\}$  is the metric-compatible and torsionless Levi-Civita connection that is computed from the metric  $g_{\mu\nu}$  (with mostly plus Lorentzian signature) as

$$\{\alpha{}_{\beta\gamma}\} \equiv \frac{1}{2}g^{\alpha\lambda}(g_{\beta\lambda,\gamma} + g_{\lambda\gamma,\beta} - g_{\beta\gamma,\lambda}). \quad (2)$$

Despite its undeniable observational success, this description comes hand in hand with the inherent conceptual and technical difficulties of working in a pseudo-Riemannian spacetime so that it is desirable to attain a simpler formulation of the theory. A step forward in this direction is taken in the teleparallel reformulation [1,2], where the geometry is simplified by the constraint  $R^\alpha{}_{\beta\mu\nu} = 0$ , which reduces the connection to the Weitzenböck form. In this work we will pursue further improvements in the simplifying sequence of geometrical frameworks, and formulate GR in a flat and torsion-free spacetime. Furthermore, we

will introduce a class of its generalizations in the trivially connected geometry, i.e., with  $\Gamma^\alpha{}_{\mu\nu} = 0$ . The standard vierbein formalism of GR has been reinterpreted in terms of nonlinear realizations of the group  $GL(4, \mathbb{R})$  [3], and teleparallelized in the metric-affine gauge theory (see Refs. [4,5], and Sec. 5.9 in Ref. [6]), but it can still be clarified whether it is natural to stipulate a trivial geometry since there, as we shall discover, the inertial connection is a translation.<sup>1</sup>

We start by recalling that the general affine connection admits the decomposition [8]

$$\Gamma^\alpha{}_{\mu\nu} = \{\alpha{}_{\mu\nu}\} + K^\alpha{}_{\mu\nu} + L^\alpha{}_{\mu\nu}, \quad (3)$$

which includes the *contortion*

$$K^\alpha{}_{\mu\nu} \equiv \frac{1}{2}T^\alpha{}_{\mu\nu} + T_{(\mu}{}^\alpha{}_{\nu)}, \quad (4)$$

due to torsion  $T^\alpha{}_{\mu\nu} \equiv 2\Gamma^\alpha{}_{[\mu\nu]}$ , and the *disformation*

$$L^\alpha{}_{\mu\nu} \equiv \frac{1}{2}Q^\alpha{}_{\mu\nu} - Q_{(\mu}{}^\alpha{}_{\nu)}, \quad (5)$$

<sup>1</sup>Compelling arguments to regard gravitation as the gauge theory of translations were presented e.g., in Ref. [7], wherein also the field-theoretical approach as an alternative to the conventional geometrical approach to gravitation was emphasized. Note that in the Weitzenböck teleparallelism [1,2], gravitation is still geometry, and despite the motivation as a translation gauge theory, the gauge connection is a (pseudo)rotation.

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due to nonmetricity  $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$ . The various tensor fields we have already introduced satisfy important identities, an example being the Bianchi identity [8]

$$R^\mu_{[\alpha\beta\gamma]} - \nabla_{[\alpha} T^\mu_{\beta\gamma]} + T^\nu_{[\alpha\beta} T^\mu_{\gamma]\nu} = 0, \quad (6)$$

which we shall need later. In the metric-affine formalism, a theory can now be defined by a scalar action of the form

$$S = \int d^n x \sqrt{-g} f(g^{\mu\nu}, R^\alpha_{\beta\mu\nu}, T^\alpha_{\mu\nu}, Q_\alpha^{\mu\nu}) + S_M, \quad (7)$$

where the independent variables are the metric and the affine connection, plus the matter fields contained in  $S_M$ . The sources introduced by the latter are the energy-momentum tensor and the hypermomentum tensor density defined as

$$\mathfrak{T}_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}}, \quad \mathfrak{S}^{\lambda\mu\nu} = -\frac{1}{2} \frac{\delta S_M}{\delta \Gamma^\lambda_{\mu\nu}}, \quad (8)$$

respectively.

## II. THE NEW GR

Before proceeding to the central result of this work, it will be interesting to rederive the so-called New GR [9] covariantly in the metric-affine formalism. If we introduce the *superpotential*

$$S_\alpha^{\mu\nu} \equiv a T_\alpha^{\mu\nu} + b T^{[\mu}_\alpha{}^{\nu]} + c \delta_\alpha^{[\mu} T^{\nu]}, \quad (9)$$

then the general even-parity quadratic theory can be defined in terms of the three-parameter quadratic form  $\mathbb{T} \equiv \frac{1}{2} S_\alpha^{\mu\nu} T^\alpha_{\mu\nu}$ . Thus, instead of imposing *a priori* the flatness and metricity conditions on the connection, we set up the gravitational action as

$$S = \int d^n x \left[ \frac{1}{2} \sqrt{-g} \mathbb{T} + \lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \lambda^\alpha_{\mu\nu} \nabla_\alpha g^{\mu\nu} \right], \quad (10)$$

where the connection is left completely arbitrary and the Lagrange multipliers that impose the teleparallelism constraints are introduced as tensor densities with the obvious symmetries  $\lambda_\alpha^{\mu\nu} = \lambda_\alpha^{(\mu\nu)}$  and  $\lambda_\alpha^{\nu\mu\rho} = \lambda_\alpha^{\nu[\mu\rho]}$ . We first compute the field equations by varying the action (10) with respect to the metric, yielding

$$\begin{aligned} T_{\alpha\beta(\mu} S^{\alpha\beta}_{\nu)} - \frac{1}{2} T_{[\mu|\alpha\beta|} S_{\nu]}^{\alpha\beta} - \frac{1}{2} g_{\mu\nu} \mathbb{T} \\ = \mathfrak{T}_{\mu\nu} + \frac{2}{\sqrt{-g}} (\nabla_\alpha + T_\alpha) \lambda^\alpha_{\mu\nu}, \end{aligned} \quad (11)$$

while the variations with respect to the connection result in

$$(\nabla_\rho + T_\rho) \lambda_\alpha^{\nu\mu\rho} + \frac{1}{2} T^\mu_{\rho\sigma} \lambda_\alpha^{\nu\rho\sigma} = \Delta_\alpha^{\mu\nu}, \quad (12)$$

where we have defined the source term

$$\Delta_\alpha^{\mu\nu} \equiv \mathfrak{S}_\alpha^{\mu\nu} + \frac{1}{2} \sqrt{-g} S_\alpha^{\nu\mu} - \lambda^{\mu\nu}_\alpha, \quad (13)$$

and used the symmetries of  $\lambda_\alpha^{\nu\mu\rho}$  and  $\lambda_\alpha^{\mu\nu}$ . Now the field equations (11) seem to have no dynamics for the torsion. However, we still need to solve for the constraints imposed by the Lagrange multipliers. Using the constraint  $R^\alpha_{\beta\mu\nu} = 0$  and the antisymmetry of the corresponding Lagrange multiplier, we can derive the divergence of the source term as

$$\frac{1}{2} \nabla_\mu T^\mu_{\rho\sigma} \lambda_\alpha^{\nu\rho\sigma} - \nabla_\mu T_\rho \lambda_\alpha^{\nu\rho\mu} - T_\rho \nabla_\mu \lambda_\alpha^{\nu\rho\mu} = \nabla_\mu \Delta_\alpha^{\mu\nu}. \quad (14)$$

Then we can use Eq. (12) to replace  $\nabla_\mu \lambda_\alpha^{\nu\rho\mu}$ , and yet take advantage of the Bianchi identity (6) to verify that

$$(\nabla_\mu + T_\mu) \Delta_\alpha^{\mu\nu} = 0. \quad (15)$$

We can then plug this result into Eq. (11) to rewrite them as

$$\begin{aligned} \mathfrak{T}_{\mu\nu} + \frac{2}{\sqrt{-g}} (\nabla_\alpha + T_\alpha) \mathfrak{S}_\mu^\alpha{}_\nu \\ = -\mathcal{D}_\alpha S_{\mu\nu}{}^\alpha - S^{\alpha\beta}{}_\nu (T_{\alpha\mu\beta} + K_{\alpha\beta\mu}) - \frac{1}{2} g_{\mu\nu} \mathbb{T}, \end{aligned} \quad (16)$$

where we have referred to the familiar Levi-Civita covariant derivative  $\mathcal{D}_\alpha$  that comes with the Christoffel symbols (2). Note that we have removed the symmetrization from Eq. (16) as a more compact manner of writing all the equations: the ten symmetric components correspond to the metric field equations and, in general, the equation also has six antisymmetric components which are nothing but the equations (15). In the standard prescription for standard matter fields, the hypermomentum decouples from the symmetric equations. The teleparallel equivalent of GR (TEGR) is reproduced by the choice of parameters  $a = \frac{1}{4}$ ,  $b = \frac{1}{2}$ ,  $c = -1$  and  $n = 4$ .

We have now seen that this theory and its generalizations can be formulated without introducing the additional structure of the frame bundle and its corresponding extra set of indices. The connection can then be reduced to its contortion component entirely determined by the torsion, which is propagating due to the restriction to a teleparallel geometry.<sup>2</sup>

<sup>2</sup>Due to the teleparallelity constraint, the connection includes the first derivative of a general linear transformation, and the second derivatives resulted in the field equation (16) from solving the multiplier that imposes the metricity constraint. The gauge freedom of the Lagrange multipliers and the number of effective components of the multipliers and equations have been investigated previously in the gauge formalism [4,5,10].

### III. A NEWER GR

We now move to the much less explored case of symmetric teleparallelisms [11] (see also Refs. [12–15]). The physical interpretation we suggest extrapolates the successful argument of teleparallelism to its logical conclusion. As it is well known, GR cannot distinguish between gravitation and inertial effects, but by resorting to frame fields, the gravitational energy can be defined covariantly in the teleparallel approach [2,16,17]. The canonical frame is now identified by the absence of curvature and torsion, and the canonical coordinates are now identified by the absence of inertial effects. This is a physical rationale for extracting quantities of interest such as the gravitational energy and the gravitational entropy, and how to proceed with the quantization and with the unification. The aim is to establish the frame and the coordinate system wherein the canonical commutation relations can be recovered for the operators corresponding to physical observables.

The nonmetricity tensor has two independent traces, which we denote as  $Q_\mu = Q_\mu^\alpha{}_\alpha$  and  $\tilde{Q}^\mu = Q_\alpha^{\mu\alpha}$ . We can then define the quadratic nonmetricity scalar as

$$Q = -\frac{1}{4}Q_{\alpha\beta\mu}Q^{\alpha\beta\mu} + \frac{1}{2}Q_{\alpha\beta\mu}Q^{\beta\mu\alpha} + \frac{1}{4}Q_\alpha Q^\alpha - \frac{1}{2}Q_\alpha \tilde{Q}^\alpha \quad (17)$$

which is special among the general quadratic combinations because, in addition to being invariant under local general linear transformations, it is also the special quadratic combination that is invariant under a translational symmetry that allows to completely remove the connection. To clarify this remarkable property, let us introduce, in analogy with the superpotential of New GR, the following *nonmetricity conjugate*

$$P^\alpha{}_{\mu\nu} \equiv c_1 Q^\alpha{}_{\mu\nu} + c_2 Q_{(\mu}{}^\alpha{}_{\nu)} + c_3 Q^\alpha g_{\mu\nu} + c_4 \delta_{(\mu}^\alpha \tilde{Q}_{\nu)} + \frac{c_5}{2} (\tilde{Q}^\alpha g_{\mu\nu} + \delta_{(\mu}^\alpha Q_{\nu)}), \quad (18)$$

and define the general quadratic form  $\mathbb{Q} = Q_\alpha^{\mu\nu} P^\alpha{}_{\mu\nu}$ . Then, one may consider the general quadratic theory

$$\mathcal{S} = \int d^n x \left[ -\frac{1}{2} \sqrt{-g} \mathbb{Q} + \lambda_\alpha^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \lambda_\alpha^{\mu\nu} T^\alpha{}_{\mu\nu} \right]. \quad (19)$$

It is noteworthy that the five terms that go into the definition of the full conjugate can be related to the squares of the four irreducible pieces of the nonmetricity and their one possible cross term [6]. The special nonmetricity scalar given in Eq. (17) corresponds to the choice of parameters  $c_4 = 0$  and  $c_1 = -\frac{1}{2}c_2 = -c_3 = \frac{1}{2}c_5 = -\frac{1}{4}$ , i.e., in that case we have  $\mathbb{Q} = Q$  and the theory simply becomes the symmetric teleparallel equivalent of GR (STTEGR) [11–15], as we will

explicitly show below. Before that, let us give the field equations for the general case. Variations with respect to the metric lead to the equations

$$\frac{2}{\sqrt{-g}} \nabla_\alpha (\sqrt{-g} P^\alpha{}_{\mu\nu}) - q_{\mu\nu} - \mathbb{Q} g_{\mu\nu} = \mathfrak{Z}_{\mu\nu}, \quad (20)$$

where the shorthand  $q_{\mu\nu}$  stands for

$$q_{\mu\nu} = c_1 (2Q_{\alpha\beta\mu} Q^{\alpha\beta}{}_\nu - Q_{\mu\alpha\beta} Q_\nu{}^{\alpha\beta}) + c_2 Q_{\alpha\beta\mu} Q^{\beta\alpha}{}_\nu + c_3 (2Q_\alpha Q^\alpha{}_{\mu\nu} - Q_\mu Q_\nu) + c_4 \tilde{Q}_\mu \tilde{Q}_\nu + c_5 \tilde{Q}_\alpha Q^\alpha{}_{\mu\nu}. \quad (21)$$

On the other hand, the connection equations are

$$\nabla_\rho \lambda_\alpha{}^{\nu\mu\rho} + \lambda_\alpha{}^{\mu\nu} = \sqrt{-g} P^\mu{}_\alpha + \mathfrak{H}_\alpha{}^{\mu\nu}. \quad (22)$$

Notice that the Lagrange multipliers do not enter the metric field equations (20), which is a technical simplification as compared to the teleparallel geometry with torsion considered in the previous section. Thus, the gauge symmetries of the multipliers [4,5,10] in this case are irrelevant (see Ref. [18]). The vanishing curvature constraint forces the connection to be purely inertial, i.e., it differs from the trivial connection by a general linear gauge transformation, while the torsionless condition further simplifies it to take the form  $\Gamma^\alpha{}_{\mu\beta} = (\partial x^\alpha / \partial \xi^\lambda) \partial_\mu \partial_\beta \xi^\lambda$  for some arbitrary  $\xi^\lambda$ . We then arrive at the crucial result that we can completely remove the connection by means of a diffeomorphism (called a ‘‘Diff’’ hereafter) and, thus, the  $\xi^\lambda$ ’s make their appearance as the Stückelberg fields restoring this gauge symmetry. If we fix the gauge  $\Gamma^\alpha{}_{\mu\nu} = 0$  and expand the Lagrangian to quadratic order in the perturbations  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ , we obtain

$$\sqrt{-g} \mathcal{L}^{(2)} = c_1 \partial_\alpha h_{\mu\nu} \partial^\alpha h^{\mu\nu} + (c_2 + c_4) \partial_\alpha h_{\mu\nu} \partial^\mu h^{\alpha\nu} + c_3 \partial_\alpha h \partial^\alpha h + c_5 \partial_\mu h^\mu{}_\nu \partial^\nu h. \quad (23)$$

As it is well known, this Lagrangian propagates more than two d.o.f. unless a linearized Diff-symmetry is imposed.<sup>3</sup> The requirement of this Diff-symmetry thus fixes the theory, up to a degeneracy of  $c_2 + c_4$  and the overall normalization, to the case of the symmetric teleparallel equivalent of GR, i.e.,  $\mathbb{Q} = Q$ . For that choice of parameters, the trivial connection is consistent with maintaining Diff-invariance because the inertial connection drops from the nonmetricity sector of the action. An extra Diff was found to be at work, in the self-dual sector [19], making possible the double-copy structure of the gravity amplitudes [20]. Let us also notice that Eq. (23) also gives

<sup>3</sup>Actually, there is a second option where only transverse Diffs are imposed together with an additional Weyl rescaling.

directly the theoretical (absence of instabilities) and phenomenological (Newtonian limit compatible with Solar System observations) constraints of the general quadratic theory.

#### IV. THE $f(\mathcal{Q})$ COSMOLOGY

The new simple geometrical formulation of GR motivates a promising new framework for studies of modified gravity. To demonstrate this with an example, we propose the following action:

$$\mathcal{S}_G = \int d^n x \left[ \frac{1}{2} \sqrt{-g} f(\mathcal{Q}) + \lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \lambda_\alpha^{\mu\nu} T^\alpha_{\mu\nu} \right], \quad (24)$$

where the special case  $f = \mathcal{Q}$  corresponds to the Einstein action as discussed above. For cosmological applications we consider the line element  $ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$  with the expansion rate defined as  $H = \dot{a}/a$ . In the coincident gauge with  $T^\alpha_{\mu\nu} = 0$  we have  $\mathcal{Q} = 6H^2$  and the cosmological equations in the presence of a perfect fluid with energy density  $\rho$  and pressure  $p$  are, with Newton's constant  $G$  restored,

$$6f'H^2 - \frac{1}{2}f = 8\pi G\rho, \quad (25)$$

$$(12f''H^2 + f')\dot{H} = -4\pi G(\rho + p). \quad (26)$$

It is straightforward to verify that the continuity equation  $\dot{\rho} = -3H(\rho + p)$  is sustained by the Bianchi identities applied to the lhs of the above equations, which is a cross-check of the consistency of our gauge choice and ansatz for the cosmological solution.<sup>4</sup>

One immediate interesting observation is that the cosmological background evolution of the theory  $f = \mathcal{Q} + \Lambda\sqrt{\mathcal{Q}}$ , where  $\Lambda$  is some scale, cannot be distinguished from GR, but the perturbation evolution could be studied to constrain the value of  $\Lambda$ .

In order to grasp some interesting phenomenological consequences of general  $f(\mathcal{Q})$  theories, let us consider  $f(\mathcal{Q}) = \mathcal{Q} - 6\lambda M^2 (\frac{1}{6}\mathcal{Q}/M^2)^\alpha$  with  $M$  some scale and  $\lambda$  a dimensionless parameter (which we will assume to be of order 1). The modified Friedmann equation reads

$$H^2 \left[ 1 + (1 - 2\alpha)\lambda \left( \frac{H^2}{M^2} \right)^{\alpha-1} \right] = \frac{8\pi G}{3}\rho. \quad (27)$$

For  $\alpha = 1/2$  we again recover the usual GR background cosmology. We then see that for  $\alpha > 1$  the GR evolution is

<sup>4</sup>Notice that we have used the Diff gauge freedom to fix the coincident gauge and, therefore, setting the lapse to 1 is not, in principle, a permitted choice. It happens however that the  $f(\mathcal{Q})$  theories retain a time-reparametrization invariance that allows to get rid of the lapse (see Ref. [18]).

recovered for  $H^2 \ll M^2$  and the modifications appear in the early Universe whenever  $H^2 \gtrsim M^2$ . The evolution in that regime becomes  $H^2 \propto \rho^{\frac{1}{\alpha}}$  which shows a potential inflationary solution for  $\alpha$  sufficiently large. Notice that this transition is only possible if  $(1 - 2\alpha)\lambda > 0$ . Interestingly, if this is not the case the evolution can lead to a maximum value for  $H^2$  and  $\rho$ . In general we expect to have several branches of solutions and it seems a reasonable condition to choose the one matching GR at low densities/curvatures. On the other hand, if  $\alpha < 1$  we expect the modifications to become relevant in the late Universe when  $H^2 \lesssim M^2$  so that these models can provide self-accelerating solutions where the Universe transits from a matter-dominated epoch to an asymptotically de Sitter universe.

These promising cosmological scenarios deserve a more detailed analysis, which we leave, together with the perturbation analysis of general  $f(\mathcal{Q})$  models, to a future work.<sup>5</sup>

#### V. CONCLUSION: THE PURIFIED GR

We hope that the presented formulation is useful in linking some concepts of generalized spacetime geometry with experimental precision science. We demonstrated such a possibility in the field of cosmology, by sketching how a strikingly simple class of models that exists in the symmetric teleparallel geometry [11,14,15] can provide potentially viable alternatives to inflation and dark energy. In the following we show further advantages of the purified framework in some specific applications.

*Gauge theory.* In the gauge with vanishing connection, the nonmetricity scalar  $\mathcal{Q}$  can be expressed in terms of the Christoffel symbols (2) as

$$\mathcal{Q} = g^{\mu\nu} (\{^{\alpha}_{\beta\mu}\} \{^{\beta}_{\nu\alpha}\} - \{^{\alpha}_{\beta\alpha}\} \{^{\beta}_{\mu\nu}\}). \quad (28)$$

The rhs of this relation is nothing but the quadratic piece of the Hilbert action<sup>6</sup> that only differs from the Ricci scalar by a total derivative so that, written in terms of the metric, both actions are equivalent. Thus, we arrive at the remarkable result that the theory described by  $\mathcal{Q}$  is equivalent to an improved version of GR where the boundary term is absent, the connection can be fully trivialized and, thus, the inertial character of gravity as an effect of the connection has completely disappeared, which represents a much simpler

<sup>5</sup>See also Refs. [21,22] for related cosmological solutions in the framework of vector distortion, where the connection (2) was amended with three vector-field-dependent terms with the intention to parametrize the effects of generalized gauge geometry.

<sup>6</sup>Which is often known as the ‘‘Einstein-Hilbert action.’’ However, some authors [23] refer to the scalar curvature action as the Hilbert action, and we can then say that the Einstein action, i.e., the ‘‘IG action’’ given by Eq. (28), differs from the Hilbert action by a total derivative. A covariant version of the ‘‘IG action’’ has also been constructed by means of a reference connection associated to a reference metric [24].

geometrical interpretation of gravity. Furthermore, the trivial connection corresponds to the unitary gauge for the Stückelberg fields  $\xi^\lambda = x^\lambda$  so that the origins of the tangent space and the spacetime coincide. Therefore we call this theory the *coincident GR*. The covariant  $\Gamma$  action realizes gravitation as a gauge theory of translations.

*Field theory.* Another notable feature of this formulation of GR is that it exactly reproduces the resummation for a self-interacting massless spin-2 field [25] (see also Refs. [24,26–28]), unlike GR where the boundary term must be added by hand, as was pointed out in Ref. [26]. The coincident GR thus provides a more robust relation with the field theory approach to gravity. In addition, the existence of a gravitational energy-momentum tensor [2,11,16,17,23] makes this approach less ambiguous (this does not get around the Weinberg-Witten obstruction though [29]). Finally, we may recall that in the loop computations, clear technical advantages of the  $\Gamma$  action with respect to the Hilbert action are already known, and as pointed out above, our covariant realization further hints at an understanding of the double-copy structure [19,20].

*Euclidean action.* The coincident GR Lagrangian may also provide new insights into the Euclidean approach to quantum gravity. For instance, in the usual GR computation of the black hole entropy, the full contribution is given by the Gibbons-Hawking-York boundary term which, in addition, needs to be properly normalized to obtain a finite result. In contrast, the coincident GR Lagrangian requires neither a boundary term nor regularization, but instead allows to identify a set of natural coordinates<sup>7</sup> which directly give a finite value (see Ref. [18] for more details).

*Matter couplings.* In non-Riemannian geometries, ambiguities arise regarding the coupling of matter. In spacetimes

with torsion, the gravitational coupling of gauge fields such as the Maxwell field can spoil the gauge invariance of the theory. This is obviously avoided by our torsion-free  $\nabla$ . In TEGR, a further problem occurs with fermions, since they couple to the axial contorsion of the Weitzenböck connection. This is not viable, and one has to *ad hoc* invoke the usual Levi-Civita connection and proclaim that it defines the matter coupling also in TEGR. In STEGR however, this problem is completely avoided due to the property that Dirac fermions only couple to the completely antisymmetric part of the affine connection and, thus, they are oblivious to any disformation piece. The viability of minimal coupling gives further support to our claim that the coincident GR represents the translation gauge theory in the unique geometry purified of inertial effects.

Conceptually, TEGR [1,2] has offered a tensor for the gravitational energy-momentum [2,11,16,17], and currently new insights are sought into holography and entropy (see e.g., Ref. [30]). Meanwhile modified nonequivalent theories are vigorously investigated in the context of cosmology [31].

Having arrived at a simpler and possibly yet more consistent formulation of GR, we may conclude this work with an open mind for a new convention in geometry.

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<sup>7</sup>Notice that Diffs are only realized up to a boundary term in the action of coincident GR and, therefore, the value of the Euclidean action will depend on the choice of coordinates (or rather on classes of coordinates nontrivially related at infinity).

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