Inflation compactification from dynamical spacetime

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(Received 24 May 2018; published 21 August 2018)

A mechanism of inflation from higher dimensions compactification is studied. An early Universe capable of providing exponential growth for some dimensions and exponential contraction for others, giving therefore an explanation for the big size of the observed four-dimensional Universe as well as the required smallness of the extra dimensions is obtained. The mechanism is formulated in the context of dynamical spacetime theory, which produces a unified picture of dark energy, dark matter, and can also provide a bounce for the volume of the Universe. A negative vacuum energy puts an upper bound on the maximum volume, and the bounce imposes a lower bound. So in the early Universe the volume oscillates, but in each oscillation the extra dimensions contract exponentially, and the ordinary dimension expand exponentially. The dynamical spacetime theory provides a natural way to exit from the inflation compactification epoch since the scalar field that drives the vacuum energy can smoothly climb into small positive values of vacuum energy, which is the end of the inflation compactification. A semianalytic solution for a step function potential is also studied, where all of these effects are shown, especially the jump of the vacuum energy effect only on the derivative of dynamical spacetime vector field, and not the volume or its derivatives, which match smoothly.

DOI: 10.1103/PhysRevD.98.043522

I. INTRODUCTION

In many interesting models, a vacuum with a negative cosmological constant is predicted, as in superstrings, supergravity [1], etc. Here, we will present a model which uses extra dimensions and a primordial negative vacuum energy. For cosmology with higher dimensions and in the presence of a noncanonical scalar field, the dynamics is governed by a dynamical spacetime vector field, which is used as a Lagrange multiplier of an energy momentum tensor of the scalar field [2]. A non-Lagrangian approach similar to this was developed by Gao and Collaborators [3].

An interesting feature of the model [2] is that it allows for bouncing solutions. This effect in higher dimensions combined with a negative cosmological constant in the early Universe leads to the existence of an "inflationary phase" for some dimensions and a simultaneous "deflationary phase" for the remaining dimensions, since the volume of the spacetime remains constant or oscillating in the early Universe. For an approximately constant volume, some dimensions will grow exponentially, where others will shrink exponentially. This effect is obtained without invoking exotic matter or quantum effects as a similar inflation compactification scenario was discussed in [4–8].

We discuss how it may be possible to exit from this inflation-compactification era by dynamically increasing the cosmological constant, until it becomes positive and small. This is possible in our model because the scalar field can evolve towards increasing values of vacuum energy, without problems. Finally, the need for trapping the extra dimensions when they become become very small to prevent their reexpansion is studied. One could use, for example, the Casimir effect for periodic extra dimensions for this purpose.

II. THE BASIS OF THE MECHANISM

A. The geometry

For understanding the basics of the mechanism, we review the formalism of cosmology with a higher dimension as developed in [9] for a "classical Kaluza-Klein cosmology for a torus space with a cosmological constant and matter". The metric we assume is the following:

$$ds^2 = -dt^2 + R(t)^2 \frac{\Sigma dx^j dx^j}{f_D(x)^2} + R(t)^2 \frac{\Sigma dx^p dx^p}{f_d(x)^2}, \quad (1)$$

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where

$$f_D = 1 + \frac{k_D}{4}\Sigma(x^i)^2, \qquad f_d = 1 + \frac{k_d}{4}\Sigma(y^p)^2, \quad (2)$$

$R(t)$ is the scale factor for the D dimensions (x^i), and $r(t)$ is the scale factor for the other d dimensions (y^p). The k_d and k_D are the special curvatures. Their Hubble constants are defined as

$$\mathcal{H}_R = \frac{\dot{R}}{R} \qquad \mathcal{H}_r = \frac{\dot{r}}{r}. \quad (3)$$

The complete volume of the Universe is defined as

$$V = R^D r^d, \quad (4)$$

which allows us to define the "volume expansion parameter"

$$\mathcal{H} = \frac{\dot{V}}{V}. \quad (5)$$

The connection between the volume expansion parameter and the Hubble parameters, using the definition (4) is

$$\mathcal{H} = D\mathcal{H}_R + d\mathcal{H}_d. \quad (6)$$

The motivation for defining the total volume is because of the ability to write down one combination of the Einstein equation which has no dependence on the individual scale factor, only through the volume, as we will see below.

B. Einstein equations

We first consider the case of a stress energy tensor which has for every individual scale factor its own pressure: p for D dimensions and p' for d dimensions

$$T^\mu_\nu = \mathbf{diag}(\rho, -p, -p, \ldots, -p', -p', \ldots). \quad (7)$$

Using the identities from the Appendix, we can obtain the solution for the Einstein equation,

$$\frac{1}{2}D(D-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_D}{R^2}\right] + \frac{1}{2}d(d-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_d}{R^2}\right] + Dd\frac{\dot{R}}{R}\frac{\dot{r}}{r} = 8\pi\rho \quad (8)$$

$$(D-1)\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r} - d\frac{\dot{R}}{R}\frac{\dot{r}}{r} - (D-1)\left[\frac{\dot{R}^2}{R^2} + \frac{k_D}{R^2}\right] = -8\pi(\rho + p) \quad (9)$$

$$D\frac{\ddot{R}}{R} + (d-1)\frac{\ddot{r}}{r} - D\frac{\dot{R}}{R}\frac{\dot{r}}{r} - (d-1)\left[\frac{\dot{r}^2}{r^2} + \frac{k_d}{r^2}\right] = -8\pi(\rho + p'). \quad (10)$$

For simplicity, we set the special curvature for all the dimensions to zero $k_d = k_D = 0$. Under the assumption of isotropy of the pressure $p = p' = (\gamma - 1)\rho$, the relation from Eqs. (8)–(10) gives

$$D\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r} = \frac{8\pi\rho}{D+d-1}[1-(D+d)\gamma]. \quad (11)$$

The properties of densities as a function of the volume are summarized in Table I. By the definition of the volume (4), the equation could be represented as

$$\frac{\ddot{V}}{V} = \frac{D+d}{D+d-1}8\pi(\rho - p). \quad (12)$$

The notation of normalized density gives a dimensionless equation of motion. By integrating the equation and using the dimensionless density $\Omega \coloneqq \frac{\rho}{\rho_c}$, we obtain as in [9],

$$E = \frac{1}{2}\dot{V}^2 - \frac{D+d}{D+d-1}\Omega V^2, \quad (13)$$

where E is the anisotropy parameter, which is an integration constant that appears in the solution. The special feature of this equation is that it depends on the total volume and not the separate scale parameters of the individual dimensions.

Using the volume definition again (4) in the energy equation (13) together with (26), we obtain the first-order differential equations for R and r in terms of the volume solution,

$$\frac{\dot{R}}{R} = \frac{1}{(D+d)V}\left[\dot{V} + \sqrt{\frac{2Ed}{D}(D+d-1)}\right] \quad (14a)$$

$$\frac{\dot{r}}{r} = \frac{1}{(D+d)V}\left[\dot{V} - \sqrt{\frac{2ED}{d}(D+d-1)}\right]. \quad (14b)$$

From those equations, we obtain that the basic condition for the existence of solution is $E \neq 0$, because of the appearance of the square root of E. After an integration,

$$R(t) = V^{\frac{1}{D+d}}\exp\left[+\frac{1}{D+d}\sqrt{\frac{2Ed(D+d-1)}{D}}\int\frac{dt}{V}\right] \quad (15a)$$

TABLE I. The properties of densities as a function of the volume.

Name	ω	ρ dependence
stiff	1	V^{-2}
matter	0	V^{-1}
radiation	$\frac{1}{D+d}$	$V^{-\frac{D+d+1}{D+d}}$
dark energy	−1	constant

$$r(t) = V^{\frac{1}{D+d}} \exp\left[-\frac{1}{D+d}\sqrt{\frac{2ED(D+d-1)}{d}}\int\frac{dt}{V}\right]. \tag{15b}$$

Those equations could be used for obtaining the solution for every individual scale parameter of any particular dimension. After calculating the solution for the total volume from the energy equation, the equations above could give us the evolution of each scale factor. Notice that for any $E > 0$, we get an anisotropic evolution.

C. Solutions with constant equation of state

Simple examples of density dependence on the volume could be given under the assumption of a constant equation of state $\omega = \frac{p}{\rho}$. Substituting the density which the Universe contains into the energy equation, would give the density multiple by the quadratic term of the volume. That means the massless scalar field $\rho_\phi V^2$ will provide a constant term into the energy equation. However, only a ghost kinetic term of the scalar will shift the potential upwards, and a physical kinetic term of a scalar will push the potential down. The contribution for dark energy will be a parabolic term V^2 into the effective potential of the energy equation. If the dark energy has negative values, the parabola will provide a barrier, which will prevent high values of V. A positive value of dark energy will provide a nonstable effective potential, which pushes the Universe to infinity. The dark matter gives a linear term in the effective potential of the energy equation with a negative slope.

D. Kasner solution

For a complete vacuum free from density and pressure, the Kasner solution automatically follows from the basic formalism. From the energy equation (13), we get $V = \sqrt{2E}t$, which means the total volume grows linearly with the time. This case leads to a well-known Kasner vacuum solution [10], which describes an anisotropic universe without matter, with different scale factors as well. Using Eq. (15), we get the powers for the scale factors $R(t) = t^p$, $r(t) = t^q$,

$$p = \frac{1}{D+d}\left(1 + \sqrt{\frac{d(D+d+1)}{D}}\right) \tag{16}$$

$$q = \frac{1}{D+d}\left(1 - \sqrt{\frac{D(D+d+1)}{d}}\right), \tag{17}$$

which obey the Kasner conditions,

$$Dp + dq = Dp^2 + dq^2 = 1. \tag{18}$$

This solution allows us to check that our formalism recovers the well-known vacuum solutions.

III. INFLATION FROM UNIFIED DE-DM

A. Unified dark energy and dark matter solution

A suggestion for an action which produces DE-DM unification takes the form of [2],

$$\mathcal{L} = -\frac{1}{2}R + \chi_{\mu;\nu}T^{\mu\nu}_{(\phi)} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi), \tag{19}$$

where R is the Ricci scalar ($8\pi G = 1$), ϕ is a quintessential scalar field with a potential $V(\phi)$, and χ_μ is a dynamical spacetime vector field which is a Lagrange multiplier enforcing the covariant conservation law of the energy momentum tensor,

$$\nabla_\mu T^{\mu\nu}_{(\phi)} = 0. \tag{20}$$

We use the same stress energy tensor as the one postulated by Gao and colleagues [3],

$$T^{\mu\nu}_{(\phi)} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}, \tag{21}$$

which they require to be conserved without an action principle. The covariant conservation of this stress energy tensor leads to unified dark energy and dark matter for a constant potential and for interacting DE-DM, a nonconstant potential $U(\phi)$. This action produces very similar effects, but also include additional effects like bouncing, which are not obtained in [3]. The action depends on three different variables: the scalar field ϕ, the dynamical space time vector χ_μ, and the metric $g_{\mu\nu}$.

B. Equations of motion

According to this ansatz, the scalar field is just a function of time $\phi(t)$ and the dynamical vector field will have only the time component $\chi_\mu = (\chi_0, 0, 0, 0)$, where χ_0 is also just a function of time. A variation with respect to the dynamical space time vector field χ_μ will force a conservation of the original stress energy tensor, which implies

$$\ddot{\phi} + \frac{1}{2}\mathcal{H}\dot{\phi} + U'(\phi) = 0. \tag{22}$$

Notice that for a standard quintessence, the equation does not contain the factor $\frac{1}{2}$, but only the "volume expansion parameter" $\mathcal{H}$, which equals to $3H$ for the case of the isotopic expansion of $3+1$ dimensions. The second variation with respect to the scalar field ϕ gives a nonconserved current,

$$\chi^\lambda_{;\lambda}U'(\phi) - V'(\phi) = \nabla_\mu j^\mu \tag{23a}$$

$$j^\mu = \frac{1}{2}\phi_{,\nu}(\chi^{\mu;\nu} + \chi^{\nu;\mu}) + \phi^{,\mu}, \tag{23b}$$

and the derivatives of the potentials are the source of the current. For constant potentials, the source becomes zero, and we get a covariant conservation of this current. For the metric we presented above, this equation of motion takes the form,

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}(\mathcal{H}(\dot{\chi}_0 - 1) + \ddot{\chi}_0) = U'(\phi)(\dot{\chi}_0 + \mathcal{H}\chi_0) - V'(\phi). \tag{24}$$

For constant potentials, the current (23) is covariantly conserved; a feature which will be used later. The last variation, with respect to the metric, gives the stress energy tensor we know from the Einstein equation,

$$G^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) + \frac{1}{2}\chi^{\alpha;\beta}\phi_{,\alpha}\phi_{,\beta} + \chi^{\lambda}\phi_{,\lambda}U'(\phi)\right) - \frac{1}{2}\phi^{,\mu}((\chi^{\lambda}_{;\lambda} + 2)\phi^{,\nu} + \chi^{\lambda;\nu}\phi_{,\lambda} + \chi^{\lambda}\phi^{;\nu}_{;\lambda}) - \frac{1}{2}(\chi^{\lambda}\phi^{;\mu}_{;\lambda}\phi^{,\nu} + \chi^{\lambda;\mu}\phi_{,\lambda}\phi^{,\nu}). \tag{25}$$

For the stress energy tensor from Eqs. (7) and (25), the relation between the energy density and the fields is

$$\rho = \left(\dot{\chi}_0 - \frac{1}{2}\right)\dot{\phi}^2 + V(\phi), \tag{26}$$

which has no dependence on the potential $U(\phi)$ or its derivatives. Those three variations are sufficient for building a complete solution of the theory, using the energy equation (13) and the integration form of the individual scale parameters (15).

C. Constant potentials solution

In order to compute the evolution of the scalar field, we have to specify a form for the potentials. For a simplified case of constant potentials,

$$U(\phi) = C, \qquad V(\phi) = \Omega_{\Lambda}. \tag{27}$$

The solution for the variation with respect the dynamical time, which is Eq. (22) can be integrated to give

$$\dot{\phi}^2 = \frac{2\Omega_m}{V}, \tag{28}$$

where Ω_m is an integration constant which represents the effective dark matter ratio. From the second variation, with respect to the scalar field ϕ a conserved current is obtained, which from Eq. (24) gives the exact solution of the dynamical time vector field,

$$\dot{\chi}_0 = 1 + \frac{\kappa}{V^2}, \tag{29}$$

where κ is another constant of integration. Together with Eqs. (28) and (29) into the density equation (26), the volume dependence of $\Omega \coloneqq \frac{\rho}{\rho_c}$ is

$$\Omega = \Omega_{\Lambda} + \frac{\Omega_m}{V} + \frac{\Omega_{\kappa}}{V^{3/2}}, \tag{30}$$

where $\Omega_{\kappa} = \kappa\Omega_m$. Using the energy equation (13) which is the way to solve Einstein equations under the "inflation compactification mechanism", we obtain the relation,

$$E = \frac{1}{2}\dot{V}^2 + U_{\mathbf{eff}}(V) \tag{31}$$

with the appropriate effective potential,

$$U_{\mathbf{eff}}(V) = -\frac{D+d}{D+d-1}(\Omega_{\Lambda}V^2 + \Omega_m V + \Omega_{\kappa}\sqrt{V}). \tag{32}$$

In Fig. 1, we can see the plot of the effective potential for $\Omega_{\Lambda}, \Omega_{\kappa} < 0$, and $\Omega_m > 0$.

From Eqs. (15), we can see terms with $\sqrt{E}$; therefore, $E > 0$ is a basic condition for the existence of solutions, where E is the measure of the anisotropy of the solution. Only for $E = 0$, we have an isotropic solution. Because of this condition, we can obtain two different cases, represented in Fig. 1: the left case, where all of the effective potential is positive everywhere and the right case, where there is a part with negative values of the potential.

In the left case, if $E = E_{\min}$, we have $V = V_C = \mathbf{const}$, which refers to a constant total volume. But from Eq. (15),

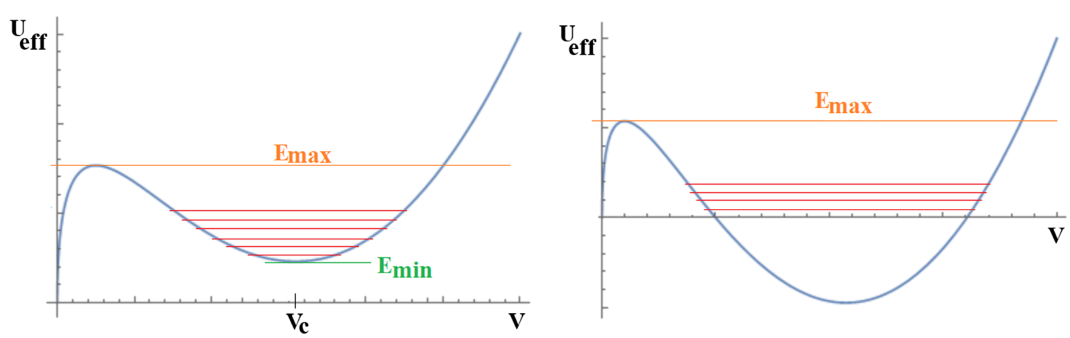

FIG. 1. The effective potential, for two cases, where $\Omega_{\Lambda}, \Omega_{\kappa} < 0$ and $\Omega_m > 0$.

we obtain that the scale parameter $R(t)$ is exponentially growing and the $r(t)$ is exponentially shrinking,

$$R(t) = V_C^{\frac{1}{D+d}} \exp\left[+\frac{1}{D+d}\sqrt{\frac{2Ed(D+d-1)}{D}}\frac{t}{V_C}\right] \quad (33a)$$

$$r(t) = V_C^{\frac{1}{D+d}} \exp\left[-\frac{1}{D+d}\sqrt{\frac{2ED(D+d-1)}{d}}\frac{t}{V_C}\right]. \quad (33b)$$

This kind of solutions holds only for the left case, because of the energy condition $E = E_{\min} > 0$ could only exist if the potential is positive at the minimum. In general, when $E > E_{\min}$, we have an oscillating volume solution. If E is slightly larger than $E_{\min}$, the oscillation will not be so large, and the expansion of the individual scale parameters will be close to an exponentially growing or decreasing, as shown in Fig. 2. On the other hand, if E is much larger than $E_{\min}$, the oscillations will be large also, and the individual scale parameters will grow and shrink modulated by an oscillatory behavior, as shown in Fig. 3. Another important condition is $E < E_{\max}$ as can be seen in Fig. 1.

For proving the existence solutions for a more nonconstant potential $V(\phi)$, where dynamically we change from negative to positive values, we study the case of the step function potential.

D. End of inflation compactification, using a step function potential

The end of the inflation compactification era will take place when the cosmological constant changes from negative values to positive values since then the effective potential does not prevent the total volume from expanding to infinity. For example, a smooth potential that interpolates from those values is

$$V(\phi) = \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\tanh(\beta\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}, \quad (34)$$

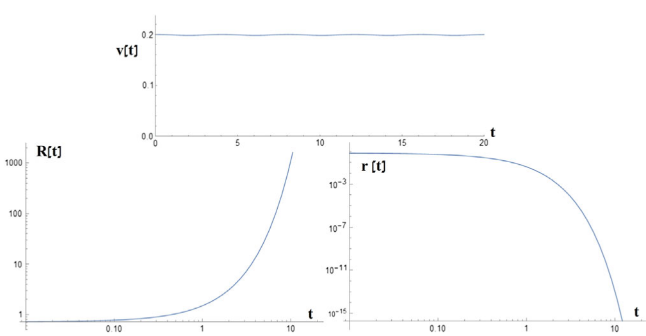

FIG. 2. A numerical solution for the volume and the scale factors in a Kaluza Klein universe, with the parameters: $\Omega_\Lambda = -0.04$, $\Omega_m = 0.24$, $\Omega_\kappa = -0.2$, with the initial condition $\dot{V} = 0.01$.

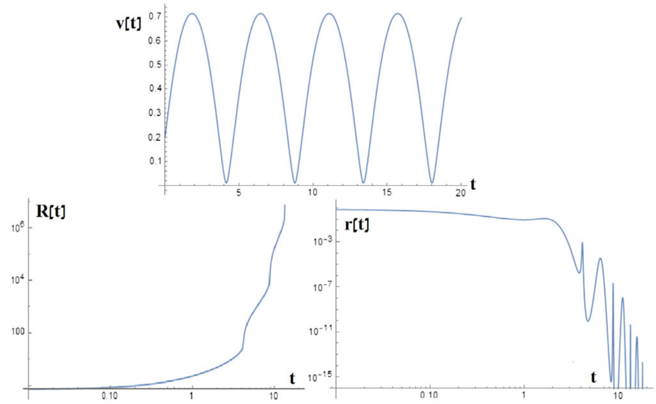

FIG. 3. A numerical solution of the volume and the scale factors for a Kaluza Klein universe, with the parameters: $\Omega_\Lambda = -0.04$, $\Omega_m = 0.24$, $\Omega_\kappa = -0.2$ with an initial condition $\dot{V} = 0.5$.

where $\Lambda_{+\infty} > 0$ is the asymptotic value of the potential for $\phi \to \infty$ and is chosen to be small. On the other hand, $\Lambda_{-\infty} < 0$ is the asymptotic value of the potential for $\phi \to -\infty$. For obtaining a partially analytic and more simple solution, we take the limit for $\beta \to \infty$, which then becomes a step function,

$$V(\phi) = \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\mathbf{Sign}(\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}. \quad (35)$$

Notice that there is no problem for the scalar field ϕ to increase and go up in the direction of increasing dark energy, since its dynamics is not determined by the potential $V(\phi)$ that determined the value of the dark energy. In general, the scalar field evolution depends on $U'(\phi)$, which is in this case zero, since the potential $U(\phi)$ is a constant. Still by choosing the positive root of Eq. (28), we get the desired effect of increasing dark energy as a function of time. For simplicity, let us define the parameter ξ,

$$\xi = \dot{\chi}_0 - 1, \quad (36)$$

which estimates the difference between the dynamical time and the cosmic time. If $\xi = 0$ then $\chi_0 = t$. The variation with respect to the scalar field ϕ, Eq. (24), takes the form,

$$\dot{\phi}\left(\dot{\xi} + \frac{1}{2}\mathcal{H}\xi\right) = -V'(\phi). \quad (37)$$

Since ϕ is a monotonic function of time, it is better to change the time dependence to the scalar dependence $\frac{d}{dt} = \dot{\phi}\frac{d}{d\phi}$. In this way, the equation is easier to analyze,

$$\frac{2\Omega_m}{V^{\frac{3}{2}}} d(\xi V^{\frac{1}{2}}) = -dV(\phi). \quad (38)$$

For the potential (35), we get the differential equation,

$$\frac{2\Omega_m}{V}\left(\frac{d}{d\phi}\xi + \frac{1}{2V}\frac{d}{d\phi}V\right) = -(\Lambda_{+\infty} - \Lambda_{-\infty})\delta(\phi), \quad (39)$$

where in the right-hand side, we obtain a source term, with the piecewise solution,

$$\xi(\phi < 0) = \frac{\kappa_-}{V^{\frac{1}{2}}_{(\phi=0)}}, \qquad \xi(\phi > 0) = \frac{\kappa_+}{V^{\frac{1}{2}}_{(\phi=0)}}. \quad (40)$$

From continuity of the geometry, we demand $V_- = V_+$; otherwise, the geometry is not defined at the junction. From an integration around an infinitesimal region that contains $\phi = 0$, we obtain the jump of the ξ,

$$\Delta\xi = -\frac{V_{(\phi=0)}}{2\Omega_m}(\Lambda_{+\infty} - \Lambda_{-\infty}). \quad (41)$$

Inserting (40) into (41) gives the discontinuity of κ,

$$\kappa_+ - \kappa_- = -\frac{V^{\frac{3}{2}}_{(\phi=0)}}{2\Omega_m}(\Lambda_{+\infty} - \Lambda_{-\infty}). \quad (42)$$

Subtitling all the known terms to the energy equation gives

$$E = \frac{1}{2}\dot{V}^2 - \frac{D+d}{D+d-1}V^2\left[\frac{2\Omega_m}{V}\left(\xi + \frac{1}{2}\right) + \frac{\Lambda_{+\infty} - \Lambda_{-\infty}}{2}\mathbf{Sign}(\phi) + \frac{\Lambda_{+\infty} + \Lambda_{-\infty}}{2}\right]. \quad (43)$$

Because of the jump of the potential and the field ξ, we can calculate the jump of $\dot{V}^2$ from the energy equation. The solution gives

$$\frac{1}{2}\Delta\dot{V}^2 = \frac{D+d}{D+d-1}\left(\frac{2\Omega_m}{V_{(\phi=0)}}\Delta\xi + \Lambda_{+\infty} - \Lambda_{-\infty}\right)V^2_{(\phi=0)} = 0, \quad (44)$$

where there is no jump in the volume and its first derivative. From Eq. (15), which gives the dependence of the metric components, we obtain that all derivatives of the scale factors are continuous. That leads to the conclusion that even when there is a large discontinuous change in the potential, still the metric and its derivative do not suffer from these discontinuities.

E. Large times behavior and extra dimensional stabilization

For obtaining the asymptotic limit of the solutions, let us take the case of the pure vacuum energy $\Omega = \text{const}$ which is the case of late time expansion; we get an upside down harmonic oscillator, which for large volumes gives the solution,

$$V(t) = V_0 \exp(\chi t), \quad (45)$$

where $\chi^2 = 2\frac{D+d}{D+d-1}\Omega$. The integration form of the scale factors in Eq. (15) leads to

$$R(t) \sim e^{\frac{\chi t}{D+d}} \exp\left[-\sqrt{\frac{2Ed(D+d-1)}{D}}\frac{V_0^{\frac{1}{D+d}}}{\chi}e^{-\chi t}\right]$$

$$r(t) \sim e^{\frac{\chi t}{D+d}} \exp\left[+\sqrt{\frac{2ED(D+d-1)}{d}}\frac{V_0^{\frac{1}{D+d}}}{\chi}e^{-\chi t}\right]. \quad (46)$$

From the fact that the solution for the integral gives $\exp(-\chi t)$ which decays for large times, we are left with the limit,

$$R(t) \to R_0 \exp\left(\frac{\chi}{D+d}t\right) \quad (47a)$$

$$r(t) \to r_0 \exp\left(\frac{\chi}{D+d}t\right). \quad (47b)$$

This represents a restoration of the isotropy in the evolution of all dimensions in the Universe. This has to be avoided, because the extra dimensions should be small also in the late Universe. One way to archive this is to generate a potential for the extra dimensions, which starts to act when the extra dimensions are very small and then freeze the extra dimensions to very small size. This can be obtained, for example, by using the Casimir effect present in periodic extra dimensions [11–13]. The stopping of the extra dimensions can also be used as a particle production mechanism, that can result in the reheating of the Universe by a field independent of the inflaton (our field ϕ) which is the extra dimension size. The extra dimension size becomes therefore a curvaton field [14–16].

IV. DISCUSSION

In this article, we studied the basics of an inflation compactification mechanism from the interplay of ordinary and higher dimensions. In the case of isotropic pressure, the solution can be obtained for the total volume and with no dependence with the individual scale factors of each dimension. Those can be calculated directly from the total volume dependence and the anisotropy constant E.

For the dynamical spacetime theory produces a unification of dark energy, dark matter, and a bounce of the volume, which naturally prevents the collapse of the Universe and obtains a lower bound for the volume of the Universe. Likewise, the presence of a negative cosmological constant prevents the volume from becoming very big in the early Universe. There is an effective potential that governs the evolution of the volume. In this case, the effective potential is positive and has a minimum, a static solution for the total volume is obtained, and exponential compactification of the extra dimensions occurs. In that case, the ordinary dimensions exponentially increase and the extra dimensions exponentially decrease. For small values of E higher than the value obtained for the case the volume sits at the minimum, the total volume oscillates and the ordinary dimensions expand exponentially with an oscillatory modulation.

The dynamical spacetime theory provides a natural way to exit from the inflation compactification epoch. The main reason for that is that the theory allows for two different potentials: $U(\phi)$ which drives directly the evolution of the scalar field ϕ and $V(\phi)$ which determines the value of the dark energy. It is therefore perfectly possible for the scalar field that drives the vacuum energy to smoothly climb into small positive values of vacuum energy, which is defined as the end of the inflation compactification. A semianalytic solution for the step function potential is also derived. In this limit, the matching of the solution at the value of the scalar field where the vacuum energy jumps, still respects the continuity of all components of the metric and also for its time derivatives.

We have showed that the exponential growth of the total volume breaks the anisotropy and all the scale factors start to expand in a similar fashion. The role of the inflation compactification mechanism, as we have explained, is to push the extra dimensions to very low sizes and the ordinary dimension to very large sizes. However, we cannot extend the model to all future time, since the vacuum energy at the end will restores the isotropy of the expansion of all dimensions. So we have to invoke a mechanism that locks the extra dimensions when they reduce to very small sizes. This could be produced from the known Casimir effect that takes place in the compact extra dimension for example. The stopping of the extra dimensions can be used also as a particle production mechanism, that can result in the reheating of the Universe by a field independent of the inflaton (our field ϕ) which is the extra dimension size. The extra dimension size becomes therefore a curvaton field.

ACKNOWLEDGMENTS

This article is supported by COST Action No. CA15117 “Cosmology and Astrophysics Network for Theoretical Advances and Training Action” (CANTATA) of the COST (European Cooperation in Science and Technology). In addition, we thank the Foundational Questions Institute FQXi for support, in particular, support for our conference BASIC2018, where this research was carried out.

APPENDIX: IDENTITIES

The covariant conservation of the energy momentum tensor gives

$$\dot{\rho} + (p+\rho)D\frac{\dot{R}}{R} + (p'+\rho)\frac{\dot{r}}{r} = 0. \tag{A1}$$

The Ricci tensor nonvanishing values, under the metric (1),

$$R_{00} = -\left(D\frac{\ddot{R}}{R} + d\frac{\ddot{r}}{r}\right) \tag{A2}$$

$$R_{DD} = \dot{H}_D + (DH_D + dH_d)H_D + (D-1)\frac{k_D}{R^2} \tag{A3}$$

$$R_{dd} = \dot{H}_d + (DH_D + dH_d)H_d + (d-1)\frac{k_d}{r^2}. \tag{A4}$$

And the Ricci scalar,

$$R = 2D\frac{\ddot{R}}{R} + 2d\frac{\ddot{r}}{r} + 2DdH_DH_d + D(D-1)\left(H_D^2 + \frac{k_D}{R^2}\right) + d(d-1)\left(H_d^2 + \frac{k_d}{R^2}\right). \tag{A5}$$

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