Anisotropic inflation in Brans-Dicke gravity with a non-Abelian gauge field

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We study anisotropic inflation in the Brans-Dicke gravity in the presence of a non-Abelian gauge field where the gauge field is nonminimally coupled to the inflaton. We consider the displaced quadratic potential for the inflation. We find out that the solution of equations of motion is an attractor in the phase space. Moreover, anisotropy grows with the number of e-folds. It may become either positive or negative in contradiction to the Abelian gauge field coupling. The anisotropy depends on the Brans-Dicke parameter and constant parameter of the coupling function of the scalar field and the non-Abelian gauge field.

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I. INTRODUCTION

Cosmological inflation is the leading candidate for the solution of several difficulties in the Big-Bang cosmology, such as horizon and flatness problems [1,2]. It also provides a background that primordial fluctuations are redshifted far outside the Hubble radius, which account for the formation of large scale structure of the universe [3–5]. These primordial fluctuations produce a nearly scale-invariant and almost statistically isotropic power spectrum with an almost Gaussian distribution. Deviation from scale invariance, statistical isotropy, and Gaussianity are quite small. They are results of violation of the temporal part, spatial part, and translational symmetry of the de Sitter symmetry, respectively. They have been confirmed by cosmological observations of WMAP and Planck [6–10].

Brocken statistical isotropy of CMB perturbation has been found in the studies of the WMAP data [11] for the first time, and later studies have confirmed it [12,13]. These studies show that the statistics of CMB do not possess full rotational invariance. Ackerman *et al.* were the first to attempt to put constraints on a preferred direction during inflation [14]. They parametrized the power spectrum as an expansion series in the limit of small anisotropy which truncated at the quadruple term. So anisotropic amplitude of power spectrum g_* is read in the parametrization

$$P(\mathbf{k}) = P(k)(1 + g_* \cos^2\theta_{\mathbf{k},\mathbf{n}}), \tag{1}$$

here $P(k)$ is the power spectrum for the primordial density perturbations $\delta(k)$ and depends only on the magnitude of the vector $\mathbf{k}$. Also, $\mathbf{n}$ is the privileged direction by which rotational invariance is broken. Moreover, g_* characterizes the deviation from the isotropy. The obtained bound using 5-year WMAP data at the nine sigma level for the a preferred direction very close to elliptic pole is $g_* = 0.29 \pm 0.031$ [15]. Since the WMAP scanning strategy is tied to the elliptic plane, this strongly suggests that the nonzero value of g_* is due to some systematic effect. Another constrain using 9-year WMAP data is $-0.046 < g_* < 0.048$ at 68% Confidence Level (CL) [16]. A latter analysis based on Planck data gave the constraint $g_* = 0.002 \pm 0.016$ at 68% CL [17]. The Planck team got very similar constraints [10]. On different scales (and marginalizing over the privileged direction, $\mathbf{n}$) Large-Scale-Structure data analysis constrain $-0.41 < g_* < 0.38$ at 95% CL [18,19], and from Baryon Oscillation Spectroscopic Survey Data Release 12 (BOSS DR12) galaxies using bipolar spherical harmonics constrain $-0.09 < g_* < 0.08$ with a 95% CL [20].

Although, vector fields during inflation are claimed as a source of such anisotropy, but an inflating solid or elastic medium [21] is another candidate. It is also interesting that the apparent breaking of statistical isotropy can actually be an artifact of non-Gaussianity [22,23]. A pioneer work on vector field driven inflation was proposed by L. H. Ford [24]. He considered a Bianchi type-I (BI) metric and showed that the universe expands anisotropically at the end of the inflationary era and this anisotropy either survives until late times or is damped out depending on the potential. The study of perturbations in a similar model was proposed by Dimopoulos [25]. A non-minimal coupling of the vector field to gravity was considered in [26,27]. None of the models have mentioned so far escapes instabilities related to negative energy of longitudinal modes. In the self-coupled model a ghost appears at small wavelengths and in the non-minimally coupled the instability concerns the region around horizon crossing [28–30]. Models with varying gauge coupling can overcome the problem of instabilities. In these models a massless vector

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field is non-minimally coupled to the inflaton field and the longitudinal mode disappears and instabilities are avoided [31–34]. Such a model is considered in Ref. [35]. The authors showed that for a suitable choice of the coupling function of the massless vector field and inflaton field, anisotropic hair is survived. Their model which is motivated by supergravity, is stable and can be regarded as a counter example of cosmic no-hair conjecture [36]. This model has been studied extensively in the literature and it has been extended to various models [37–51].

The prototype of an alternative to Einstein's general relativity was done by Brans and Dicke [52]. The primary motivation for their theory comes from Mach's principle, that the phenomenon of inertia ought to arise from accelerations with respect to the general mass distribution of the universe [53]. Brans-Dicke (BD) theory is an important branch of the extended theories of gravity in the scalar-tensor theories. In BD theory, however, the gravitational coupling is variable. It is determined by all matter in the universe, accordingly, a scalar field is considered to couple to the Ricci curvature nonminimally. In spite of declining of interest in BD gravity in the 1970s, a surge interest has raised owing to the new importance of scalar fields in unified theories, in particular string theory. Another reason for this interest is discovering plausible mechanisms that allow the parameter ω (a variable in the BD gravity) to get values of order unity in the early universe and diverge later [54]. Finally, the using of scalar tensor gravity theories in inflationary scenarios of the universe, has renewed interest in BD gravity [55–57].

We extend the model considered in [35] to the BD gravity in our previous work [58] where the inflaton field coupled to the Abelian gauge field. In this paper, we extend our work to the non-Abelian gauge field. We consider the non-Abelian gauge field belong to the SU(2) subgroup, for instance we consider the Yang-Mills gauge field. In particle physics models, we deal with non-Abelian gauge fields. They offer a richer amount of predictions compared to the Abelian case. Also, they have multi-gauge-components and nonlinear self-couplings.

This paper is organized as follows. In Sec. II, the action of anisotropic inflation in the BD Gravity is considered. Then, the equations of motion are obtained. In Sec. III, numerical calculation is performed for a specific potential. It shows that, there is an attractor solution, phase transition occurs and anisotropy grows in this model. In Sec. IV, analytical calculation is performed and anisotropy is obtained in the terms of the slow roll parameters. Conclusion remarks are given in Sec. V.

II. ANISOTROPIC BRANS-DICKE INFLATION WITH YANG-MILLS GAUGE FIELD

In order to generate anisotropic effect during inflation, we add a non-Abelian gauge kinetic term to the action of Brans-Dicke model. This term is coupled to the inflaton field ϕ through the gauge coupling function $f^2(\phi)$. So the action is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\phi R - \frac{1}{2}\frac{\omega_{\mathrm{BD}}}{\phi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{4}f^2(\phi)\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})\right], \tag{2}$$

where ω_{BD} is the BD parameter which is a constant, and hereafter we drop out its subscript and write it as ω. $U(\phi)$ is the potential of the inflaton field. The coupling function $f(\phi)$ will be specified later. The non-Abelian gauge field belongs to the Yang-Mills gauge field. Using Pauli matrices σ^a, the generators of SU(2) is defined by $T^a = \sigma^a/2$ $(a = 1, 2, 3)$ satisfying the following algebra,

$$[T^a, T^b] = i\epsilon^{abc}T^c, \qquad \mathrm{tr}(T^aT^b) = \frac{1}{2}\delta^{ab}, \tag{3}$$

where ϵ^{abc} is a Levi-Civita symbol and δ^{ab} is a Kronecker delta. The Yang-Mills gauge field is defined as $A = A^a_\mu T^a dx^\mu$ with three gauge components $A^a/2$ $(a = 1, 2, 3)$ corresponding to three generators T^a. The field strength $F^{\mu\nu}$ of the SU(2)-gauge field is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig_Y[A_\mu, A_\nu]$ where g_Y is the Yang-Mills coupling constant. The action (2) is invariant under the local SU(2) gauge transformation. We have also set the Planck scale $M_p^2 = 1$ for convenience. We focus on the BI metric, given by

$$ds^2 = -dt^2 + e^{2\alpha(t)-4\sigma(t)}dx^2 + e^{2\alpha(t)+2\sigma(t)}(dy^2 + dz^2), \tag{4}$$

where $\alpha(t)$ measures the number of e-folds of average isotropic expansion of the universe and $\sigma(t)$ is spatial shear which represents deviation from the isotropy. For the average isotropic expansion rate H, we would have

$$H = \frac{H_a + 2H_b}{3} = \dot{\alpha}, \qquad H_a = \frac{\dot{a}(t)}{a(t)}, \qquad H_b = \frac{\dot{b}(t)}{b(t)}, \tag{5}$$

where $a = e^{\alpha-2\sigma}$ and $b = e^{\alpha+\sigma}$. We will work in temporal gauge $A^a_0 = 0$. Imposing the rotational symmetry in the y–z plane on the gauge field A, it is reduced into the following form

$$A(x^\mu) = v_1(t)T^1dx + v_2(t)(T^2dy + T^3dz), \tag{6}$$

where the gauge field is parametrized by the functions $v_1(t)$ and $v_2(t)$. The equations of motion can be written down as

$$\frac{1}{2}\phi R g_{\mu\nu} - [R_{\mu\nu} + g_{\mu\nu}\nabla_\lambda\nabla^\lambda - \nabla_\mu\nabla_\nu]\phi - g_{\mu\nu}U(\phi) + \frac{\omega}{\phi}\left[\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\nabla_\lambda\phi\nabla^\lambda\phi\right] + \frac{1}{2}f^2(\phi)\frac{\partial(F_{\lambda\rho}F^{\lambda\rho})}{\partial g_{\mu\nu}} - \frac{1}{4}g_{\mu\nu}f^2F_{\lambda\rho}F^{\lambda\rho} = 0 \tag{7}$$

$$\nabla_\lambda\nabla^\lambda\phi + \frac{\phi}{2\omega}\left(\frac{-\omega}{\phi^2}\nabla_\lambda\phi\nabla^\lambda\phi - 2U'(\phi) + R + f(\phi)f'(\phi)F_{\lambda\rho}F^{\lambda\rho}\right) = 0 \tag{8}$$

$$D_\mu(f^2(\phi)F^{\mu\nu}) = 0 \tag{9}$$

where ∇_μ represents a covariant derivative with respect to the metric $g_{\mu\nu}$ and a prime denotes a derivative with respect to ϕ and we have defined the gauge covariant derivative as $D_\mu = \nabla_\mu + ig_Y[A_\mu, \ldots]$. From Eqs. (7), (8) and using the metric (4) and the gauge potential (6), we obtain constraint, evolution, and inflaton field equations in the BI space as

$$(3\dot{\alpha}^2 - 3\dot{\sigma}^2)\phi = -3\dot{\alpha}\dot{\phi} + \frac{1}{2}\omega\dot{\phi}^2 + U + \frac{1}{2}f^2[\dot{v}_1^2e^{-2\alpha+4\sigma} + 2\dot{v}_2^2e^{-2\alpha-2\sigma} + 2g_Y^2\dot{v}_1^2\dot{v}_2^2e^{-4\alpha+2\sigma} + g_Y^2\dot{v}_2^4e^{-4\alpha-4\sigma}], \tag{10}$$

$$(\ddot{\alpha} + 3\dot{\alpha}^2)\phi = -\frac{5}{2}\dot{\alpha}\dot{\phi} - \ddot{\phi} + U + \frac{1}{6}f^2[\dot{v}_1^2e^{-2\alpha+4\sigma} + 2\dot{v}_2^2e^{-2\alpha-2\sigma} + 2g_Y^2\dot{v}_1^2\dot{v}_2^2e^{-4\alpha+2\sigma} + g_Y^2\dot{v}_2^4e^{-4\alpha-4\sigma}], \tag{11}$$

$$(\ddot{\sigma} + 3\dot{\alpha}\dot{\sigma})\phi = -\dot{\sigma}\dot{\phi} + \frac{1}{3}f^2[\dot{v}_1^2e^{-2\alpha+4\sigma} - \dot{v}_2^2e^{-2\alpha-2\sigma} - 2g_Y^2\dot{v}_1^2\dot{v}_2^2e^{-4\alpha+2\sigma} + g_Y^2\dot{v}_2^4e^{-4\alpha-4\sigma}], \tag{12}$$

$$\ddot{\phi} + 3\dot{\alpha}\dot{\phi} = -\frac{\phi}{2\omega}\left(-\frac{\omega}{\phi^2}\dot{\phi}^2 - R + 2U'\right) + \frac{\phi}{2\omega}ff'[\dot{v}_1^2e^{-2\alpha+4\sigma} + 2\dot{v}_2^2e^{-2\alpha-2\sigma} - 2g_Y^2\dot{v}_1^2\dot{v}_2^2e^{-4\alpha+2\sigma} - g_Y^2\dot{v}_2^4e^{-4\alpha-4\sigma}]. \tag{13}$$

Using (6), the equations of motion of the gauge field are obtained as

$$\ddot{v}_1 + 2\frac{f'}{f}\dot{v}_1\dot{\phi} + (\dot{\alpha} + 4\dot{\sigma})\dot{v}_1 + 2g_Y^2v_1v_2^2e^{-2\alpha-2\sigma} = 0, \tag{14}$$

$$\ddot{v}_2 + 2\frac{f'}{f}\dot{v}_2\dot{\phi} + (\dot{\alpha} - 2\dot{\sigma})\dot{v}_2 + 2g_Y^2v_1^2v_2e^{-2\alpha+2\sigma} + g_Y^2v_2^3e^{-2\alpha-2\sigma} = 0. \tag{15}$$

Equation (13) could be written in the following form

$$\ddot{\phi} + 3\dot{\alpha}\dot{\phi} = \frac{2}{2\omega+3}(2U - \phi U') + \frac{2}{2\omega+3}(ff'\phi - 2f^2)[\dot{v}_1^2e^{-2\alpha+4\sigma} + 2\dot{v}_2^2e^{-2\alpha-2\sigma} - 2g_Y^2\dot{v}_1^2\dot{v}_2^2e^{-4\alpha+2\sigma} - g_Y^2\dot{v}_2^4e^{-4\alpha-4\sigma}]. \tag{16}$$

When the non-Abelian gauge field goes to zero, and $\sigma \to 0$ (i.e., the metric is spatially flat Friedmann-Robertson-Walker (FRW) universe), the model is reduced to that of the [56,57,59]. Moreover, in the limit $v_2 = 0$ and $\dot{v}_2 = 0$, it is reduced to the Abelian case [58]. Considering the slow roll conditions $|\dot{\phi}| \ll |H\phi|$ and $|\ddot{\phi}| \ll |3H\dot{\phi}|$, and additionally $\sigma \ll \alpha$, $\dot{\sigma} \ll \dot{\alpha}$ hold, then (11) and (16) reduce to

$$3\phi\dot{\alpha}^2 \simeq U(\phi), \tag{17}$$

$$3\dot{\phi}\,\dot{\alpha} \simeq \frac{2}{2\omega+3}[2U(\phi) - \phi U'(\phi)]. \tag{18}$$

It should be noted that the second lines of Eqs. (11) and (16), which is proportional to the energy density of the gauge field, do not appear in Eqs. (17) and (18), because in this scenario, the anisotropy is restricted to the condition that the gauge field is negligible. Beside in [58] we show that the necessary condition for inflation is satisfied if U and U' overcome shear $\Sigma = \dot{\sigma}$, energy density of the gauge field and $\dot{\phi}^2$. Using Eqs. (17) and (18), and following the same process as [58], the coupling function is obtained as

$$f(\phi) = e^{-c(2\omega+3)\int\frac{U}{\phi(2U-\phi U')}d\phi}, \tag{19}$$

where $c > 1$ is a constant parameter. In this paper, we consider displaced quadratic inflationary potential as follows

$$U(\phi) = \frac{1}{2}m^2(\phi - \phi_0)^2, \tag{20}$$

where $m = 10^{-5}$ and ϕ_0 is a shift in the potential. This potential is a generalized version of the Starobinsky R^2 inflation in the Einstein frame. Consistency of this potential with the Planck 2015 data in BD gravity has been investigated by [57] using the Jordan frame, and with Planck 2013 data by [60] in the Einstein frame. Using (20), Eq. (19) becomes

$$f(\phi) = \phi^{-c\frac{(2\omega+3)}{2}} e^{c\frac{(2\omega+3)}{2}\frac{\phi}{\phi_0}}. \tag{21}$$

This coupling function is used to solve the equations of motion numerically in the next section. If we ignore the effect of the non-Abelian gauge field, the slow-roll parameters which have been introduced for this model [56,61] are as follows

$$\epsilon_1 \equiv -\frac{\dot{H}}{H^2} = \frac{(U - \phi U')(2U - \phi U')}{(2\omega + 3)U^2}, \tag{22}$$

$$\epsilon_2 \equiv \frac{\ddot{\phi}}{H\dot{\phi}} = \epsilon_1 + \frac{2\phi(U' - \phi U'')}{(2\omega + 3)U}, \tag{23}$$

$$\epsilon_3 \equiv \frac{\dot{\phi}}{2H\phi} = \frac{2U - \phi U'}{(2\omega + 3)U} = \frac{U\epsilon_1}{(U - \phi U')}, \tag{24}$$

$$\epsilon_4 \equiv \frac{\dot{E}}{2HE} = 0, \tag{25}$$

where the parameter E is defined as

$$E \equiv \phi\left[\frac{\omega}{\phi} + \frac{3\dot{\phi}}{2\phi}\right]. \tag{26}$$

III. NUMERICAL ANALYSIS

Solving Eqs. (11)–(16) numerically for the potential (20) and the coupling function (21), and considering $\phi_0 = 1$, the phase-plane in $\dot{\phi} - \phi$ is obtained (Fig. 1). Figure 1 shows that the behavior of phase-plane is similar to that shown for an anisotropic inflation in the case of $U(1)$ gauge field [58]. It shows that anisotropic inflation in the BD gravity with a non-Abelian gauge field is an attractor solution in the phase-plane.

Evolution of the anisotropy parameter $\Sigma/H = \dot{\sigma}/\dot{\alpha}$ with respect to the e-folding number N for $c = 3$, $\omega = 6$ and different value of $\dot{v}_2/\dot{v}_1$ is presented in Fig. 2. It shows that we have two phases of inflation—isotropic and anisotropic phases. The trajectory starts with an isotropic inflation during horizon crossing of the CMB and soon becomes anisotropic in the second slow roll stage. During the anisotropic stage the energy density of the gauge field is increasing and, after sufficient e-folding, it is decreased. Moreover, the anisotropy can be either positive or negative depending on the initial ratio $\dot{v}_2/\dot{v}_1$. For $0 < \dot{v}_2/\dot{v}_1 < 1$ the anisotropy is positive and for $\dot{v}_2/\dot{v}_1 > 1$ the anisotropy is negative. But, this feature is in contrast to the Abelian case, where the generated anisotropy is always positive [58]. Thus, anisotropy in our model depends on the initial condition of the gauge field. If the sign of Σ/H is negative, then the sign of g_* is positive. This point was implied in [62,63], where the authors showed that $g_* \propto -\Sigma$ and g_* may be positive or negative depending on the model. Thus, the sign of g_* is consistent with the observed one [16–20] for $SU(2)$ gauge field. It should be noted that the massive vector field produces the negative anisotropy too [64]. Moreover, anisotropy is suppressed for $\dot{v}_2/\dot{v}_1 = 1$. All of these features are similar to the corresponding evolution of anisotropy, induced by the $SU(2)$ gauge field in the Einstein gravity [65] and in the Gauss-Bonnet set up [66]. The effect of changing the parameter c is shown in Fig. 3. This figure shows that by increasing c, anisotropy is

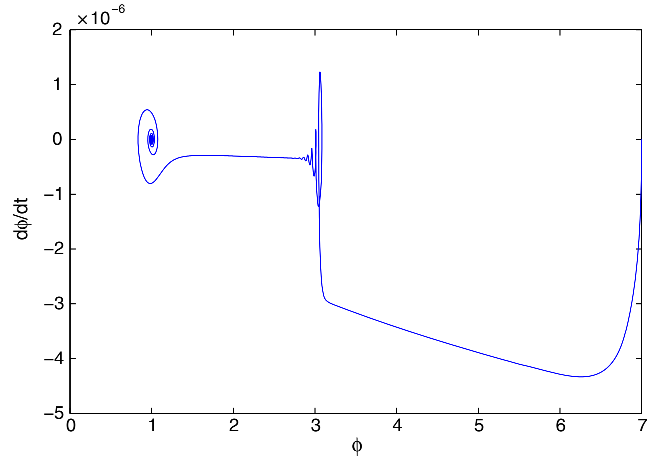

FIG. 1. Phase flow for ϕ is depicted, parameters are assumed $c = 3$, $\omega = 3$, initial conditions have been taken as $\alpha_i = \sigma_i = \dot{\sigma}_i = \dot{\phi}_i = 0$, $\phi_i = 7$, $v_1 = v_2 = 0$, $\dot{v}_1 = 10^{-75}$ and $\dot{v}_2/\dot{v}_1 = 0.5$.

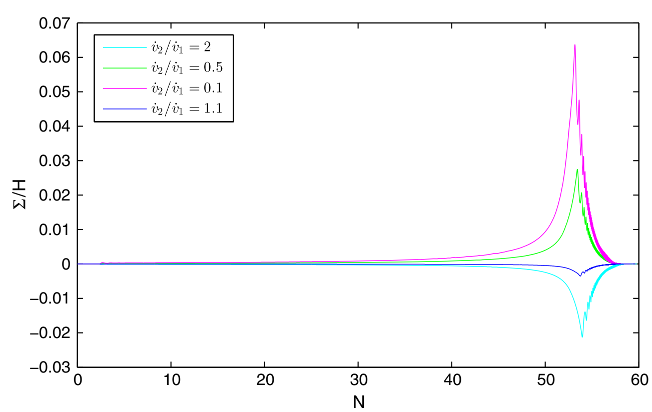

FIG. 2. Σ/H as function of N for $c = 3$, $\omega = 6$, with different values of $\dot{v}_2/\dot{v}_1$. Other initial values are the same as Fig. 1 except for the ratio $\dot{v}_2/\dot{v}_1$.

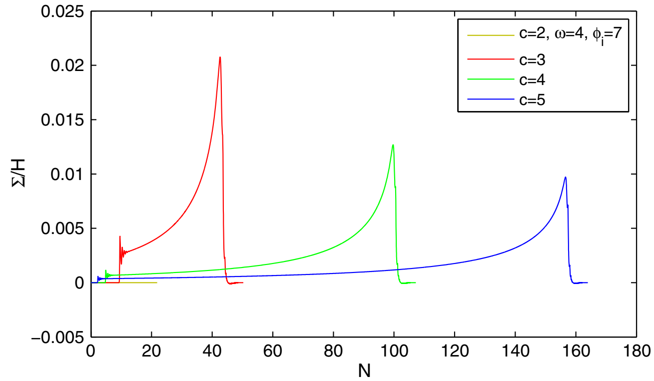

FIG. 3. Σ/H as function of N for $\omega = 4$ and $\dot{v}_2/\dot{v}_1 = 0.5$, with different values of c. Other initial values are the same as Fig. 1.

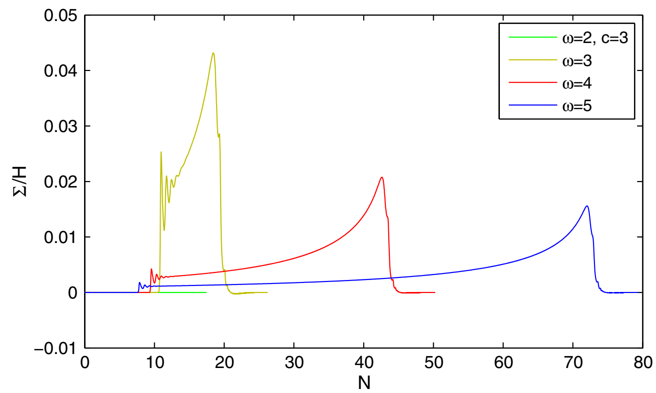

FIG. 4. Σ/H as function of N for $c = 3$ and $\dot{v}_2/\dot{v}_1 = 0.5$, with different values of ω. Other initial values are the same as Fig. 1.

shifted to the bigger e-folding and the value of anisotropy Σ/H, is decreased. These behaviors are in contradiction to the non-Brans-Dicke gravity, in which the anisotropy was independent of the parameter c. Moreover, the effect of changing the parameter ω is shown in Fig. 4. It shows that the anisotropy is very sensitive to the value of ω. By increasing ω, anisotropy is shifted to the bigger e-folding number. Thus, the range of N for which Σ/H is constant, will be increased. On the other hand, the value of the anisotropy is decreased.

IV. ANALYTICAL ANALYSIS

In Sec. III, we solved equations of motion numerically. In this section, we try to solve them using slow roll approximation. During the slow roll inflationary phase the initial value of the inflaton field is chosen to be $\phi_i \sim 7$, the Brans-Dicke parameter is assumed as $\omega \sim 3$ and constant parameter of the coupling function is taken as $c \sim 3$. Using (21), the gauge coupling function becomes $f(\phi) \sim 10^{20}$. Thus, the effective gauge coupling $g_Y/f(\phi)$ in the action (2) becomes very small $g_Y/f(\phi) \sim 10^{-20}$ during slow roll inflation. Therefore, we can ignore the Yang-Mills gauge coupling during inflation. Then, the equation of motion of the gauge field (14) can be integrated to obtain

$$\dot{v}_1 = f^{-2}(\phi)e^{-\alpha-4\sigma}p_{A1},$$
$$\dot{v}_2 = f^{-2}(\phi)e^{-\alpha+2\sigma}p_{A2}, \tag{27}$$

where p_{A1} and p_{A2} are constants of integration. Ignoring σ, Eq. (12) can be written as

$$(\ddot{\sigma} + 3\dot{\alpha}\dot{\sigma})\phi = -\dot{\sigma}\dot{\phi} + \frac{1}{3}f^{-2}e^{-4\alpha}(p_{A1}^2 - p_{A2}^2). \tag{28}$$

From this equation, it is clear that anisotropy will grow only when the last term will be a dominant term. Hence, anisotropy starts to grow at least for $f(\phi) = e^{-2\alpha}$, or more generally for $f(\phi) = e^{-2c\alpha}$, where $c > 1$. Using (19), this means that the necessary condition in order to commence the anisotropic inflation is

$$\frac{f'}{f}\frac{\phi(\phi U' - 2U)}{(2\omega+3)U} > 1. \tag{29}$$

Moreover, we can use Eqs. (17) and (18) in the slow-roll approximation to obtain number of e-folds as follows

$$N = \frac{2\omega+3}{2}\int_{\phi_i}^{\phi_e}\frac{U}{\phi(2U - \phi U')}d\phi, \tag{30}$$

where ϕ_e is the value of the scalar field at the end of inflation, ϕ_i is the value of the scalar field at the horizon crossing. Equation (30) for the potential (20) reads

$$N = \frac{2\omega+3}{4}\left[\frac{(\phi_i - \phi_e)}{\phi_0} - \ln\frac{\phi_i}{\phi_e}\right]. \tag{31}$$

We will now use slow roll approximations to estimate Σ/H. With the approximation $\sigma \ll \alpha$ the Σ satisfies the equation of motion

$$(\dot{\Sigma} + 3H\Sigma)\phi = -\Sigma\dot{\phi} + \frac{1}{3}g(\alpha)^{-1}(p_{A1}^2 - p_{A2}^2). \tag{32}$$

where $g(\alpha)$ is

$$g(\alpha) = f^2 e^{4\alpha}, \tag{33}$$

In (33) σ is neglected. By substituting (17) and (18) in (32) and by assuming $\dot{\Sigma}$ is negligible in (32) (Because Σ/H is proportional to g_*, and g_* does not change on different scales [10,20].), we have

$$\frac{\Sigma}{H} \simeq \left(\frac{3(2\omega+3)(p_{A1}^2 - p_{A2}^2)}{(2\omega+3)3U + 2(2U - \phi U')}\right)g(\alpha)^{-1}. \tag{34}$$

In order to obtain $g(\alpha)$, we use (16) in the slow roll approximation, which after substituting (27), reads

$$d\phi/d\alpha = \frac{2}{2\omega+3}\frac{\phi(2U-\phi U')}{U} + \frac{2\phi(\frac{f'}{f}\phi-2)}{U(2\omega+3)}(p_{A1}^2+2p_{A2}^2)g(\alpha)^{-1}. \tag{35}$$

We find that this equation reduces to

$$\frac{dg(\alpha)}{d\alpha} + 4(c-1)g(\alpha) = \Omega(\phi). \tag{36}$$

Finally,

$$g(\alpha) = g(\alpha_0)e^{-4(c-1)(\alpha-\alpha_0)} + \frac{\Omega(\phi)}{4(c-1)}, \tag{37}$$

where

$$\Omega(\phi) = 4c(p_{A1}^2+2p_{A2}^2)\left(\frac{(2(2U-\phi U')+c(2\omega+3))U}{(2U-\phi U')^2}\right). \tag{38}$$

In the limit of $\alpha \to -\infty$, the first term of Eq. (37) is dominated and $g(\alpha)$ diverges to infinity. Consequently, anisotropy goes to zero, i.e., $\Sigma/H \to 0$ and Eq. (35) will become

$$d\phi/d\alpha = \frac{2}{2\omega+3}\frac{\phi(2U-\phi U')}{U}. \tag{39}$$

In this stage the slow-roll parameters are same as Eqs. (22)–(25), that we call isotropic inflation. On the other hand, with $\alpha \to \infty$, the second term of (37) is dominated, and $g(\alpha) \to \Omega/4(c-1)$. Therefore anisotropy parameter reads

$$\frac{\Sigma}{H} \to \left(\frac{3(2\omega+3)(p_{A1}^2-p_{A2}^2)}{(2\omega+3)3U+2(2U-\phi U')}\right)\frac{4(c-1)}{\Omega} = \frac{3(c-1)(p_{A1}^2-p_{A2}^2)(2\omega+3)(2U-\phi U')^2}{c(p_{A1}^2+2p_{A2}^2)((2\omega+3)3U+2(2U-\phi U'))((2(2U-\phi U')+c(2\omega+3))U)}, \tag{40}$$

and Eq. (35) reads

$$d\phi/d\alpha = \frac{1}{c}\frac{2}{(2\omega+3)}\frac{\phi(2U-\phi U')}{U}. \tag{41}$$

Therefore, the slow-roll parameters are obtained as follows:

$$\varepsilon_1 = \frac{1}{c}\frac{(U-\phi U')(2U-\phi U')}{(2\omega+3)U^2}, \tag{42}$$

$$\varepsilon_2 = \varepsilon_1 + \frac{2\phi(U'-\phi U'')}{(2\omega+3)U}, \tag{43}$$

$$\varepsilon_3 = \frac{1}{c}\frac{2U-\phi U'}{(2\omega+3)U}. \tag{44}$$

We can write Σ/H in the terms of the slow roll-roll parameter (44) as follows:

$$\frac{\Sigma}{H} = \frac{3(c-1)(p_{A1}^2-p_{A2}^2)(2\omega+3)}{2(p_{A1}^2+2p_{A2}^2)}\left(\frac{\varepsilon_3^2}{(3+c\varepsilon_3)(1+\varepsilon_3)}\right). \tag{45}$$

It should be noticed that the result of [58] can be recovered if we put $p_{A2} = 0$ in (45). The anisotropy can be either positive or negative depending on the ratio $\dot{v}_2/\dot{v}_1 \sim p_{A2}/p_{A1}$. The anisotropy becomes negative if $p_{A2} > p_{A1}$ as observed in Fig. 2, but it exactly vanishes, when $p_{A2} = p_{A1}$. However, as a matter of fact, we know from the previous section that by increasing the parameters ω and c, the value of the anisotropy is decreased and the range of N for constant anisotropy is increased. By substituting ε_3 in (45), we find out $\Sigma/H \propto 1/(c(2\omega+3))$. Thus, the value of anisotropy, Σ/H, is decreased by increasing in ω and c. Increasing the range of N for constant anisotropy by increasing ω and c is understood from Eq. (31). Equation (31) determines that increasing ω leads to the enhancement of the number of e-folds. Therefore, the range of N for constant Σ/H lasts for more e-folds. The anisotropy Σ/H, is proportional to g_* [67]. For $g_* < 10^{-2}$, anisotropy must be of order $< 10^{-9}$ [67]. Anisotropic inflation in the Einstein gravity leads the order of $\sim 10^{-3}$ for anisotropy [35]. Anisotropic inflation in the Brans-Dicke gravity provides a variable value for Σ/H, which can be controlled by parameters ω and c. For $2 < \omega < 3$ and $c = 3$, anisotropy is $10^{-20} < \Sigma/H < 10^{-4}$, where Σ/H is closed to its observational value.

V. DISCUSSION AND CONCLUSIONS

In the present work, we studied anisotropic inflation in the Brans-Dicke gravity with the $SU(2)$ Yang-Mills gauge field coupled to the inflaton field. We saw that there is a phase transition in this model. There are two phases of inflation—the isotropic and anisotropic phases. The trajectory starts with an isotropic inflation during the horizon crossing of the CMB and soon becomes anisotropic in the second slow roll stage. The slow roll parameters in the first stage are different from the slow roll parameters in the second stage. Numerical calculations show that the anisotropy may become either positive or negative depending on the initial value of $\dot{v}_2/\dot{v}_1$. This feature is consistent with the observational value of g_*. Increasing c leads N to be increased, and the range of N for consistent Σ/H lasts for more e-folds. This behavior is in contradiction to the non-Brans-Dicke gravity, in which the anisotropy was independent of the parameter c. Moreover, by increasing ω, anisotropy is shifted to bigger e-folds. The reason can be understood from the analytical calculations. The relationship which obtained for N shows that the number of e-fold is directly proportional to ω. N is increased by increasing ω. Moreover, the value of anisotropy is decreased by increasing ω and c. Therefore, anisotropic inflation in the Brans-Dicke gravity provides a variable value for Σ/H, which can be controlled by the parameters ω and c. We can choose the parameters in a way that Σ/H is closer to its observational value.

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