

Anisotropic pressure in strange quark matter in the presence of a strong nonuniform magnetic field

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Thermodynamic properties of strange quark matter (SQM) in a nonuniform magnetic field are studied within the phenomenological MIT bag model under the charge neutrality and beta equilibrium conditions, relevant to the interior of strange quark stars. The spatial dependence of the magnetic field strength is modeled by the dependence on the baryon chemical potential in the exponential and power forms. The total energy density, longitudinal and transverse pressures in magnetized SQM are found as functions of the baryon chemical potential. It is clarified that the central magnetic field strength in a strange quark star is bound from above by the critical value at which the derivative of the longitudinal pressure with respect to the baryon chemical potential vanishes first somewhere in the interior of a star under varying the central field. Above this upper bound, the instability along the magnetic field is developed in magnetized SQM. The change in the form of the dependence of the magnetic field strength on the baryon chemical potential between the exponential and power ones has a nonnegligible effect on the critical magnetic field strength while the variation of the bag pressure within the absolute stability window for magnetized SQM has a little effect on the critical field.

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I. INTRODUCTION. BASIC EQUATIONS

It was conjectured some time ago that for a certain range of model QCD-related parameters strange quark matter (SQM), consisting of deconfined u , d , and s quarks, can be the true ground state of matter [1–3]. In that case, at zero external pressure and temperature, the energy per baryon of SQM is less than that for the most stable ^{56}Fe nucleus. If this hypothesis holds true, then the formation of strange quark stars, composed of SQM and self-bound by strong interactions, is possible [4–7]. This conjecture got recently support from the observations of massive compact stars with $M \sim 2 M_{\odot}$ and the indications that some neutron stars may be very compact (with the radii smaller than 10 km). While the former implies that the equation of state (EoS) of strongly interacting matter should be stiff, the latter requires the soft EoS. A possible explanation of these contradictory requirements could be the existence of two separate families of compact stars: quark stars which can be very massive, according to the perturbative QCD calculations, and hadronic stars which can be very compact [8].

Another important peculiarity, related to compact stars, is that they can possess strong magnetic fields. For example, for magnetars—strongly magnetized neutron stars [9], the magnetic field strength can reach values of about 10^{14} – 10^{15} G at the surface [10,11], and can be even larger, up to 10^{19} G, in the core of a star [12,13].

Usually, such estimates of the possible interior magnetic field strengths are based on the virial theorem [14] while general relativistic calculations, based on the Einstein–Maxwell equations, lead to the more modest estimate $H \lesssim (1\text{--}3) \times 10^{18}$ G [15]. Such strong magnetic fields can result in the large pulsar kick velocities because of the asymmetric neutrino emission in direct Urca processes in the dense core of a magnetized compact star [16]. The mechanism responsible for generation of strong magnetic fields of magnetars is still to be clarified, and, among other possibilities, this can be due to the turbulent dynamo amplification mechanism in a star with the rapidly rotating core [10], or because of the spontaneous ordering of nucleon [17–20], or quark [21] spins in the dense matter inside a compact star.

Strong magnetic fields can have significant impact on thermodynamic properties of strongly interacting matter in the dense interior of a compact star [22–27]. In particular, because of the breaking of the rotational symmetry, the pressure becomes essentially anisotropic in strongly magnetized matter [28–33]. The longitudinal pressure p_l (along the magnetic field direction) gets the negative contribution from the magnetic field given by the Maxwell term $\frac{H^2}{8\pi}$. Under increasing the magnetic field strength, the longitudinal pressure decreases and, eventually, becomes negative, resulting in the appearance of the longitudinal instability in a strongly magnetized matter. For the uniform magnetic field, the onset of the longitudinal instability

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corresponds to the critical magnetic field strength, at which the longitudinal pressure vanishes. The estimates show that the critical magnetic field has the upper bound of about 10^{19} G for quark matter [29,30,33], neutron matter [31,32] and strange baryonic matter [34]. For strange quark stars, self-bound by strong interactions, the condition of absolute stability of magnetized SQM, with account of the pressure anisotropy, sets the constraint on the allowable magnetic field strength $H \lesssim (1-3) \times 10^{18}$ G [35,36]. For hybrid stars, based on the energy conservation arguments, the possible magnetic field strength in the quark core is estimated as $H \sim 10^{20}$ G [29]. For a nonuniform magnetic field, with allowance for the inhomogeneous mass distribution, the application of the virial theorem gives the estimate for the central field in a neutron star $H \sim 10^{19}$ G [29].

In this research, we study thermodynamic properties of SQM in a nonuniform magnetic field, taking into account that in strange quark stars the magnetic field strength can change by several orders of magnitude from the core to the surface of a star. The spatial dependence of the magnetic field strength is modeled by its dependence on the baryon chemical potential μ_B . As will be shown in this study, the longitudinal instability in a nonuniform magnetic field is associated with the appearance of the negative derivative $p'_l(\mu_B) < 0$, unlike to the case of a uniform magnetic field where the longitudinal instability occurs at $p_l < 0$.

As a theoretical framework to study strongly magnetized SQM, we will utilize the MIT bag model. The details of a theoretical formalism are presented in Ref. [33]. The domain of absolute stability of magnetized SQM within the MIT bag model with account of the effects of the pressure anisotropy was determined in [35,36]. To mimic the spatial dependence of the magnetic field, we will parametrize the magnetic field strength in terms of the baryon chemical potential μ_B in the exponential form [37–39]:

$$H(\mu_B) = H_s + H_{\text{cen}}(1 - e^{-\beta(\frac{\mu_B - \mu_{B0}}{\mu_{B0}})^\gamma}). \quad (1)$$

Here μ_{B0} and H_s are the baryon chemical potential and the magnetic field strength at the surface of a strange quark star, respectively. In Eq. (1), the quantity H_{cen} is given by $H_{\text{cen}} \approx H(\mu_B \gg \mu_{B0})$, assuming that $H_{\text{cen}} \gg H_s$; β and γ are the model parameters. Also, in numerical calculations we will adopt the power parametrization:

$$H(\mu_B) = H_s + H_{\text{cen}} \left[1 - \left(\frac{\mu_B^c - \mu_B}{\mu_B^c - \mu_{B0}} \right)^\alpha \right], \quad (2)$$

where μ_B^c is the baryon chemical potential in the center of a star, α is the model parameter. One can see that $H(\mu_{B0}) = H_s$ and $H(\mu_B^c) \approx H_{\text{cen}}$. Further we consider bare strange quark stars (without a thin layer of nuclear matter above the quark surface). In order to determine the baryon

chemical potential μ_{B0} at the surface, we will use the conditions of charge neutrality

$$2q_u - q_d - q_s - 3q_e = 0, \quad (3)$$

and chemical equilibrium with respect to weak processes in SQM:

$$\mu_d = \mu_u + \mu_e, \quad (4)$$

$$\mu_d = \mu_s, \quad (5)$$

where q_i and μ_i are the number density and chemical potential for fermions of i th species ($i = u, d, s, e$). Further we assume, analogously to Ref. [40], a spherically symmetric radial distribution of the magnetic field inside a star. Then the longitudinal pressure (along the magnetic field direction) should vanish at the surface of a star:

$$p_l = -\sum_i \Omega_i - \frac{H^2}{8\pi} - B = 0. \quad (6)$$

Here Ω_i is the thermodynamic potential for free relativistic fermions of i th species in a magnetic field [26,33]:

$$\Omega_i = -\frac{|q_i|g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}^i} (2 - \delta_{\nu,0}) \times \left\{ \mu_i k_{F,\nu}^i - \bar{m}_{i,\nu}^2 \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}, \quad (7)$$

and

$$\bar{m}_{i,\nu} = \sqrt{m_i^2 + 2\nu|q_i|H}, \quad k_{F,\nu}^i = \sqrt{\mu_i^2 - \bar{m}_{i,\nu}^2}. \quad (8)$$

In Eq. (7), summation on Landau levels runs up to

$$\nu_{\text{max}}^i = I \left[\frac{\mu_i^2 - m_i^2}{2|q_i|H} \right], \quad (9)$$

$I[\dots]$ being an integer part of the argument, the factor $(2 - \delta_{\nu,0})$ takes into account the spin degeneracy of Landau levels, and g_i is the remaining degeneracy factor ($g_i = 3$ if $i = u, d, s$ and $g_i = 1$ if $i = e$).

The chemical potentials of quarks and electrons at the surface of a quark star can be determined from Eqs. (3)–(6) with account of the relationship between the particle number densities $q_i = -(\frac{\partial \Omega_i}{\partial \mu_i})_H$ and respective chemical potentials μ_i :

$$q_i = \frac{|q_i|g_i H}{2\pi^2} \sum_{\nu=0}^{\nu_{\text{max}}^i} (2 - \delta_{\nu,0}) k_{F,\nu}^i. \quad (10)$$

Since

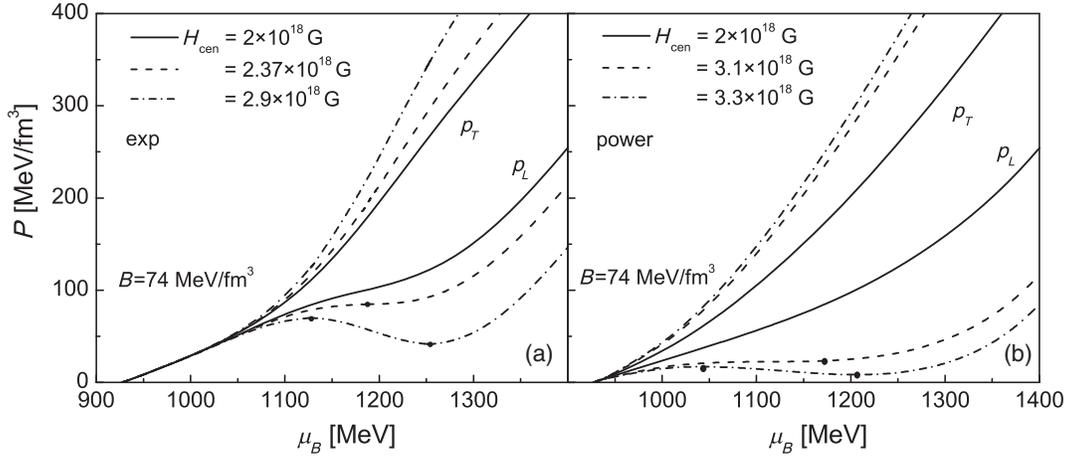


FIG. 1. Transverse p_t and longitudinal p_l pressures in magnetized SQM as functions of the baryon chemical potential, corresponding to: (a) the exponential parametrization (1) with $\beta = 45$, $\gamma = 3$, and (b) the power parametrization (2) with $\alpha = \frac{3}{2}$, $\mu_B^c = 1400$ MeV for $H_s = 10^{15}$ G and variable central field H_{cen} . The full dots correspond to the points where $p'_l(\mu_B) = 0$.

$$\mu_B = \mu_u + \mu_d + \mu_s, \quad (11)$$

then one can find the baryon chemical potential μ_{B0} at the surface of a star. In numerical calculations, we will use the model parameters $\beta = 45$ and $\gamma = 3$ for the exponential parametrization, $\alpha = \frac{3}{2}$ for the power parametrization, and $H_s = 10^{15}$ G. The bag pressure is set $B = 74$ MeV/fm³, which is slightly smaller than the upper bound $B_u \simeq 75$ MeV/fm³ from the absolute stability window for the quark current masses $m_u = m_d = 5$ MeV, and $m_s = 150$ MeV [35]. Then one can numerically determine the baryon chemical potential $\mu_{B0} \approx 927.4$ MeV.

II. NUMERICAL RESULTS AND DISCUSSION

In the MIT bag model, the total energy density E and the transverse pressure p_t in magnetized SQM read

$$E = \sum_i (\Omega_i + \mu_i Q_i) + \frac{H^2}{8\pi} + B, \quad (12)$$

$$p_t = -\sum_i \Omega_i - HM + \frac{H^2}{8\pi} - B, \quad (13)$$

where $M = -\sum_i (\frac{\partial \Omega_i}{\partial H}) \mu_i$ is the magnetization of the system. In order to study the impact of a strong nonuniform magnetic field, parametrized by Eq. (1), or by Eq. (2), on the anisotropic pressure and the equation of state (EoS) of the system, we will fix the baryon chemical potential in the center of a strange quark star, $\mu_B^c = 1400$ MeV (that corresponds to the baryon number density ρ_B^c of about eight times nuclear saturation density—densities of such magnitude are expected to occur in the center of strange quark stars [30,37,38]), and will vary the central magnetic field strength H_{cen} .

Figure 1 shows the dependence of the transverse p_t and longitudinal p_l pressures in the system on the baryon chemical potential μ_B for several values of the central magnetic field strength H_{cen} . Let us discuss, first, the case of the exponential parametrization of the magnetic field strength, represented in Fig. 1(a). It is seen that, under increasing the central field H_{cen} , the transverse pressure p_t increases while the longitudinal pressure p_l decreases. Also, the transverse pressure p_t always remains the increasing function of the baryon chemical potential μ_B while the dependence of the longitudinal pressure p_l on μ_B can be different. At not too strong central fields (e.g., at $H_{\text{cen}} = 2 \times 10^{18}$ G), the longitudinal pressure p_l remains the increasing function of μ_B . However, with the increase of H_{cen} , the curve $p_l(\mu_B)$ bends down in its middle part, and there exists such central field H_{cen} , at which the derivative $p'_l(\mu_B)$ vanishes first somewhere in the interior of a strange quark star. For a given set of the model parameters, this happens for $H_{\text{cen}} \approx 2.37 \times 10^{18}$ G at $\mu_B \approx 1188.4$ MeV (the corresponding point on the curve is marked by the full dot). Under further increasing the central field H_{cen} , there appears the part on the curve $p_l(\mu_B)$, characterized by $p'_l(\mu_B) < 0$ (e.g., at $H_{\text{cen}} \approx 2.9 \times 10^{18}$ G, this part of the curve on the figure is contained between two full dots). This contradicts the thermodynamic constraint $p'_l(\mu_B) > 0$. Hence, such states of magnetized SQM are unstable, and instability is developed along the magnetic field direction. The strength of the central field $H_{\text{cen}} \approx 2.37 \times 10^{18}$ G, at which the derivative $p'_l(\mu_B)$ vanishes first, is the critical field for the onset of the longitudinal instability. This value represents the upper bound on the central magnetic field strength in a strange quark star.

Concerning the criterion $p'_l(\mu_B) < 0$ for the appearance of the longitudinal instability, it is important to note that, according to rigorous microscopic derivations [29,41], the

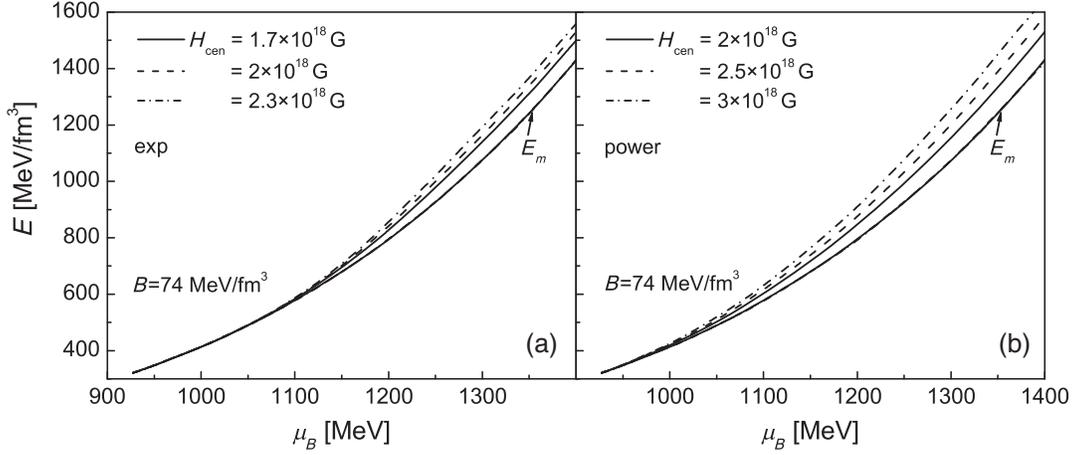


FIG. 2. Same as in Fig. 1, but for the energy density E and the matter energy density $E_m \equiv E - \frac{H^2}{8\pi}$ (the most lower curve) of magnetized SQM.

total parallel pressure p_l contains both matter and field contributions [cf. Eq. (6)]. While the derivative of the matter part of the longitudinal pressure with respect to the baryon chemical potential is positive, the magnetic field contributes negatively to the derivative $p'_l(\mu_B)$, and, at a strong enough central field, the field contribution overcomes the matter contribution, making the derivative $p'_l(\mu_B)$ negative.

For the power parametrization (2) of the magnetic field strength [cf. Fig. 1(b)], the behavior of the curves $p_l(\mu_B)$, $p_t(\mu_B)$ under varying the central field H_{cen} is qualitatively similar to that for the exponential parametrization (1). The derivative $p'_l(\mu_B)$ vanishes first for $H_{\text{cen}} \approx 3.1 \times 10^{18}$ G at $\mu_B \approx 1172.1$ MeV, and, hence, the change in the form of the parametrization $H(\mu_B)$ from exponential to the power one has the nonnegligible effect on the critical magnetic field strength.

We repeated also the calculations for the bag pressure $B = 58$ MeV/fm³ which is slightly above the lower bound $B_l \approx 57$ MeV/fm³ from the absolute stability window [35], but this variation of the bag pressure has a little effect on the results, e.g., for the power parametrization the critical magnetic field strength increases till $H_{\text{cen}} \approx 3.17 \times 10^{18}$ G.

Note that for an uniform magnetic field the longitudinal instability in magnetized matter is associated with the appearance of the negative longitudinal pressure $p_l < 0$ [28–33]. Since vanishing of the derivative $p'_l(\mu_B)$ occurs at smaller central magnetic field strength H_{cen} than vanishing of p_l , for a nonuniform magnetic field, parametrized by Eq. (1), or by Eq. (2), the corresponding criterion is the occurrence of the negative derivative $p'_l(\mu_B) < 0$. The last criterion for the appearance of the longitudinal instability in a nonuniform magnetic field sets a stronger constraint on the upper bound of the central magnetic field in strongly magnetized strange quark star than the criterion $p_l < 0$.

Figure 2 shows the energy density E of magnetized SQM and its matter part $E_m \equiv E - \frac{H^2}{8\pi}$ (without the pure magnetic field contribution) as functions of the baryon chemical potential. With increasing the central magnetic field strength, the energy density E increases while the matter part E_m remains practically unchanged. In particular, the curves for E_m are almost indistinguishable for the different values of the central magnetic field H_{cen} , used in calculations, and look as one curve. This figure allows to estimate the relative role of the matter E_m and magnetic field $\frac{H^2}{8\pi}$ contributions to the total energy density E . It is seen that the matter part dominates over the field part at such baryon chemical potentials and magnetic field strengths H_{cen} for both the exponential and power parametrizations of the magnetic field strength.

In strong magnetic fields the total pressure in magnetized SQM becomes essentially anisotropic. Therefore, EoS of the system becomes also highly anisotropic. Figure 3, showing the dependence of the transverse p_t and longitudinal p_l pressures on the energy density E of magnetized SQM, explicitly demonstrates this moment. In the given cases, when the values of the central field are smaller than the critical field for the appearance of the longitudinal instability, the pressures p_t , p_l , and the energy density E are the increasing functions of the baryon chemical potential μ_B . Hence, after excluding μ_B , one gets the anisotropic EoS in the form of two distinct increasing functions $p_t(E)$ and $p_l(E)$.

In conclusion, we have considered the impact of a strong magnetic field on thermodynamic properties of SQM at zero temperature under conditions relevant to the interior of magnetized strange quark stars. The spatial dependence of the magnetic field strength is modeled by the dependence on the baryon chemical potential in the exponential and power forms. The total energy density E , transverse p_t and longitudinal p_l pressures in magnetized SQM have been

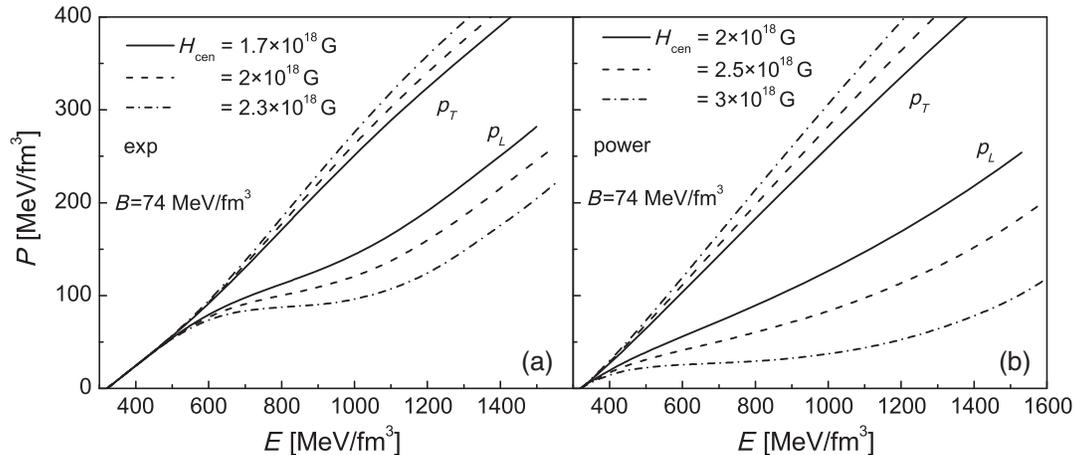


FIG. 3. The transverse p_t and longitudinal p_l pressures in magnetized SQM as functions of the total energy density E , corresponding to: (a) the exponential parametrization (1), and (b) the power parametrization (2) of the magnetic field strength.

calculated as functions of the baryon chemical potential. Also, the highly anisotropic EoS has been determined in the form of $p_t(E)$ and $p_l(E)$ dependences. It has been clarified that the central magnetic field in a strange quark star is bound from above by the critical value, at which the derivative of the longitudinal pressure $p'_l(\mu_B)$ vanishes first somewhere in the interior of a star under varying the central field. Above this upper bound, the instability along the magnetic field direction is developed in magnetized SQM. The change in the form of the dependence $H(\mu_B)$ between the exponential and power ones leads to the noticeable quantitative differences, in particular, it has the non-negligible effect on the critical magnetic field strength. While the variation of the bag pressure within the absolute stability window for magnetized SQM has a little effect on the results, in particular, the critical field remains almost unaltered under such a change.

Based on the criterion of the longitudinal instability $p'_l(\mu_B) < 0$, the possible central magnetic field strength $H_{\text{cen}} \lesssim (2-3) \times 10^{18}$ G has been estimated to be more than three orders of magnitude larger than the surface field. In some of the previous calculations, based on the Einstein–Maxwell equations, the central magnetic field was estimated to be only five times larger than the surface value [42]. Nevertheless, as discussed in Ref. [15], where solution of the Einstein–Maxwell equations gives the estimate on the possible interior magnetic field $H \lesssim (1-3) \times 10^{18}$ G, other choices of the nonuniform current function, or the relaxation of the condition of axial symmetry of magnetic field distribution, which can influence the shape of a star, could lead to even stronger interior magnetic fields.

In this research, all consideration has been done within the phenomenological MIT bag model, which is quite popular and frequently used in various astrophysical applications (just some of recent references include, e.g., [26,38,43–45]). Despite its relative simplicity, it allows to

qualitatively describe the appearance of the longitudinal instability in strongly magnetized SQM and to get the correct order of magnitude of the upper bound on the magnetic field strength in strange quark stars. The MIT bag model establishes the baseline for more advanced calculations and further improvement of the obtained estimates is possible with more elaborated models.

It is worthy to note that the proposed mechanism for the appearance of the longitudinal instability in magnetized matter in a nonuniform magnetic field parametrized in terms of the baryon chemical potential is universal and does not depend on the specific type of a compact star whether it is a quark star, or a neutron star, or a hybrid star. The specific type of a compact star will be reflected in the underlying model for the EoS of matter in the interior of compact stellar object, depending on whether it is a quark phase, or a hadronic phase in the given inner region of a star. Inevitably, the longitudinal instability will occur in a strong enough magnetic field as soon as the derivative $p'_l(\mu_B)$ becomes negative in the field beyond the critical one.

The formulation of the problem in terms of the energy density E as an independent variable would be the other possible way to consider the longitudinal instability in a strong nonuniform magnetic field. This would lead to the criterion of the longitudinal instability in the form $p'_l(E) < 0$. Under such an approach, it would be consistent to parametrize the magnetic field strength H in terms of the energy density E as well, $H = H(E)$. Nevertheless, taking into account the possible applications of the criterion of the longitudinal instability to other types of compact stars, such as, e.g., hybrid stars, the most flexible way to tackle the problem is to formulate it in terms of the baryon chemical potential. If the energy density were used as the independent variable, then, because the energy density is discontinuous at the phase boundary of a first order quark-hadron phase transition, the magnetic field strength $H(E)$ would

experience the unphysical jump across the phase transition boundary, which is missing for the parametrization $H(\mu_B)$.

Note that, although the presence of a magnetic field leads to the appearance of the local pressure anisotropy, magnetized strange quark stars, considered in this study, are spherically symmetric because of the radial distribution of the magnetic field inside a star. There can be other sources

of the local pressure anisotropy, like superfluid states with the finite orbital momentum of Cooper pairs [46–50], or finite superfluid momentum [51,52], which, nevertheless, lead to a spherically symmetric star. It would be of interest to extend this research by incorporating the effects of the pressure anisotropy within the framework of general relativity.

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