Editors' Suggestion

Measuring *CP* violation in $b \to c\tau^-\bar{\nu}_\tau$ using excited charm mesons

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There is growing evidence for deviation from the standard model predictions in the ratios between semitauonic and semileptonic B decays, known as the $R(D^{(*)})$ puzzle. If the source of this nonuniversality is new physics, it is natural to assume that it also breaks CP symmetry. In this paper we study the possibility of measuring CP violation in semitauonic B decays, exploiting interference between excited charm mesons. Given the current values of $R(D^{(*)})$, we find that our proposed CP-violation observable could be as large as about 10%. We discuss the experimental advantages of our method and propose carrying it out at Belle II and LHCb.

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I. INTRODUCTION

Within the standard model (SM) of particle physics, the electroweak (EW) interactions obey flavor symmetry and hence exhibit lepton flavor universality (LFU). Observation of LFU breaking beyond that of the Yukawa interactions would be a clear sign of physics beyond the SM. In recent years, there have been accumulating experimental indications for possible LFU violation in the ratios of branching fractions

$$R(D^{(*)}) \equiv \frac{\mathrm{BR}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau)}{\mathrm{BR}(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell)}, \tag{1}$$

where ℓ denotes an electron or muon. An average of *BABAR* [1,2], Belle [3–5] and LHCb [6,7] measurements, calculated by the Heavy Flavor Averaging Group [8], yields

$$R(D) = 0.407 \pm 0.046, \quad R(D^*) = 0.304 \pm 0.015, \tag{2}$$

with a correlation coefficient of -0.2 between the $R(D)$ and $R(D^*)$ measurements. SM predictions have been calculated in Refs. [9–13] (for consistency, we follow the predictions considered in Ref. [8]),

$$R_{\mathrm{SM}}(D) = 0.299 \pm 0.011 \ [9],$$
$$R_{\mathrm{SM}}(D) = 0.300 \pm 0.008 \ [10], \tag{3}$$

$$R_{\mathrm{SM}}(D^*) = 0.252 \pm 0.003 \ [13]. \tag{4}$$

The combination of these results deviates by 4.1σ from the SM [8]. More recent calculations of $R(D^*)$ [14–16] reduce this tension somewhat, but do not solve the puzzle.

Another $b \to c\tau\bar{\nu}$ ratio was recently measured by LHCb [17],

$$R(J/\psi) = \frac{\mathrm{BR}(B_c^- \to J/\psi\tau^-\bar{\nu}_\tau)}{\mathrm{BR}(B_c^- \to J/\psi\mu^-\bar{\nu}_\mu)} = 0.71 \pm 0.25. \tag{5}$$

Although the SM predictions are in the range $R(J/\psi) = 0.25$–0.28, the absence of systematic estimation of the uncertainty and lattice calculations make it debatable whether this measurement increases the tension with respect to the SM.

The $R(D^{(*)})$ anomaly is puzzling and has received a great deal of attention. Future measurements, mostly by LHCb and Belle II, will greatly reduce the experimental uncertainties. If the disagreement with the SM becomes significant, it will constitute a clear signal of physics beyond the SM. New physics (NP) explanations for this puzzle have been widely discussed in the literature, where the most popular framework is that of effective field theory (EFT) with new dimension-six operators that enhance the taunic decays by about 30% (for a review, see, e.g., [18] and references therein). In order to explain the enhancement of the central value, all NP solutions introduce hard breaking of lepton flavor symmetry.

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A priori, there is no reason for NP models that solve the $R(D^{(*)})$ puzzle and break lepton flavor symmetry to not break CP at $\mathcal{O}(1)$ as well. Since the SM predicts unobservably small CP violation (CPV) in semileptonic B decays, looking for CPV in $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ can be a clean way to probe physics beyond the SM.

A naive observable of such CP violation is a direct asymmetry in $\bar{B} \to D^{(*)}\tau^-\nu$ transitions, i.e.,

$$\mathcal{A}_{CP}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau) = \frac{\mathrm{BR}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau) - \mathrm{BR}(B \to \bar{D}^{(*)}\tau^+\nu_\tau)}{\mathrm{BR}(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau) + \mathrm{BR}(B \to \bar{D}^{(*)}\tau^+\nu_\tau)}. \quad (6)$$

However, even if there is a NP amplitude with a new weak (i.e., CP-violating) phase, this asymmetry is very small due to the absence of a significant strong (i.e., CP conserving) phase between the interfering amplitudes in this process.

The object of this paper is to introduce and explore a new observable that incorporates strong phases, and thus is sensitive to CP violation in models that break lepton universality in $b \to c\tau\nu$ transitions. Other CPV observables have been suggested in Refs. [19,20]. The main idea in both cases was to use four-body decay kinematics to construct a triple product, thus avoiding the need for an explicit strong phase. To obtain a four-body decay from $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$, one can utilize the subsequent decay of the D^* into $D\pi$ [19], or the decay of the τ^- [20]. Construction of such observables requires knowing the momentum vectors of the τ^- and the $\bar{\nu}_\tau$ in the $\bar{B}$ rest frame [19], or is limited to use of semihadronic τ^- decays [20].

Here we discuss an alternative that is applicable for both leptonic and semihadronic τ^- decays. Furthermore, it does not require measurement of angular variables, although does benefit from even partial angular information that is experimentally obtainable. Our suggestion is to exploit interference between excited charm mesons. As can be easily understood in the Breit-Wigner approximation, interference between overlapping resonances gives rise to strong phases with known phase-space dependence.

The paper is organized as follows: In Sec. II we lay out our formalism and explain the basic mechanism for generating the strong-phase difference. In Sec. III we describe the asymmetry observable we suggest to measure. In Sec. IV we construct a simplified model to illustrate our method. Section V is dedicated to a discussion of differences between our simplified model and a realistic experiment. We conclude in Sec. VI.

II. FORMALISM AND BASIC MECHANISM

It is well known that observation of a CP asymmetry requires at least two interfering amplitudes with different weak and strong phases. Any new physics that modifies $b \to c\tau\nu$ transitions and breaks CP naturally provides a second amplitude with a weak-phase difference relative to the SM amplitude. However, the source of a strong phase is less trivial.

TABLE I. The spin, parity, mass, width, and decay modes of interest of the D^{**} mesons [21].

Particle	J^P	M (MeV)	Γ (MeV)	Decay Modes
D_0^*	0^+	2349	236	$D\pi$
D_1^*	1^+	2427	384	$D^*\pi$
D_1	1^+	2421	31	$D^*\pi$
D_2^*	2^+	2461	47	$D^*\pi$, $D\pi$

In our proposal, the strong-phase difference arises from overlapping excited charm-meson resonances. We consider the decays $\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$, where D^{**} is a generic name for the first four excited charm mesons, D_0^*, D_1^*, D_1, and D_2^*. The parameters of these states and some of their allowed decays are listed in Table I. The intermediate states D_0^* and D_2^*, as well as their interference, are selected by reconstructing the decay $D^{**} \to D\pi$. Similarly, the decay $D^{**} \to D^*\pi$ selects the states D_1^*, D_1, and D_2^*. Generally, the D_1 and D_2^* states are easier to study experimentally, since their widths are smaller.

Reference [22] shows a *BABAR* study of the semileptonic decays $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ with $D^{**} \to D^{(*)}\pi$. The integrated luminosity of Belle II will be more than 100 times larger, allowing precision measurements of the properties of the D^{**} states, as well as the $\bar{B} \to D^{**}$ form factors needed for interpretation of the $\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$ results. Similar measurements can be performed at LHCb. Additional studies of the D^{**} states can be performed with $\bar{B} \to D^{**}\pi^-$, and in some cases also with inclusive D^{**} production. We refer to these measurements as control studies, and note that similar studies were performed with $\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell$ and $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ as part of the measurements of $R(D^{(*)})$ [1–7]. Such studies are also necessary for a $R(D^{**})$ measurement predicted in [21,23].

In developing our formalism, we make three simplifying assumptions.

(1) The nonresonant $D^{(*)}\pi$ contribution to the $\bar{B} \to D^{(*)}\pi\tau^-\bar{\nu}_\tau$ decay is relatively small over the narrow $D^{(*)}\pi$ invariant-mass range of interest. While this contribution should be studied within an experimental analysis, it can be safely ignored for the purpose of the current discussion. Therefore, we write the amplitude for this decay as a sum over the intermediate D^{**} resonances denoted by the index i:

$$\mathcal{A} \equiv \mathcal{A}(\bar{B} \to D^{(*)}\pi\tau^-\bar{\nu}_\tau) = \sum_i \mathcal{A}(\bar{B} \to D_i^{**}(\to D^{(*)}\pi)\tau^-\bar{\nu}_\tau). \quad (7)$$

(2) We use the narrow-width approximation for the D^{**} mesons. Then the amplitude for the state D_i^{**} is

$$\mathcal{A}(\bar{B}\to D_i^{**}(\to D^{(*)}\pi)\tau^-\bar{\nu}_\tau) = \sum_\lambda \frac{i\mathcal{A}(\bar{B}\to D_i^{**}(\lambda)\tau^-\bar{\nu}_\tau)\mathcal{A}(D_i^{**}(\lambda)\to D^{(*)}\pi)}{m^2_{D^{(*)}\pi} - M^2_{D_i^{**}} + i\Gamma_{D_i^{**}}M_{D_i^{**}}}, \tag{8}$$

where $m^2_{D^{(*)}\pi}$ is the invariant mass of the $D^{(*)}\pi$ system, $M^2_{D_i^{**}}$ and $\Gamma_{D_i^{**}}$ are the mass and width of the intermediate D_i^{**} resonance, λ indicates the helicity of the D_i^{**}, and $\mathcal{A}(D_i^{**}(\lambda)\to D^{(*)}\pi)$ is the D_i^{**} decay amplitude.

(3) We further assume that there are no NP contributions in $\mathcal{A}(D_i^{**}(\lambda)\to D^{(*)}\pi)$, and that there is one NP $\bar{B}\to D^{**}\tau^-\bar{\nu}_\tau$ amplitude with a new weak phase $\varphi^{\rm NP}$. Therefore, we parametrize the total $\bar{B}\to D^{**}\tau^-\bar{\nu}_\tau$ amplitude as

$$\mathcal{A}(\bar{B}\to D_i^{**}(\lambda)\tau^-\bar{\nu}_\tau) = r_i^{\rm SM}e^{i\delta_i^{\rm SM}} + r_i^{\rm NP}e^{i(\varphi^{\rm NP}+\delta_i^{\rm NP})}. \tag{9}$$

Here

$$r_i^{\rm SM} = |C^{\rm SM}||\langle D_i^{**+}\tau^-\bar{\nu}_\tau|\mathcal{O}^{\rm SM}|\bar{B}^0\rangle|, \tag{10}$$

$$r_i^{\rm NP} = |C^{\rm NP}||\langle D_i^{**+}\tau^-\bar{\nu}_\tau|\mathcal{O}^{\rm NP}|\bar{B}^0\rangle|, \tag{11}$$

$$\varphi^{\rm NP} = \arg(C^{\rm NP}), \tag{12}$$

$$\delta_i^{\rm SM} = \arg(\langle D_i^{**+}\tau^-\bar{\nu}_\tau|\mathcal{O}^{\rm SM}|\bar{B}^0\rangle), \tag{13}$$

$$\delta_i^{\rm NP} = \arg(\langle D_i^{**+}\tau^-\bar{\nu}_\tau|\mathcal{O}^{\rm NP}|\bar{B}^0\rangle), \tag{14}$$

where $\mathcal{O}^{\rm SM}$ and $\mathcal{O}^{\rm NP}$ are the SM and NP operators contributing to the transition, respectively, and $C^{\rm SM}$ and $C^{\rm NP}$ are the corresponding Wilson coefficients. We neglect the tiny *CP* violation in the SM amplitude, and by redefinition of the states we set the SM weak phase to be $\varphi^{\rm SM}=0$.

In principle, the strong phases $\delta_i^{\rm SM}$ and $\delta_i^{\rm NP}$ depend on the kinematics of the event. This dependence is expected to be small, and we neglect it at that stage. Up to this small phase-space dependence, we redefine the states so as to set $\delta_i^{\rm SM}=0$. Furthermore, the strong phases $\delta_i^{\rm SM}$ and $\delta_i^{\rm NP}$ are equal in the heavy quark symmetry limit [24]. Thus, we cannot count on their difference to be large enough to make it possible to probe *CP* violation. Therefore, we adopt a conservative and simplifying approach, setting all these strong phases to zero. We elaborate on this in Sec. V A.

A known and large relative strong phase arises from interference between different overlapping D^{**} meson amplitudes in Eq. (7), particularly in the kinematic region $m^2_{D^{(*)}\pi}\sim M^2_{D^{**}}$. This is the source of strong-phase difference in our proposal. Using it to generate a sizable *CP* asymmetry requires that $\bar{B}\to D^{**}\tau^-\bar{\nu}_\tau$ amplitudes involving different D^{**} resonances also have different weak phases. As can be seen from Eqs. (9) through (11), such a weak-phase difference arises only if

$$\frac{r_i^{\rm NP}}{r_i^{\rm SM}} \neq \frac{r_j^{\rm NP}}{r_j^{\rm SM}}, \tag{15}$$

i.e., if the interfering resonances have different sensitivities to the NP operator relative to the SM operator. This happens only if the resonances have different spins and the SM and NP operators have different Dirac structures, i.e., $\mathcal{O}_{\rm NP}\neq\mathcal{O}_{\rm SM}$

We emphasize that in the case $\mathcal{O}_{\rm NP}=\mathcal{O}_{\rm SM}$, even if $\varphi^{\rm NP}\neq\varphi^{\rm SM}$, there is no relative weak phase between amplitudes involving different D^{**} mesons, and hence no *CP* asymmetry. Moreover, in this case, the τ angular distributions originating from the SM and NP operators are identical, so that the previously proposed methods [19,20] also become insensitive to CPV.

III. OBSERVABLE *CP* ASYMMETRY

A *CP* asymmetry is obtained by comparing the rate coming from Eq. (7) to its *CP* conjugate,

$$\mathcal{A}_{CP} = \frac{\int d\Phi(|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2)}{\int d\Phi(|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2)}, \tag{16}$$

where $\int d\Phi$ stands for partial phase-space integration, which is the main issue of this section. A four-body decay, such as $\bar{B}\to D^{**}(\to D^{(*)}\pi)\tau^-\bar{\nu}_\tau$, depends on five kinematical variables. We choose these to be

(i) q^2—the invariant mass of the $\tau^-\bar{\nu}_\tau$ system,
(ii) $m_{D^{(*)}\pi}$—the invariant mass of the $D^{(*)}\pi$ system,
(iii) θ_τ—the angle between the τ momentum and the direction opposite the $\bar{B}$ momentum in the $\tau^-\bar{\nu}_\tau$ rest frame,
(iv) θ_D—the angle between the $D^{(*)}$ momentum and the direction opposite the $\bar{B}$ momentum in the $D^{(*)}\pi$ rest frame.
(v) ϕ—the angle between the plane defined by the momenta of the $D^{(*)}$ and the π and the plane defined by the momenta of the τ and $\bar{\nu}_\tau$ in the $\bar{B}$ rest frame.

In general, choices regarding the $\int d\Phi$ integral need to balance two requirements. On the one hand, performing the analysis in terms of several phase-space variables is experimentally daunting. This is partly due to the complex modeling of correlated background distributions, but also due to the difficulty of measuring all the variables in the presence of unobservable neutrinos. On the other hand, integration leads to cancellation of opposite-sign contributions to the *CP* asymmetry in different regions of phase space. Thus, our goal is to optimize the phase-space integration with these considerations in mind.

First, we identify the integrals that make the asymmetry vanish. Since the main source of strong phases in our method is interference between excited charm mesons, integrals that reduce these interference terms are undesirable. In particular, since the phase of the Breit-Wigner amplitude varies as a function of $m^2_{D^{(*)}\pi}$, the distribution of this variable is critical for the analysis and must not be integrated over. Experimentally, $m^2_{D^{(*)}\pi}$ is straightforward to evaluate, since the 4-momenta of the $D^{(*)}$ and the π are directly measured. Furthermore, the $m^2_{D^{(*)}\pi}$ measurement resolution is much smaller than the widths $\Gamma_{D^{**}_i}$ and the mass differences $M_{D^{**}_i} - M_{D^{**}_j}$, which set the mass scale over which the strong-phase difference varies significantly.

Next, we consider the angular variables. A well-known fact in quantum mechanics is that while the angular-momentum operator does not commute with the momentum operator, $[\hat{\mathbf{P}}, \hat{\mathbf{L}}^2] \neq 0$, it does commute with its square, $[\hat{\mathbf{P}}^2, \hat{\mathbf{L}}^2] = 0$. Therefore, as long as we keep track of the directions of the D^{**} daughter particles, we do not know the spin of the D^{**}, allowing interference to take place. On the other hand, in a gedanken experiment that cannot measure momentum eigenstates but does measure the $\hat{\mathbf{L}}^2$ quantum number of the $D^{(*)}\pi$ two-particle wave function, interference between intermediate states of different spins is forbidden by selection rules. Mathematically, this can be understood from the orthogonality of the Y_{lm} spherical harmonic functions. Thus, we conclude that we must not integrate over the entire $D^{(*)}\pi$ angular range defined by both θ_D and ϕ. In the general case, integration over one of the phases does not completely cancel the asymmetry.

Since the strong phases come from the hadronic part of the decay, integrating over the leptonic phase-space variables θ_τ and q^2 does not in principle cancel the CP asymmetry. This is encouraging, since it is experimentally difficult to measure θ_τ and ϕ. In practice, however, integration over these angular variables does reduce the asymmetry. We study this effect, as well as the extent to which it can be mitigated, in the following section.

To summarize, we find that the experimentally simplest, nonvanishing CP asymmetry is

$$\mathcal{A}_{CP}((m_{D^{(*)}\pi}, \theta_D) \equiv \frac{\int d\theta_\tau d\phi dq^2(|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2)}{\int d\theta_\tau d\phi dq^2(|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2)}. \quad (17)$$

In the case of $D^{**} \to D^*\pi$ decays, we assume that integration over the decay angle of the $D^* \to D\pi$ or $D^* \to D\gamma$ decay is performed, and hence implicitly sum over the D^* helicity states.

IV. TOY MODEL

We consider a toy model in order to illustrate our method and obtain a rough estimation of the asymmetry. In what follows we make the following assumptions:

(1) The solution to the $R(D^{(*)})$ puzzle originates from new degrees of freedom that are heavier than the EW scale, and their effect can be represented by non-renormalizable terms in the Lagrangian.
(2) The NP modifies only $b \to c\tau^-\bar{\nu}_\tau$ transitions, while $b \to c\ell^-\bar{\nu}_\ell$ is given by the SM.
(3) We neglect EW breaking effect in the NP operators, i.e., we assume that the NP terms are invariant under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry of the SM. In practice, this means that we ignore the vector operator $(\bar{\tau}_L\gamma^\mu\nu_{\tau_L})(\bar{c}_R\gamma_\mu b_R)$.
(4) There are no new states that are lighter than the weak scale. In particular, there are no light right-handed neutrinos.

Under these assumptions, one finds that only four operators can break lepton universality in $b \to c\ell^-\bar{\nu}_\ell$ transitions [18]:

$$\mathcal{O}_{V_L^{(3)}} = (\bar{L}\gamma^\mu\tau^a L)(\bar{Q}\gamma_\mu\tau^a Q), \quad \mathcal{O}_{S_R} = (\bar{e}L)(\bar{Q}d), \quad (18)$$

$$\mathcal{O}_{S_L} = (\bar{e}L)(\bar{u}Q), \qquad \mathcal{O}_T = (\bar{e}\sigma^{\mu\nu}L)(\bar{u}\sigma_{\mu\nu}Q). \quad (19)$$

Following standard notation, here L and e are the $SU(2)_L$ doublet and singlet lepton fields, and Q, u and d are the $SU(2)_L$ doublet, up-singlet and down-singlet quark fields. Since the SM operator is $\mathcal{O}_{\rm SM} = \mathcal{O}_{V_{L^{(3)}}}$, the CP asymmetry is not sensitive to a NP phase in the Wilson coefficient of this operator. Therefore, we do not consider this operator further. At the scale of the B meson, it is more convenient to work in the broken phase with a different linear combination of the remaining operators. We use the following basis:

$$\mathcal{O}_S = (\bar{\tau}_R\nu_{\tau_L})(\bar{c}b), \qquad \mathcal{O}_P = (\bar{\tau}_R\nu_{\tau_L})(\bar{c}\gamma^5 b),$$
$$\mathcal{O}_T = (\bar{\tau}_R\sigma^{\mu\nu}\nu_{\tau_L})(\bar{c}\sigma_{\mu\nu}b). \quad (20)$$

We remark that our study is purely phenomenological, and we do not attempt to address solutions to the $R(D^{(*)})$ anomaly. Nevertheless, it is known in the literature (e.g., Ref. [18] and references therein) that if a single mediator is responsible for the anomaly, then there are four possible candidate mediators, labeled $W'_\mu \sim (1,3)_0$, $U_\mu \sim (3,1)_{2/3}$, $S \sim (3,1)_{-1/3}$, $V_\mu \sim (3,2)_{-5/6}$. The main role of those mediators is to generate a significant contribution to $\mathcal{O}_{V_L^{(3)}}$, which tends to solve the $R(D^{(*)})$ anomaly by means of new physics. Except for the case of W'_μ, integrating out the mediator generically leads to one or more of the operators $\mathcal{O}_{S_R}$, $\mathcal{O}_{S_L}$, $\mathcal{O}_T$ being of the same order of magnitude as the NP contribution to $\mathcal{O}_{V_L^{(3)}}$.

In what follows we study the $D^*\pi\tau^-\bar{\nu}_\tau$ final state of the D^{**} decays. We discuss the $D\pi\tau^-\bar{\nu}_\tau$ final state in Sec. VI. For the purpose of this proof-of-principle discussion, we make several simplifications. While some have been mentioned above, we collect them all here for completeness:

(1) We assume that the observation of D^* includes integration over the D^* decay angle. Therefore, we neglect interference of different D^* helicity states.
(2) We neglect CPV in the SM. This is exact up to tiny higher-order corrections to the tree-level SM process.
(3) We consider interference only between the narrow D_1 and D_2^* resonances, ignoring the broad D_1^*, which also decays to $D^*\pi$. As in the case of the ignored nonresonant amplitude discussed in Sec. II, the broad resonance contributes little over the small mass range covered by the narrow resonances.
(4) We use the Breit-Wigner approximation for the resonances, as shown explicitly in Eq. (8). We expect this to be a good approximation close to the resonance peak, and become less precise farther from the peak. Corrections to this limit can be accommodated if needed. See details in, e.g., the resonances section of [25] and references therein.
(5) We assume factorization of the hadronic current and the leptonic currents in Eqs. (10)–(11), i.e., $\langle D^{**+}\tau^-\bar{\nu}_\tau|\mathcal{O}|\bar{B}^0\rangle \simeq \langle D^{**+}|\mathcal{O}_q|\bar{B}^0\rangle\langle\tau^-\bar{\nu}_\tau|\mathcal{O}_\ell|0\rangle$.
(6) We calculate the leptonic currents to leading order (LO) in perturbation theory.
(7) We calculate the $\bar{B}\to D^{**}$ transition to LO in the heavy quark limit, namely, neglecting corrections of order $\Lambda_{\rm QCD}/m_c$. This assumption has two implications. First, as discussed above, we set the non-Breit-Wigner strong phases to zero. Second, we set all form factors to be the same and equal to a single Isgur-Wise function. The hadronic matrix elements $\langle\rangle D_i^{**}|\mathcal{O}|\bar{B}$ are given explicitly in Appendix A. Subleading $1/m_Q$ and α_s corrections are given in [21].
(8) For the D^{**} decay amplitude we use an approximate model inspired by leading-order heavy quark effective theory (HQET). Details are given in Appendix B.

It is important to note that these simplifications do not change the major conclusions of our study. Furthermore, as discussed in Sec. V, they pose no limitation for actual analysis of experimental data.

A. Results and cross-checks

Using the above simplified model, we calculate the *CP* asymmetry of Eq. (17). For this purpose, we set the Wilson coefficient of one of the operators in Eq. (20) to $C_A^{\rm NP} = 0.15(1+i)C_{\rm SM}$ (where $A = S/P/T$), while setting the others to zero. The choice of this value is arbitrary, but motivated by the ∼30% enhancement of the central values of $R(D^{(*)})$ with respect to the SM expectation. We choose $(1+i)$ to obtain an arbitrary order-one value for $\varphi^{\rm NP}$.

It is unrealistic that the UV physics that solves the $R(D^{(*)})$ anomaly generates just a single operator as is assumed in our phenomenological study. However, we explicitly checked that using a generic linear combination of those operators, particularly either one of the motivated combinations $\mathcal{O}_{S_R}$ or $\mathcal{O}_{S_L}$, does not significantly modify our results.

In Fig. 1 we plot the asymmetry of Eq. (17) as a function of the $D^*\pi$ invariant mass $m_{D^{(*)}\pi}$ and the D^{**} decay angle θ_D for the three NP operators. We find the asymmetry to be of order one percent.

In addition, we study the implication of having partial knowledge of the τ angular distribution. To simplify this study, we take a representative value of the $D^*\pi$ invariant mass, fixing it to be between the peaks of the two resonances, $m_{D^*\pi} = (M(D_1) + M(D_2^*))/2$, where the Breit-Wigner phase difference is large. We then plot the asymmetry in the plane of θ_D vs either θ_τ (Fig. 2) or ϕ (Fig. 3), after integrating over the remaining variables. As shown in these figures, retaining the θ_τ or ϕ dependence leads to up to an order of magnitude enhancement in the asymmetry. As mentioned in the Introduction, one objective of our study is to propose a *CP*-violation analysis which, in contrast to previous proposals, does not require a full angular analysis. Nonetheless, one can slightly relax this requirement and observe a larger asymmetry when measuring two of the three phase-space angles. We discuss the experimental aspects of this approach in Sec. V B.

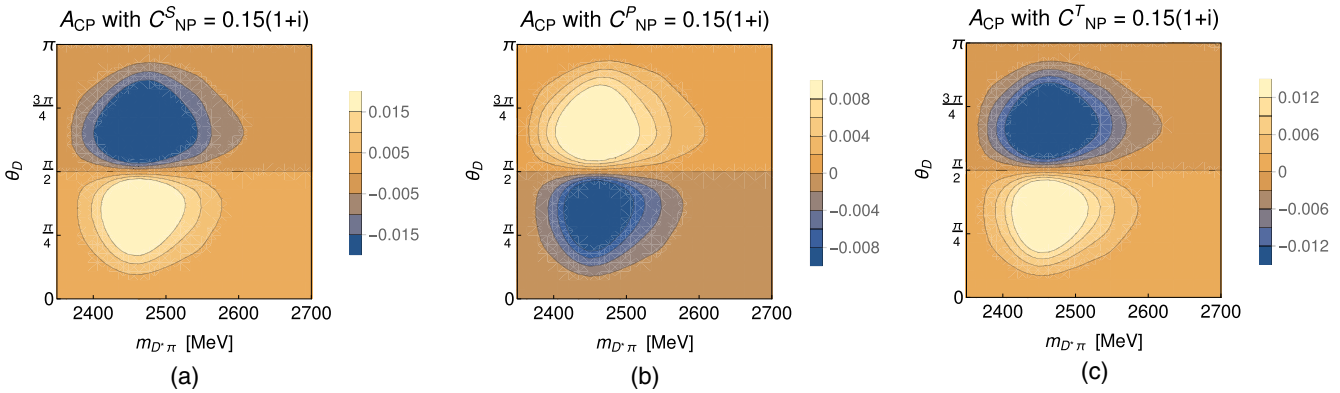

FIG. 1. The *CP* asymmetry of Eq. (17) as a function of the $D^*\pi$ invariant mass $m_{D^{(*)}\pi}$ and the D^{**} decay angle θ_D for a (a) scalar, (b) pseudoscalar, and (c) tensor NP operator.

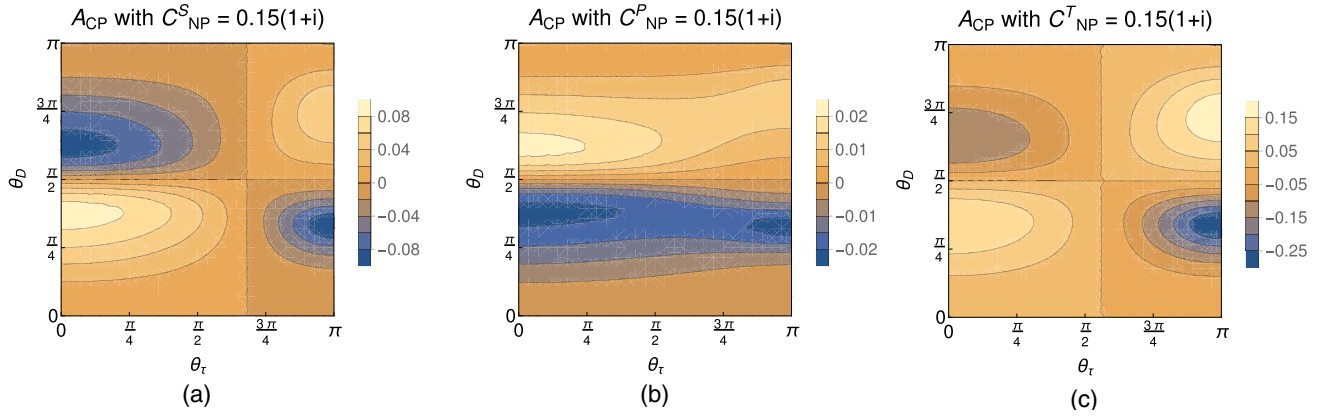

FIG. 2. The CP asymmetry of Eq. (17) as a function of the angles θ_D and θ_τ for a fixed value of the $D^*\pi$ invariant mass, for a (a) scalar, (b) pseudoscalar, and (c) tensor NP operator.

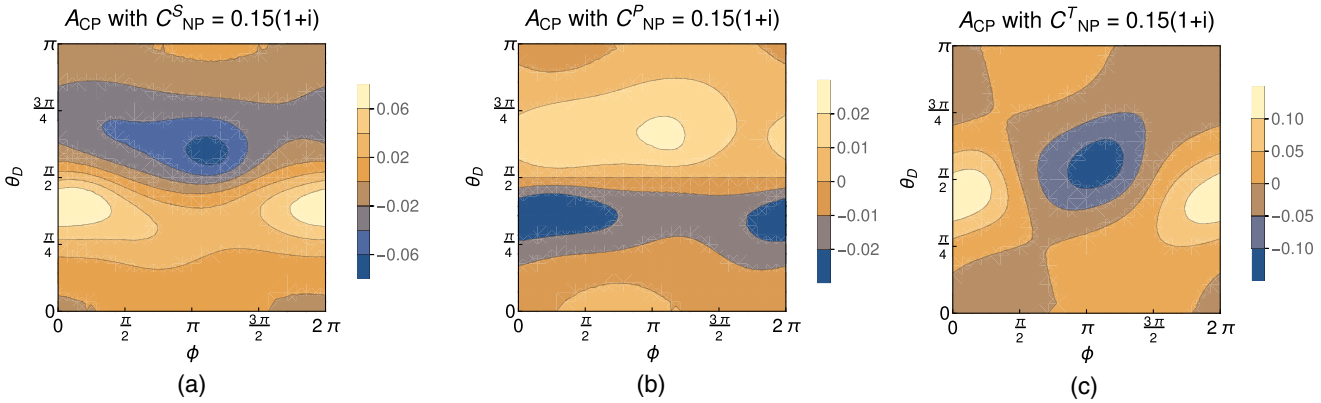

FIG. 3. The CP asymmetry of Eq. (17) as a function of the angles θ_D and ϕ for a fixed value of the $D^*\pi$ invariant mass, for a (a) scalar, (b) pseudoscalar, and (c) tensor NP operator.

In order to verify the validity of our numerical results, we performed a set of cross-checks. For a random point in phase space, we verified that the CP asymmetry vanishes when the NP Wilson coefficients are set to be real, i.e., $\varphi^{NP} = 0$. This is shown in Fig. 4(a) for the scalar operator, and similar results are obtained for the pseudoscalar and tensor cases. For complex Wilson coefficients, we verified that before phase-space integration there is a ϕ-dependent CP asymmetry even in the case of a single mediator, as in Ref. [19]. A remnant of this effect is seen in the off-

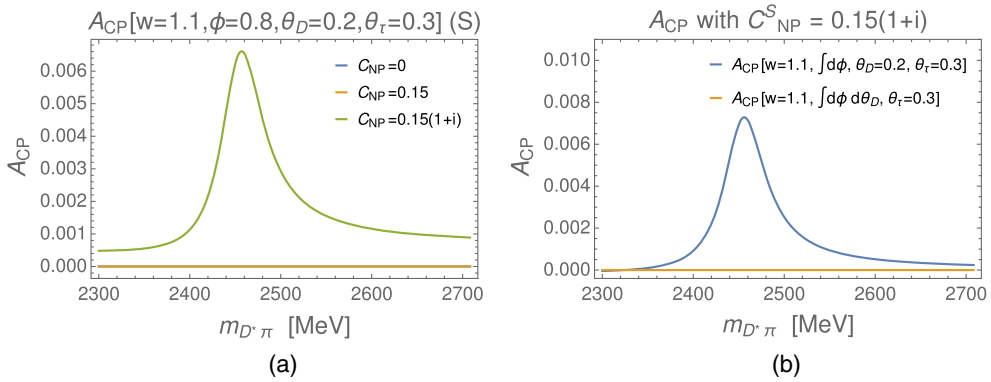

FIG. 4. Representative cross-checks, showing the CP asymmetry in an arbitrary point in phase space for the case of a scalar NP operator. (a) Results obtained with (blue) no NP, (orange) NP without CP violation, and (green) NP with CP violation. (b) Results obtained with NP with CP violation, after integration over (blue) ϕ or (orange) ϕ and θ_D.

resonance sidebands of the green curve in Fig. 4(a). As can be seen in Fig. 4(b), this asymmetry vanishes after integrating over ϕ (blue curve). Finally, as discussed above, we have verified that for a fixed arbitrary angle θ_τ, the asymmetry vanishes after integration over the hadronic angles θ_D and ϕ. This is shown by the orange curve in Fig. 4(b).

V. A DETAILED EXTENSION TO REAL LIFE EXPERIMENT

In the previous section we listed the assumptions used in our proof-of-principle study. In an actual analysis of collider data, obtaining precise measurement of the asymmetry requires replacing these assumptions with experimental results with properly evaluated uncertainties. In Sec. V A we discuss the validity of these assumptions and explain that they do not change the nature of our conclusions, particularly the general magnitude of the asymmetry for given values of the complex NP Wilson coefficients. Experimental considerations are discussed in Sec. V B.

A. Theoretical considerations

(1) As long as we integrate over the $D^* \to D\pi$ angular distribution, there is no interference between different D^* helicity states. Therefore, the implicit incoherent sum over different helicities in Eq. (17) is precise. Nevertheless, retaining some information about the $D\pi$ angular distribution would generally give rise to interference between different helicities of the D^*. As D_2^* decays only to transverse D^* (see Appendix B), this would result in a somewhat larger asymmetry. Another effect related to the D^* angular distribution that might enhance the asymmetry somewhat, and which we do not study here, is similar to that of the D angular distribution studied in Ref. [19].

(2) The assumption of tiny CPV within the SM is straightforward, and does not require additional discussion.

(3) In our calculations of the asymmetry we considered only the D_1 and D_2^* resonances. The only additional resonance that can contribute to the $D^*\pi$ final state is D_1^*. Since this state is very broad, its contribution to the observed final state across the narrow $m_{D^{(*)}\pi}$ range defined by the D_1 and D_2^* is small, and its phase varies only little. As a result, it does not affect our study significantly. It is advisable, however, to account for the D_1^*, along with nonresonant and background contributions, in the experimental data analysis. The experimental analysis would anyway obtain the parameters of all the D^{**} resonances and the relevant form factors from the control studies described in Sec. II, particularly with $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ [22]. In addition, matrix elements for the $\bar{B} \to D_1^*$ transition can be taken from [21].

(4) Although we use the Breit-Wigner approximation, the shape of the resonance, and in particular the $M_{D^*\pi}$ dependence of the strong phase, should be measured as one of the control studies.

(5) To LO in HQET, all strong phases $\delta_i^{\rm SM}$, δ_i^{NP} are equal. Moreover, strong phases are expected to be small when the final state has only one hadron, as in this case, due to the absence of rescattering. In any case, the phase-space dependence of these phases is small, and thus does not lead to cancellation of the asymmetry. Finally, we note that these phases can be measured in $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$. Therefore, deviations from the assumptions outlined here do not change the conclusions of our work.

(6) Corrections to the factorization assumption and Next to leading order (NLO) corrections to the leptonic currents are $\alpha_{\rm EM}$ suppressed and thus negligible.

(7) NLO corrections to $\bar{B} \to D^{**}$ transition form factors are as large as tens of percent. Therefore $\mathcal{O}(1)$ corrections to the expected asymmetry arising from these terms are expected. These corrections can be taken from Ref. [21] and studied in $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ decays. In any case, they are not expected to significantly change the global picture that arises from our study. For instance, as pointed out in [21] LO fails to predict the ratio $\mathrm{BR}(\bar{B} \to D_2^*\ell\bar{\nu})/\mathrm{BR}(\bar{B} \to D_1\ell\bar{\nu})$. We checked explicitly that correcting for this discrepancy modifies our main results by $\mathcal{O}(10\%)$.

(8) We use LO HQET for modeling the D^{**} decay. Large corrections to this approximation are expected in purely charmed systems. In particular, it is predicted [26,27] that $D_1 - D_1^*$ mixing leads to significant S-wave contribution. As can be seen from Eq. (B3), this leads to enhancement of the helicity-amplitude ratio $|\mathcal{A}_{10}^1|/|\mathcal{A}_{00}^1|$ (see Appendix B). Since different helicity amplitudes do not interfere, and $\mathcal{A}_{00}^2 = 0$ due to selection rules, a larger value of $|\mathcal{A}_{10}^1|/|\mathcal{A}_{00}^1|$ enhances the interference of D_1 and D_2^*, increasing the asymmetry. Thus, our use of LO HQET leads to a conservative estimate of the *CP*-violating signal. In any case, the modeling of the D^{**} decay will be improved by precise measurements of the helicity amplitudes at Belle II and LHCb, as part of the control studies.

B. Experimental considerations

(1) First, it is desirable to estimate the achievable uncertainty on the asymmetry. Such an estimate is bound to be highly inaccurate, due to lack of any experimental studies of $\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$. Nonetheless, one can gain some insight from a study of $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ by *BABAR* [22], performed with full hadronic reconstruction of the other B meson in the event.

For example, *BABAR* finds 165 ± 18 events in the channel $B^- \to D_1^0 \ell^- \bar{\nu}_\ell$. At Belle II, the integrated luminosity will be about 100 times larger. However, the branching fractions for $\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$ are expected to be about 10 times smaller than those of $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ [28]. Hence, a naive scaling of the *BABAR* result to Belle II yields a $B^- \to D_1^0 \ell^- \bar{\nu}_\ell$ signal of $(165 \times 10) \pm (18 \times \sqrt{10})$ events. This assumes that the signal efficiency and signal-to-background ratio remain as in Ref. [22]. There is no reason to think that these assumptions are correct, since the two detectors, integrated luminosities, and analysis optimization procedures are very different. The different signal-to-background ratios can be approximately corrected for. For this purpose, we note that Fig. 1(a) of Ref. [22] indicates a background yield of about 30 events under the D_1 and D_2^* peaks. If we naively assume that this background has negligible impact on the signal yield uncertainty in Ref. [22] and that it will become 3000 events in the Belle II analysis due to the 100-fold increase in integrated luminosity, then the expected uncertainty on the $B^- \to D_1^0 \tau^- \bar{\nu}_\tau$ signal yield at Belle II becomes about $\sqrt{18^2 \times 10 + 3000} \approx 80$ events. From this, one finds that the uncertainty on a phase-space-integrated asymmetry would be 5%.

While keeping in mind the caveats about the large inaccuracy of this uncertainty estimate, we note that full exploitation of Belle II data would include also $\bar{B}^0$ decays, additional D^{**} resonances, and additional methods for reconstruction of the other B meson in the event, reducing the overall uncertainty. Furthermore, our estimate pertains only to Belle II, while LHCb is also likely to contribute significantly to this measurement.

(2) In Sec. IV A we showed that analyzing the CP asymmetry in terms of θ_τ, in addition to θ_D, helps avoid cancellations and results in a large increase of the asymmetry. Thus, it is important to understand whether θ_τ can be determined with sufficient precision despite the unobservable neutrinos. We show here that this can be done at Belle II, using the momentum $\vec{p}_\ell$ of the observed light lepton produced in the leptonic decay $\tau^- \to \ell^- \nu_\tau \bar{\nu}_\ell$. The kinematic constraints of the $e^+e^- \to B\bar{B}$ production process provide information about the 3-momentum $\vec{p}_B$ of the $\bar{B}$ meson is known. As a result, the 3-momentum $\vec{q}$ of the $\tau^-\bar{\nu}$ system is determined to within about 30 or 300 MeV, depending on whether the other B meson in the event is fully reconstructed via a hadronic decay [1–3,5] or partially reconstructed in a semileptonic decay [4]. LHCb has also demonstrated the ability to measure $\vec{q}$ with some precision [6,7]. Knowledge of $\vec{p}_\ell$ and $\vec{q}$ enables measurement of θ_ℓ, the angle between $\vec{p}_\ell$ and $-\vec{p}_B$ in the $\tau^-\bar{\nu}$ rest frame.

By simulating the kinematics of the full decay chain using EvtGen [29] within the Belle II software framework, we find that knowledge of θ_ℓ gives θ_τ to within an uncertainty of about $\pi/4$. As can be seen from Fig. 2, this precision is sufficient for significantly reducing the cancellation that would otherwise arise from integration over θ_τ. Thus, observation of large asymmetry values, close to those seen in Fig. 2 for the scalar and tensor cases, is in principle possible.

In the semihadronic decays $\tau^- \to \pi^-(n\pi^0)\nu_\tau$ and $\tau^- \to \pi^-\pi^+\pi^-(n\pi^0)\nu_\tau$ [where $(n\pi^0)$ stands for additional neutral pions that may or may not be reconstructed], the corresponding angles θ_π and $\theta_{3\pi}$, respectively, should give a somewhat more precise estimate of θ_τ than that obtained from θ_ℓ, thanks to the presence of only one neutrino in the decay. In $\tau^- \to \pi^-\pi^+\pi^-(n\pi^0)\nu_\tau$, vertexing provides additional information on θ_τ. At Belle II, the τ flies an average distance of 50 μm before decaying, while the position resolutions on the B decay vertex and on the $\pi^-\pi^+\pi^-$ production vertex are about 25 μm each [30,31]. This too enables an estimate of θ_τ with an uncertainty of order $\pi/4$.

The locations of the B and τ decay vertices also allow an estimate of the angle ϕ with similar precision. We note that LHCb has already demonstrated successful use of the $\tau^- \to \pi^-\pi^+\pi^-(n\pi^0)\nu_\tau$ decay vertex to study $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ [7]. Thus, LHCb may be able to determine ϕ and θ_τ in this way with better precision than Belle II.

(3) Finally, we note that as for any multibody final state, one has to account for the dependence of the reconstruction efficiency on the phase-space variables. This applies to both the measured and the integrated variables.

VI. SUMMARY AND CONCLUSIONS

In this paper we suggest a new method to study CP violation in $\bar{B} \to D^{**}(\to D^{(*)}\pi)\tau^-\bar{\nu}_\tau$ decays. Our motivation is based on the so-called $R(D^{(*)})$ anomaly of lepton flavor nonuniversality in $\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau$ decays. If this anomaly is indeed a result of physics beyond the SM, it is natural to assume that the new physics amplitude may also have an order-one CP-violating phase with respect to the SM amplitude.

The source of strong-phase difference in our scheme is interference between intermediate D^{**} resonances. In the Breit-Wigner approximation, it is transparent that this phase difference obtains large values when the invariant mass of the D^{**} final state is in the range between the interfering resonance peaks.

We study in detail the case of interference between the narrow resonances D_1 and D_2^*, which decay to the common final state $D^*\pi$. While the D_1^* also decays to $D^*\pi$, we do not expect it to contribute significantly to the asymmetry due to its large width, which results in small overlap with the narrow resonances. The $D\pi$ final state may also be used for this measurement. In this case, interference takes place only between the D_2^* and the D_0^*. The large width of the D_0^* again leads to a small expected asymmetry.

We find that an order-one CP-violating phase in the new-physics amplitude results in an order-percent CP asymmetry [Eq. (17)] when integrating over the $\tau^-\bar{\nu}_\tau$ kinematics. We also observe that partial measurement of the direction of the τ^- momentum leads to an order-of-magnitude enhancement of the asymmetry. We outline how such a measurement can be performed at Belle II and LHCb. Our main results are summarized in Figs. 1, 2, and 3.

We make several approximations that help clarify the physical effect and its kinematical dependence while also giving the correct order of magnitude for the asymmetry. We discuss the use of control studies, particularly with $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ and $\bar{B} \to D^{**}\pi^-$, for the purpose of studying these approximations, improving upon them, and obtaining the related systematic uncertainties.

Our main goal is to study an observable that can probe NP by observing a nonvanishing CP asymmetry. However, we note that a full analysis, which includes measurement of $R(D^{**})$, the strong phases, and form factors, yields the value of the underlying CPV phase $\varphi^{\rm NP}$. Clearly, initial observation of an asymmetry would provide the motivation for such an analysis.

We discuss the uncertainty with which the proposed CP asymmetry can be measured at Belle II, using a *BABAR* study of $B^- \to D_1^0\ell^-\bar{\nu}_\ell$ with hadronic reconstruction of the other B meson in the event. We find that for this channel alone, the uncertainty is about 5%. Nonetheless, we caution that such an estimate is highly inaccurate, and encourage more detailed experimental studies to obtain a better estimate, at both Belle II and LHCb.

As a last remark, we note that while current measurements motivate searching for new physics and CP violation in $\bar{B} \to D^{**}\tau^-\bar{\nu}_\tau$, our method can also be applied to the search for a CP asymmetry in any other $\bar{B}$ decay involving D^{**} mesons, including $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$. In this case, one loses the benefit of using $\bar{B} \to D^{**}\ell^-\bar{\nu}_\ell$ for control studies. This is likely to result in reduced sensitivity, but cannot create a fake CP asymmetry.[1] Such a search can provide a powerful test for CP violation in semileptonic $\bar{B}$ decays.

[1]As in any measurement of a CP-odd observable, systematic effects related to the CP asymmetry of the detector need to be accounted for.

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APPENDIX A: $B \to D^{**}$ HQET

For this paper to be self-contained, we give the leading-order HQET terms for the $B \to D^{**}$ matrix elements. For the D_1 meson we have

$$\frac{\langle D_1(v',\epsilon)|S|B(v)\rangle}{\sqrt{M_D M_B}} = -\sqrt{\frac{2}{3}}\tau(w)(1+w)\epsilon^*\cdot v, \tag{A1}$$

$$\frac{\langle D_1(v',\epsilon)|P|B(v)\rangle}{\sqrt{M_D M_B}} = 0, \tag{A2}$$

$$\frac{\langle D_1(v',\epsilon)|V^\mu|B(v)\rangle}{\sqrt{M_D M_B}} = \frac{\tau(w)}{\sqrt{6}}\{(1-w^2)\epsilon^{*\mu} - [3v^\mu + (2-w)v'^\mu]\epsilon^*\cdot v\}, \tag{A3}$$

$$\frac{\langle D_1(v',\epsilon)|A^\mu|B(v)\rangle}{\sqrt{M_D M_B}} = -\frac{i\tau(w)}{\sqrt{6}}(1+w)\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v_\rho v'_\sigma, \tag{A4}$$

$$\frac{\langle D_1(v',\epsilon)|T^{\mu\eta}|B(v)\rangle}{\sqrt{M_D M_B}} = -\frac{i\tau(w)}{\sqrt{6}}[(1+w)(v-v')^{[\mu}\epsilon^{*\eta]} + 3v^{[\eta}v'^{\mu]}\epsilon^*\cdot v]. \tag{A5}$$

Similarly, for D_2^* we find

$$\frac{\langle D_2^*(v',\epsilon)|S|B(v)\rangle}{\sqrt{M_D M_B}} = 0, \tag{A6}$$

$$\frac{\langle D_2^*(v',\epsilon)|P|B(v)\rangle}{\sqrt{M_D M_B}} = \tau(w)\epsilon^*_{\mu\nu}v^\mu v^\nu, \tag{A7}$$

$$\frac{\langle D_2^*(v',\epsilon)|V^\mu|B(v)\rangle}{\sqrt{M_D M_B}} = -i\tau(w)\varepsilon^{\mu\nu\rho\sigma}\epsilon^*_{\nu\alpha}v^\alpha v_\rho v'_\sigma, \tag{A8}$$

$$\frac{\langle D_2^*(v',\epsilon)|A^\mu|B(v)\rangle}{\sqrt{M_D M_B}} = -\tau(w)\epsilon^*_{\nu\alpha}v^\alpha[(1+w)g^{\mu\nu} - v^\nu v'^\mu], \tag{A9}$$

$$\frac{\langle D_2^*(v',\epsilon)|T^{\mu\eta}|B(v)\rangle}{\sqrt{M_D M_B}} = -\tau(w)\varepsilon^{\mu\eta\rho\sigma}\epsilon^*_{\rho\alpha}v^\alpha(v+v')_\sigma. \tag{A10}$$

Above v, v' are, respectively, the velocities of the B, D^{**} mesons, $A^{[\mu}, B^{\nu]}$ stands for the antisymmetrization $A^\mu B^\nu - A^\nu B^\mu$, $\varepsilon^{\alpha\beta\mu\nu}$ is the completely antisymmetric tensor, and ϵ^μ and $\epsilon^{\mu\nu}$ are spin-one and spin-two polarization

tensors. For the D_2^* meson moving in the $\hat{z}$ direction, we use the massive spin-two polarization tensors

$$\epsilon^{\mu\nu}(0) = \frac{1}{\sqrt{6}} \begin{pmatrix} -\frac{2p^2}{m^2} & 0 & 0 & -\frac{2pE}{m^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{2pE}{m^2} & 0 & 0 & -\frac{2E^2}{m^2} \end{pmatrix}, \tag{A11}$$

$$\epsilon^{\mu\nu}(\pm 1) = \frac{1}{2} \begin{pmatrix} 0 & \frac{p}{m} & \mp i\frac{p}{m} & 0 \\ \frac{p}{m} & 0 & 0 & \frac{E}{m} \\ \mp i\frac{p}{m} & 0 & 0 & \mp i\frac{E}{m} \\ 0 & \frac{E}{m} & \mp i\frac{E}{m} & 0 \end{pmatrix}, \tag{A12}$$

$$\epsilon^{\mu\nu}(\pm 2) = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \mp i & 0 \\ 0 & \mp i & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A13}$$

Finally, for the Isgur-Wise function $\tau(\omega)$ we follow the leading-order result of [21], $\tau(w) \simeq 2 - 0.9\omega$. We checked that our results are insensitive to $\mathcal{O}(1)$ modification of this function.

APPENDIX B: MODELING OF THE D^{**} DECAY

In order to model the D^{**} decay, we relate helicity amplitudes to partial waves. Using the helicity formalism (e.g., [32,33]) the $D^{**} \to D^*\pi$ amplitude is given by

$$\mathcal{A}(D^{**}(\lambda) \to D^{*+}(\lambda')\pi^-) = \sqrt{\frac{2J+1}{4\pi}} e^{-i\varphi(\lambda-\lambda')} d^J_{\lambda\lambda'}(\theta_D) \mathcal{A}^J_{\lambda' 0}, \tag{B1}$$

where J is the spin of the D^{**} meson, λ and λ' are the helicities of the D^{**} and D^* mesons, respectively, and $d^J_{\lambda\lambda'}$ are the Wigner functions [25]. In the helicity amplitude $\mathcal{A}^J_{\lambda' 0}$ the zero stands for the pion helicity. In the case of D^{**} mesons, the projection of the helicity amplitudes to partial waves is given by [34]

$$\mathcal{A}^J_{\lambda' 0} = (-1)^{1-J} \sqrt{\frac{1}{2J+1}} C_{10}(J, \lambda'; \lambda', 0) S + \sqrt{\frac{5}{2J+1}} C_{21}(J, \lambda'; 0, \lambda') D, \tag{B2}$$

where S, D are partial wave functions, and $C_{j_1 j_2}(J, M; m_1, m_2)$ are Clebsch-Gordan coefficients. It follows that

$$\mathcal{A}^1_{00} = \sqrt{\frac{1}{3}} S - \sqrt{\frac{2}{3}} D, \quad \mathcal{A}^1_{10} = \mathcal{A}^1_{-10} = \sqrt{\frac{1}{3}} S + \sqrt{\frac{1}{6}} D, \tag{B3}$$

$$\mathcal{A}^2_{00} = 0, \qquad \mathcal{A}^2_{10} = -\mathcal{A}^2_{-10} = -\sqrt{\frac{1}{2}} D. \tag{B4}$$

We emphasize that the above results are exact. In order to proceed, we use leading-order HQET for D^{**} decays [26]. To LO D_1 does not decay through S wave, thus $\mathcal{A}^1_{00} = -2\mathcal{A}^1_{10}$. Another prediction of leading-order HQET is that the ratio of the decay rates of $D_1 \to D^*\pi$ and $D_2^* \to D^*\pi$ is 5/3. We, therefore, find

$$\frac{\Gamma(D_1 \to D^*\pi)}{\Gamma(D_2^* \to D^*\pi)} \simeq \frac{|\mathcal{A}^1_{00}|^2 + 2|\mathcal{A}^1_{10}|^2}{2|\mathcal{A}^2_{10}|^2} \Rightarrow \frac{|\mathcal{A}^1_{10}|^2}{|\mathcal{A}^2_{10}|^2} = \frac{5}{9}. \tag{B5}$$

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