Determination of the J/ψ chromoelectric polarizability from lattice data

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(Received 8 February 2018; revised manuscript received 13 July 2018; published 30 August 2018)

The chromoelectric polarizability of J/ψ is extracted from lattice QCD data on the nucleon-J/ψ potential in the heavy quark limit. The value of $\alpha(1S) = (1.6 \pm 0.8)\ \mathrm{GeV}^{-3}$ is obtained. This value may have a systematic uncertainty due to lattice artifacts which cannot be estimated at present, but will become controllable in future studies. We also comment on the possibility of hadrocharmonia.

DOI: 10.1103/PhysRevD.98.034030

I. INTRODUCTION

The chromoelectric polarizability α of a hadron describes the hadron's effective interaction with soft gluonic fields. This property is analogous to the electric polarizability quantifying the response of a neutral atom placed in an external electric field, which describes the emergence of induced dipole moments and van der Waals forces.

The chromoelectric polarizabilities of charmonia are important quantities in the heavy quark effective theory. Among their most interesting applications are studies of hadrocharmonia: when the compact charmonium penetrates a light hadron, its interaction with the soft gluon fields inside the hadron is systematically described in terms of a multipole expansion [1,2]. The strength of the effective interaction between a charmonium and a light hadron is determined by the chromoelectric polarizability of the charmonium [3,4]. If this effective interaction is strong enough, bound states emerge: hadrocharmonia [4–7]. The binding of J/ψ in nuclear medium and nuclei was also studied [8–11].

Other important applications of chromoelectric polarizabilities include the description of hadronic transitions between charmonium resonances [12,13] and the interaction of slow charmonia with a nuclear medium. The chromoelectric polarizabilities also play a vital role for the understanding of photoproduction and hadroproduction of charmonia and charmed hadrons on nuclear targets with important applications for the diagnostics of the creation of quark gluon plasma in heavy-ion collisions; see [14–18] and references therein.

Despite their importance little is known about the phenomenological values of these nonperturbative charmonium properties. Only on the transitional chromoelectric polarizability $\alpha(2S \to 1S)$ is some information available [4]. The value of $\alpha(1S)$ could in principle be inferred from the rare decay $J/\psi \to \pi\pi\ell^+\ell^-$ with soft pions [19], but such an analysis is challenging and $\alpha(1S)$ is not yet known.

A nonperturbative determination of $\alpha(1S)$ is therefore of great importance. In this work we present a method to determine $\alpha(1S)$ from lattice QCD calculations of the J/ψ-nucleon potential. We estimate conservatively the theoretical uncertainties which are associated with underlying assumptions, and discuss critically how these assumptions can be tested with future lattice data. We also comment on the possibility of nucleon-$\psi(2S)$ bound states.

II. THE EFFECTIVE QUARKONIUM-BARYON INTERACTION

The interaction of a heavy quarkonium with a baryon is dominated in the heavy quark limit by the emission of two virtual color-singlet chromoelectric dipole gluons [3,4] and described, for S-wave quarkonia, by an effective potential in terms of the quarkonium chromoelectric polarizability α and energy-momentum tensor (EMT) densities of the baryon as [7]

$$V_{\mathrm{eff}}(r) = -\alpha \frac{4\pi^2}{b} \frac{g_c^2}{g_s^2}\left(\nu T_{00}(r) - 3p(r)\right),$$
$$\nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}, \qquad (1)$$

where $b = (\frac{11}{3}N_c - \frac{2}{3}N_f)$ is the leading coefficient of the Gell-Mann–Low function, g_c (g_s) is the strong coupling constant renormalized at the scale μ_c (μ_s) associated with the heavy quarkonium (baryon) state. The parameter ξ_s denotes the fraction of the baryon energy carried by gluons at the scale μ_s [13]. In Eq. (1) $T_{00}(r)$ and $p(r)$ are the energy density and pressure inside the baryon [20], which satisfy respectively

$$\int d^3r T_{00}(r) = M_B, \qquad \int d^3r p(r) = 0, \tag{2}$$

where M_B denotes the mass of the baryon. The derivation of Eq. (1) is justified in the limit that the ratio of the quarkonium size is small compared to the effective gluon wavelength [4], and a numerically small term proportional to the current masses of the light quarks is neglected.

Due to Eq. (2) the effective potential has the following normalization and mean square radius

$$\int d^3r V_{\rm eff}(r) = -\alpha \frac{4\pi^2}{b}\frac{g_c^2}{g_s^2}\nu M_B,$$
$$\langle r^2_{\rm eff}\rangle \equiv \frac{\int d^3r r^2 V_{\rm eff}(r)}{\int d^3r V_{\rm eff}(r)} = \langle r_E^2\rangle - \frac{12 d_1}{5\nu M_B^2} \tag{3}$$

with the mean square radius of the energy density $\langle r_E^2\rangle = \int d^3r r^2 T_{00}(r)/M_B$ and the D-term $d_1 = \frac{5}{4}M_B \int d^3r r^2 p(r)$ [20,21]. Using the normalization condition for $V_{\rm eff}$ in Eq. (3) to eliminate the ratio $(g_c/g_s)^2$ from Eq. (1) and exploring the large-r behavior of $T_{00}(r)$ and $p(r)$ derived in [22] we obtain the following expression for the long-distance behavior of $V_{\rm eff}(r)$ in the chiral limit, which is convenient for our purposes:

$$V_{\rm eff}(r) = \frac{27}{16\pi^2}\frac{1+\nu}{\nu}\frac{g_A^2}{M_B F_\pi^2}\frac{1}{r^6}\int d^3r' V_{\rm eff}(r') \quad \text{for } r \text{ large}, \tag{4}$$

where $F_\pi = 93$ MeV is the pion decay constant, and g_A is the axial coupling constant with $g_A = 1.26$ for the nucleon. Notice that this result refers to the leading order of the expansion in a large number of colors N_c [22] with $N_c \to \infty$ taken first, and $m_\pi \to 0$ taken second (in general these limits do not commute). For finite m_π the behavior is $V_{\rm eff}(r) \propto \exp(-2m_\pi r)/r^2$ at $r \gg 1/m_\pi$ [22].

III. CHROMOELECTRIC POLARIZABILITIES

The chromoelectric polarizabilities α are important properties of quarkonia. Little is known about them especially for charmonia, except that the chromoelectric polarizabilities of J/ψ and ψ', $\alpha(1S)$ and $\alpha(2S)$, are real and positive, and satisfy the Schwarz inequality $\alpha(1S)\alpha(2S) \geq \alpha(2S\to 1S)^2$ [4]. The chromoelectric polarizabilities were calculated in the large-N_c limit in the heavy quark approximation [23]. Applying the results to the charmonium case yields [7]

$$\alpha(1S)_{\rm pert} \approx 0.2\ {\rm GeV}^{-3}, \tag{5a}$$

$$\alpha(2S)_{\rm pert} \approx 12\ {\rm GeV}^{-3}, \tag{5b}$$

$$\alpha(2S\to 1S)_{\rm pert} \approx -0.6\ {\rm GeV}^{-3}. \tag{5c}$$

Independent phenomenological information on the value of the $2S \to 1S$ transition polarizability is available from analyses of data on the decay $\psi' \to J/\psi\pi\pi$ [4]

$$|\alpha(2S\to 1S)| \approx 2\ {\rm GeV}^{-3}(\text{phenomenology}). \tag{6}$$

In the heavier bottomonium system $1/N_c$ corrections to $\alpha(1S)$ are of $\mathcal{O}(5\%)$ [24]. In the charmonium system presently no information is available on the chromoelectric polarizabilities besides the perturbative estimates [23] and the phenomenological value for the $2S \to 1S$ polarizability [4] which is only in rough agreement with the perturbative prediction; see Eq. (5c) vs (6) [notice that $\pi\pi$ final state interactions [25] may reduce the value in Eq. (6)].

In this situation, independent information on the chromoelectric polarizabilities of charmonia is of importance.

IV. EXTRACTION OF THE CHROMOELECTRIC POLARIZABILITY OF J/ψ

The recent lattice QCD data on the effective charmonium-nucleon interaction [26] put us in the position to extract the chromoelectric polarizability $\alpha(1S)$ of J/ψ. This nonperturbative determination of $\alpha(1S)$ warrants a study, even though the lattice data [26] (published in a conference proceeding) may have unestimated systematic uncertainties. The results of Ref. [26] were obtained using $2+1$ flavor full QCD gauge configurations which were simulated with a Wilson clover quark action on a $16^3 \times 32$ lattice with lattice spacing $a = 0.1209$ fm. Using this action for heavy quarks "may bring large discretization errors" as stressed in [26]. Another concern are the unphysical light quark masses used in [26] which correspond to a pion mass of $m_\pi = 875$ MeV. The results of [26] are in qualitative agreement with earlier studies in quenched lattice QCD [27]. Until future lattice QCD studies performed with physical light quark masses on finer lattices or with relativistic heavy quark action for charm, we have to keep these points in mind as unestimated potential systematic uncertainties in our extraction.

The extraction assumes that the charm-quark mass is sufficiently large to neglect heavy quark mass corrections, which can be tested with future lattice QCD data. Although below we will see that the lattice data are compatible with this assumption, presently also this point has to be kept in mind as a potential uncontrolled systematic uncertainty.

From Eq. (3) we obtain (here M_N denotes the nucleon mass)

$$\alpha = -\frac{b}{4\pi^2\nu M_N}\frac{g_s^2}{g_c^2}\int d^3r V_{\rm eff}(r). \tag{7}$$

Let us discuss the different factors which play a role in the extraction of α and their uncertainties.

The coefficient ν introduced in Eq. (1) was estimated on the basis of the instanton liquid model of the QCD vacuum

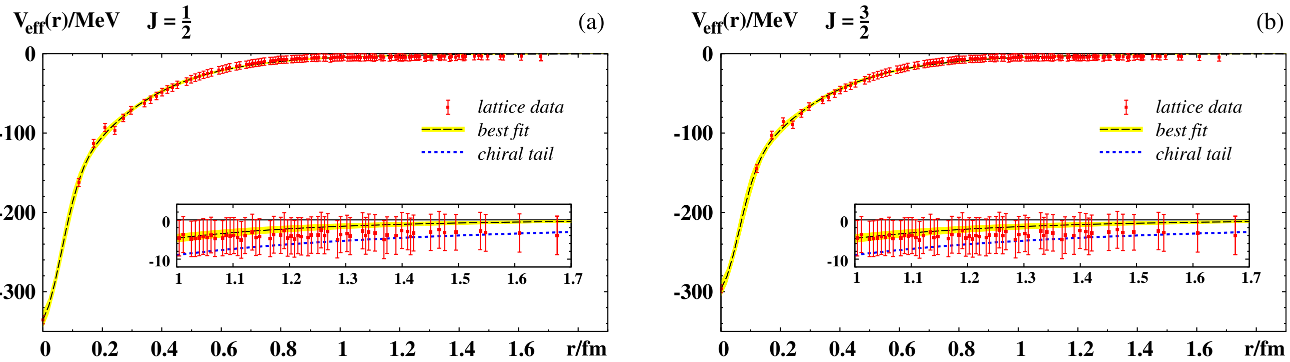

FIG. 1. Effective J/ψ-nucleon potential $V_{\mathrm{eff}}(r)$ as function of r from the lattice QCD calculation [26] and the best fits (10)–(12) in the channels: (a) $J=\frac{1}{2}$, and (b) $J=\frac{3}{2}$. The shaded areas show the 1-σ regions of the fits. The insets show the regions of 1 fm $< r <$ 1.7 fm where the available lattice data are compatible within error bars also with zero or with chiral predictions.

and the chiral quark soliton model, where the strong coupling constant freezes at a scale set by the nucleon size at $g_s^2/(4\pi)\approx 0.5$. Assuming $\xi_s\approx 0.5$ as suggested by the fraction of nucleon momentum carried by gluons in DIS at scales comparable to μ_s one obtains the value $\nu\approx 1.5$ [7]. This is supported by the analysis of the nucleon mass decomposition [28] with $\xi_s\approx\frac{1}{3}$ leading to $\nu\approx 1.4$. Based on these results we will use

$$\nu\approx 1.5\pm 0.1 \tag{8}$$

in this work. Let us remark that a similar result $\nu = (1.45\ldots 1.6)$ was obtained for the pion in Ref. [13].

In order to estimate the factor g_s^2/g_c^2 we use two extreme approaches. One estimate is based on effective nonperturbative methods. For that we use the nonperturbative result $g_s^2/(4\pi)\approx 0.5$ from the instanton vacuum model mentioned above which refers to a low scale of the nucleon; see above. Interestingly, phenomenological calculations of charmonium properties require $g_c^2/(4\pi) = 0.5461$ at a scale associated with charmonia [29]. This indicates that $g_s^2/g_c^2\sim 1$ is a reasonable assumption [7]. Another "extreme" result is provided by the leading-order QCD running coupling constant. We follow Ref. [30] where the description of the strong coupling constant was optimized to guarantee perturbative stability down to a low initial scale $\mu_{\mathrm{LO}}^2 = 0.26\ \mathrm{GeV}^2$ of the parametrizations for the unpolarized parton distribution functions. In this way we obtain $g_s^2/(4\pi) = 0.46$ at a scale set by the nucleon mass, while $g_c^2/(4\pi) = (0.27\ldots 0.36)$ depending on whether one evaluates the running coupling constant at the scale m_c or $2m_c$ [the leading-order derivation of Eq. (1) does not fix the scale, and both choices are equally acceptable]. In this way we obtain the "leading-order perturbative estimate" $g_s^2/g_c^2\sim(1.3\ldots 1.7)$. This indicates that this quantity is associated with a substantial theoretical uncertainty. In order to cover both extreme cases, we will assume that

$$\frac{g_s^2}{g_c^2}\approx 1.37\pm 0.37. \tag{9}$$

The information on $\int \mathrm{d}^3 r V_{\mathrm{eff}}(r)$ is obtained from the lattice QCD calculation [26] performed with unphysical light quark masses such that $m_\pi = 875$ MeV and $M_N = 1816$ MeV but with a physical value of m_c. In the heavy quark limit the effective potential factorizes in the chromoelectric polarizability α and nucleonic properties, and we may expect the extracted value of α to be weakly affected by the unphysical light quark masses. (The heavy quark mass corrections might be sensitive to light quark masses. This is part of the currently uncontrolled systematic uncertainties, which can be revisited in the future when lattice calculations with physical light quark masses will become available for V_{eff}.)

In the lattice calculation $V_{\mathrm{eff}}(r)$ was computed in the region $0\le r\le 1.7$ fm in the angular momentum channels $J=\frac{1}{2}$ and $J=\frac{3}{2}$ as shown in Fig. 1. The lattice data in both channels can be fitted with functions of the form

$$V_{\mathrm{eff}}(r) = C_0 e^{-\frac{r}{r_0}}\frac{1}{1+\frac{r^2}{r_1^2}} + C_2 e^{-\frac{r^2}{r_2^2}}, \tag{10}$$

where the first term is defined such that at large r it has the form dictated by chiral symmetry [22], while the second term constrains the parametrization in the small-r region. (We are not aware of a deep physical reason why the second term should be Gaussian, besides the fact that among the *Ansätze* we explored it yields the lowest χ^2; see below.) The best fit parameters in the channel $J=\frac{1}{2}$ are as follows:

$$\begin{aligned} C_0^{(1/2)} &= -(178.2\pm 3.7)\ \mathrm{MeV},\\ r_0^{(1/2)} &= (0.573\pm 0.065)\ \mathrm{fm},\\ r_1^{(1/2)} &= (0.429\pm 0.041)\ \mathrm{fm},\\ C_2^{(1/2)} &= -(157.4\pm 4.3)\ \mathrm{MeV},\\ r_2^{(1/2)} &= (0.091\pm 0.003)\ \mathrm{fm},\\ \chi^2_{\mathrm{d.o.f.}} &= 0.18. \end{aligned} \tag{11}$$

The best fit parameters in the channel $J=\frac{3}{2}$ are as follows:

$$\begin{aligned} C_0^{(3/2)} &= -(160.4 \pm 3.3)\ \text{MeV}, \\ r_0^{(3/2)} &= (0.619 \pm 0.073)\ \text{fm}, \\ r_1^{(3/2)} &= (0.426 \pm 0.039)\ \text{fm}, \\ C_2^{(3/2)} &= -(136.0 \pm 3.9)\ \text{MeV}, \\ r_2^{(3/2)} &= (0.088 \pm 0.004)\ \text{fm}, \\ \chi^2_{\text{d.o.f.}} &= 0.17. \end{aligned} \tag{12}$$

The fits are shown in Fig. 1. Several remarks are in order.

First, the potentials in both channels are very similar, and agree with each other within ±5% relative accuracy. In fact, except for the point at $r = 0$ both lattice data sets are compatible with each other within error bars. Let us remark that, if heavy quark mass corrections play a role, one should expect them to have an impact especially in the region of small $r \lesssim 1/m_c \approx 0.13$ fm. The independence of $V_{\text{eff}}(r)$ of $J = \frac{1}{2}$ or $\frac{3}{2}$ is an important consistency check of our approach. The effective potential is universal in our approach, and differences due to different J are expected to be suppressed in the heavy quark limit, as we observe. Thus, we have no indication that heavy quark mass corrections are significant for $V_{\text{eff}}(r)$ in the charmonium system. As mentioned above, this point can be tested quantitatively with future lattice data.

Second, chiral symmetry dictates $r_0 = (2m_\pi)^{-1} = 0.11$ fm. The fits are a factor of 5 off. Notice, however, that the lattice data clearly constrain $V_{\text{eff}}(r)$ in both channels only up to about $r \lesssim 1$ fm. It is likely that this limited r-region does not extend far enough to see the chiral asymptotics. Indeed, for 1 fm $< r <$ 1.7 fm the lattice data on $V_{\text{eff}}(r)$ are actually compatible with zero within error bars; see the insets in Fig. 1. Notice, however, that a fit with the fixed parameter $r_0 = (2m_\pi)^{-1}$ (with $m_\pi = 875$ MeV here) has still an excellent χ^2 per degree of freedom of $\chi^2_{\text{d.o.f.}} = 0.4$ for both channels. This is remarkable and indicates that the lattice data are compatible with chiral symmetry.

Third, we explored also other shapes for the fit functions with practically no difference in the region $r \lesssim 1$ fm where the lattice data have the strongest constraining power. We will comment below on the region $r > 1$ fm.

In order to evaluate $\int d^3r V_{\text{eff}}(r)$ we consider separately the region $r < 1$ fm where the lattice data are clearly nonzero, and $r \geq 1$ fm where the lattice data are compatible with zero within error bars (including the region $r > 1.7$ fm with no available lattice data); see the inset in Fig. 1. In the region $r < 1$ fm the fits in Eqs. (10)–(12) yield

$$\int_{r<1\ \text{fm}} d^3r V_{\text{eff}}(r) = \begin{cases} (-9.3 \pm 0.8)\ \text{GeV}^{-2} & \text{for } J = \frac{1}{2}, \\ (-8.9 \pm 0.8)\ \text{GeV}^{-2} & \text{for } J = \frac{3}{2}. \end{cases} \tag{13}$$

The uncertainty of these results is due to the statistical uncertainty of the lattice data. We tried several other fit *Ansätze* which all had larger $\chi^2_{\text{d.o.f.}}$, and gave results compatible with (13) within statistical error bars. The systematic uncertainty due to the choice of fit *Ansatz* is therefore negligible compared to the statistical uncertainty of the fits.

In the region $r > 1$ fm systematic uncertainties due to the choice of fit *Ansatz* are not negligible. The form (10) of the best fit is well motivated by chiral symmetry. But the lattice data [26] have a modest constraining power for 1 fm $<$ $r <$ 1.7 fm, and no lattice data are available beyond that. To proceed we assume that the fits (10)–(12) give useful estimates for the central values of contributions from $r > 1$ fm to the integrals over $V_{\text{eff}}(r)$, and assign a systematic error by using two extreme estimates. For the first estimate we approximate $V_{\text{eff}}(r) = 0$ for $r \geq 1$ fm, which fits the lattice data in the region 1 fm $< r <$ 1.7 fm with a $\chi^2_{\text{d.o.f.}} = 0.7$, and certainly leads to overestimates of the contributions from the large-r region to $\int d^3r V_{\text{eff}}(r)$ in both channels. For the second extreme estimate we assume $V_{\text{eff}}(r) \propto 1/r^6$ with the coefficient given by Eq. (4). Notice that the coefficient strictly speaking needs the full result for $\int d^3r V_{\text{eff}}(r)$ which we do not yet know. At this point one could design an iterative procedure, but for our purposes it is sufficient to assume that $\int d^3r V_{\text{eff}}(r) \approx -(10\ldots20)\ \text{GeV}^{-2}$. This is also compatible with the lattice data [a fit assuming $\int d^3r V_{\text{eff}}(r) = -15\ \text{GeV}^{-2}$ has $\chi^2_{\text{d.o.f.}} = 0.20$ and is shown in Fig. 1] and certainly leads to an underestimate of the large-r contribution to the integral. To summarize, in the large-r region we obtain

$$\int_{r\geq 1\ \text{fm}} d^3r V_{\text{eff}}(r) = \begin{cases} 0 & J = \frac{1}{2}, \frac{3}{2}\ \text{extreme estimate (i)}: V_{\text{eff}}(r) = 0 \ \ \text{for } r > 1\ \text{fm}, \\ -(4.9 \pm 3.4)\text{GeV}^{-2} & J = \frac{1}{2} \rightarrow \text{extrapolation based on the best fit in Eqs. (10) and (11)}, \\ -(5.4 \pm 3.9)\text{GeV}^{-2} & J = \frac{3}{2} \rightarrow \text{extrapolation based on the best fit in Eqs. (10) and (12)} \\ -(3.3\ldots6.6)\text{GeV}^{-2} & J = \frac{1}{2}, \frac{3}{2}\ \text{extreme estimate (ii)}: V_{\text{eff}}(r)\ \text{with "chiral tail"} \ \ \text{for } r > 1\ \text{fm}. \end{cases} \tag{14}$$

We use the best fit results as central values and the extreme estimates to assign a systematic uncertainty as follows:

$$\int_{r\geq 1\ \text{fm}} d^3r V_{\text{eff}}(r) = \begin{cases} -4.9 \pm 3.4^{+4.9}_{-1.7}\ \text{GeV}^{-2} & J = \frac{1}{2}, \\ -5.4 \pm 3.9^{+5.4}_{-1.2}\ \text{GeV}^{-2} & J = \frac{3}{2}. \end{cases} \tag{15}$$

Combining Eqs. (13) and (15) the final result for the full integral of the effective potential is

$$\int \mathrm{d}^3 r V_{\mathrm{eff}}(r) = \begin{cases} (-14.2 \pm 0.8^{+6.0}_{-3.8})\ \mathrm{GeV}^{-2} & J=\frac{1}{2}, \\ (-14.3 \pm 0.8^{+6.7}_{-4.1})\ \mathrm{GeV}^{-2} & J=\frac{3}{2}, \end{cases} \quad (16)$$

where the first error is due to the statistical accuracy of the lattice data in the region $r < 1$ fm and the second error is due to the systematic uncertainty in the extrapolation for $r > 1$ fm (with the uncertainties from Eq. (15) combined in quadrature).

From Eqs. (8), (9), and (16) we obtain the value for the chromoelectric polarizability

$$\alpha(1S) = \begin{cases} (1.63 \pm 0.09^{+0.69}_{-0.44} \pm 0.44 \pm 0.11 \pm 0.01)\ \mathrm{GeV}^{-3} & J=\frac{1}{2}, \\ (1.64 \pm 0.09^{+0.76}_{-0.47} \pm 0.44 \pm 0.11 \pm 0.01)\ \mathrm{GeV}^{-3} & J=\frac{3}{2}, \end{cases} \quad (17)$$

with the errors due to the following uncertainties (in this order): statistical accuracy of the lattice data in the region $r < 1$ fm, systematic uncertainty of $\int \mathrm{d}^3 r V_{\mathrm{eff}}(r)$ due to extrapolation in the region $r > 1$ fm, uncertainty of the ratio $(g_c/g_s)^2$ and that of ν, and uncertainty of the lattice value for M_N [the latter was not quoted in [26] but is estimated to be of the order of $\mathcal{O}(10\ \mathrm{MeV})$ [31]]. Combing the uncertainties in quadrature we obtain

$$\alpha(1S) = \begin{cases} (1.63 \pm 0.09^{+0.82}_{-0.63})\ \mathrm{GeV}^{-3} & J=\frac{1}{2}, \\ (1.64 \pm 0.09^{+0.89}_{-0.65})\ \mathrm{GeV}^{-3} & J=\frac{3}{2}. \end{cases} \quad (18)$$

The agreement of the $\alpha(1S)$ values extracted from $V_{\mathrm{eff}}(r)$ in the $J=\frac{1}{2}$ and $\frac{3}{2}$ channels supports the assumption that heavy quark mass corrections do not play a dominant role in our analysis. Rounding off and combing all sources (statistical and systematic) of uncertainties, we obtain for both channels

$$\alpha(1S) = (1.6 \pm 0.8)\ \mathrm{GeV}^{-3}. \quad (19)$$

We stress that this result has very little sensitivity to the shape of V_{eff} at small r since we need the integral $\int \mathrm{d}^3 r V_{\mathrm{eff}}(r)$ where the volume element suppresses the small-r region. The result is much more sensitive to the large-r dependence of V_{eff}. We have conservatively estimated the pertinent systematic uncertainty by assuming extreme limiting cases in Eq. (14). It is important to keep in mind that the result (19) may have further systematic uncertainties inherent to the lattice data (discretization effects, unphysical light quark masses) which cannot be estimated at this point.

V. POSSIBILITY FOR HADROCHARMONIA

The charmonium-nucleon potential is attractive and we can study the possibility of a bound state—hadrocharmonium [5]. A candidate for such a state with a mass around 4450 MeV was recently observed by LHCb [32]. To do this we rescale the lattice effective potential by the factor $M_N^{\mathrm{phys}}/M_N^{\mathrm{lattice}}$, where $M_N^{\mathrm{phys}} = 940$ MeV is the physical nucleon mass and $M_N^{\mathrm{lattice}} = 1816$ MeV the nucleon mass obtained in lattice measurements of [26]. We need this rescaling to ensure the physical normalization condition (3) for the effective potential.

Solving the Schrödinger equation for the rescaled potential we confirm the conclusion of Ref. [26] that J/ψ does not form the bound state with the nucleon. Now we can study the possibility of a nucleon bound state with $\psi(2S)$. To do this we note that according to Eq. (1) the shape of the nucleon-$\psi(2S)$ potential is the same as for the corresponding potential for J/ψ; the only difference is the overall normalization factor due to chromoelectric polarizability.

Using the results for the shape of the effective potential extracted here from the lattice data and treating $\alpha(2S)$ as a free parameter, we obtain the following results:

- (i) The nucleon-$\psi(2S)$ bound states can form if $\alpha(2S) \geq \alpha_{\mathrm{crit}}(2S) = (8 \pm 4)\ \mathrm{GeV}^{-3}$, where error bars are due to the statistical and systematic error of our fit, and due to the uncertainty of $(g_s/g_c)^2$; see Eq. (9). Note that in the ratio $\alpha(2S)/\alpha(1S)$ many systematic uncertainties are canceled. For this ratio we obtain $\alpha_{\mathrm{crit}}(2S)/\alpha(1S) = (5.0 \pm 0.5)$. The values of $\alpha_{\mathrm{crit}}(2S)$ from the $J = 1/2$ and $J = 3/2$ potentials are indistinguishable within error bars. The obtained value of $\alpha_{\mathrm{crit}}(2S)$ is compatible with those obtained in Refs. [7,33] in completely different frameworks.
- (ii) For $\alpha(2S) = (24 \pm 12)\ \mathrm{GeV}^{-3}$ the bound state with mass 4450 MeV is formed. It may correspond to the narrow LHCb pentaquark $P_c(4450)$. Again we have a good agreement with the findings of Refs. [7,33]. In terms of the ratio $\alpha(2S)/\alpha(1S)$ the hadrocharmonium $P_c(4450)$ exists for $\alpha(2S)/\alpha(1S) = (15 \pm 1)$. Such a value of $\alpha(2S)$ and the results in Eqs. (6) and (19) satisfy the Schwarz inequality $\alpha(1S)\alpha(2S) \geq \alpha(2S \to 1S)^2$ [4].
- (iii) From the data [26] for $J = 1/2$ and $J = 3/2$ effective potentials we are able to estimate the hyperfine splitting between $\frac{3}{2}^-$ and $\frac{1}{2}^-$ components of $P_c(4450)$. We find the hyperfine mass splitting

(30 ± 30) MeV with a tendency for $J = 3/2$ to be heavier. This is compatible with both zero and with the estimate of 5–10 MeV obtained in [7].

We see that the lattice data of [26] confirm the conclusions about nucleon-$\psi(2S)$ bound state made in Refs. [7,33]. It would be very interesting to make an independent lattice measurement of the nucleon-$\psi(2S)$ effective potential.

VI. CONCLUSIONS

The chromoelectric polarizability $\alpha(1S)$ of J/ψ was extracted on the basis of the formalism [3] from the lattice QCD data [26] on the effective nucleon-J/ψ potential $V_{\rm eff}$. The final result is $\alpha(1S) = (1.5 \pm 0.6)$ GeV^{-3}.

The quoted error bar includes uncertainties due to strong coupling constants at nucleon and charmonium scales, parameter ξ_s describing the fraction of baryon energy carried by gluons, and statistical error bars of the lattice data [26] on $V_{\rm eff}$ for $r \leq 1$ fm. In this region the systematic uncertainty due to choosing a specific fit *Ansatz* for $V_{\rm eff}$ is negligible because only the integral $\int d^3 r V_{\rm eff}(r)$ is needed for the extraction. Exploring guidance from chiral symmetry (which dictates the behavior of $V_{\rm eff}$ at large r) we were able to provide a conservative estimate of the systematic uncertainty due to extrapolation beyond $r > 1$ fm where the lattice data for $V_{\rm eff}$ are compatible with zero or not available.

The extracted $\alpha(1S)$-value may have further systematic uncertainties which cannot be estimated at this point, one of which concerns our approach and the assumption of the heavy quark limit. The compatibility of lattice data for $V_{\rm eff}$ in the angular momentum channels $J = \frac{1}{2}$ and $J = \frac{3}{2}$ [26] provides an encouraging hint (but not more than that) that heavy quark mass corrections to $V_{\rm eff}$ might be within statistical error bars of the lattice data [26]. Unestimated potential systematic uncertainties pertain also to the lattice data (discretization effects, unphysical light quark masses) [26]. Future lattice QCD studies will allow us to test whether the charm quark mass is large enough for the validity of our approach, and allow us to assess systematic uncertainties inherent to lattice simulations.

The obtained value $\alpha(1S) = (1.6 \pm 0.8)$ GeV^{-3} is larger than the perturbative prediction $\alpha(1S)_{\rm pert} \approx 0.2$ GeV^{-3} [7,23] which was so far basically the only available information on the chromoelectric polarizability of J/ψ. The larger value obtained here is in line with the suspicion $\alpha(1S) \gtrsim |\alpha(2S \to 1S)|$ [6] with the value $|\alpha(2S \to 1S)| \approx 2$ GeV^{-3} from $\psi' \to J/\psi\pi\pi$ decays [4] (which may be reduced [25] by final state interaction effects). This argument is not rigorous but based on the intuitive assumption that off-diagonal matrix elements may be naturally expected to be smaller than diagonal ones [6].

We also studied the possibility of the nucleon-$\psi(2S)$ bound state. We came to conclusions which are similar to those in Refs. [7,33], and support the interpretation of $P_c(4450)$ as a $\psi(2S)$-nucleon bound state if $\alpha(2S)/\alpha(1S) \approx 15$. Our result is compatible with the value of $\alpha(2S) \approx 17$ GeV^{-3} obtained in Refs. [7,33] in completely different frameworks. This is remarkable, considering that in Refs. [7,33] chiral models were used with massless [7] and physical [33] pion masses, while here we used lattice data obtained at large unphysical m_π. The results for the $\psi(2S)$ chromoelectric polarizability obtained in Refs. [7,33] and here are based on the interpretation of $P_c(4450)$ as a hadrocharmonium. Our analysis also provides independent support for this interpretation.

The results obtained in this work contribute to a better understanding of the chromoelectric polarizabilities of charmonia, and will have interesting applications for the phenomenology of hadrocharmonia.

ACKNOWLEDGMENTS

We thank Takuya Sugiura for valuable correspondence and providing us with the data of Ref. [26]. This work was supported in part by the National Science Foundation (Contract No. 1406298), the Wilhelm Schuler Stiftung, and by CRC110 (Deutsche Forschungsgemeinschaft).

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