

Novel methods of constraining the parton distribution function uncertainty in the measurement of the W -boson mass

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(Received 22 April 2018; published 30 August 2018)

The measurement of the mass of the W boson is one of the most precise and impactful measurements to test the standard model (SM). Following the discovery of the Higgs boson and the measurement of its mass, all SM parameters are sufficiently well known in order to make precise predictions for electroweak observables such as the W boson mass. These observables receive corrections in extensions of the SM. Experimentally, the dominant uncertainty in the W -boson mass measurement at hadron colliders arises due to parton distribution functions (PDFs). We present novel *in situ* methods of constraining the PDF variation and of reducing the PDF impact on the W -mass measurement uncertainty at hadron colliders. These methods can exploit the very high-statistics samples of W -boson candidate events at the LHC.

DOI: [10.1103/PhysRevD.98.033008](https://doi.org/10.1103/PhysRevD.98.033008)

The discovery [1] of the Higgs boson in 2012 has confirmed the Higgs mechanism [2] of the standard model [3] for generating masses for the electroweak gauge bosons. The Yukawa couplings between the Higgs field and some of the fermions have also been observed. Even so, the Higgs sector of the standard model (SM) continues to contain riddles. The Higgs potential in the SM is empirical without a dynamical or symmetry-based explanation. The values of the fermion Yukawa couplings vary widely between generations. Finally, unlike other dimensionless SM parameters which are multiplicatively renormalized and depends logarithmically on the ultraviolet cutoff, the dimensionful parameter that defines the observable Higgs boson mass and the field's vacuum expectation value is additively renormalized with a quadratic dependence on the ultraviolet cutoff. This leads to the well-known “fine-tuning” problem of the Higgs boson mass if there is a large hierarchy between the electroweak scale and the energy scale of new physics, e.g., the Planck scale. To render the Higgs potential parameters “natural,” it has been postulated that new physics should provide an ultraviolet cutoff for loop diagrams that is not much higher than the electroweak scale.

As the Higgs boson mass M_H is parametrically defined, its value was historically constrained by measurements of the weak-boson masses and other precision electroweak observables through the Higgs-loop induced corrections. For example, loops in the W -boson propagator contribute to the correction Δr , defined in the following expression for the W -boson mass M_W in the *on-shell* scheme [4]:

$$M_W^2 = \frac{\hbar^3 \pi}{c} \frac{\alpha_{\text{EM}}}{\sqrt{2} G_F (1 - M_W^2/M_Z^2)(1 - \Delta r)}, \quad (1)$$

where α_{EM} is the electromagnetic coupling at $Q = M_Z c^2$, G_F is the Fermi weak coupling extracted from the muon lifetime, M_Z is the Z -boson mass, and $\Delta r = 3.58\%$ [5] includes all radiative corrections. Δr is dominated by quark loops while depending logarithmically on M_H . The directly-measured M_H is consistent with the fit to the electroweak observables, where the fit precision is dominated by the uncertainty on M_W and the weak mixing angle, with smaller contributions from the uncertainties on the top quark mass and α_{EM} .

The direct measurement of M_H enables precise predictions for the precision electroweak observables at the level of 0.01%. Measuring these observables at this level of precision provides a stringent test of new physics through loop effects [6]. A discrepancy between the measured and predicted values of M_W can be interpreted in the framework of effective field theory as a measurement of the energy scale of new physics. Evidence of physics beyond the SM would be revolutionary for the advancement of fundamental science, and information on the next energy scale would be hugely beneficial for the planning of future experiments. Hence, higher-precision measurements of M_W are motivated.

Following the discovery of the W boson in 1983 at the UA1 and UA2 experiments [7], M_W has been measured with increasing precision at both lepton and hadron colliders; using $p\bar{p}$ collisions at the CDF [8–10] and D0 [11–13] experiments (Tevatron Run I and Run II), e^+e^- collisions at the ALEPH [14], DELPHI [15], L3 [16], and OPAL [17] experiments (LEP), and pp collisions at the ATLAS [18] experiment (LHC). The LEP and Tevatron average is $M_W = 80385 \pm 15 \text{ MeV}/c^2$ [19,20].

With the steady improvement in the experimental precision of M_W , the requirements for more precise theoretical

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and model inputs have grown. The largest source of production model uncertainty in hadron collisions is the uncertainty in the parton distribution functions. The most recent measurements from the ATLAS, CDF and D0 experiments quote the PDF-induced uncertainty as 10 MeV, 10 MeV and 11 MeV respectively, which is correlated among the measurements and dominates the total uncertainty in the world average. In this article, we explore novel methods of constraining the parton distribution functions *in situ* from the hadron collider data.

In hadron-hadron collisions, W boson production can be described at the Born level via s -channel annihilation of quarks and antiquarks. In the $p\bar{p}$ collisions at the Tevatron, the initial-state partons are primarily valence quarks, with a smaller contribution from sea-quark annihilation. In the higher-energy pp collisions at the LHC, the contribution from sea-quark annihilation is substantial. The W bosons are accompanied by hadronic jets which arise from the fragmentation of radiated gluons and (anti)quarks produced by initial-state gluon splitting. The W boson decays either to a quark-antiquark pair ($q\bar{q}'$) or to a charged lepton and neutrino ($\ell\nu$). The hadronic decays cannot be used for a precise M_W measurement because the jets from the W boson decay cannot be calibrated to sufficient precision, and because this signal is overwhelmed by multijet background. The leptonic decays, $W \rightarrow \ell\nu$ ($\ell = e, \mu$), which have about 22% total branching fraction, are used for the mass measurement.

The hadron collider experiments use a coordinate system in which the z axis is the axis of symmetry of the cylindrical detector and points along the beam direction. Defining conventional units with $\hbar = c \equiv 1$, the rapidity $\zeta = \frac{1}{2} \ln[(E + p_z)/(E - p_z)]$ is additive under boosts along the z axis. In the case of massless particles, ζ equals the pseudorapidity $\eta = -\ln[\tan(\theta/2)]$, where θ is the polar angle with respect to the z axis. Transverse quantities such as transverse momentum \vec{p}_T are projections onto the plane perpendicular to the z axis. The transverse momentum of the beam particles is considered negligible. From \vec{p}_T conservation, the transverse momentum of the neutrino in W -boson decay is inferred as $\vec{p}_T^\nu \equiv -\vec{p}_T^\ell - \vec{u}_T$, where \vec{p}_T^ℓ is the transverse momentum of the charged lepton and \vec{u}_T denotes the measurement of the transverse momentum of all particles recoiling against the W boson.

At a hadron collider, the longitudinal momentum p_z of a given W boson is not measurable because beam remnants are scattered very close to the beam axis, carrying an unknown amount of longitudinal momentum. The p_z distribution of the W bosons is predicted by convolving the p_z distributions of the interacting partons. The latter are provided by the parton distribution functions (PDFs) describing the fraction x of a hadron's momentum carried by a given interacting parton.

Since the neutrino p_z is unmeasured, M_W is measured by fitting three transverse quantities: p_T^ℓ , p_T^ν , and the

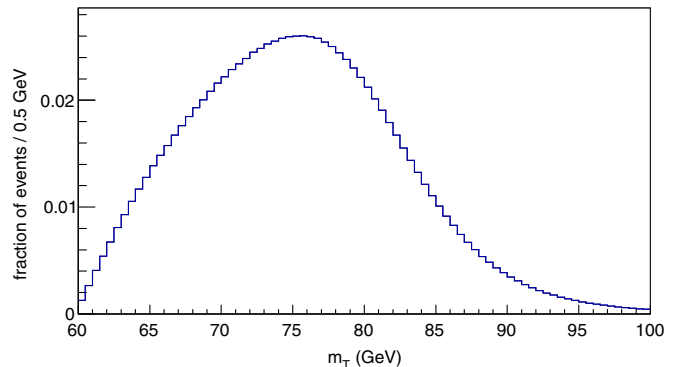


FIG. 1. The observable m_T distribution from simulation.

transverse mass $m_T = \sqrt{2p_T^\ell p_T^\nu (1 - \cos \Delta\phi)}$ [21], where $\Delta\phi$ is the azimuthal angle between the charged lepton and neutrino momenta. Events are selected with $u_T \ll p_T^\ell$, so that $p_T^\ell \approx p_T^\nu$ and the Jacobian edge in the fitted distributions is not significantly smeared by the boson's transverse boost [22].

A parametrized Monte Carlo detector simulation is used to process generator-level events in order to model the observed line shapes of the p_T^ℓ , p_T^ν , and m_T distributions. The simulated m_T distribution is shown in Fig. 1. Templates of these distributions are produced from the simulated events at fixed input values of the boson mass and a fixed PDF set. Additional simulated events are used as pseudo-data in which the input PDFs are varied. The templates are normalized to the pseudodata and a maximum-likelihood fit is used to extract the observable M_W corresponding to each set of PDFs. We consider the PDF parametrizations from the CTEQ [23], MSTW/MMHT [24], and NNPDF [25] collaborations.

The p_z of the W boson should not affect the fit to the distributions of the transverse kinematics which should be invariant under longitudinal boosts. This invariance is destroyed by the limited acceptance for the charged lepton, which is approximately $|\eta_\ell| < 1$ at the Tevatron and $|\eta_\ell| < 2.5$ at the LHC. The effect of this limited acceptance in the laboratory frame is best understood in the rest frame of the W boson, where the decay polar angle θ^* determines $p_T^\ell = \frac{M_W}{2} \sin \theta^*$. Thus, events with large p_T^ℓ are the ones in which the W boson decays centrally in the rest frame, while the events with small p_T^ℓ are the ones in which the decay leptons are produced in the forward-backward direction in the rest frame. Equivalently, the low p_T^ℓ events correspond to large $|\eta_\ell^*|$ and vice-versa.

The mapping from the rest-frame $|\eta_\ell^*|$ to the laboratory-frame $|\eta_\ell|$ is determined by the longitudinal boost of the W boson and hence by the PDFs. The result of the M_W fit depends on the fraction of events with low and high p_T^ℓ , and hence the fraction of events with high and low $|\eta_\ell^*|$ respectively, that enter the $|\eta_\ell| < 1(2.5)$ acceptance of the

Tevatron (LHC) detector. This ratio of events varies with PDF and captures the most important aspect of the PDF variation that induces the systematic uncertainty in the M_W measurement.

The CTEQ and MSTW collaborations determine a set of eigenvectors in PDF parameter space to form an orthogonal basis. The PDF uncertainty is obtained from a quadrature sum of all eigenvector contributions in a given PDF set,

$\delta M_W^{\text{PDF}} = \frac{1}{2} \sqrt{\sum_i (M_W^{i+} - M_W^{i-})^2}$, where $M_W^{i\pm}$ represents the fitted mass obtained using the $\pm\sigma$ shifts in the i th eigenvector. When the M_W deviations have the same sign, the maximum deviation between the nominal M_W and M_W^{i+} or M_W^{i-} is used as the contribution from this eigenvector. For the NNPDF set, the M_W uncertainty is given by the *rms* of the fitted M_W values resulting from the 100 PDF replicas.

Using events generated with PYTHIA [26], we study the variation in the shape of the m_T distribution due to the different PDF eigenvectors in a given set. We select simulated events passing the following kinematic requirements: $p_T^\ell > 30$ GeV, $p_T^\nu > 30$ GeV, $u_T < 15$ GeV and $|\eta_\ell| < 1$. For each PDF eigenvector, we normalize the m_T distribution to unit area and subtract from it the normalized m_T distribution corresponding to the default PDF for that set. These residual differences are shown for the CT10 and CT10W (both providing 90% confidence level (CL) uncertainties), MSTW2008 (68% CL and 90% CL), MMHT2014 (68% CL) and NNPDF3.0 PDF sets in Fig. 2. The residuals for the PDF sets which provide 90% CL uncertainties have been scaled down in order to compare to the others with 68% CL uncertainties. The NNPDF3.0 residuals have been scaled down by a factor of 10 so that the variances may be summed rather than averaged over the replicas, providing a correspondence with the sum over eigenvectors for the other PDF sets.

The pattern of the residuals as a function of m_T reveals a number of features. The difference of normalized distributions must integrate to zero, requiring the presence of at least one node. Furthermore, as the m_T distributions vanish near the bounds of the histogram, the residuals must also vanish near the bounds. The two striking and unexpected features are that the pattern of residuals shows only one node, and that the position of the node is almost universal for all PDF sets and all eigenvectors / replicas within a given PDF set.

Figure 2 shows that the salient feature of all PDF variations is simply to induce a variation in the fraction of events above and below a common threshold at $m_T \approx 73$ GeV. The threshold value depends on kinematic requirements and the chosen domain [60, 100] GeV of the m_T histogram. The same feature is also observed in the residuals of the p_T^ℓ and p_T^ν distributions, indicating the universality of the underlying mechanism. This is the first important conclusion of this paper.

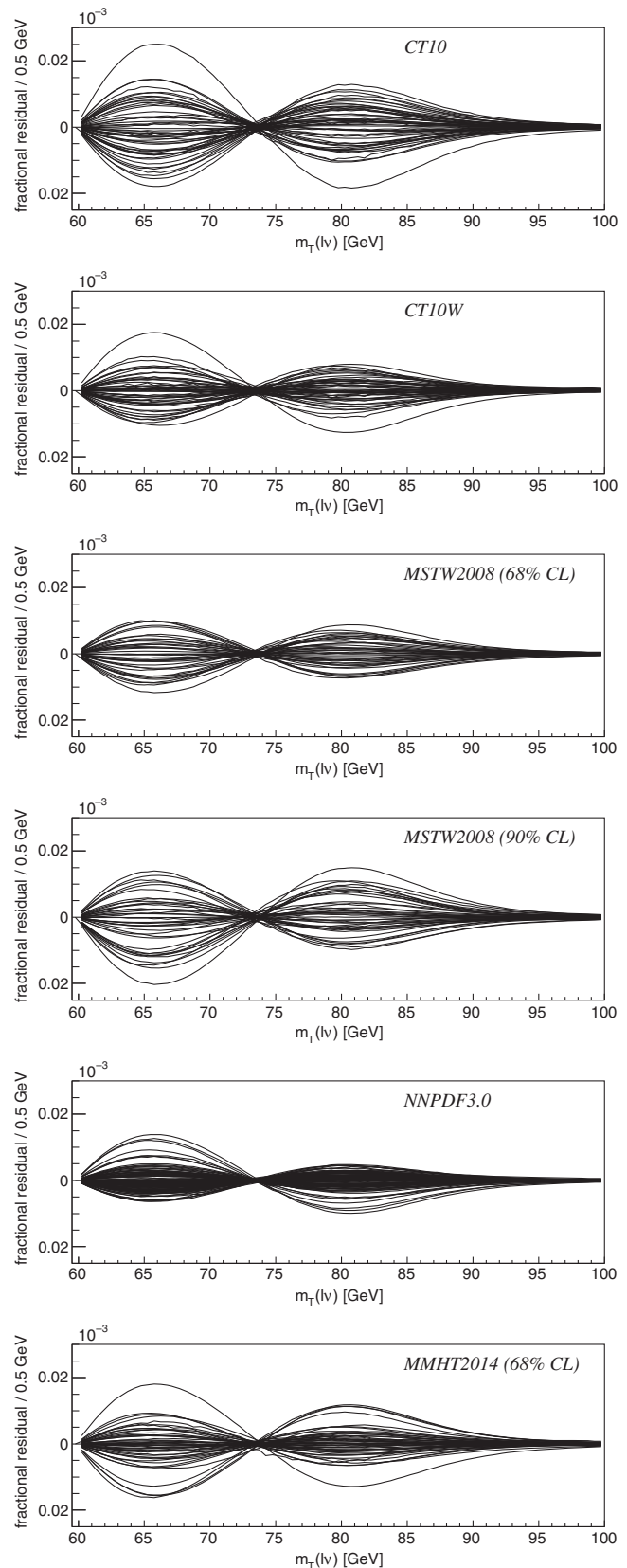


FIG. 2. Residual differences between the normalized m_T distributions for six PDF sets. For each PDF set, the residuals from the default PDF are overlaid for all eigenvectors. In the case of the NNPDF3.0 set, the residuals for the 100 replicas are shown.

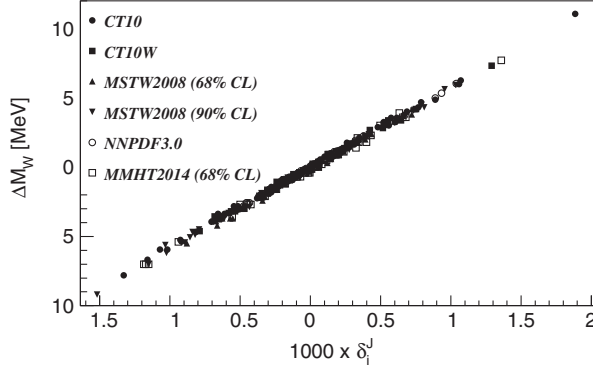


FIG. 3. Correlation between the m_T distribution shape variation parameter δ_i^J and the M_W shift Δ_i^J , for all eigenvectors i and PDF sets J . In the case of the NNPDF3.0 PDF set, i denotes the replica index. The scale factors described in the text have been applied as for Fig. 2.

As the shapes of all residual distributions are very similar, the variation between eigenvectors is captured by the ratio $r = I_a/I_b$ where I_a (I_b) represents the integral of the normalized m_T distribution above (below) the threshold value of 73 GeV. The PDF variation parameter δ_i^J for eigenvector i in PDF set J is defined as $\delta_i^J = r_i^J - r_0^J$, where r_0 is computed for the default (central) PDF in the set J . Simultaneously, we compute the shift $\Delta_i^J = M_W(i) - M_W(0)$, where $M_W(i)$ and $M_W(0)$ refer to the fitted M_W from pseudo-data generated with eigenvector i and with the central PDF, respectively. As shown in Fig. 3, there is almost perfect correlation between δ_i^J and Δ_i^J , indicating that the PDF systematic uncertainty in the measurement of M_W is driven almost entirely by the δ_i^J observable. This is the second important conclusion of this paper.

The universality of the variation in the transverse kinematic distributions induced by the PDF parameter variations may be understood as follows. In the boson rest frame, $m_T = M_W \sin \theta_\ell^*$, in the approximation that the boson has small transverse boost. The pseudorapidity difference between the leptons with low and high m_T respectively may be estimated using the relation $\sin \theta_\ell = \text{sech } \eta_\ell$. The typical m_T variation is ≈ 7.3 GeV about the 73 GeV threshold, implying the typical values of $|\eta_\ell^*| = 0.65$ ($\eta_\ell^* = 0$) for the low (high) m_T sub-samples. In the absence of any boson longitudinal boost, both the low- m_T and high- m_T sub-samples would be fully contained within the detector acceptance $|\eta_\ell| < \eta_{\text{cut}}$, where η_{cut} is 1.0 (2.5) at the Tevatron (LHC). However, the typical boson boost y_W causes the preferential loss of low- m_T (high $|\eta_\ell^*|$) events, when $0.65 + y_W > \eta_{\text{cut}}$. At the Tevatron (LHC), events with $y_W > 0.35$ (1.85) will be preferentially lost from the low- m_T subsample, and the PDF-induced variation in the rate of high- y_W events ultimately causes the M_W uncertainty.

The energy of the collider and the rapidity-acceptance of the detector work in opposite directions with regards to the

sensitivity to the y_W distribution. The higher the energy of the collider, the wider the actual y_W distribution. The rapidity plateau for W bosons extends to $y_W \approx 1.5$ (3.5) at the Tevatron (LHC). A given lepton-rapidity acceptance therefore sculpts the observed rapidity and p_T distributions more heavily and increases the sensitivity to the PDFs. On the other hand, at a given collider energy, a greater lepton-rapidity coverage increases the minimum y_W needed to influence the low- m_T sub-sample, thus reducing the fraction of events affected by the acceptance cut. Hence, as expected, increasing the lepton rapidity coverage reduces the sensitivity to the PDFs.

For a given collider energy and lepton acceptance, we can use $y_W \approx 0.65$ as an estimate of the boson boost differential between the low- m_T and high- m_T subsamples in order to analyze the PDF sensitivity, with the understanding that the collider energy and detector acceptance will determine its typical value in a specific experiment. As we see below, the PDF sensitivity depends quadratically on the boson boost differential. Therefore, there is motivation to increase the lepton rapidity coverage such that η_{cut} approaches the width of the W boson rapidity plateau and the boost differential reduces.

The W boson's longitudinal kinematics are related to the momentum fractions $x_{1,2}$ of the interacting partons by $E(x_1 - x_2) = p_z^W = M_W \sinh y_W$ and $E_W = E(x_1 + x_2)$, where E is the hadron beam energy, and p_z^W and E_W denote the longitudinal momentum and energy respectively of the W boson. The labels 1(2) refer to the hadron moving in the positive (negative) z direction. The boson's rapidity $y_W = \frac{1}{2} \ln \frac{E_W + p_z^W}{E_W - p_z^W}$ reduces to $y_W = \frac{1}{2} (\ln x_1 - \ln x_2)$. Defining $\bar{x} = M_W/(2E)$ as the partonic momentum fraction corresponding to W bosons produced at rest, we find $y_W = \ln x_1 - \ln \bar{x} = \Delta \ln x$. Thus, the high m_T subsample is generated from partons whose x values are close to \bar{x} , and the low m_T subsample is generated from partons whose x values vary from \bar{x} by the ratio $e^{0.65} \sim 2$, in the test case where the observed lepton has $|\eta_\ell| \approx 0$.

As the production rate is proportional to the product of PDFs $f(x_1) \cdot f(x_2)$, one finds $I_a \propto f^2(\bar{x})$ and $I_b \propto f(2\bar{x}) \cdot f(\bar{x}/2)$. Defining $\xi = \ln x$ and $p(\xi) = \ln f(x)$ yields $\ln r \simeq 2p(\bar{\xi}) - p(\bar{\xi} + \Delta\xi) - p(\bar{\xi} - \Delta\xi) \approx -\partial^2 p / \partial \xi^2 \cdot (\Delta\xi)^2$, where $\Delta\xi \approx 0.65$. Therefore, the PDF variation induced in $\ln r$, which is given by δ_i^J , is characterized by the variation of the second derivative of p with respect to ξ , scaled by $(\Delta\xi)^2$.

The PDFs $p(\xi)$ are smooth functions which can be expanded in the vicinity of $\bar{\xi}$ as the Taylor expansion $p(\xi) = \sum_n p^{(n)}(\bar{\xi}) (\Delta\xi)^n / n!$, where $p^{(n)}$ denotes the n th derivative with respect to ξ . The first term representing the overall normalization of the PDF is irrelevant for the shape variable r and the shape fit for M_W . These shape variables are also insensitive to the first derivative, which leaves the parton luminosity product $f(x_1) \cdot f(x_2)$ invariant.

By extension, all odd derivatives of p would leave the shape variables invariant. Thus the first relevant derivative is $p^{(2)}$, followed by $p^{(4)}$. Over the relevant ξ interval with small width of $\mathcal{O}(\Delta\xi)$, the PDF variation would be dominated by the variation in a single parameter, $p^{(2)}$.

Any significant contribution from $p^{(4)}$ would be visible in Fig. 2 as a shift in the nodal point or as the presence of an additional node in the residual distribution. The absence of these features indicates that the contribution from $p^{(4)}$ and higher derivatives is substantially smaller than the dominant contribution from $p^{(2)}$.

We note that the dominant x -dependence of PDFs at low x is given by $f(x) \sim x^{-\alpha}$, yielding $p^{(1)} = -\alpha$ and $p^{(2)} = 0$. Thus, there is no uncertainty induced in r and the M_W measurement by the uncertainty in α . By isolating $p^{(2)}$ as the relevant PDF parameter, the universality of the PDF-induced uncertainty in r and M_W is understood. Furthermore, we obtain the insight that the PDF-induced uncertainty is caused by the variation in the subleading terms of the PDF parametrization at low x . This is the third important conclusion of this paper.

Figure 3 prompts the development of two strategies to reduce the PDF uncertainty in the measurement of the W boson mass. First, a direct measurement of the ratio r from the collider data can be used to constrain the PDF-induced variation in δ_i^j , which will directly translate into a reduced uncertainty in M_W . As seen in Fig. 3, a measurement of r at the level of 0.1% precision would be required. Achieving this statistical error requires in excess of two million candidate events. The full Tevatron data set has surpassed this level of statistics, and the LHC data sets are already substantially larger and growing. A strength of this method is that the observable r , being a ratio of the number of events in two kinematic ranges, is not sensitive to overall normalization uncertainties such as the integrated luminosity and global efficiencies. The systematic uncertainties affecting the r measurement include the kinematics-dependent background contamination and the lepton identification efficiency. More than one million Z -boson candidate events would be required to measure the p_T -dependent lepton efficiencies.

The second approach to reduce the PDF-induced systematic uncertainty is to restrict the kinematic range of the sample used for fitting [27]. Figure 4 shows the reduction in the PDF-induced uncertainty as the lower edge L of the fitting window $[L, 90]$ GeV is raised from 60 GeV to 75 GeV. The reduction in the uncertainty is substantial as the variation in δ_i^j is progressively limited by the restricted domain. This is the fourth important conclusion of this paper. The reduction by the factor of two in the PDF-induced uncertainty would significantly improve the promise of future M_W measurements.

As the fitting domain is restricted, there is a corresponding loss of statistical precision in the fit. The tradeoff

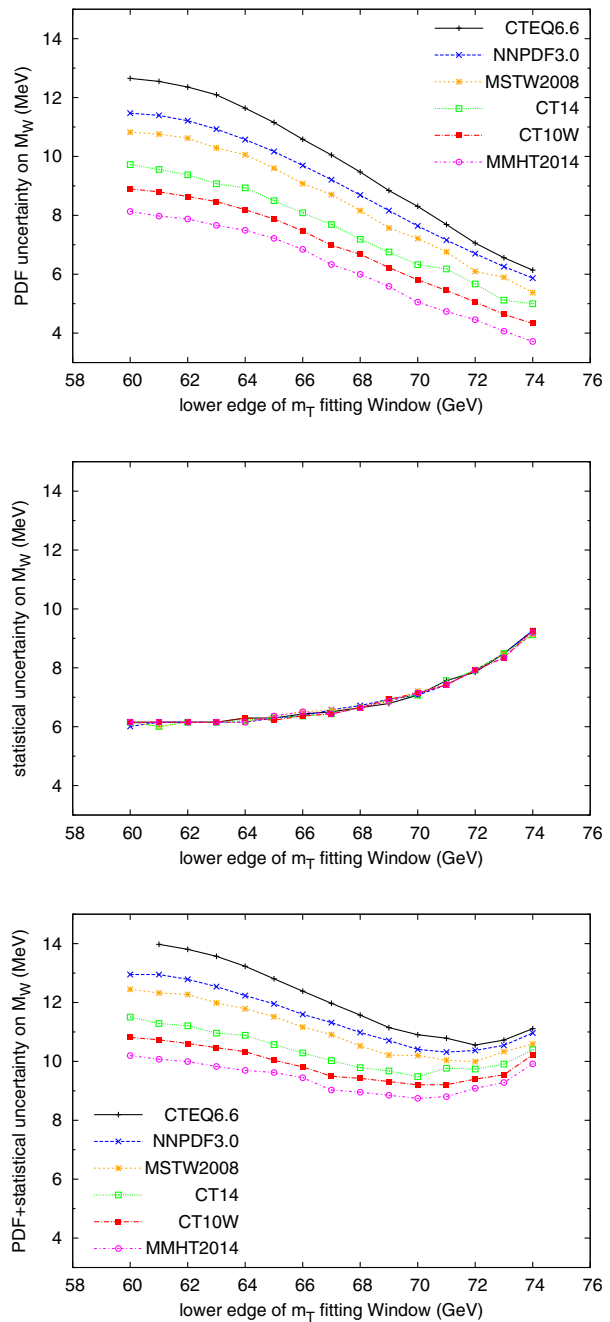


FIG. 4. The PDF-induced M_W uncertainty (top), the M_W statistical error (middle) and their combination in quadrature (bottom) as a function of the lower edge of the m_T fitting window.

between the decreasing PDF uncertainty and the increasing statistical error is investigated in Fig. 4 for a sample of four million W boson events. The combination in quadrature of the PDF-induced uncertainty and the statistical error is also shown in Fig. 4, illustrating the opportunity for minimizing the total uncertainty in the measurement by a judicious choice of L . As the sample statistics increase, raising the value of L is motivated.

In summary, we have identified the salient feature of PDF variation that induces the uncertainty δM_W^{PDF} in the

measurement of the W -boson mass. We have shown how the PDF-induced variation in the m_T distribution can be characterized by a single, observable shape parameter, whose precise measurement directly translates into a reduction in δM_W^{PDF} . We have also shown that δM_W^{PDF} may be reduced substantially by a judicious choice of the kinematic-fitting window, and optimized with respect to the statistical uncertainty of the fit. These methods pave the way for higher-precision M_W measurements at hadron colliders.

ACKNOWLEDGMENTS

We thank Sourav Sen for his help in accessing the LHAPDF [28] library. We thank Pavel Murat, Christopher Hays, Konstantinos Vellidis, Bodhitha Jayatilaka, William Ashmanskas, Willis Sakumoto, Giorgio Belletini, Jeffrey Appel, David Toback and colleagues on the CDF experiment for helpful discussions. We thank Fermilab for providing the computing resources. This work was supported by the U.S. Department of Energy.

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