Semileptonic decays of B_c meson to *P*-wave charmonium states

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Inspired by a series of unexpected measurements of semileptonic decays mediated via $b \rightarrow c$ charged current interactions, we explore semileptonic B_c decays to the four lightest *P*-wave charmonium states, $\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$, by the recently developed improved perturbative QCD formalism, in which the charm quark mass effect is included both in the Sudakov factor and the hard kernels. We first directly evaluate the concerned transition form factors with vector and axial-vector currents in the region of small momentum transfer squared and then recast them to the full kinematical region by adopting the exponential parametrization. The obtained form factors are used to evaluate the semileptonic decay branching ratios, which can reach the order of 10^{-3} , letting the corresponding measurement appear feasible. For a better analysis, a comparison of our results with the predictions of other models is provided. We also present the ratios between the tau and light lepton branching ratios and the polarization contributions in the relevant processes, which still need experimental tests in the ongoing and forthcoming experiments. Any significant deviations from the Standard Model results may provide some hints of new physics effects.

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I. INTRODUCTION

Recently, a number of experimental measurements involving semitauonic decays of the charged current $b \rightarrow c\tau\nu_{\tau}$ transitions have shown interesting deviations from their Standard Model (SM) expectations, though the significance of the excess is low due to the large statistical uncertainties. For example, the measured values for $\mathcal{R}(D^{(*)})$ corresponding to the ratios of branching fractions $\mathcal{B}(B \rightarrow D^{(*)}\tau\bar{\nu}_{\tau})/\mathcal{B}(B \rightarrow D^{(*)}l\bar{\nu}_l)$, with *l* either an electron or muon, by the *BABAR* [1,2], Belle [3–6], and LHCb [7–9] Collaborations show a significant excess over the SM expectation [10–13]. The most statistically significant deviation at the 4σ level [13] is seen in the combination of $\mathcal{R}(D)$ and $\mathcal{R}(D^*)$. Very recently, the corresponding measurement regarding $b \rightarrow c\tau\nu_{\tau}$ in B_c had also been reported by LHCb [14]

$$\mathcal{R}(J/\psi) = \frac{\mathcal{B}(B_c^+ \to J/\psi\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to J/\psi\mu^+\nu_\mu)}$$

= 0.71 ± 0.17(stat) ± 0.18(syst). (1)

The yield value lies at about 2σ above the range of existing predictions in the SM [15–17]. These ratios have been calculated to high precision due to the cancellation of numerous uncertainties common to the numerator and denominator. Within the SM, the deviation from unity is mainly caused by the massive τ lepton, which also increases the sensitivity to new physics (NP) in these decays. Then, the possible NP effects in the semileptonic decays have been discussed recently in several papers [18–26]. To maximize future sensitivity to NP contributions, measuring and understanding the semileptonic modes involving various P-wave orbitally excited charmonium $X(X \in {\chi_{c0}, \chi_{c1}, \chi_{c2}, h_c})$ in the final state for the same flavor content are important and necessary, not only as they can give additional and complementary information on the NP but also as they constitute backgrounds to the $\mathcal{R}(J/\psi)$ measurements.

Experimentally, many nonleptonic decays with J/ψ or $\psi(2S)$ as the final charmonium have been detected [27], and the first evidence for the decay $B_c \rightarrow \chi_{c0}\pi$ is found at 4.0σ significance by the LHCb experiment [28]. However, for the semileptonic decays, so far, only the $B_c \rightarrow J/\psi$ transitions have recently been observed by the LHCb Collaboration [14,29]. As the LHC accumulates more and more data, the semileptonic B_c decays to the *P*-wave charmonium will have more possibilities to be detected. Theoretically, essential to the study of the semileptonic decays is the calculation of the invariant form factors describing the corresponding hadronic transitions. In the literature, a wide range of various approaches has been used

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to compute the $B_c \rightarrow X$ transition form factors, such as the QCD sum rules (QCDSR) [30,31], the covariant light-front quark model (LFQM) [32], the renormalization group method (RGM) [33], the relativistic constituent quark model (RCQM) [34], relativistic quark model (RQM) [35], the nonrelativistic quark model (NRQM) [36], the Bethe-Salpeter approach (BS) [37], the relativistic quark model based on the quasipotential approach (RQMQP) [38], and the Isgur-Scora-Grinstein-Wise II model (ISGW II) [39]. More recently, the relativistic corrections to the form factors of the B_c meson into *P*-wave orbitally excited charmonium have been investigated using the nonrelativistic QCD effective theory (NRQCD) [40].

As a successive work of [15,41,42], in this paper, we do not attempt to resolve the $\mathcal{R}(J/\psi)$ anomaly beyond the SM but provide more reliable calculations of those orbitally excited state modes within the SM. A future improvable measurement might reveal whether a similar anomaly also exists in $\mathcal{R}(X)$. In order to meet the measurements for charmonium B_c decays with good precision, we adopt the socalled improved perturbative QCD formalism [43] recently developed by Xin Liu *et al.* The charmonium B_c decays are a multiscale process, which contain three scales: the bottom quark mass m_b , the charm quark mass m_c , and the QCD scale Λ_{OCD} . Under the hierarchy of $m_b \gg m_c \gg \Lambda_{\text{OCD}}$, the charm quark effect enters the Sudakov exponent through an additional large infrared logarithm log (m_b/m_c) , which should be resummed. For the detailed derivation of the k_T resummation technique with the finite charm quark mass, the reader is referred to [43].

The outline of the paper is as follows: In Sec. II, we define kinematics and describe the meson distribution amplitude of the initial and final states. In Sec. III, we give the factorization formulas for the $B_c \rightarrow X$ form factors in the PQCD approach. Subsequently, we present the general formalism for the semileptonic differential decay widths with the lepton-helicity states. Section IV is devoted to the numerical analysis of the form factors, branching ratios, polarizations, and comparison of our results with the other approaches. A summary is given in Sec. V.

II. KINEMATICS AND MESON DISTRIBUTION AMPLITUDES

For simplicity, we work in the rest frame of the B_c meson and use light-cone coordinates. The momentum of the B_c meson and charmonium can be denoted as [15,16,44]

$$P_1 = \frac{M}{\sqrt{2}}(1, 1, \mathbf{0}_{\rm T}), \qquad P_2 = \frac{M}{\sqrt{2}}(r\eta^+, r\eta^-, \mathbf{0}_{\rm T}), \quad (2)$$

with the ratio r = m/M, and m(M) is the mass of the charmonium (B_c) meson. The factors $\eta^{\pm} = \eta \pm \sqrt{\eta^2 - 1}$ are defined in terms of the velocity transfer $\eta = v_1 \cdot v_2$ with $v_1 = P_1/M$ and $v_2 = P_2/m$ [44]. For the momentum



FIG. 1. The leading-order Feynman diagrams for the semileptonic decays $B_c^+ \rightarrow X l^+ \nu_l$ with $l = (e, \mu, \tau)$.

transfer $q = P_1 - P_2$, there exists $\eta = \frac{1+r^2}{2r} - \frac{q^2}{2rM^2}$. The momentum of the valence quarks $k_{1,2}$, whose notation are displayed in Fig. 1, are parametrized as

$$k_1 = x_1 P_1 + (0, 0, \mathbf{k}_{1T}), \quad k_2 = x_2 P_2 + (0, 0, \mathbf{k}_{2T}), \quad (3)$$

where the $\mathbf{k}_{1T,2T}$, $x_{1,2}$ represent the transverse momentum and longitudinal momentum fraction of the charm quark inside the meson, respectively.

As the direct analogue of the vector charmonium [15], for an axial-vector charmonium, the longitudinal (transverse) polarization vectors $\epsilon(0(\pm))$ can be defined as

$$\epsilon(0) = \frac{1}{\sqrt{2}} (\eta^+, -\eta^-, \mathbf{0}_{\mathrm{T}}), \qquad \epsilon(\pm) = (0, 0, \mathbf{1}_{\mathrm{T}}), \qquad (4)$$

which satisfy the normalization $e^2(0) = e^2(\pm) = -1$ and the orthogonality $e(0) \cdot P_2 = 0$.

For the tensor charmonium, since the polarization tensor $\epsilon_{\mu\nu}(\lambda)$ with helicity λ is traceless, symmetric, and satisfies the condition $\epsilon_{\mu\nu}(\lambda)P_2^{\nu} = 0$, it can be constructed via the polarization vector $\epsilon(0, \pm)$ [45,46]:

$$\epsilon_{\mu\nu}(\pm 2) = \epsilon_{\mu}(\pm)\epsilon_{\nu}(\pm),$$

$$\epsilon_{\mu\nu}(\pm 1) = \frac{1}{\sqrt{2}}[\epsilon_{\mu}(\pm)\epsilon_{\nu}(0) + \epsilon_{\nu}(\pm)\epsilon_{\mu}(0)],$$

$$\epsilon_{\mu\nu}(0) = \frac{1}{\sqrt{6}}[\epsilon_{\mu}(+)\epsilon_{\nu}(-) + \epsilon_{\mu}(-)\epsilon_{\nu}(+)] + \sqrt{\frac{2}{3}}\epsilon_{\mu}(0)\epsilon_{\nu}(0).$$
(5)

As usual, it is convenient to define another two polarization vectors $e_{T\mu}$ and e_{μ} corresponding to the transition form factors and light-cone distribution amplitudes (LCDAs), respectively, which are related to the polarization tensor by [47]

$$\epsilon_{T\mu}(\lambda) = \frac{\epsilon_{\mu\nu}(\lambda)P_1^{\nu}}{M}, \qquad \epsilon_{\bullet\mu}(\lambda) = m\frac{\epsilon_{\mu\nu}(\lambda)v^{\nu}}{P_2 \cdot v}, \quad (6)$$

with the unit vectors $v = (0, 1, \mathbf{0}_{T})$ on the light cone. Combining Eqs. (2) and (4)–(6), we further have

$$\epsilon_{T\mu}(\pm 2) = 0,$$

$$\epsilon_{T\mu}(\pm 1) = \sqrt{\frac{1}{2}} \frac{\epsilon(0) \cdot P_1}{M} \epsilon_{\mu}(\pm) = \sqrt{\frac{1}{2}} \sqrt{\eta^2 - 1} \epsilon_{\mu}(\pm),$$

$$\epsilon_{T\mu}(0) = \sqrt{\frac{2}{3}} \frac{\epsilon(0) \cdot P_1}{M} \epsilon_{\mu}(0) = \sqrt{\frac{2}{3}} \sqrt{\eta^2 - 1} \epsilon_{\mu}(0),$$

$$\epsilon_{\cdot\mu}(\lambda) = \frac{\epsilon_{T\mu}(\lambda)}{\sqrt{\eta^2 - 1}}.$$
(7)

Note that both ϵ_T and ϵ_{\bullet} above have the same energy scaling as the usual polarization vector ϵ . It makes the calculations of the B_c decays into a tensor meson similar to those of the vector analogues. The only difference is that the polarization vector ϵ is replaced by ϵ_{\bullet} in the LCDAs but by ϵ_T in the transition form factors.

In the course of the PQCD calculations, the necessary inputs contain the LCDAs, which are constructed via the nonlocal matrix elements. The B_c meson is a heavy-light system, whose light-cone matrix element can be decomposed as

$$\int d^4 z e^{ik_1 \cdot z} \langle 0|\bar{b}_{\alpha}(0)c_{\beta}(z)|B_c(P_1)\rangle$$
$$= \frac{i}{\sqrt{2N_c}} [(P_1 + M)\gamma_5\phi_{B_c}(k_1)]_{\beta\alpha}, \qquad (8)$$

where $N_c = 3$ is the color factor. Here, we only consider one of the dominant Lorentz structures. In coordinate space, the distribution amplitude ϕ_{B_c} with an intrinsic *b* (the conjugate space coordinate to k_T) dependence is adopted in a Gaussian form as [43]

$$\phi_{B_c}(x,b) = N_{B_c} x(1-x) \exp\left[-\frac{(1-x)m_c^2 + xm_b^2}{8\omega^2 x(1-x)} - 2\omega^2 b^2 x(1-x)\right],$$
(9)

with the shape parameter $\omega = 1.0 \pm 0.1$ GeV related to the factor N_{B_c} by the normalization

$$\int_0^1 \phi_{B_c}(x,0) dx = 1.$$
 (10)

For the *P*-wave charmonium states, their LCDAs were recently analyzed in Ref. [41] and are defined by

$$\langle S(P_2) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [I\!\!\!/_2 \psi_S^v(x) + m \psi_S^s(x)]_{\beta\alpha},$$

$$\langle A(P_2, \epsilon(0)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \gamma_5 \not\!\!/_5 \not\!\!/_5 (0) \psi_A^L(x) + \gamma_5 \not\!\!/_5 \not\!\!/_5 (0) I\!\!\!/_2 \psi_A^T(x)]_{\beta\alpha},$$

$$\langle A(P_2, \epsilon(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \gamma_5 \not\!\!/_5 \not\!\!/_5 (\pm) \psi_A^V(x) + \gamma_5 \not\!\!/_5 \not\!\!/_5 (\pm) I\!\!\!/_2 \psi_A^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(0)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!\!/_{\bullet}(0) \psi_T(x) + \not\!\!/_{\bullet}(0) I\!\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!\!/_{\bullet}(\pm) I\!\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!\!/_{\bullet}(\pm) I\!\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!\!/_{\bullet}(\pm) I\!\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!/_{\bullet}(\pm) I\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!/_{\bullet}(\pm) I\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

$$\langle T(P_2, \epsilon_{\bullet}(\pm)) | \bar{c}_{\alpha}(z) c_{\beta}(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2 \cdot z} [m \not\!/_{\bullet}(\pm) \psi_T^V(x) + \not\!/_{\bullet}(\pm) I\!\!/_2 \psi_T^T(x)]_{\beta\alpha},$$

where the abbreviations *S*, *A*, and *T* correspond to scalar, axial-vector, and tensor charmonium states, respectively. $\psi_S^v, \psi_A^{L,T}$, and $\psi_T^{(T)}$ are of twist-2, while $\psi_S^s, \psi_A^{t,V}$, and $\psi_T^{t,V}$ are of twist-3. For their expressions, the same form and parameters are adopted as in [41].

III. FORM FACTORS IN THE PQCD APPROACH

Based on the k_T factorization theorem, the transition form factors can be expressed as the convolution of a hard kernel with the distribution amplitudes of those mesons involved in the decays in the heavy-quark and large-recoil limits. For a review of this approach, please see Ref. [48]. The hard kernel can be treated by PQCD at the leading order in an α_s expansion (single gluon exchange as depicted in Fig. 1). Below, we will derive the general formulas of the $B_c \rightarrow S, A, T$ transition form factors in the PQCD approach.

A. $B_c \rightarrow \chi_{c0}$ form factors

The $B_c \rightarrow \chi_{c0}$ form factors are defined by [32,49]

$$\begin{split} \langle S(P_2) | \bar{c} \gamma^{\mu} \gamma_5 b | B_c(P_1) \rangle &= \left[(P_1 + P_2)^{\mu} - \frac{M^2 - m^2}{q^2} q^{\mu} \right] \\ &\times F_+(q^2) + \frac{M^2 - m^2}{q^2} q^{\mu} F_0(q^2). \end{split}$$
(12)

It is conventional to define two auxiliary form factors $f_1(q^2)$ and $f_2(q^2)$, which are related to $F_+(q^2)$ and $F_0(q^2)$ by

$$F_{+}(q^{2}) = \frac{1}{2}(f_{1}(q^{2}) + f_{2}(q^{2})), \qquad F_{0}(q^{2}) = \frac{1}{2}f_{1}(q^{2})\left[1 + \frac{q^{2}}{M^{2} - m^{2}}\right] + \frac{1}{2}f_{2}(q^{2})\left[1 - \frac{q^{2}}{M^{2} - m^{2}}\right].$$
(13)

After standard calculations, we obtain their factorization formulas as follows:

$$f_{1}(q^{2}) = 4\sqrt{\frac{2}{3}}\pi M^{2}f_{B}C_{f}r\int_{0}^{1}dx_{1}dx_{2}\int_{0}^{\infty}b_{1}b_{2}db_{1}db_{2}\phi_{B_{c}}(x_{1},b_{1})[\psi_{S}^{v}(x_{2},b_{2})r(x_{2}-1)-\psi_{S}^{s}(x_{2},b_{2})(r_{b}-2)]\alpha_{s}(t_{a})S_{ab}(t_{a})$$

$$\times h(\alpha_{e},\beta_{a},b_{1},b_{2})S_{t}(x_{2}) - [\psi_{S}^{v}(x_{2},b_{2})(r-2\eta x_{1})+\psi_{S}^{s}(x_{2},b_{2})2(x_{1}+r_{c})]\alpha_{s}(t_{b})S_{ab}(t_{b})h(\alpha_{e},\beta_{b},b_{2},b_{1})S_{t}(x_{1}),$$

$$(14)$$

$$f_{2}(q^{2}) = 4\sqrt{\frac{2}{3}}\pi M^{2}f_{B}C_{f}\int_{0}^{1}dx_{1}dx_{2}\int_{0}^{\infty}b_{1}b_{2}db_{1}db_{2}\phi_{B_{c}}(x_{1},b_{1})[\psi_{S}^{v}(x_{2},b_{2})(2r_{b}-1-2r\eta(x_{2}-1))+\psi_{S}^{s}(x_{2},b_{2})$$

$$\times 2r(x_{2}-1)]\alpha_{s}(t_{a})S_{ab}(t_{a})h(\alpha_{e},\beta_{a},b_{1},b_{2})S_{t}(x_{2}) - [\psi_{S}^{v}(x_{2},b_{2})(-r_{c}+x_{1})-\psi_{S}^{s}(x_{2},b_{2})2r]$$

$$\times \alpha_{s}(t_{b})S_{ab}(t_{b})h(\alpha_{e},\beta_{b},b_{2},b_{1})S_{t}(x_{1}), \qquad (15)$$

with $r_{b,c} = \frac{m_{b,c}}{M}$. α_e and $\beta_{a,b}$ are the virtuality of the internal gluon and quark, respectively. Their expressions are

$$\begin{aligned} \alpha_e &= -M^2 [x_1 + \eta^+ r(x_2 - 1)] [x_1 + \eta^- r(x_2 - 1)], \\ \beta_a &= m_b^2 - M^2 [1 + \eta^+ r(x_2 - 1)] [1 + \eta^- r(x_2 - 1)], \\ \beta_b &= m_c^2 - M^2 [\eta^+ r - x_1] [\eta^- r - x_1], \end{aligned}$$
(16)

where the explicit expressions of the functions S_t , h, and the scales $t_{a,b}$ are referred to [50]. The modified Sudakov factor S_{ab} , which includes the charm quark mass effect, can be found in [43].

B. $B_c \rightarrow \chi_{c1}, h_c$ form factors

Following Ref. [15], the $B_c \rightarrow \chi_{c1}$, h_c transition induced by the vector and axial-vector currents is parametrized by

$$\langle A(P_2) | \bar{c} \gamma^{\mu} b | B_c(P_1) \rangle = 2m \frac{\epsilon^* \cdot q}{q^2} q^{\mu} V_0(q^2) + (M-m) \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^{\mu} \right] V_1(q^2) - \frac{\epsilon^* \cdot q}{M-m} \left[(P_1 + P_2)^{\mu} - \frac{M^2 - m^2}{q^2} q^{\mu} \right] V_2(q^2),$$

$$\langle A(P_2) | \bar{c} \gamma^{\mu} \gamma_5 b | B_c(P_1) \rangle = \frac{2iA(q^2)}{M-m} \epsilon^{\mu\nu\rho\sigma} \epsilon^*_{\nu} P_{2\rho} P_{1\sigma},$$

$$(17)$$

where the convention $e^{0123} = +1$ is taken. Compared with the $B_c \rightarrow J/\psi$ transition, here the behavior of the vector and axial-vector currents is interchanged, and the factor M + m is replaced by M - m. The relation $2rV_0(0) = (1 - r)A_1(0) - (1 + r)A_2(0)$ is obtained to smear the singularity at $q^2 = 0$.

The factorization formulas are acquired as

$$V_{0}(q^{2}) = -2\sqrt{\frac{2}{3}}\pi M^{2}f_{B}C_{f}\int_{0}^{1}dx_{1}dx_{2}\int_{0}^{\infty}b_{1}b_{2}db_{1}db_{2}\phi_{B_{c}}(x_{1},b_{1})[\psi^{L}(x_{2},b_{2})(1-2r_{b}-r(x_{2}-1)(r-2\eta)) -\psi^{t}(x_{2},b_{2})r(2x_{2}-r_{b})]\alpha_{s}(t_{a})S_{ab}(t_{a})h(\alpha_{e},\beta_{a},b_{1},b_{2})S_{t}(x_{2}) -\psi^{L}(x_{2},b_{2})[-r_{c}+r^{2}+x_{1}(1-2r\eta)] \times\alpha_{s}(t_{b})S_{ab}(t_{b})h(\alpha_{e},\beta_{b},b_{2},b_{1})S_{t}(x_{1}),$$
(18)

$$V_{1}(q^{2}) = 4\sqrt{\frac{2}{3}} \frac{r}{1-r} \pi M^{2} f_{B} C_{f} \int_{0}^{1} dx_{1} dx_{2} \int_{0}^{\infty} b_{1} b_{2} db_{1} db_{2} \phi_{B_{c}}(x_{1}, b_{1}) [\psi_{A}^{V}(x_{2}, b_{2})(-2r_{b} + \eta r(x_{2} - 1) + 1) \\ + \psi_{A}^{T}(x_{2}, b_{2}) [\eta r_{b} - 2(\eta + r(x_{2} - 1))]] \alpha_{s}(t_{a}) S_{ab}(t_{a}) h(\alpha_{e}, \beta_{a}, b_{1}, b_{2}) S_{t}(x_{2}) - \psi_{A}^{V}(x_{2}, b_{2})[-r_{c} - x_{1} + \eta r] \alpha_{s}(t_{b}) S_{ab}(t_{b}) \\ \times h(\alpha_{e}, \beta_{b}, b_{2}, b_{1}) S_{t}(x_{1}),$$
(19)

$$V_{2}(q^{2}) = -A_{1} \frac{(1-r)^{2}(r-\eta)}{2r(\eta^{2}-1)} - 2\pi M^{2} f_{B} C_{f} \sqrt{\frac{2}{3}} \frac{1-r}{\eta^{2}-1} \int_{0}^{1} dx_{1} dx_{2} \int_{0}^{\infty} b_{1} b_{2} db_{1} db_{2} \phi_{B_{c}}(x_{1}, b_{1}) \\ \times [\psi_{A}^{t}(x_{2}, b_{2})(r_{b}(1-\eta r) + 2r^{2}(x_{2}-1) - 2\eta r(x_{2}-2) - 2) \\ -\psi_{A}^{L}(x_{2}, b_{2})(2r_{b}(\eta-r) - \eta + r(\eta r(x_{2}-1) - 2\eta^{2}(x_{2}-1) + x_{2}))]\alpha_{s}(t_{a})S_{ab}(t_{a})h(\alpha_{e}, \beta_{a}, b_{1}, b_{2})S_{t}(x_{2}) \\ +\psi_{A}^{L}(x_{2}, b_{2})[-r_{c}(r-\eta) + \eta r^{2} + r(-2\eta^{2}x_{1} + x_{1} - 1) + \eta x_{1}]\alpha_{s}(t_{b})S_{ab}(t_{b})h(\alpha_{e}, \beta_{b}, b_{2}, b_{1})S_{t}(x_{1}),$$
(20)
$$A(q^{2}) = 2\sqrt{\frac{2}{3}}\pi M^{2}f_{B}C_{f}(1-r)\int_{0}^{1} dx_{1}dx_{2}\int_{0}^{\infty} b_{1}b_{2}db_{1}db_{2}\phi_{B_{c}}(x_{1}, b_{1}) \\ \times [\psi_{A}^{V}(x_{2}, b_{2})r(1-x_{2}) + \psi_{A}^{T}(x_{2}, b_{2})(r_{b} - 2)]\alpha_{s}(t_{a})S_{ab}(t_{a})h(\alpha_{e}, \beta_{a}, b_{1}, b_{2})S_{t}(x_{2}) \\ -\psi_{A}^{V}(x_{2}, b_{2})r\alpha_{s}(t_{b})S_{ab}(t_{b})h(\alpha_{e}, \beta_{b}, b_{2}, b_{1})S_{t}(x_{1}).$$
(21)

It should be stressed that the nonlocal matrix element for the axial-vector and scalar charmonium meson in Eq. (11) can be related to the vector and pseudoscalar ones [15], respectively, by multiplying by the structure $(-i)\gamma_5$ from the left-hand side. The factorization formulas $f_{1,2}$, $V_{0,1,2}$, and A here are similar to the corresponding ones in [15] with the r_c term flipping signs and the replacement $1 + r \rightarrow 1 - r$.

C. $B_c \rightarrow \chi_{c2}$ form factors

In analogy with $B_c \rightarrow J/\psi$ form factors, we parametrize the $B_c \rightarrow \chi_{c2}$ form factors induced by the vector and axial-vector currents as

$$\langle T(P_2) | \bar{c} \gamma^{\mu} b | B_c(P_1) \rangle = \frac{2iV(q^2)}{M+m} \epsilon^{\mu\nu\rho\sigma} \epsilon^*_{T\nu} P_{2\rho} P_{1\sigma},$$

$$\langle T(P_2) | \bar{c} \gamma^{\mu} \gamma_5 b | B_c(P_1) \rangle = 2m \frac{\epsilon^*_T \cdot q}{q^2} q^{\mu} A_0(q^2) + (M+m) \left[\epsilon^{*\mu}_T - \frac{\epsilon^*_T \cdot q}{q^2} q^{\mu} \right] A_1(q^2) - \frac{\epsilon^*_T \cdot q}{M+m} \left[(P_1 + P_2)^{\mu} - \frac{M^2 - m^2}{q^2} q^{\mu} \right] A_2(q^2).$$

(22)

Note that the structure of above form factors is analogous to the J/ψ case with the replacement $\epsilon \rightarrow \epsilon_T$. In addition, as mentioned before, the LCDAs of a tensor meson are also similar to the vector ones except that the ϵ is replaced by ϵ_{\bullet} . So, the factorization formulas here can be straightforwardly obtained by replacing the twist-2 or twist-3 LCDAs of the J/ψ with the corresponding twists of the χ_{c2} one in Eq. (11). After multiplying by the different definitions of the polarization vector, we have [47]

$$\mathcal{F}^{B_c \to \chi_{c2}} = \frac{\epsilon_{\bullet}}{\epsilon_T} \mathcal{F}^{B_c \to J/\psi}|_{\psi_V \to \psi_T} = \frac{1}{\sqrt{\eta^2 - 1}} \mathcal{F}^{B_c \to J/\psi}|_{\psi_V \to \psi_T}.$$
(23)

D. The semileptonic differential decay rates

As is well known, the above form factors are reliable only in the small q^2 region in the PQCD framework [47,51]. In order to estimate the semileptonic differential decay rates, we need to know the q^2 -dependent form factors in the full kinematical region. Our form factors are truncated at about $q^2 = m_{\tau}^2$ with m_{τ} the mass of the τ lepton. We first perform the PQCD calculations on them in the range of $0 < q^2 < m_{\tau}^2$, while the momentum dependence of the form factors in the $m_{\tau}^2 < q^2 < (M - m)^2$ region is determined by fitting through a three-parameter function. The following fit parametrization is chosen for the form factors with respect to q^2 [15]:

$$\mathcal{F}_i(q^2) = \mathcal{F}_i(0) \exp\left[a\frac{q^2}{M^2} + b\left(\frac{q^2}{M^2}\right)^2\right],\tag{24}$$

where \mathcal{F}_i denotes any one of the form factors, and *a*, *b* are the fitted parameters.

After integrating out the off-shell W boson, the effective Hamiltonian for the $b \rightarrow c l \nu_l$ transition is written as [52]

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cb}^* \bar{b} \gamma_\mu (1 - \gamma_5) c \otimes \bar{\nu}_l \gamma^\mu (1 - \gamma_5) l, \qquad (25)$$

where $G_F = 1.16637 \times 10^{-5}$ GeV⁻² is the Fermi coupling constant and V_{cb} is one of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The differential decay rate of the exclusive processes $B_c \rightarrow (S, A) l\nu$ can be expressed in terms of the form factors as [32]

$$\frac{d\Gamma}{dq^2}(B_c \to Sl\nu) = \frac{G_F^2 |V_{cb}|^2}{384\pi^3 M^3 q^2} \sqrt{\lambda(q^2)} \left(1 - \frac{m_l^2}{q^2}\right)^2 [3m_l^2 (M^2 - m^2)^2 |F_0(q^2)|^2 + (m_l^2 + 2q^2)\lambda(q^2)|F_+(q^2)|^2], \tag{26}$$

$$\frac{d\Gamma_L}{dq^2} (B_c \to A l\nu) = \frac{G_F^2 |V_{cb}|^2}{384\pi^3 M^3 q^2} \sqrt{\lambda(q^2)} \left(1 - \frac{m_l^2}{q^2}\right)^2 \left\{ 3m_l^2 \lambda(q^2) |V_0(q^2)|^2 + \frac{m_l^2 + 2q^2}{4m^2} |(M^2 - m^2 - q^2)(M - m)V_1(q^2) - \frac{\lambda(q^2)}{M - m} V_2(q^2)|^2 \right\},$$
(27)

$$\frac{d\Gamma_{\pm}}{dq^2}(B_c \to Al\nu) = \frac{G_F^2 |V_{cb}|^2}{384\pi^3 M^3} \lambda^{3/2}(q^2) \left(1 - \frac{m_l^2}{q^2}\right)^2 (m_l^2 + 2q^2) \left|\frac{A(q^2)}{M - m} \mp \frac{(M - m)V_1(q^2)}{\sqrt{\lambda(q^2)}}\right|^2,\tag{28}$$

where m_l is the lepton mass and $\lambda(q^2) = (M^2 + m^2 - q^2)^2 - 4M^2m^2$. The subscripts L, +, and – denote the longitudinal, positive, and negative polarizations of the final state, respectively. As stated before, the decay width of $B_c \rightarrow \chi_{c2} l\nu$ can be related to the J/ψ one [15] by making the following replacement:

$$\frac{d\Gamma_L}{dq^2} (B_c \to \chi_{c2} l\nu) = \frac{2(\eta^2 - 1)}{3} \frac{d\Gamma_L}{dq^2} (B_c \to J/\psi l\nu) |_{\mathcal{F}^{B_c \to J/\psi} \to \mathcal{F}^{B_c \to \chi_{c2}}},$$

$$\frac{d\Gamma_{\pm}}{dq^2} (B_c \to \chi_{c2} l\nu) = \frac{\eta^2 - 1}{2} \frac{d\Gamma_{\pm}}{dq^2} (B_c \to J/\psi l\nu) |_{\mathcal{F}^{B_c \to J/\psi} \to \mathcal{F}^{B_c \to \chi_{c2}}},$$
(29)

where the factors $\frac{2(\eta^2-1)}{3}$ and $\frac{\eta^2-1}{2}$ come from Eq. (7). The total differential widths for the axial-vector and tensor charmonium modes can be written as

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_+}{dq^2} + \frac{d\Gamma_-}{dq^2}.$$
(30)

IV. NUMERICAL ANALYSIS AND DISCUSSION

For numerical evaluation, we collect the input parameters such as the masses and the meson decay constants in Table I, while the CKM matrix elements and B_c lifetime are set as $V_{cb} = 0.0405$ [27] and $\tau_{B_c} = 0.507$ ps [27], respectively. In the fitting procedure, the form factors in the lower region, namely, $q^2 \in [0, m_{\tau}^2]$, are computed in the PQCD framework. The numerical results of the relevant form factors at the scale $q^2 = 0$ as well as the fitted parameters *a* and *b* are presented in Table II, and here the uncertainties for our results are estimated including three aspects. The first type of error comes from the shape parameter ω of the B_c meson distribution amplitude; the second one is from the charm quark mass; the last one is caused by the decay constants of the charmonium states. In the evaluation, these uncertainties are obtained by simply taking a $\pm 10\%$ uncertainty on the central value. The combined uncertainties can reach 25%. In addition, the uncertainties from the CKM matrix elements and the hard scale *t* are very small and have been neglected.

It is found that the form factors of the P-wave modes are smaller than those of the S-wave ones in our previous study [15]. This phenomenon can be understood from the wave functions of the two states. The additional nodes in the

TABLE I. The quark masses and the B_c meson decay constant are taken from [43], while the decay constants of the *P*-wave charmonium states are adopted from the recent updated values evaluated from the QCD sum rules at the scales $\mu = m_c$ [53]. Other parameters are from PDG 2016 [27].

Mass (GeV)	M = 6.277	$m_{ au} = 1.777$	$m_b = 4.8$	$m_c = 1.5$
	$m_{\chi_{c0}} = 3.415$	$m_{\chi_{c1}} = 3.511$	$m_{\chi_{c2}} = 3.556$	$m_{h_c} = 3.525$
Decay constants (MeV)	$f_{B_c} = 0.489 \ f_{\chi_{c2}} = 0.177$	$f_{\chi_{c0}} = 0.0916 \ f_{\chi_{c2}}^\perp = 0.128$	$f_{\chi_{c1}} = 0.185$ $f_{h_c} = 0.127$	$f_{\chi_{c1}}^{\perp} = 0.0875$ $f_{h_c}^{\perp} = 0.133$

$\overline{\mathcal{F}_i}$	This work	QCDSR [31]	LFQM [32]	NRQCD [40] ^a	ISGW II [39]	а	b
$F_0^{B_c \to \chi_{c0}}$	$0.41\substack{+0.09+0.01+0.04\\-0.07-0.02-0.04}$	0.673 ± 0.195	$0.47\substack{+0.03\\-0.06}$	$1.25^{+0.15}_{-0.12}$		2.6	2.8
$F_{+}^{B_c \to \chi_{c0}}$	$0.41\substack{+0.09+0.01+0.04\\-0.07-0.02-0.04}$	0.673 ± 0.195	$0.47\substack{+0.03\\-0.06}$	$1.25^{+0.15}_{-0.12}$		3.6	2.6
$A^{B_c \to \chi_{c1}}$	$0.18\substack{+0.03+0.01+0.02\\-0.03-0.02-0.02}$	0.13 ± 0.04	$0.36\substack{+0.02\\-0.04}$	$0.99\substack{+0.19\\-0.15}$	$-0.36\substack{+0.01\\-0.01}$	2.4	13.8
$V_0^{B_c \to \chi_{c1}}$	$0.18\substack{+0.02+0.01+0.02\\-0.02-0.02-0.02}$	0.03 ± 0.01	$0.13\substack{+0.01 \\ -0.01}$	$0.12\substack{+0.01\\-0.01}$	$-0.55\substack{+0.00\\-0.01}$	4.8	-0.2
$V_1^{B_c \to \chi_{c1}}$	$0.86\substack{+0.14+0.06+0.09\\-0.10-0.08-0.09}$	0.30 ± 0.09	$0.85\substack{+0.02\\-0.04}$	$2.34_{-0.22}^{+0.21}$	$-0.42\substack{+0.02\\-0.02}$	2.7	-11.0
$V_2^{B_c \to \chi_{c1}}$	$0.11\substack{+0.02+0.01+0.01\\-0.01-0.00-0.01}$	0.06 ± 0.02	$0.15\substack{+0.01 \\ -0.01}$	$0.47\substack{+0.07 \\ -0.06}$	$0.28\substack{+0.01\\-0.01}$	5.3	-1.8
$V^{B_c \to \chi_{c2}}$	$1.15\substack{+0.15+0.00+0.11\\-0.13-0.03-0.10}$		$1.36\substack{+0.12\\-0.19}$	$5.89^{+1.60}_{-1.30}$		5.1	12.9
$A_0^{B_c \to \chi_{c2}}$	$0.83\substack{+0.13+0.04+0.08\\-0.11-0.05-0.08}$		$0.86\substack{+0.14\\-0.13}$	$1.80\substack{+0.40\\-0.33}$		7.0	15.3
$A_1^{B_c \to \chi_{c2}}$	$0.55\substack{+0.08+0.01+0.06\\-0.06-0.02-0.06}$		$0.81\substack{+0.10 \\ -0.10}$	$1.95^{+0.43}_{-0.35}$		4.3	7.3
$A_2^{B_c \to \chi_{c2}}$	$-0.14\substack{+0.06+0.07+0.01\\-0.09-0.08-0.01}$		$0.68\substack{+0.06\\-0.00}$	$2.24^{+0.51}_{-0.42}$		37.1	-96.1
$A^{B_c \to h_c}$	$0.10\substack{+0.02+0.00+0.01\\-0.01-0.00-0.01}$	0.13 ± 0.04	$0.07\substack{+0.01 \\ -0.01}$	$0.07\substack{+0.00 \\ -0.01}$	$0.05\substack{+0.00\\-0.00}$	3.0	-0.2
$V_0^{B_c \to h_c}$	$0.22\substack{+0.03+0.02+0.03\\-0.02-0.01-0.02}$	0.03 ± 0.01	$0.64\substack{+0.10\\-0.02}$	$1.63^{+0.25}_{-0.19}$	$0.78\substack{+0.01 \\ -0.01}$	3.1	1.8
$V_1^{B_c \to h_c}$	$0.46\substack{+0.05+0.01+0.05\\-0.05-0.03-0.05}$	0.30 ± 0.09	$0.50\substack{+0.05\\-0.08}$	$0.46\substack{+0.07\\-0.03}$	$0.61\substack{+0.01 \\ -0.01}$	2.6	-1.1
$V_2^{B_c \to h_c}$	$-0.03\substack{+0.00+0.00+0.00\\-0.00-0.01-0.00}$	0.06 ± 0.02	$-0.32\substack{+0.06\\-0.05}$	$-0.75\substack{+0.17\\-0.17}$	$-0.39\substack{+0.01\\-0.01}$	7.5	41.1

TABLE II. The form factors of the B_c meson decay to *P*-wave charmonium evaluated by PQCD and by other methods in the literature. We also show theoretical uncertainties induced by the shape parameter ω , charm quark mass m_c , and the decay constants of charmonium states, respectively. The last two columns correspond to the fit parameters *a* and *b* in this work.

^aWe quote the leading-order results of NRQCD.

wave functions of the orbital excited charmonium state cause the overlap between the initial and final state wave functions to become smaller. In addition, the smaller decay constants of P-wave charmonium states also suppress the corresponding values. Comparing the form factors of $B_c \rightarrow \chi_{c1}$ with $B_c \rightarrow h_c$ in Table. II, one can find the large differences between them. The main reason is the different DAs and the decay constants for the two kinds of axialvector charmonium. Because of the G parity, the DAs for χ_{c1} and h_c mesons exhibit the different asymptotic behaviors [41]. Moreover, the longitudinal and transverse decay constants (see Table I) in the two axial-vector mesons can also contribute to different values. The $B_c \rightarrow T$ transition form factor is somewhat larger since the prefactor in Eq. (23) is roughly $2r/(1-r^2) \approx 1.7$ at the maximally recoiling point, which enhanced the numbers accordingly.

So far, several authors have calculated the form factors of the concerned decays via different frameworks. To compare the results, we should rescale them according to the form factor definitions in Eqs. (12), (17), and (22). For example, comparing the definitions of the $B_c \rightarrow T$ transition form factor of Ref. [32] with ours, we have the following relations at the maximal recoil point:

$$V = M(M+m)h, \quad A_1 = \frac{M}{M+m}k, \quad A_2 = -M(M+m)b_+,$$
(31)

where the values of h, k, and b_+ can be found in [32]. Note that we have dropped an overall phase factor i which is

irrelevant for the calculation of the decay widths. Other results, such as QCDSR [31], ISGW II [39], and NRQCD [40], are also converted into the numbers according to our definitions in this paper and are listed in Table II. As indicated in Table II, the results evaluated in the different models are roughly comparable. Our results are generally close to those of the LFQM [32] and the QCDSR [31], while some of the results for the $B_c \rightarrow \chi_{c1}$ transition from the ISGW II model possess a sign that is the opposite of ours. The recent NRQCD predictions in [40] are obviously larger for most of the decay channels.

Based on the values of the transition form factors at $q^2 = 0$ and the fit parameters *a* and *b* listed in Table II, we can plot the momentum transfer squared dependence of these form factors in Fig. 2 for the four processes in the whole accessible kinematical range. The difference of the curve behavior for the various *P*-wave charmonium states is the consequence of their different LCDAs. The form factors for the $B_c \rightarrow \chi_{c2}$ transition have a relatively stronger momentum dependence than others. The main reason is that the $B_c \rightarrow \chi_{c2}$ form factors received additional q^2 dependence, as can be seen from the factorization formulas in Eq. (23), which provide an enhancement to the corresponding values with the increase of q^2 .

With the form factors at hand, one can directly obtain the partial decay width by integrating the corresponding differential decay rates over q^2 in Eqs. (26)–(29). We are now ready to calculate the respective semileptonic decay branching ratios. The numerical results are shown in Table III together with the numbers obtained in other



FIG. 2. The q^2 dependence of the transition form factors for the decay modes (a) $B_c \to \chi_{c0}$, (b) $B_c \to \chi_{c1}$, (c) $B_c \to \chi_{c2}$, and (d) $B_c \to h_c$. A minus sign has been added to $A_2^{B_c \to \chi_{c2}}$ and $V_2^{B_c \to h_c}$ so that the corresponding curves show in the upper panels.

model calculations for comparison. In general, it is observed that the branching ratios have close values within the error bars in all models. In particular, our results match very well with those of QCDSR [31]. Since the electron and muon are very light compared with the heavy tau lepton, we neglect their masses in the calculations. It is seen that the semitauonic decays' branching ratios fall short by a large factor compared with the

TABLE III. Branching ratios (in units of 10^{-3}) of semileptonic B_c decays evaluated by PQCD and by other methods in the literature. The errors are induced by the same sources as in Table II.

Modes	This work	QCDSR [31]	LFQM [32]	RGM [33]	RCQM [34]	RQM [35]	NRQM [36]	BS [37]	RQMQP [38]
$\overline{B_c \to \chi_{c0} e \nu_e}$	$2.22^{+1.08+0.11+0.47}_{-0.69-0.21-0.42}$	1.82 ± 0.51	$2.1^{+0.2}_{-0.4}$	1.2	1.7	1.8	1.1	1.3 ± 0.3	0.87
$B_c \to \chi_{c0} \tau \nu_{\tau}$	$0.48\substack{+0.23+0.02+0.10\\-0.15-0.05-0.09}$	0.49 ± 0.16	$0.24\substack{+0.01\\-0.03}$	0.17	0.13	0.18	0.13	0.16 ± 0.08	0.075
$B_c \to \chi_{c1} e \nu_e$	$1.53\substack{+0.57+0.24+0.32\\-0.35-0.30-0.29}$	1.46 ± 0.42	$1.4^{+0.0}_{-0.1}$	1.5	0.92	0.98	0.66	1.1 ± 0.3	0.82
$B_c \to \chi_{c1} \tau \nu_\tau$	$0.20\substack{+0.08+0.03+0.04\\-0.04-0.03-0.04}$	0.147 ± 0.044	$0.15\substack{+0.01 \\ -0.02}$	0.24	0.089	0.12	0.072	0.097 ± 0.065	0.092
$B_c \to \chi_{c2} e \nu_e$	$2.68^{+1.23+0.50+0.56}_{-0.80-0.57-0.51}$		$1.7\substack{+0.5\\-0.7}$	1.9	1.7	2.0	1.3	1.0 ± 0.3	1.6
$B_c \to \chi_{c2} \tau \nu_\tau$	$0.22\substack{+0.09+0.03+0.04\\-0.06-0.04-0.04}$		$0.096\substack{+0.027\\-0.036}$	0.29	0.082	0.14	0.093	0.082 ± 0.048	0.093
$B_c \to h_c e \nu_e$	$1.06^{+0.19+0.12+0.22}_{-0.21-0.13-0.20}$	1.42 ± 0.40	$3.1_{-0.8}^{+0.5}$	1.8	2.7	3.1	1.7	2.8 ± 0.8	0.96
$B_c \to h_c \tau \nu_{\tau}$	$0.13\substack{+0.02+0.01+0.03\\-0.03-0.02-0.02}$	0.137 ± 0.038	$0.22\substack{+0.02 \\ -0.04}$	0.25	0.17	0.27	0.15	0.19 ± 0.13	0.077

Modes	f_0	f_+	f_{-}
$B_c \to \chi_{c1} e \nu_e$	$0.34\substack{+0.01+0.01+0.00\\-0.00-0.01-0.00}$	$0.04\substack{+0.01+0.00+0.00\\-0.00-0.00-0.00}$	$0.62\substack{+0.00+0.00+0.00\\-0.02-0.01-0.00}$
$B_c \to \chi_{c1} \tau \nu_{\tau}$	$0.33\substack{+0.00+0.00+0.00\\-0.01-0.01-0.00}$	$0.06\substack{+0.01+0.00+0.00\\-0.00-0.00-0.00}$	$0.61\substack{+0.01+0.01+0.00\\-0.01-0.00-0.00}$
$B_c \to \chi_{c2} e \nu_e$	$0.77\substack{+0.03+0.04+0.00\\-0.02-0.03-0.00}$	$0.03\substack{+0.01+0.01+0.00\\-0.00-0.00-0.00}$	$0.20\substack{+0.03+0.04+0.00\\-0.02-0.02-0.02}$
$B_c \to \chi_{c2} \tau \nu_{\tau}$	$0.72^{+0.02+0.03+0.00}_{-0.03-0.04-0.00}$	$0.05\substack{+0.01+0.01+0.00\\-0.00-0.00-0.00}$	$0.23\substack{+0.02+0.02+0.00\\-0.02-0.03-0.00}$
$B_c \to h_c e \nu_e$	$0.68\substack{+0.01+0.03+0.00\\-0.02-0.00-0.00}$	$0.04\substack{+0.00+0.00+0.00\\-0.01-0.00-0.00}$	$0.28\substack{+0.02+0.01+0.00\\-0.01-0.02-0.00}$
$B_c \to h_c \tau \nu_{\tau}$	$0.60\substack{+0.00+0.02+0.00\\-0.02-0.00-0.00}$	$0.07\substack{+0.00+0.00+0.00\\-0.00-0.00-0.00}$	$0.33\substack{+0.02+0.00+0.00\\-0.00-0.02-0.00}$

TABLE IV. The PQCD predictions for the polarization fractions. The errors are induced by the same sources as in Table II.

corresponding values of the *e* and μ channels due to suppression from the phase space. In order to reduce the theoretical uncertainties from the hadronic parameters, we define four ratios between the branching fractions of semitauonic decays of B_c mesons relative to the decays involving lighter lepton families,

$$\mathcal{R}(X) = \frac{\mathcal{B}(B_c^+ \to X\tau^+\nu_\tau)}{\mathcal{B}(B_c^+ \to Xe^+\nu_e)}.$$
(32)

From the numbers in Table III, we have

$$\begin{aligned} \mathcal{R}(\chi_{c0}) &= 0.22^{+0.00}_{-0.01}, \qquad \mathcal{R}(\chi_{c1}) = 0.13^{+0.01}_{-0.00}, \\ \mathcal{R}(\chi_{c2}) &= 0.08^{+0.01}_{-0.00}, \qquad \mathcal{R}(h_c) = 0.12^{+0.01}_{-0.00}, \end{aligned}$$
(33)

where all uncertainties are added in quadrature. The central values lie between 0.08 and 0.22, which are typically smaller than our previous prediction for that of J/ψ with $\mathcal{R}(J/\psi) = 0.29$ [15] because the heavy *P*-wave charmonium states bring a smaller phase space than the *S*-wave ones. More recently, the LHCb Collaboration [14] published a measurement $\mathcal{R}(J/\psi) = 0.71$ that shows the discrepancy with the prediction of the SM. It would be interesting to see whether the similar anomalies also exist independently in these *P*-wave charmonium modes. Therefore, the measurements of various ratios such as $\mathcal{R}(X)$ in the future will give an additional hint for the NP effect in the $b \rightarrow cl\nu$ transition.

Next, we made a comprehensive polarization analysis of the axial-vector and tensor channels. Since the initial state B_c is a spinless particle, the final state axial-vector/tensor charmonium and lepton pair carry spin degrees of freedom. According to the angular momentum conservation, the semileptonic decays of $B_c \rightarrow A/T l \nu_l$ contain three different polarizations. It is meaningful to define three polarization fractions $f_{L,\pm} = \Gamma_{L,\pm}/(\Gamma_L + \Gamma_+ + \Gamma_-)$. Their individual polarization fractions are shown in Table IV, where the sources of the errors in the numerical estimates have the same origin as in Table II. We made the following observations. First, the minus polarization fractions have larger magnitudes in comparison to the plus components, and the latter are only at the percent level. From Table II, one can see the form factors A and V_1 have the

same sign, which gives constructive contributions to the minus polarized decay width but destructive contributions to the plus partners, as can be seen in Eq. (28). Second, for $B_c \rightarrow \chi_{c1}$ decays, the transverse polarization contributions dominated the branching ratio due to a destructive interference between V_1 and V_2 in the longitudinally polarized decay width. However, in the case of the $B_c \rightarrow h_c$ transition, the value of V_2 is a negative number, which reverses the constructive or destructive interference situation. The dramatically different polarization contributions between the two axial-vector decay channels are similar to the explanation in [32]. Finally, for each charmonium channel, the longitudinal, plus, and minus polarization fractions of the τ are roughly equal to the corresponding values of e, which reflects that the relative polarization contributions still favor the lepton flavor universality. These results will be tested at the ongoing and forthcoming hadron colliders.

V. CONCLUSION

Semileptonic charmonium decays of B_c mesons play a critical role in the determination of the magnitudes of the CKM matrix elements V_{cb} and in the test of the lepton flavor universality which is a basic assumption of the SM. The investigation of the corresponding P-wave charmonium modes is of special interest and further provides complementary information on physics beyond the SM. In this paper, we first calculated the $B_c \rightarrow \chi_{c0}, \chi_{c1}, \chi_{c2}, h_c$ transition form factors at the small momentum region within the improved PQCD framework. By fitting an auxiliary threeparameter exponential function, we obtained the momentumsquared-dependent form factors in the full kinematical region. We used them to estimate the branching ratios of the considered semileptonic. The order of branching ratios shows that these channels are accessible in the near future experiments. We also gave predictions on the ratio between the tau and light lepton branching ratio $\mathcal{R}(X)$, which are smaller than our previous calculation of $\mathcal{R}(J/\psi)$ due to the suppression from the phase space. A future improvable measurement might reveal whether a similar anomaly exists in these ratios. Three polarization contributions were also investigated in detail for the axial-vector and tensor modes. The approximately equal polarization fractions between the tau and light lepton with the same charmonium in the final ZHOU RUI, JIE ZHANG, and LI-LI ZHANG

states may indicate that the lepton flavor universality violation is negligible in the relative polarization contributions. These results and findings will be further tested by the LHCb and Belle II experiments in the near future.

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