Local field theory construction of very special conformal symmetry

Yu Nakayama
Department of Physics, Rikkyo University, Toshima, Tokyo 171-8501, Japan

(Received 3 April 2018; published 9 July 2018)

Cohen and Glashow argued that very special conformal field theories of a particular kind [i.e., with HOM(2) or SIM(2) invariance] cannot be constructed within the framework of local field theories. We, however, show examples of local construction by using nonlinear realization. We further construct linear realization from the topological twist at the cost of unitarity. To demonstrate the ubiquity of our idea, we also present corresponding holographic models.

DOI: 10.1103/PhysRevD.98.025007

I. INTRODUCTION

The assumption of locality plays an essential role in relativistic quantum field theories. In particle physics, it is usually argued that locality is necessary to guarantee the causal structure that is compatible with the special relativity: "nothing can travel faster than the speed of light." This, however, implies that the motivation to impose locality comes from the more sacred principle of causality and may not be fundamental. Indeed, with extended objects such as branes or strings, the interaction may take place in a nonlocal way, but still is compatible with the relativistic causality. In this sense, we may say that locality is tied up with the notion of particles under the assumption of the special relativity.

If we abandon the special relativity, the role of locality becomes less obvious. However, Cohen and Glashow argued that the locality may also play a significant role in very special relativity [1,2], which is a certain subgroup of Lorentz symmetry that preserves a particular null direction. They claim that if they impose the locality in field theories that obey a certain class of the very special relativity, they must be fully Lorentz invariant. The claim yields a direct connection between the violation of Lorentz symmetry and the violation of locality, which makes the very special relativity more predictive and interesting. In addition, the speed of light is constant in every direction even though we have a particular null direction, so such theories are phenomenologically viable.

Their argument was based on the spurion analysis. Suppose we begin with a relativistic field theory and consider its local deformation to break the symmetry down to particular subgroups of the Lorentz symmetry [technically known as SIM(2) or HOM(2) invariant very special relativity to be defined below]. Cohen and Glashow found that there are no such local operators available from the representation theory of Lorentz algebra. Therefore, they argue that there are no local field theories that realize SIM(2) and HOM(2) invariant very special relativity without symmetry enhancement to the full Lorentz symmetry. Alternatively, they proposed a way to achieve this by violating the assumption of locality at the same time [2].[1]

In this paper, we, however, point out that there is a loophole in their argument. When the original theory possesses a further global symmetry, one may construct the deformation that preserves the very special relativity without violating the locality. We show some examples in the context of very special conformal field theories [4] for definiteness, but a similar construction is possible and obviously easier without imposing the conformal symmetry.

II. VERY SPECIAL CONFORMAL SYMMETRY

To discuss very special relativity as well as very special conformal field theories, it is convenient to introduce light-cone coordinates $x^+ = \frac{1}{\sqrt{2}}(x^0 + x^1)$, $x^- = \frac{1}{\sqrt{2}}(x^0 - x^1)$, and x^i $(i = 2,\ 3)$ [or $x_+ = -\frac{1}{\sqrt{2}}(x^0 - x^1)$, $x_- = -\frac{1}{\sqrt{2}}(x^0 + x^1)$, and $x_i = x^i$]. The light-cone tensors are defined in a similar manner.

The very special relativity is based on the algebra spanned by P_+, P_-, P_i, and J_{+i}, where $P_\mu = \{P_+, P_-, P_i\}$ are spacetime translations and J_{+i} is a Lorentz transformation that preserves a particular null direction. In [1], they proposed four different algebras of very special relativity, but in this paper, we focus either on the HOM(2) invariant case by

[1] See, e.g., an explicit background-field origin of this non-locality in QED [3].

TABLE I. The commutation relation $i[X,Y]$ of very special conformal generators.

	P_+	P_-	P_i	J_{ij}	J_{+i}	J_{+-}	K_+	$\tilde{D}$
P_+	0	0	0	0	0	$-P_+$	0	0
P_-	0	0	0	0	P_i	P_-	$-\tilde{D}$	$2P_-$
P_i	0	0	0	P_j	P_+	0	J_{+i}	P_i
J_{ij}	0	0	$-P_j$	J_{kl}	J_{+i}	0	0	0
J_{+i}	0	$-P_i$	$-P_+$	$-J_{+i}$	0	$-J_{+i}$	0	$-J_{+i}$
J_{+-}	P_+	$-P_-$	0	0	J_{+i}	0	K_+	0
K_+	0	$\tilde{D}$	$-J_{+i}$	0	0	$-K_+$	0	$-2K_+$
$\tilde{D}$	0	$-2P_-$	$-P_i$	0	J_{+i}	0	$2K_+$	0

adding J_{+-} or mainly on the SIM(2) invariant case by adding J_{+-} and J_{ij}. If we abandon J_{+-} in each case, we have E(2) or T(2) invariant very special conformal field theories respectively.[2]

The conformal extension of the algebra of very special relativity was discussed in [4]. The gist is that we can only add the dilatation $\tilde{D}$ and a particular special conformal transformation K_+ (as a subgroup of the Poincaré conformal algebra). The schematic form of the commutation relation for the SIM(2) invariant very special conformal algebra is summarized in Table I. For the HOM(2) invariant case, one can just ignore the column and row of J_{ij}. The relevant fact that we will use later is that J_{+-} does not appear in Table I as a result of the commutator.

III. LOCAL FIELD THEORY EXAMPLES

Let us first recall the argument that very special conformal field theories with SIM(2) or HOM(2) invariance cannot be constructed from the spurion method. Suppose we have a conformal field theory and try to deform it by adding local operators that preserve SIM(2) symmetry. In order to preserve the E(2) invariant very special conformal symmetry, which is a subgroup of SIM(2) invariant very special conformal symmetry, we try to add a vector primary operator

$$S = S_0 + \int d^4x \lambda^\mu J_\mu, \tag{1}$$

where S_0 is the action of a Poincaré conformal field theory, and λ^μ has only nonzero components in λ^+ so that it preserves J_{+i} and J_{ij}.[3] In order to preserve the very special conformal symmetry $\tilde{D}$ and K_+, we further assume that the Poincaré scaling dimension of J_μ is five. This gives us a local field theory construction of E(2) invariant very special conformal field theory. The problem here is that the spurion vector λ^μ is not invariant under J_{+-}, and therefore, we cannot preserve the SIM(2) invariant very special conformal symmetry. This is essentially the reasoning made in [1] to claim that there is no SIM(2) invariant (not necessarily conformal) field theories from the spurion method.[4]

Nevertheless we do find a way to avoid this no-go argument by demanding that the spurion λ^μ transforms as a vector under J_{ij} and J_{+i}, but transforms as a "scalar" under J_{+-}. Since J_{+-} does not appear in the right-hand side of commutation relations of the very special conformal algebra, this causes no inconsistency at the level of the algebra. Of course, originally the spurion λ^μ was a vector under the full Lorentz transformation $J_{\mu\nu}$, so we need a trick to implement this idea.

The easiest way to do this is to use the concept of "topological twist" [7,8]. Suppose the original theory possesses an additional noncompact global $U(1)$ symmetry Q. Suppose also that it has a vector operator J_+ which transforms as $e^{-i\theta Q} J_+ e^{i\theta Q} = e^\theta J_+$ under the global symmetry Q. Then we see $\int dt d^3x J_+$ is invariant under $\tilde{J}_{+-} = J_{+-} + Q$ (while it was not invariant under J_{+-}). Now, we deform the action by the interaction

$$S = S_0 + \int d^4x \lambda^\mu J_\mu. \tag{2}$$

By construction, it is invariant under J_{+i} and J_{ij} as well as $\tilde{J}_{+-}$ as discussed above. The commutation relations among J_{+i}, J_{ij}, and $\tilde{J}_{+-}$ are the same as the ones in the very special relativity, so we may well regard $\tilde{J}_{+-}$ as J_{+-} in the very special conformal algebra. In addition, if the Poincaré scaling dimension of J_μ is five, it preserves $\tilde{D}$ and K_+. In this way, we have constructed a very special conformal field theory with the SIM(2) invariance in a local fashion.[5]

Let us show a couple of concrete examples to demonstrate the construction. First, we consider a field theory with two real fields A and B, which is defined by the action

[2]The total very special relativity algebra has various names in the literature. The combination of E(2) and P_μ is sometimes called the Bargmann algebra or massive Galilean algebra (see, e.g., [5] and references therein). The combination of SIM(2) and P_μ is called ISIM(2) algebra in [6].

[3]More generically, we could add the tensor operators with only + components, but the discussions below do not change.

[4]The argument does not rely on the conformal invariance, but note that it is based on the assumption that one can turn off the deformation such that the Lorentz invariance is recovered. This argument alone did not exclude the isolated examples if any.

[5]A similar idea to use the Poincaré dilatation rather than the global symmetry to twist J_{+-} was discussed in [9]. While the Lorentz part of the symmetry algebra is SIM(2), the commutator with the translation is different from the ones in Table I. We, therefore, called $\tilde{D}$ rather than J_{+-} in our discussions.

$$S = \int d^3xdt(\partial_+ A\partial_- B + \partial_+ B\partial_- A - \partial_i A\partial_i B + \lambda A^2(A\partial_+ B - B\partial_+ A)). \quad (3)$$

Here the last term $\lambda A^2(A\partial_+ B - B\partial_+ A)$ plays the role of $\lambda^\mu J_\mu$ above. It is obviously invariant under P_+, P_-, and P_i as well as J_{ij} and J_{i+}. It is invariant under the dilatation $\tilde{D}$,

$$i[\tilde{D}, A(0)] = A(0), \quad i[\tilde{D}, B(0)] = B(0), \quad (4)$$

as well as under the "twisted" Lorentz boost J_{+-},

$$i[J_{+-}, A(0)] = \frac{1}{2}A(0), \quad i[J_{+-}, B(0)] = -\frac{1}{2}B(0), \quad (5)$$

where we omit the orbital part by setting $x^\mu = 0$ because the invariance is trivial. Since the deformation is given by a vector primary operator, it is invariant under a particular special conformal transformation K_+,

$$i[K_+, A(x)] = (2x^- + 2(x^-)^2\partial_- + 2x^- x^i\partial_i + x_i^2\partial_+)A(x),$$
$$i[K_+, B(x)] = (2x^- + 2(x^-)^2\partial_- + 2x^- x^i\partial_i + x_i^2\partial_+)B(x). \quad (6)$$

Therefore, this model is a concrete example of very special conformal field theories with the SIM(2) invariance.

Let us, however, mention one caveat of this model. The theory is nonunitary because of the wrong sign in the kinetic term. The underlying reason we needed the non-unitarity is that we have to introduce the global noncompact $U(1)$ symmetry under which real fields change their absolute values rather than the phases. This typically requires the kinetic term with the negative signature. In other words, it must be $SO(1,1)$ rather than $SO(2)$.

On the other hand, at the level of effective field theories, one may also construct a unitary field theory with the SIM(2) invariant very special conformal symmetry realized in a nonlinear way. As an example, let us consider a field theory with a complex scalar ϕ and a real scalar φ with the action

$$S = \int d^3xdt(\partial_+\phi^*\partial_-\phi + \partial_+\phi\partial_-\phi^* - \partial_i\phi^*\partial_i\phi + |\phi|^2\left(\partial_+\varphi\partial_-\varphi - \frac{1}{2}\partial_i\varphi\partial_i\varphi\right) + i\lambda e^\varphi(\phi^*\partial_+\phi - \phi\partial_+\phi^*)). \quad (7)$$

To see how J_{+-} symmetry is realized, we make φ transform nonlinearly under the dilatation $\tilde{D}$ and the Lorentz transformation J_{+-}:

$$i[\tilde{D}, \phi] = \phi, \quad i[\tilde{D}, \varphi] = 2, \quad i[J_{+-}, \phi] = 0, \quad i[J_{+-}, \varphi] = 1, \quad (8)$$

so that the interaction $\int dtd^3xe^\varphi(\phi^*\partial_+\phi - \phi\partial_+\phi^*)$ is invariant under J_{+-} (as well as $\tilde{D}$). Note that the kinetic term is also invariant under the shift of φ. While the action is invariant, this model breaks the dilatation and special conformal transformation spontaneously by choosing the vacuum expectation values of $\phi \neq 0$ to avoid the singular kinetic term for φ.

The similar construction is possible for the HOM(2) invariant case. Consider the action

$$S = \int d^3xdt(\partial_+\phi^*\partial_-\phi + \partial_+\phi\partial_-\phi^* - \partial_i\phi^*\partial_i\phi + |\phi|^2\left(\partial_+\varphi\partial_-\varphi - \frac{1}{2}\partial_i\varphi\partial_i\varphi\right) + i\lambda^{\mu\nu}e^\varphi(\partial_\mu\phi^*\partial_\nu\phi - \partial_\nu\phi^*\partial_\mu\phi)), \quad (9)$$

where $\lambda^{\mu\nu} = -\lambda^{\nu\mu}$ has only a nonzero component in $\lambda^{+2} = -\lambda^{2+}$. We immediately see that the action is invariant under P_μ and J_{+i} (but not under J_{ij}). Invariance under J_{+-} is again guaranteed by the shift transformation of the φ field as $i[J_{+-}, \varphi] = 1$ so that the interaction term $\int dtd^3xi\lambda^{\mu\nu}e^\varphi (\partial_\mu\phi^*\partial_\nu\phi - \partial_\nu\phi^*\partial_\mu\phi)$ becomes invariant. The theory is unitary, but it breaks the very special conformal symmetry spontaneously. The linear construction at the cost of unitarity is also possible with the action similar to (3).

In [4], a holographic model for the E(2) invariant very special conformal field theory was discussed. Here, we, for the first time to our knowledge, present a holographic model for SIM(2) invariant very special conformal field theories. Let us consider the five-dimensional Einstein gravity coupled with two real vector fields A_M and B_M with the action

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2}R - \Lambda - \frac{1}{2}F_{MN}G^{MN} - m^2A_MB^M\right), \quad (10)$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ and $G_{MN} = \partial_M B_N - \partial_N B_m$. We set $\Lambda = -6$ and $m^2 = 8$. Then we find a particular solution of the equations of motion with the metric given by

$$ds^2 = g_{MN}d^M dx^N = \frac{-2dx^+dx^- + dx^idx^i + dz^2}{z^2} \quad (11)$$

and the vector fields

$$A = A_M dx^M = -\frac{dx^-}{z^2}, \qquad B = 0. \tag{12}$$

Invariance under P_μ, J_{+i}, J_{ij}, $\tilde{D}$, and K_+ can be checked along the same line of discussions in [4], where the holographic models for E(2) invariant very special conformal field theories are studied. Note, however, that in contrast with the model in [4], the energy-momentum tensor from the vector fields here is zero, so the geometry is not that of the Schrödinger holography [10,11] but it is just the anti–de Sitter spacetime.

Our claim is that this is a holographic dual description of a SIM(2) invariant very special conformal field theory. Naively, the vector condensation (12) is not invariant under the isometry of J_{+-} while the metric is. Nevertheless, the crucial point is that the theory has a noncompact $U(1)$ global symmetry

$$\delta_\lambda A = e^\lambda A, \qquad \delta_\lambda B = e^{-\lambda} B, \tag{13}$$

and the condensation becomes invariant under the new "J_{+-}" if we define it by a combined transformation of the coordinate transformation $dx^- \to e^{-\lambda} dx^-$ and the noncompact global $U(1)$ symmetry $\delta_\lambda A = e^\lambda A$. This mechanism is essentially the holographic counterpart of what we used in the field theory construction of conserved J_{+-} from the idea of the "topological twist".[6] Here the condensation of A_M is equivalent to adding J_μ to the action. Similarly, the holographic theory is not unitary because of the wrong signs in the kinetic terms for the vector fields A_M and B_M much as the field theory construction discussed at the beginning of this section.

IV. DISCUSSIONS

In this paper, we have constructed a local field theory example of SIM(2) or HOM(2) invariant very special conformal field theories, which was believed to be impossible within local quantum field theories. Our construction is either a nonunitary or a nonlinear realization. We may regard these examples as counterexamples of the no-go argument in [1] with a little bit of a caveat. Now we are going to discuss what the caveat would imply.

The very special relativity was originally introduced from the motivations in elementary particle physics, but the existence of a particular null direction may have its origin from the other spacetime physics. For example, let us imagine quantum field theories near a black hole (or black brane) horizon. There, the existence of a horizon may be associated with a particular null direction in spacetime, and one may locally approximate the symmetry of the spacetime by the very special relativity.

More precisely, if one takes the near horizon limit of a nonextremal black hole, e.g., the Schwartzshild black hole, then it is described by the Rindler space

$$ds^2 = -2\frac{e^{-r}}{r}(-dx^+ dx^-) + r^2 d\Omega^2 \to -2dx^+ dx^- + dy^2 + dz^2, \tag{14}$$

which is locally the same as the Minkowski space. However, a crucial difference here is that the null direction $x^- = 0$ is special because it represents the location of the event horizon. On the other hand, the other null directions including $x^+ = 0$ are less sacred and can be broken by the boundary condition imposed by the black hole background (rather than the white hole background with horizon located at $x^+ = 0$). Thus, the symmetry that is compatible with the black hole system near the horizon region is given by one of the very special symmetry.

We may even speculate that the difficulty of constructing SIM(2) or HOM(2) invariant field theories has its origin in black hole physics. On the one hand, we have to abandon locality to construct unitary theories. On the other hand, we have to abandon unitarity to construct local field theories. The locality vs unitarity in the black hole information puzzle has been a hot debate these days, and our discussions may be related to these studies in a deep manner.

We have also shown the local field theory construction of HOM(2) and SIM(2) very special conformal field theories with its nonlinear realization by the spontaneous breaking. How does such nonlinear realization appear in physics? We imagine that the very special relativity itself may originate from the spontaneous symmetry breaking of the full Lorentz symmetry. In the black hole case above, this is what is precisely happening: the gravitational physics spontaneously breaks the Lorentz symmetry. Then we expect that the similar nonlinear realization of the very special conformal symmetry may occur naturally.

Finally, beyond the spurion analysis in [1], there is no strict argument that very special conformal field theories with HOM(2) or SIM(2) invariance are impossible without violation of unitarity, violation of locality, or spontaneous breaking of the symmetry. It would be very important to prove or disprove this point. Such a no-go theorem (i.e., unitary Poincaré invariant field theories with $\tilde{D}$ and K_+ must be fully conformal invariant) does exist in $d = 2$ dimensions [14], and the analysis there suggests that we should understand the properties of correlation functions, in particular those of the energy-momentum tensor.[7]

ACKNOWLEDGMENTS

This work is in part supported by JSPS KAKENHI Grant No. 17K14301.

[6] The topological twist in the context of holography has been studied, e.g., in [12,13].

[7] Some correlation functions in very special conformal field theories are currently studied by using the embedding method in [15].

[1] A. G. Cohen and S. L. Glashow, Phys. Rev. Lett. **97**, 021601 (2006).
[2] A. G. Cohen and S. L. Glashow, arXiv:hep-ph/0605036.
[3] A. Ilderton, Phys. Rev. D **94**, 045019 (2016).
[4] Y. Nakayama, Phys. Rev. D **97**, 065003 (2018).
[5] R. Andringa, E. Bergshoeff, S. Panda, and M. de Roo, Classical Quantum Gravity **28**, 105011 (2011).
[6] G. W. Gibbons, J. Gomis, and C. N. Pope, Phys. Rev. D **76**, 081701 (2007).
[7] E. Witten, Commun. Math. Phys. **117**, 353 (1988).
[8] E. Witten, Commun. Math. Phys. **118**, 411 (1988).
[9] A. J. Hariton and R. Lehnert, Phys. Lett. A **367**, 11 (2007).
[10] D. T. Son, Phys. Rev. D **78**, 046003 (2008).
[11] K. Balasubramanian and J. McGreevy, Phys. Rev. Lett. **101**, 061601 (2008).
[12] Y. Nakayama, Phys. Rev. D **95**, 066010 (2017).
[13] Y. Nakayama, Phys. Rev. D **95**, 046006 (2017).
[14] J. Polchinski, Nucl. Phys. **B303**, 226 (1988).
[15] Y. Nakayama (to be published). Some results were presented at the KIAS-YITP joint workshop 2017 "Strings, Gravity and Cosmology".