

Superradiant instabilities in scalar-tensor Horndeski theory

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We study the superradiance effect in a class of scalar-tensor Horndeski theory. We first study the dynamics of a massive charged scalar wave scattered off the horizon of a Reissner-Nordström black hole which, in addition to its canonical coupling to gravity, is also coupled kinetically to curvature. We find that a trapping potential is formed outside the horizon of a Reissner-Nordström black hole, due to this coupling, and as the strength of the new coupling is increased, the scattered wave is superradiantly amplified, resulting in the instability of the Reissner-Nordström spacetime. We then consider the backreacting effect of the scalar field coupled to curvature interacting with the background metric, and we study the superradiant effect and find the superradiance conditions of a massive charged wave scattered off the horizon of a Horndeski black hole.

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I. INTRODUCTION

It was shown by Penrose [1] that if a particle accretes into a Kerr black hole, this process may result in the extraction of energy from the black hole. This process suggests that energy is extracted from the black hole. The same process can be realized if you scatter waves off the black hole. It was found by Misner [2] that this happens if the relation

$$\omega < m\Omega \quad (1.1)$$

is satisfied, where ω is the frequency of the incident wave and Ω is the rotational frequency of the black hole. Later, it was shown by Teukolsky [3] that this amplification mechanism also works in the case of electromagnetic and gravitational waves if the condition (1.1) is satisfied. Further, Bekenstein showed [4] that the Misner process can also be realized by extracting charge and electrical energy from a charged black hole. He showed that this is a consequence of Hawking's theorem that the surface area of a black hole cannot decrease.

Introducing a reflecting mirror, it was suggested by Press and Teukolsky [5] that the wave will be amplified as a result of the bounce back and forth between the black hole and the mirror. In the case of a rotating black hole, the role of the reflecting mirror can be played by the mass of the scalar field because the gravitational force binds the massive field and keeps it from escaping to infinity once the condition (1.1) is satisfied. As a consequence, the rotational energy extracted from the black hole by the incident field grows

exponentially over time. A detailed investigation of the superradiant amplification of a wave scattered off a Kerr black hole surrounded by a mirror, was carried out in [6]. In the case of extracting charge and electric energy from a Reissner-Nordström black hole, the Misner condition (1.1) is modified [4] to

$$\omega < q\Phi, \quad (1.2)$$

where q is the charge coupling constant of the field and Φ is the electric potential of the charged black hole. The superradiant scattering of charged scalar waves in the regime (1.2) may lead to an instability of the Reissner-Nordström spacetime. Recently there is a study of the superradiance effect of a black hole immersed in an expanding Universe [7]. Sufficient evidence was found that there is extraction of energy from the neutral McVittie black hole because of the shrinking of its apparent horizon as the Universe expands.

The stability of Reissner-Nordström black holes under neutral gravitational and electromagnetic perturbations was established by Moncrief [8,9]. Evidence was provided in [10–13] for the stability of charged Reissner-Nordström black holes under charged scalar perturbations. The stability of extremal braneworld charged holes was studied in [14], where it was shown that if the spacetime dimension is higher than four, the superradiant amplification can occur. In addition, the Einstein-Maxwell-Klein-Gordon equations for a spherically symmetric scalar field scattering off a Reissner-Nordström black hole in asymptotically flat spacetime were considered in [15] and a superradiant instability was found. The superradiance instability of charged black holes placed in a cavity was studied in [16–19] (for a recent review on superradiance, see [20]).

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The recent developments in AdS/CFT correspondence [21] introduce an AdS spacetime as a natural reflecting boundary on which the reflecting wave keeps amplifying, making the system unstable. This results in the formation of hairy black holes with a charged scalar field trapped outside the black hole in which the electric repulsion balances the scalar condensate against gravitational collapse. Hairy black holes were constructed in global AdS₅ spacetime in [22–25]. The superradiant amplification of a wave packet may result in the destabilization of the AdS spacetime itself [26–29]. If we perturb the AdS spacetime with a scalar field, the system evolves towards the formation of a black hole [30–33]. The stability of near-extremal and extremal charged hairy black hole solutions under charged massive scalar field perturbations was studied in [34].

The application of the AdS/CFT correspondence to condensed matter systems (for a review, see [35]) revived interest in the dynamics of a scalar field outside a black hole horizon and its stability. The transition of a metallic state to a superconducting state, which is a strongly coupled problem in condensed matter physics, can be described by its dual weakly coupled gravity problem using the AdS/CFT correspondence [21]. It was shown that the effective mass of the scalar field which is trapped just outside the horizon of a charged black hole [36,37] becomes negative, breaking in this way an Abelian gauge symmetry outside the horizon of the Reissner-Nordström black hole, resulting in an instability of the Reissner-Nordström spacetime.

In the case of de Sitter charged black holes, instabilities were found in higher dimensions. It was showed in [38,39] that higher dimensional Reissner-Nordström-de Sitter black holes are gravitationally unstable for large values of the electric charge in $D7$ spacetime dimensions. The existence of such instability was proved analytically in the near-extremal limit [40]. In four dimensions a superradiance instability was found [41] in the Reissner-Nordström-de Sitter black holes against charged scalar perturbations with vanishing angular momentum, $l = 0$.

In this work, we study the effect of charged scalar perturbations on the stability of local solutions of theories in which a scalar field is kinetically coupled to curvature. These theories belong to a general class of scalar-tensor gravity theories resulting from the Horndeski Lagrangian [42] which recently rediscovered [43], give second-order field equations and contain as a subset a theory which preserves classical Galilean symmetry [44–46].

This derivative coupling because it has the dimensions of length squared, redefines other possible scales present in the theory like the cosmological constant, giving in this way various black hole solutions [47–50], while if one considers the gravitational collapse of a scalar field coupled to the Einstein tensor then a black hole is formed [51]. The presence of the derivative coupling has different behaviors during the cosmological evolution. It acts as a friction term

in the inflationary period [52–54] and also it gives self-acceleration or self-tuning cosmological solutions [55,56]. Moreover, it was found that at the end of inflation in the preheating period, there is a suppression of heavy particle production as the derivative coupling is increased. This was attributed to the fast decrease of kinetic energy of the scalar field due to its wild oscillations [57]. This change of the kinetic energy of the scalar field to Einstein tensor allowed to holographically simulate the effects of a high concentration of impurities in a material [58].

The above discussion indicates that one of the main effects of the kinetic coupling of a scalar field to Einstein tensor is that gravity influences strongly the propagation of the scalar field compared to a scalar field minimally coupled to gravity. We will use this property of the charged scalar field coupled to Einstein tensor to study its behavior outside the horizon of charged black holes. First we will study the case of a test scalar field coupled to Einstein tensor scattered off the horizon of a Reissner-Nordström black hole. Will show that this new dimensional derivative coupling provides a scale for a confining potential, which is an effect similar to the AdS radius provided by the cosmological constant, and in the same time modifies the Bekenstein’s superradiance condition (1.2) with the derivative coupling appearing explicitly in the superradiance condition. Then for a wide range of parameters satisfying the superradiant condition, the charged scalar field will be trapped in this confining potential leading to a superradiant instability of the Reissner-Nordström black hole. The stability of Reissner-Nordström and Kerr spacetimes was discussed in [59–61] in a different context, calculating the quasinormal frequencies and the greybody factors in the presence of the derivative coupling.

These results of the superradiance instabilities of the Reissner-Nordström black hole indicate that the background black hole may acquire scalar hair [29] and therefore we will study the backreacted effect. We will allow the scalar field coupled to Einstein tensor to backreact to a charged spherical symmetric background. This will lead to the generation of hairy charge Galileon black holes. In these solutions, the derivative coupling appears as a parameter in these hairy solutions and then we will study how the superradiance effect and the superradiance conditions will be modified in the presence of this derivative coupling in the cases of a time dependent scalar hair [62] and of a static one [63].

The work is organized as follows. In Sec. II, we will review the stability of a Reissner-Nordström black hole under charge scalar perturbations of a scalar field minimally coupled to gravity. In Sec. III, we introduce the derivative coupling of a scalar field to the Einstein tensor and we discuss the stability of the background Reissner-Nordström black hole under charge scalar field perturbations. In Sec. IV, we study the superradiance effect of the charged Galileon black hole with time-dependent scalar

hair in Sec. IV, the superradiance in the charged Galileon black hole with static hair in Sec. V, and our conclusions are in Sec. VI.

II. SUPERRADIANT STABILITY OF THE REISSNER-NORDSTRÖM BLACK HOLE

In this section, we will review the superradiant stability of the Reissner-Nordström black hole discussed in [10,11]. Consider a Reissner-Nordström black hole of mass M and electric charge Q with a metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (2.1)$$

where

$$f(r) \equiv 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \quad (2.2)$$

A massive charged scalar field was considered in the Reissner-Nordström spacetime and its dynamics was described by the Klein-Gordon equation

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]\Psi = 0, \quad (2.3)$$

where $A_\nu = -\delta_\nu^0 Q/r$ is the electromagnetic potential of the black hole. Here q and μ are the charge and mass of the field, respectively. Consider a scalar wave of the form

$$\Psi_{lm}(t, r, \theta, \phi) = e^{-i\omega t} R_{lm}(r) Y(\Omega), \quad (2.4)$$

where the angular part is given by

$$Y(\Omega) = e^{im\phi} S_{lm}(\theta), \quad (2.5)$$

where ω is the conserved frequency of the mode, l is the spherical harmonic index, and m is the azimuthal harmonic index with $-l \leq m \leq l$. The frequency mode ω is in general complex consisting of real part ω_R and an imaginary part ω_I . Then, if $\omega_I > 0$ we expect a growing mode to develop, resulting in an instability [64–66]. In the corresponding holographic picture of a fluid undergoing a phase transition, ω_I defines the relaxation time $T = 1/\omega_I$ which is needed for this instability to reach thermal equilibrium [67].

In the above decomposition, R and S denote the radial and angular part and the radial Klein-Gordon equation is given by [11]

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + UR = 0, \quad (2.6)$$

where

$$\Delta \equiv r^2 - 2Mr + Q^2, \quad (2.7)$$

and

$$U \equiv (\omega r^2 - qQr)^2 - \Delta[\mu^2 r^2 + l(l+1)]. \quad (2.8)$$

To solve the radial equation (2.6) we impose boundary conditions of purely ingoing waves at the black-hole horizon and a decaying solution at spatial infinity

$$R \sim e^{-i(\omega - qQ/r_H)y} \quad \text{as } r \rightarrow r_H \quad (y \rightarrow -\infty), \quad (2.9)$$

and

$$R \sim y^{-iqQ} e^{-\sqrt{\mu^2 - \omega^2}y} \quad \text{as } r \rightarrow \infty \quad (y \rightarrow \infty), \quad (2.10)$$

where the ‘‘tortoise’’ radial coordinate y is defined by $dy = (r^2/\Delta)dr$. Because the energy of the incident wave is positive, the frequencies are restricted to the superradiant regime (1.2), and the boundary condition (2.9) describes an outgoing flux of energy and charge from the charged black hole [4,5]. As it was shown in [68] these boundary conditions lead to a discrete set of resonances $\{\omega_n\}$ which correspond to the bound states of the charged massive field.

Defining a new ψ function

$$\psi \equiv \Delta^{1/2} R, \quad (2.11)$$

Eq. (2.6) can be written in the form of a Schrödinger-like wave equation

$$\frac{d^2\psi}{dr^2} + (\omega^2 - V)\psi = 0, \quad (2.12)$$

where

$$\omega^2 - V = \frac{U + M^2 - Q^2}{\Delta^2}. \quad (2.13)$$

Then it was shown in [10,11] that there is no superradiant amplification for the charged Reissner-Nordström black holes because mainly two necessary conditions cannot be satisfied simultaneously namely, the condition of superradiant amplification (1.2) and the existence of a trapping potential well.

III. SUPERRADIANCE INSTABILITY OF THE REISSNER-NORDSTRÖM BLACK HOLE WITH THE DERIVATIVE COUPLING

In the previous section, we discussed the superradiance effect of a massive charge scalar field coupled minimally to Gravity resulting from the action

$$S_0 = \int dx^4 \sqrt{-g} \left[\frac{R}{16\pi G} - g_{\mu\nu} D^\mu \Psi (D^\nu \Psi)^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \mu^2 |\Psi|^2 \right], \quad (3.1)$$

where $D_\mu \equiv \nabla_\mu - iqA_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. In what follows, $G = c = 1$. We note here that the only scale in the theory is provided by the mass of the scalar field.

We want to generalize this action adding a coupling of the scalar field to Einstein tensor

$$S_0 = \int dx^4 \sqrt{-g} \left[\frac{R}{16\pi G} - (g^{\mu\nu} - \beta G_{\mu\nu}) D^\mu \Psi (D^\nu \Psi)^* - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \mu^2 |\Psi|^2 \right]. \quad (3.2)$$

The motivation for introducing this coupling is twofold. First of all, the coupling of the scalar field to the Einstein tensor $G_{\mu\nu}$ captures the strong gravity effects. As it was discussed in the introduction this coupling modifies the kinetic properties of the scalar field. The scalar field feels the strong gravity force and the net result is its kinetic

energy to be reduced [51,54,57,58]. Therefore it would be interesting to see what is the effect of this coupling to the superradiance effect.

Varying the action (3.2) we get the Einstein equations

$$G_{\mu\nu} = 8\pi T_{\mu\nu}, \quad T_{\mu\nu} = T_{\mu\nu}^{(\Psi)} + T_{\mu\nu}^{(EM)} - \beta \Theta_{\mu\nu}, \quad (3.3)$$

where $T_{\mu\nu}^{(\Psi)}$, $T_{\mu\nu}^{(EM)}$ are the energy-momentum tensors of the scalar and electromagnetic fields

$$T_{\mu\nu}^{(\Psi)} = \Psi_{,\mu} \Psi_{,\nu} + \Psi_{,\nu} \Psi_{,\mu} - g_{\mu\nu} (g^{ab} \Psi_{,ab} + \mu^2 |\Psi|^2), \quad (3.4)$$

$$T_{\mu\nu}^{(EM)} = F_{\mu}{}^{\alpha} F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \quad (3.5)$$

while $\Theta_{\mu\nu}$ is the contribution to the energy-momentum tensor of the $G_{\mu\nu}$ term

$$\begin{aligned} \Theta_{\mu\nu} = & -g_{\mu\nu} R^{ab} \Psi_{,ab} + R_{\nu}{}^a (\Psi_{,\mu a} + \Psi_{,a\mu}) + R_{\mu}{}^a (\Psi_{,\nu a} + \Psi_{,a\nu}) - \frac{1}{2} R (\Psi_{,\mu\nu} + \Psi_{,\nu\mu}) \\ & - G_{\mu\nu} \Psi - \frac{1}{2} \nabla^a \nabla_{\nu} (\Psi_{,a\nu} + \Psi_{,\nu a}) - \frac{1}{2} \nabla^a \nabla_{\nu} (\Psi_{,\mu a} + \Psi_{,a\mu}) + \frac{1}{2} \square (\Psi_{,\mu\nu} + \Psi_{,\nu\mu}) \\ & + \frac{1}{2} g_{\mu\nu} \nabla_a \nabla_b (\Psi^{ab} + \Psi^{ba}) + \frac{1}{2} (\nabla_{\mu} \nabla_{\nu} + \nabla_{\nu} \nabla_{\mu}) \Psi - g_{\mu\nu} \square \Psi. \end{aligned} \quad (3.6)$$

The Klein-Gordon equation is

$$(\partial_\mu - iqA_\mu) [\sqrt{-g} (g^{\mu\nu} - \beta G^{\mu\nu}) (\partial_\nu - iqA_\nu) \Psi] = \sqrt{-g} \mu^2 \Psi, \quad (3.7)$$

and the Maxwell equations are

$$\begin{aligned} \nabla_{\nu} F^{\mu\nu} + (g^{\mu\nu} - \beta G^{\mu\nu}) [2q^2 A_{\nu} |\Psi|^2 \\ + iq(\Psi^* \nabla_{\nu} \Psi - \Psi \nabla_{\nu} \Psi^*)] = 0. \end{aligned} \quad (3.8)$$

For convenience we had set

$$\Psi_{,\mu\nu} \equiv D_{\mu} \Psi (D_{\nu} \Psi)^*, \quad (3.9)$$

$$\Psi \equiv g^{\mu\nu} \Psi_{,\mu\nu}. \quad (3.10)$$

This action if $\beta = 0$ represents a massive charged scalar field with canonical kinetic terms coupled to gravity. If $\beta \neq 0$ then this scalar field is coupled directly to curvature through its coupling to the Einstein tensor $G_{\mu\nu}$. We have to solve the Einstein-Maxwell-scalar field equations (3.3), (3.7), and (3.8). To get an insight to the superradiance effect, we will first solve this system in the probe limit, assuming that the scalar field does not backreact to the metric. This means that the Reissner-Nordström metric solves the Einstein equations and the scalar field acts like a

test field. In the next two sections, we will study the superradiance effect of the backreacting solutions [47–50] resulting from the action (3.2).

Before we proceed we will discuss the motivation for considering the action (3.2). As we discussed in the previous section, if we scatter a scalar field with canonical kinetic terms ($\beta = 0$ in the action (3.2) we considered) off the horizon of a Reissner-Nordström black hole, we do not see any superradiance amplification. The main reason for this is that because asymptotically the space is flat there is no any barrier or trapping potential to capture the reflected wave and inspite that the Bekenstein superradiance condition is satisfied we do not have an amplification of the scalar wave which would have to result in an instability of the Reissner-Nordström background metric [10,11]. As we will discuss in the following, the presence of the derivative coupling modifies the Bekenstein superradiance condition and in the same time generates a confining potential outside the horizon of the Reissner-Nordström black hole and this will lead to an instability of the background metric.

The action (3.2) we considered is part of the shift-symmetric Horndeski action [42]. The new information it introduces the derivative coupling β of the scalar field to Einstein tensor, which has the dimensions of length squared, is a scale in the theory which effectively on short distances acts as a cosmological constant [47–50]. Therefore, the space is not any more flat and as it was shown in [49] a potential well is formed near the horizon of

a Reissner-Nordström. Having this result, in this paper, we will study the superradiance effect of a massive charge scalar field coupled to Curvature in the background of a Reissner-Nordström black hole.

Considering the Reissner-Nordström metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad (3.11)$$

with $f(r) = 1 - 2M/r + Q^2/r^2$ and an electromagnetic potential $A_t = -\frac{Q}{r}$ we decompose the scalar field as in (2.4)

$$\Psi = e^{-i\omega t} e^{im\varphi} S(\theta) \frac{\psi(r)}{r}. \quad (3.12)$$

Then the radial equation resulting from the Klein-Gordon equation (3.7) using (3.12) can be written in a Schrödinger-like form

$$\frac{d^2\psi}{dr_*^2} + U_{\text{eff}}(r)\psi(r_*) = 0, \quad (3.13)$$

where we have defined the ‘‘tortoise’’ coordinate

$$\frac{dr_*}{dr} = \frac{r^2}{f(r)(r^2 + \beta(1 - f - rf'))} \quad (3.14)$$

and the effective potential is given by

$$\begin{aligned} U_{\text{eff}} = & \left(\omega - \frac{qQ}{r} \right)^2 - f \left[\frac{\ell(\ell+1)}{r^2} + \mu^2 + \frac{f'}{r^2} \right] \\ & + \beta \left(-\frac{(f-1)(2f^2 - f\ell(\ell+1) + 2q^2Q^2)}{r^4} + \frac{(f\mu^2 - 2\omega^2)f'}{r} \right. \\ & + \frac{4(f-1)qQ\omega + 2(f[f-1 + \ell(\ell+1)] - q^2Q^2)f'}{r^3} \\ & + \left. \frac{2(f-1)(f\mu^2 - 2\omega^2) + 8qQ\omega f' + 4f(f')^2 + f[2f + \ell(\ell+1)]f''}{2r^2} \right) \\ & + \beta^2 \left(\frac{(f-1)^2(2f^2 + q^2Q^2)}{r^6} + \frac{\omega^2(f')^2}{r^2} \right. \\ & + \frac{(f-1)(-2(f-1)qQ\omega + (f(1+f - \ell(\ell+1)) + 2q^2Q^2)f')}{r^5} \\ & - \frac{f'(-4(f-1)\omega^2 + 4qQ\omega f' + 2f(f')^2 + f(2f + \ell(\ell+1))f'')}{2r^3} \\ & + \frac{(f-1)(2(f-1)\omega^2 - f(2f + \ell(\ell+1))f'')}{2r^4} \\ & \left. - \frac{8(f-1)qQ\omega f' + 2(f(2(f-1) + \ell(\ell+1)) - q^2Q^2)(f')^2}{2r^4} \right). \end{aligned} \quad (3.15)$$

Note that the effective potential because of the presence of the derivative coupling β has the lapse function at second order and this introduces high order terms in the radial coordinate r . Then the effective potential depends on seven parameters $U_{\text{eff}}(M, Q, \mu, q, \omega, l, \beta)$ and even its numerical study is difficult.

The effective potential at infinity goes like

$$U_{\infty} \sim \omega^2 - \mu^2, \quad (3.16)$$

while near the horizon can be approximated as

$$U_{r_H} \sim \frac{(r_H^2 + \beta\Phi^2)^2(\omega - q\Phi)^2}{r_H^4} + O(r - r_H), \quad (3.17)$$

where $\Phi = Q/r_H$ is the electric potential at the horizon.

In a scattering experiment, the Klein-Gordon equation (3.13) has the following asymptotic behavior,

$$\psi \sim \begin{cases} T e^{-i\sigma r_*} & , \text{ as } r \rightarrow r_H \\ e^{-ir_*\sqrt{\omega^2 - \mu^2}} + R e^{ir_*\sqrt{\omega^2 - \mu^2}} & , \text{ as } r \rightarrow \infty \end{cases} \quad (3.18)$$

where we have set

$$\sigma \equiv \frac{(r_H^2 + \beta\Phi^2)(\omega - q\Phi)}{r_H^2}. \quad (3.19)$$

These boundary conditions correspond to an incident wave of unit amplitude from spatial infinity, giving rise to a

reflected wave of amplitude R and a transmitted wave of amplitude T at the horizon.

Since the effective potential is real, there exists another solution $\bar{\psi}$ to (3.13) which satisfies the complex conjugate boundary conditions [69]. The solutions ψ and $\bar{\psi}$ are linearly independent and, thus, their Wronskian $W = \psi \frac{d}{dr_*} \bar{\psi} - \bar{\psi} \frac{d}{dr_*} \psi$ is independent of r_* . Evaluating the Wronskian at the horizon and infinity, respectively, we get

$$W(r \rightarrow r_H) = 2i\sigma|T|^2, \quad (3.20)$$

$$W(r \rightarrow \infty) = -2i(|R|^2 - 1)\sqrt{\omega^2 - \mu^2}, \quad (3.21)$$

and by equating the two values, we get

$$|R|^2 = 1 - \frac{\sigma}{\sqrt{\omega^2 - \mu^2}}|T|^2. \quad (3.22)$$

We can see that if $\sigma < 0$, the wave is superradiantly amplified, $|R|^2 > 1$ [70]. So, the superradiant condition (1.2) in the presence of the derivative coupling is modified to

$$(r_H^2 + \beta\Phi^2)(\omega - q\Phi) < 0. \quad (3.23)$$

To see whether the superradiance will cause the instability of the Reissner-Nordström black hole, we need to check whether there exists a potential well outside the horizon to trap the reflected wave. If the potential well exists, the superradiant instability will occur, and the wave will grow exponentially over time near the black hole to make the background Reissner-Nordström black hole unstable. We are interested in solutions of the radial equation (3.13) with the physical boundary conditions of purely ingoing waves at the horizon and a decaying solution at spatial infinity. A bound state decaying exponentially at spatial infinity is characterized by $\omega^2 < \mu^2$. We choose the parameters $M, Q, \mu, q, \beta, \omega$ to meet this condition together with the superradiant condition (3.23). In what follows we will set $\ell = 0$ and fix the horizon at $r_H = 1$.

A. Case I: $\omega < q\Phi$ and $r_H^2 + \beta\Phi^2 > 0$

We will first examine the case where $\omega < q\Phi$ and $\beta > 0$ so that (3.23) is satisfied. Our aim is to see if a trapping potential is formed outside the horizon of the black hole. Since the effective potential (3.15) is a high-order polynomial in the radial coordinate r and therefore its analytical treatment is difficult, we will rely on its numerical investigation. We intend to study the effective potential (3.15) systematically, varying its parameters to get mainly the following physical information. First, what is the range of parameters Q and M for which a trapping potential is formed and, second, if the scalar field is strongly coupled to

TABLE I. The first and third sections correspond to a ratio less than $8/9$, while the second greater than $8/9$.

M	Q	$(Q/M)^2$
0.505	0.1	0.039
0.52	0.2	0.148
0.545	0.3	0.303
0.58	0.4	0.476
0.625	0.5	0.640
0.68	0.6	0.779
0.745	0.7	0.883
0.82	0.8	0.952
0.905	0.9	0.989
1.	1.	1.000
1.105	1.1	0.991
1.22	1.2	0.967
1.345	1.3	0.934
1.48	1.4	0.895
1.625	1.5	0.852
1.78	1.6	0.808
1.945	1.7	0.764
2.12	1.8	0.721
2.305	1.9	0.679
2.5	2.	0.640

curvature (large β) does the potential well depends? To do that, we will fix the parameters q and ω, μ as the relation $\omega^2 < \mu^2$ to be satisfied and leave the others free to vary.

In Table I, we give the ratio $(Q/M)^2$ for various values of Q and M . Our criterion for building up the values that appear in Table I was that we looked among the various values of the ratio $(Q/M)^2$ to such values of the ratio approaching one (near-extremal case), one (extremal limit), and far way from one. For a characteristic value of Q and M in the first part of Table I, we depict the effective potential as a function of r in Fig. 1, while in Fig. 2 we depict the

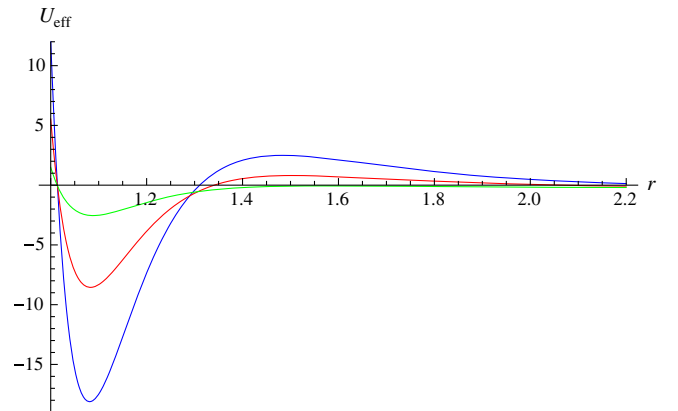


FIG. 1. The potential as a function of the radial coordinate for coupling constant $\beta = 150, \beta = 100, \beta = 50$ (blue, red, green), mass and charge for the black hole and the scalar field $M = 0.625, Q = 0.5, \mu = 0.63, q = 0.86$, respectively, and $\omega = 0.34$.

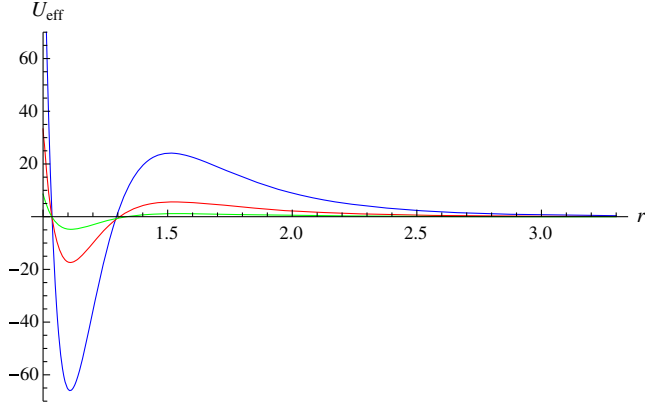


FIG. 2. The potential as a function of the radial coordinate for coupling constant $\beta = 200$, $\beta = 100$, $\beta = 50$ (blue, red, green), mass and charge for the black hole and the scalar field $M = 0.82$, $Q = 0.8$, $\mu = 0.63$, $q = 0.86$, respectively, and $\omega = 0.6$.

effective potential as a function of r for a characteristic value of Q and M in the second part of the Table I. We observe that irrespective of the value of the ratio Q/M a trapped potential is formed outside the horizon of the black hole and most importantly as the derivative coupling β is increased and the ratio $(Q/M)^2$ approaching the extremal limit the potential well gets deeper. This suggests that as the strength of the coupling of the scalar field to curvature is increased and the black hole approaches its extremal limit, the superradiant instability is amplified.

Finally, in Fig. 3, we depict the effective potential as a function of r for fixed values of ω , μ , q , β and for different values of Q and M belonging to the third part of Table I. We observe that the potential well is deeper as the ratio Q/M gets smaller. This corresponds to larger values of the mass M and charge Q since we are in the third part of the Table I.

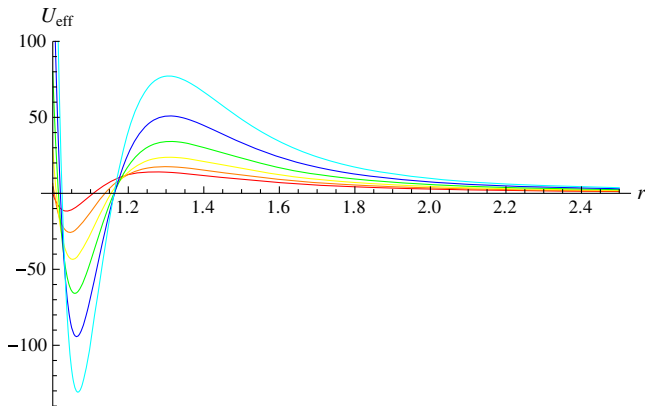


FIG. 3. The potential as a function of the radial coordinate for coupling constant $\beta = 10$, mass and charge for the scalar field $q = 0.86$, $\mu = 1.63$, $\omega = 1.28$. The ratio $(Q/M)^2$ is 0.852, 0.808, 0.760, 0.720, 0.679, 0.64 (red, orange, yellow, green, blue, cyan), respectively.

This suggests that a highly charged massive black hole radiates more.

We note that the behavior shown in the above figures are characteristic of the non zero value of the derivative coupling. If $\beta = 0$ then we can see from (3.15) that the effective potential does not develop a well and we do not expect superradiance amplification [11].

B. Case II: $\omega > q\Phi$ and $r_H^2 + \beta\Phi^2 < 0$

We consider now the case where $\omega > q\Phi$ and $\beta < -(r_H/\Phi)^2$, so again (3.23) is satisfied. This case is interesting. In [47,49] fully backreacted black hole solutions were found in the presence of the derivative coupling β . However, these solutions exist only in the case of positive coupling while if the coupling constant β is negative, then the system of Einstein-Maxwell-Klein-Gordon equations is unstable and no solutions were found. In [57], a very small window of negative β was shown to be allowed. For negative derivative coupling the stability of the Galileon black holes was investigated but there is no conclusive result. For example, in [71,72], the black hole quasinormal modes in a scalar-tensor theory with field derivative coupling to the Einstein tensor were calculated.

In our study in this section, the scalar field coupled to Einstein tensor does not backreact on the fixed Reissner-Nordström background metric. Therefore, the physical propagating degrees of freedom do not interact with the background metric to alter it. The same is happening in cosmology. It is known that the effect of the derivative coupling in a fixed FRW background metric, has the effect that for β positive the Universe contracts and if β is negative the Universe expands [53]. The reason for this behavior is that the derivative coupling defines an effective cosmological constant. Also the stability of a static Universe was

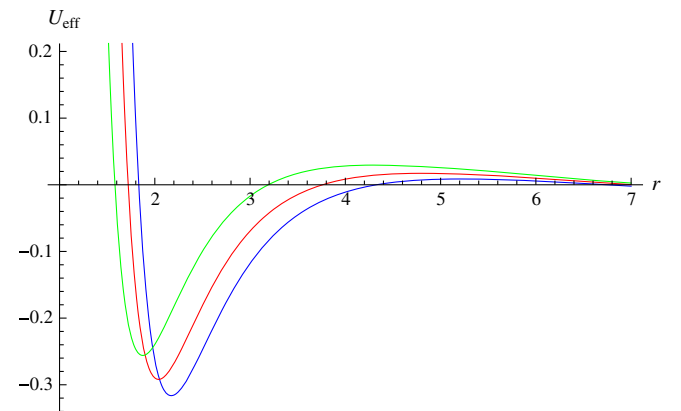


FIG. 4. The potential as a function of the radial coordinate for coupling constant $\beta = -45$, $\beta = -35$, $\beta = -25$ (blue, red, green), mass and charge for the black hole and the scalar field $M = 0.625$, $Q = 0.5$, $\mu = 1.63$, $q = 1.6$, respectively, and $\omega = 1.6$.

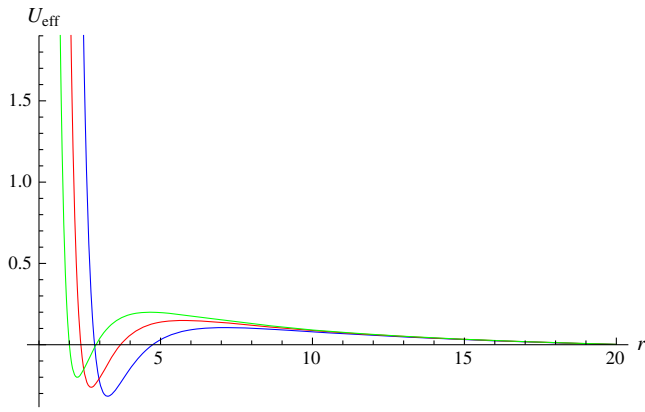


FIG. 5. The potential as a function of the radial coordinate for coupling constant $\beta = -100$, $\beta = -50$, $\beta = -25$ (blue, red, green), mass and charge for the black hole and the scalar field $M = 0.82$, $Q = 0.8$, $\mu = 1.63$, $q = 0.9$, and $\omega = 1.6$.

studied in [72] where conditions for the coupling was found for both signs of β . In the next sections where we study the backreacting effects, we take the derivative coupling to be positive (opposite sign to the usual kinetic term).

In Figs. 4 and 5, we depict the effective potential as a function of r , again for two characteristic values of the ratio Q and M belonging to the first and second part of Table I. As before, in both cases we found that there exists a negative minima of the potential well which gets deeper for larger (absolute) values of the derivative coupling β . Therefore, for a range of parameters satisfying relation (3.23), for $\omega > q\Phi$ and a negative β there is a confining potential, indicating that there should be an instability in the system of a charged scalar field scattered off a Reissner-Nordström black hole and this is due to superradiant amplification.

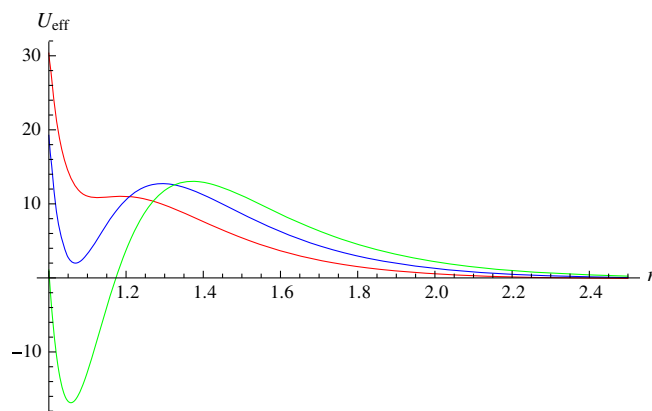


FIG. 6. The potential as a function of the radial coordinate for coupling constant $\beta = -120$, mass and charge for the scalar field $q = 0.86$, $\mu = 0.63$, $\omega = 0.62$. The ratio $(Q/M)^2$ is 0.64, 0.779, 0.883 (red, blue, green), respectively.

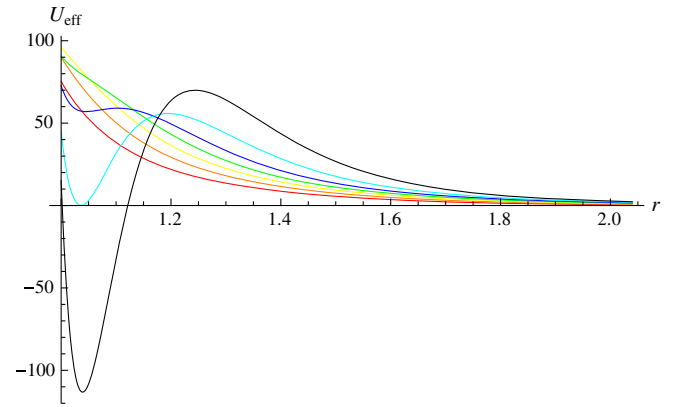


FIG. 7. The potential as a function of the radial coordinate for coupling constant $\beta = -50$, mass and charge for the scalar field $q = 0.4$, $\mu = 0.63$, $\omega = 0.6$. The ratio $(Q/M)^2$ is 0.952, 0.989, 1.000, 0.991, 0.967, 0.939, 0.895 (red, orange, yellow, green, blue, cyan, black), respectively.

An interesting behavior is observed if we fix the value of the derivative coupling β . Then, in Fig. 6, we depict the effective potential as a function of r for fixed values of ω , μ , q , β and we vary the values of change Q and mass M belonging to the first part of Table I. We see that as the ratio is increased eventually a trapping potential is formed. Note that this happening as the charge Q is increased. In Fig. 7, we depict the effective potential for values of Q and M which they belong to the second part of the Table I. We observe that a trapping potential is formed only for values of Q and M which they give a ratio that it is far away from the extremal limit or the near extremal limit. Finally in Fig. 8, we see the formation of a trapping potential for values of Q and M belonging to the third part of Table I. We observe that as the mass M and charge Q in increased the potential well gets deeper.

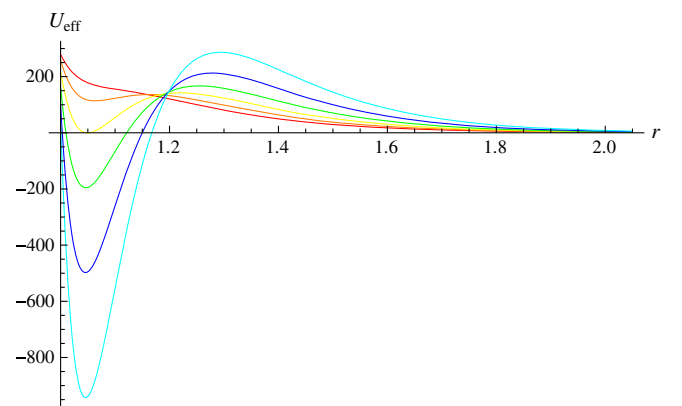


FIG. 8. The potential as a function of the radial coordinate for coupling constant $\beta = -20$, mass and charge for the scalar field $q = 0.6$, $\mu = 1.63$, $\omega = 1.28$. The ratio $(Q/M)^2$ is 0.852, 0.808, 0.760, 0.720, 0.679, 0.64 (red, orange, yellow, green, blue, cyan), respectively.

IV. SUPERRADIANCE IN THE CHARGED GALILEON BLACK HOLE WITH TIME DEPENDENT HAIR

In the previous sections, we have investigated the instability of the Reissner-Nordström black hole, when the incident wave is described by a charged scalar field, coupled to gravity and to the Einstein tensor. In what follows, we will examine the case where the scalar field ϕ

coupled to Einstein tensor backreacts to the background metric. As shown in [50], a hairy black hole is generated, the charge Galileon black hole. Then we will consider a complex, massive, charged scalar field ϕ of mass μ and charge q scattered off the horizon of the Galileon black hole and we will study if superradiance radiation.

We consider a general action in which we have also included a coupling of the gauge field to the scalar field ϕ

$$S = \int d^4x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \eta g_{\mu\nu} \partial\phi^\mu \partial\phi^\nu + \beta G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - \gamma T_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi - \eta' (D_\mu \phi)^* (D^\mu \phi) - \mu^2 \phi \phi^* \right], \quad (4.1)$$

where $D_\mu = \nabla_\mu - iqA_\mu$ and $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$. Varying the action (4.1), we get the Einstein equations,

$$G_{\mu\nu} = T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} + T_{\mu\nu}^{(3)} + T_{\mu\nu}^{(4)} + T_{\mu\nu}^\phi, \quad (4.2)$$

where we have defined

$$\begin{aligned} T_{\mu\nu}^{(1)} &= -\Lambda g_{\mu\nu} + T_{\mu\nu}^{(M)}, \\ T_{\mu\nu}^{(2)} &= \beta \left\{ \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi R - 2 \nabla_\lambda \phi \nabla_{(\mu} \phi R_{\nu)}^\lambda - \nabla^\lambda \phi \nabla^\rho \phi R_{\mu\lambda\nu\rho} - (\nabla_\mu \nabla^\lambda \phi) (\nabla_\nu \nabla_\lambda \phi) + (\nabla_\mu \nabla_\nu \phi) \square \phi, \right. \\ &\quad \left. + \frac{1}{2} G_{\mu\nu} (\nabla \phi)^2 - g_{\mu\nu} \left[-\frac{1}{2} (\nabla^\lambda \nabla^\rho \phi) (\nabla_\lambda \nabla_\rho \phi) + \frac{1}{2} (\square \phi)^2 - \nabla_\lambda \phi \nabla_\rho \phi R^{\lambda\rho} \right] \right\}, \\ T_{\mu\nu}^{(3)} &= \frac{1}{2} \gamma \left[F_{\mu\sigma} F_{\nu\rho} \nabla^\sigma \phi \nabla^\rho \phi + (F_{\mu\sigma} F^{\beta\sigma} \nabla_\beta \phi \nabla_\nu \phi + F_{\nu\sigma} F^{\beta\sigma} \nabla_\beta \phi \nabla_\mu \phi) - \frac{1}{2} g_{\mu\nu} F_{\beta\sigma} F_\tau{}^\sigma \nabla^\beta \phi \nabla^\tau \phi \right. \\ &\quad \left. + \frac{1}{8} g_{\mu\nu} \nabla^\rho \phi \nabla_\rho \phi F_{\tau\beta} F^{\tau\beta} - \frac{1}{2} F_{\mu\sigma} F_\nu{}^\sigma \nabla^\rho \phi \nabla_\rho \phi - \frac{1}{4} \nabla_\mu \phi \nabla_\nu \phi F_{\tau\beta} F^{\tau\beta} \right], \\ T_{\mu\nu}^{(4)} &= \eta \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\sigma \phi \partial^\sigma \phi \right), \end{aligned}$$

with

$$T_{\mu\nu}^{(M)} \equiv \frac{1}{2} \left(F_{\mu\sigma} F_\nu{}^\sigma - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right), \quad (4.3)$$

and

$$\begin{aligned} T_{\mu\nu}^\phi &= -\frac{\mu^2}{2} g_{\mu\nu} |\phi|^2 + \frac{\eta'}{2} (-g_{\mu\nu} \nabla_\kappa \phi^* \nabla^\kappa \phi + \nabla_\mu \phi^* \nabla_\nu \phi + \nabla_\mu \phi \nabla_\nu \phi^* - 2q^2 A_\mu A_\nu |\phi|^2 \\ &\quad - q^2 g_{\mu\nu} A_\kappa A^\kappa |\phi|^2 - iq A^\kappa g_{\mu\nu} (\phi^* \nabla_\kappa \phi - \phi \nabla_\kappa \phi^*)). \end{aligned}$$

Assuming that the scalar field ϕ is a test field and it does not backreact to the metric (that is, keeping linear terms of ϕ) we end up with the coupled system of Einstein-Maxwell equations,

$$G_{\mu\nu} = T_{\mu\nu}^{(1)} + T_{\mu\nu}^{(2)} + T_{\mu\nu}^{(3)} + T_{\mu\nu}^{(4)}, \quad (4.4)$$

$$\partial_\mu \left\{ \sqrt{-g} \left[F^{\mu\nu} - \gamma \left(\frac{1}{2} F^{\mu\nu} \nabla_\sigma \phi + (F_\sigma{}^\mu \nabla^\nu \phi - F_\sigma{}^\nu \nabla^\mu \phi) \right) \nabla^\sigma \phi \right] \right\} = 0, \quad (4.5)$$

while the scalar field equation reads

$$\nabla_\mu[(\beta G^{\mu\nu} - \eta g^{\mu\nu} - \gamma T_{(M)}^{\mu\nu})\nabla_\nu\varphi] = 0. \quad (4.6)$$

A static spherically symmetric metric of the form,

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (4.7)$$

was assumed in [62] and the scalar field was taken to be linearly time dependent as

$$\varphi(t, r) = \mathbf{q}_s t + \psi(r), \quad (4.8)$$

while the electromagnetic potential turned out to be

$$A_\mu dx^\mu = A(r)dt - P \cos(\theta)d\varphi. \quad (4.9)$$

A solution to the above equations in the special case where $h(r) = f(r)$ was found,

$$\begin{aligned} h(r) &= 1 - \frac{\mu}{r} + \frac{\eta r^2}{3\beta} + \frac{\gamma(Q^2 + P^2)}{4\beta r^2}, \\ (\psi'(r))^2 &= \frac{1 - f(r)}{f(r)^2} \mathbf{q}_s^2, \\ F_{tr} = F(r) &= \frac{Q}{r^2}, \quad F_{\theta\varphi} = C(\theta) = P \sin(\theta). \end{aligned} \quad (4.10)$$

The coupling constants, along with the integration constants and the parameter \mathbf{q}_s are related as

$$\begin{aligned} P^2\beta(\Lambda\gamma + \eta) &= Q^2\eta(\gamma - \beta), \quad \mathbf{q}_s^2 = \frac{\eta + \Lambda\beta}{\beta\eta}, \\ C_0 &= \frac{1}{\eta}(\eta - \beta\Lambda). \end{aligned} \quad (4.11)$$

Recently the stability of the black hole solution (4.10) was discussed in details in [73]. It was found that the couplings η and β should have the opposite signs to guarantee the stability of the formed black hole solution.

A. The dynamics of the scalar field ϕ outside the charge Galileon black hole

We study now the scattering of the test massive charged scalar field ϕ off the horizon of the charge Galileon black hole. The dynamics is described by the Klein-Gordon equation (we redefine $\frac{\mu^2}{\eta} \equiv \mu_s^2$)

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu_s^2]\phi = 0. \quad (4.12)$$

We decompose $\phi(t, r, \theta, \varphi)$ as

$$\phi(t, r, \theta, \varphi) = e^{-i\omega t} R(r) Y(\theta, \varphi). \quad (4.13)$$

In the above decomposition R and Y denotes the radial and angular part of the solution. Then the Klein-Gordon equation takes the form

$$\begin{aligned} -\frac{1}{f(r)} \frac{\partial^2 \phi}{\partial t^2} + f \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \phi}{\partial \varphi^2} + \frac{\cos \theta}{r^2 \sin \theta} \frac{\partial \phi}{\partial \theta} + \frac{2iqP \cos \theta}{r^2 \sin^2 \theta} \frac{\partial \phi}{\partial \varphi} - \frac{q^2 P^2 \cos^2 \theta}{r^2 \sin^2 \theta} \phi \\ + \left(\frac{rf'(r) + 2f}{r} \right) \frac{\partial \phi}{\partial r} - \frac{2iqQ}{rf(r)} \frac{\partial \phi}{\partial t} + \left(\frac{q^2 Q^2 - \mu_s^2 r^2 f(r)}{r^2 f(r)} \right) \phi = 0, \end{aligned} \quad (4.14)$$

while the angular part takes the form

$$\frac{1}{r^2} \left[\frac{\partial^2}{\partial \theta^2} - \frac{m^2}{\sin^2 \theta} + \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \theta} + \frac{2i^2 m q P \cos \theta}{\sin^2 \theta} - \frac{q^2 P^2 \cos^2 \theta}{\sin^2 \theta} \right] Y(\theta) = \lambda Y(\theta). \quad (4.15)$$

Applying the transformations $Y \rightarrow \frac{1}{\sqrt{\sin \theta}} \tilde{Y}$, this equation can be written in the form of the Jacobi differential equation, and it admits the corresponding solution with eigenvalues

$$\lambda = -(n+1)(n+qP). \quad (4.16)$$

When the magnetic charge P is zero, the Jacobi equation reduces to Legendre differential equation, with eigenvalues $n(n+1)$, while the eigenfunctions turn out to be the spherical harmonics Y_{lm} . Meanwhile, when $P = 0$ and $\beta = 0$ the metric reduces to the Reissner-Nordström spacetime, as discussed in Sec. II.

Now we can go back to Klein-Gordon of the full solution, which, after substitutions, reduces to the radial Klein-Gordon equation

$$f(r) \frac{d^2 R(r)}{dr^2} + \left(\frac{rf'(r) + 2f(r)}{r} \right) \frac{dR(r)}{dr} + \left[\frac{(\omega r - qQ)^2 - ((n+1)(n+qP) + \mu_s^2 r^2)f(r)}{r^2 f(r)} \right] R(r) = 0. \quad (4.17)$$

When $r \rightarrow r_H$ we have $f(r) = 0$ and then the third term is divergent at the horizon. To avoid this, we take the coordinate transformation to the tortoise coordinate r^* , $dr^* = \frac{1}{f(r)} dr$. Then the radial Klein-Gordon equation becomes

$$R''(r^*) + \frac{2f(r)R'(r^*)}{r} + \left[\frac{(\omega r - qQ)^2 - ((n+1)(n+qP) + \mu_s^2 r^2)f(r)}{r^2} \right] R(r^*) = 0. \quad (4.18)$$

We can bring this equation in a Schrödinger-like form

$$\frac{d^2 R(r^*)}{dr^{*2}} + \frac{U_{\text{eff}}(r)}{r^2} R(r^*) = 0, \quad (4.19)$$

where the U_{eff} is the effective potential, given by the relation

$$U_{\text{eff}}(r) = (\omega r - qQ)^2 - f(r)[(n+1)(n+qP) + rf'(r) + m^2 r^2]. \quad (4.20)$$

As we can see, the effective potential encodes all the information about the background and the test scalar field. More specifically, the $f(r)$ metric function has the derivative coupling constant β as a parameter that interest us.

Now we have to determine the solutions of the radial component of the scalar field at the boundaries. Let us assume that the potential is real. Then, since the background is stationary, the field equations are invariant under the transformations $t \rightarrow -t$ and $\omega \rightarrow -\omega$. Thus, there exists another solution \bar{R} which satisfies the complex conjugate boundary conditions. The solutions R and \bar{R} are linearly independent and thus their Wronskian is independent of r^* .

At the horizon, $f(r) = 0$ and $U_{\text{eff}}(r) = (r\omega - qQ)^2$, so we get the solution

$$R(r^*) = C_{1,h} e^{i \frac{(r_h \omega - qQ) r^*}{r_h}} + C_{2,h} e^{-i \frac{(r_h \omega - qQ) r^*}{r_h}}, \quad (4.21)$$

where the presence of a horizon compels the amplitude $C_{1,h}$ to be zero, while $C_{2,h}$ is the amplitude of the transmitted wave.

To study the equation's behavior at large r , we first have to notice that in this case, the fact that our metric is not asymptotically flat, affects the form of the effective potential which at large distances is,

$$U_{\text{eff}} = \omega^2 - \mu_s^2 - \frac{2\eta}{3\beta} - \frac{\eta(n+1)(n+qP)}{3\beta} - r^2 \left(\frac{\mu_s^2 \eta}{3\beta} + \frac{2\eta^2}{9\beta^2} \right). \quad (4.22)$$

If we want to have a Schrodinger-like equation for the radial component of the scalar field at the $r \rightarrow \infty$ limit, we have to impose the following relation between the coupling constants

$$\eta = -\frac{3\mu_s^2 \beta}{2\eta'}. \quad (4.23)$$

Then U_{eff} equals now a constant and the resulting solution reads

$$R(r^*) = C_{1,\infty} e^{i \left(\sqrt{\omega^2 - \mu_s^2 - \frac{2\eta}{3\beta} - \frac{\eta(n+1)(n+qP)}{3\beta}} \right) r^*} + C_{2,\infty} e^{-i \left(\sqrt{\omega^2 - \mu_s^2 - \frac{2\eta}{3\beta} - \frac{\eta(n+1)(n+qP)}{3\beta}} \right) r^*}, \quad (4.24)$$

where $C_{2,\infty}$ corresponds to the amplitude of an incident wave from spatial infinity and $C_{1,\infty}$ to the amplitude of the reflected wave.

1. Superradiance condition

Now that we have determined the solutions of the R component of the scalar field at the boundaries, we are able to evaluate the Wronskians of two linearly independent solutions at the boundaries

$$W(r \rightarrow r_H) = -2ik_H |C_{2,H}|^2, \\ W(r \rightarrow \infty) = 2ik_\infty (|C_{1,\infty}|^2 - |C_{2,\infty}|^2). \quad (4.25)$$

Taking the two expressions to be equal, we find

$$|C_{1,\infty}|^2 = |C_{2,\infty}|^2 - \frac{k_H}{k_\infty} |C_{2,H}|^2. \quad (4.26)$$

For $\frac{k_H}{k_\infty} < 0$ the wave is superradiantly amplified. In this case, $k_H = \frac{\omega r_H - qQ}{r_H}$ and $k_\infty = \sqrt{\omega^2 - \mu_s^2 - \frac{2\eta}{3\beta} - \frac{\eta(n+1)(n+qP)}{3\beta}}$, so we end up with the inequality

$$\frac{(r_H\omega - qQ)}{\sqrt{\omega^2 - \mu_s^2 - \frac{2\eta}{3\beta} - \frac{\eta(n+1)(n+qP)}{3\beta}}} < 0, \quad (4.27)$$

which leads us to the superradiant condition

$$r_H\omega < qQ. \quad (4.28)$$

This condition is the Bekenstein superradiance condition for the Galileon black hole with a time dependent scalar field backreacting to the metric. The information of the derivative coupling enters the final inequality via the r_H , affecting the superradiance condition. We also have to underline the fact that, only waves with

$$\omega^2 > \mu_s^2 + \frac{2\eta}{3\beta} + \frac{\eta(n+1)(n+qP)}{3\beta}, \quad (4.29)$$

propagate to infinity restricting in this way the allowed values of the frequency ω of the incitant wave.

Before ending this section, we note that the constraint (4.23) which it is imposed in order to have superradiance, connects the scattered test wave parameters of its mass and its kinetic energy with the parameters of the background Galileon black hole.

V. SUPERRADIANCE IN THE CHARGED GALILEON BLACK HOLE WITH STATIC HAIR

We will study the superradiance effect of a specific model of a hairy Galileon black hole [63]. In this model the scalar field is coupled to the background only with the Einstein tensor. Under the presence of an electric field, an asymptotically locally flat hairy black hole solution was obtained in the presence of a cosmological constant. The action is given by

$$I[g_{\mu\nu}, \varphi] = \int \sqrt{-g} d^4x \left[\kappa(R - 2\Lambda) + \frac{\beta}{2} G_{\mu\nu} \nabla^\mu \varphi \nabla^\nu \varphi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right], \quad (5.1)$$

where $\kappa = \frac{1}{16\pi G}$. Then the following solution was found

$$ds^2 = -F(r)dt^2 + \frac{15[4\kappa r^2(2 - \Lambda r^2) - q^2]^2}{r^4} \frac{dr^2}{F(r)} + r^2 d\Omega^2, \quad (5.2)$$

where

$$F(r) = 48\kappa^2 \Lambda^2 r^4 - 320\kappa^2 \Lambda r^2 + 120\kappa(8\kappa + \Lambda q^2) - \frac{\mu}{r} + 240\kappa \frac{q^2}{r^2} - 5 \frac{q^4}{r^4}, \quad (5.3)$$

$$\psi(r)^2 = -\frac{15(4\kappa\Lambda r^4 + q^2)(4\kappa r^2(2 - \Lambda r^2) - q^2)^2}{2r^6\beta} \frac{1}{F(r)}, \quad (5.4)$$

$$A_0(r) = \sqrt{15} \left(\frac{q^3}{3r^3} - 8\kappa \frac{q}{r} - 4\kappa\Lambda r q \right), \quad (5.5)$$

where $\psi(r) = \varphi'(r)$. Observe that the derivative coupling β does not appear in the metric function but only in the scalar field solution.

As before, we consider a massive test scalar wave $Y = Y_{lm}(t, r, \theta, \phi)$ in the background of the above black hole solution. The dynamics of this wave is described by the Klein-Gordon equation

$$[(\nabla^\nu - iqA^\nu)(\nabla_\nu - iqA_\nu) - \mu^2]Y = 0, \quad (5.6)$$

where electromagnetic potential A_ν is given by (5.5). Then writing the scalar field as

$$Y_{lm} = e^{im\phi} S(\theta) R(r) e^{-i\omega t}, \quad (5.7)$$

where ω is the conserved frequency, l is the spherical harmonic, and m is the azimuthal harmonic. R and S denote the radial and angular part, and the radial Klein-Gordon equation is

$$\Delta \left(\frac{d}{dr} \Delta \frac{dR}{dr} \right) + UR = 0, \quad (5.8)$$

where

$$\Delta = \frac{Fr^2}{\sqrt{X}}, \quad (5.9)$$

and

$$U = (\omega r^2 - qAr^2)^2 - \Delta(\sqrt{X}l(l+1) + \mu^2\sqrt{X}r^2). \quad (5.10)$$

For our convenience, we have define $X = \frac{15[4\kappa r^2(2 - \Lambda r^2) - Q^2]^2}{r^4}$. Defining a new function Ψ ,

$$\Psi = \Delta^{\frac{1}{2}} R, \quad (5.11)$$

Eq. (5.11) can be written in the form of a Schrodinger-like wave equation

$$\frac{d^2\Psi}{dr^2} + V\Psi = 0, \quad (5.12)$$

where

$$V = \frac{1}{4\Delta^2} \left(\left(\frac{d\Delta}{dr} \right)^2 - 2\Delta \frac{d^2\Delta}{dr^2} + 4U \right). \quad (5.13)$$

To solve the radial Klein-Gordon equation (5.8) we define the ‘‘tortoise’’ coordinate

$$\frac{dr_*}{dr} = \frac{\sqrt{X(r)}}{F(r)}. \quad (5.14)$$

Then the equation (5.8) takes the form

$$\frac{d^2 R}{dr_*^2} + \frac{U}{r^4} R = 0. \quad (5.15)$$

A. Superradiance condition

We impose boundary conditions of purely ingoing waves at the black-hole horizon and a decaying solution at spatial infinity. We calculate the frequencies

$$k_H^2 = V_{\text{eff}}(r \rightarrow r_+) = (\omega - qA(r_H))^2, \quad (5.16)$$

$$k_\infty^2 = V_{\text{eff}}(r \rightarrow \infty) = \infty. \quad (5.17)$$

At spatial infinity $k_\infty^2 \rightarrow \infty$, so there is no solution for the wave at infinity.

To overcome this problem, we have to impose conditions on the parameters as in the case of the Galileon black hole we studied in Sec. IV. The easiest choice is to put the cosmological constant $\Lambda = 0$. Then the metric (5.2) takes the form

$$ds^2 = -F(r)dt^2 + \frac{3(8\kappa r^2 - Q^2)^2}{r^4} \frac{dr^2}{F(r)} + r^2 d\Omega^2, \quad (5.18)$$

where $F(r)$ and $X(r)$ are

$$F(r) = 192\kappa^2 - \frac{\mu}{r} + 48\kappa \frac{Q^2}{r^2} - \frac{Q^4}{r^4}, \quad (5.19)$$

$$X(r) = \frac{3(8\kappa r^2 - Q^2)^2}{r^4}. \quad (5.20)$$

The electromagnetic potential of the black hole is

$$A_0(r) = \sqrt{15} \left(\frac{Q^3}{3r^3} - 8\kappa \frac{Q}{r} \right), \quad (5.21)$$

and it goes to zero at infinity, as expected.

Following the same procedure, with the new functions X , F , A_0 , we solve the equation the radial equation (5.8) and take purely ingoing waves at the black hole horizon and a decaying solution at spatial infinity

$$R \sim \exp -i(\omega - qA(r_+))r_*, \quad r \rightarrow r_+(r_* \rightarrow -\infty), \quad (5.22)$$

and

$$R \sim \exp \left\{ -i\sqrt{\omega^2 - 192\kappa^2\mu^2 r_*} \right\}, \quad r \rightarrow \infty (r_* \rightarrow \infty). \quad (5.23)$$

Evaluating the Wronskian at the horizon and infinity, respectively, and equating the two values, we get

$$|R|^2 = |I|^2 - \frac{\omega - qA(r_+)}{\sqrt{\omega^2 - 192\kappa^2\mu^2}} (|T|^2). \quad (5.24)$$

We can see that if

$$\omega < q\sqrt{15} \left(\frac{Q^3}{3r_H^3} - 8\kappa \frac{Q}{r_H} \right), \quad (5.25)$$

the wave is superradiantly amplified.

VI. CONCLUSIONS

In this work we studied the superradiant effect in a class of scalar-tensor Horndeski theory. We considered first a massive charge scalar field coupled to Einstein tensor with a derivative coupling, scattered off the horizon of a Reissner-Nordström black hole. We showed that this derivative coupling provides a scale for a confining potential, and in the same time modifies the Bekenstein's superradiance condition (1.2) with the derivative coupling appearing explicitly in the superradiance condition. We found that for a wide range of parameters the superradiant condition is satisfied and in the same time a trapping potential is formed outside the horizon of a Reissner-Nordström black hole. As the strength of the coupling of the scalar field to curvature is increasing the depth of the potential is increasing indicating the amplification of the superradiant instability. The same amplification occurs for constant coupling as the mass and charge of the black hole is increasing.

This superradiance instability indicates that the background Reissner-Nordström black hole may require scalar hair. Then we studied the backreacted effect. We allowed the scalar field coupled to Einstein tensor to backreact to a charged spherical symmetric background. This leads to the generation of hairy charge Galileon black holes. The basic property of these solutions is that the derivative coupling appears as a parameter in these hairy solutions. We studied the superradiance effect and found the superradiance conditions of a massive charged scalar wave scattered off the horizon of these Galileon black holes. We discussed two specific solutions. The first solution is a charge Galileon black hole solution with a time dependent scalar hair and in the second solution the scalar hair is static.

In both cases the derivative coupling, which shows how strong the scalar field is coupled to curvature, influences the superradiant radiation. In the case of a fixed Reissner-Nordström black hole background, the recover the Bekenstein superradiance condition for any value of the

derivative coupling β if $\beta > 0$, while if $\beta < 0$ then the Bekenstein superradiance condition is modified. In the case that the massive charge scalar field is scattered off the horizon of the charge Galileon black hole the derivative coupling appears explicitly in the superradiance condition the strength of which influences the superradiance radiation in the case the scalar hair of the Galileon black hole is time-dependent, while the superradiance condition is

independent of β if the scalar hair is static for the particular solution we considered.

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