Unified dark energy and dark matter from dynamical spacetime

David Benisty[1,2,3,*] and Eduardo I. Guendelman[1,3,4,†]

[1]*Frankfurt Institute for Advanced Studies (FIAS), Ruth-Moufang-Strasse 1, 60438 Frankfurt am Main, Germany*
[2]*Goethe-Universität, Max-von-Laue-Strasse 1, 60438 Frankfurt am Main, Germany*
[3]*Physics Department, Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel*
[4]*Bahamas Advanced Study Institute and Conferences, 4A Ocean Heights, Hill View Circle, Stella Maris, Long Island, The Bahamas*

(Received 28 February 2018; published 6 July 2018)

A unification of dark matter and dark energy based on a dynamical spacetime theory is suggested. By introducing a dynamical spacetime vector field χ_μ as a Lagrange multiplier, conservation of an energy momentum tensor $T^{\mu\nu}_{(\chi)}$ is implemented. This Lagrangian generalizes the "unified dark energy and dark matter from a scalar field different from quintessence" [Phys. Rev. D **81**, 043520 (2010)], which did not consider a Lagrangian formulation. This generalization allows the solutions which were found previously, but in addition to that also nonsingular bouncing solutions that rapidly approach to the ΛCDM model. The dynamical time vector field exactly coincides with the cosmic time for the a ΛCDM solution and suffers a slight shift (advances slower) with respect to the cosmic time in the region close to the bounce for the bouncing nonsingular solutions. In addition, we introduce some exponential potential which could enter into the $T^{\mu\nu}_{(\chi)}$ stress energy tensor or be coupled directly to the measure $\sqrt{-g}$, giving a possible interaction between dark energy and dark matter, and could explain the coincidence problem.

DOI: 10.1103/PhysRevD.98.023506

I. INTRODUCTION

Dark energy (DE) and dark matter (DM) constitute most of the observable Universe. Yet the true nature of these two phenomena is still a mystery. One fundamental question with respect to these phenomena is the coincidence problem, which is trying to explain the relation between dark energy and dark matter densities. In order to solve this problem, one approach claims that the dark energy is a dynamical entity and hopes to exploit solutions of the scaling or tracking type to remove dependence on the initial conditions. Others left this principle and tried to model the dark energy as a phenomenological fluid which exhibits a particular relation with the scale factor [1], the Hubble constant [2], or even the cosmic time itself [3].

Unifications between dark energy and dark matter from an action principle were obtained from K-essence-type actions [4] or by introducing a complex scalar field [5]. Beyond those approaches, a unified description of dark energy and dark matter using a new measure of integration has been formulated [6–10]. Also, a diffusive interaction of dark energy and dark matter models was introduced in [11,12], and it has been found that diffusive interacting dark energy–dark matter models can be formulated in the context of an action principle based on a generalization of those two-measures theories in the context of quintessential scalar fields [13,14], although these models are not equivalent to the previous diffusive interacting dark energy–dark matter models [11,12].

One has to take now into consideration the measurements on 17 August, 2017, of multimessenger gravitational wave astronomy which are in contradiction to many modified theories of gravity predictions. These observations commenced with the detection of the binary neutron star merger GW170817 and its associated electromagnetic counterparts [15]. Both signals place an exquisite bound on the speed of gravity to be the same as the speed of light. This constraint rejected many modifications to general relativity [16–21] and also many unifications between dark energy and dark matter.

A model, which also continues to be valid after the GW170817 event, for a unification of dark energy and dark matter from a single scalar field ϕ was suggested by Gao, Kunz, Liddle, and Parkinson [22]. Their model is close to traditional quintessence and gives dynamical dark energy and dark matter but introduces a modification of the equations of motion of the scalar field that apparently are impossible to formulate in the framework of an action principle. The basic stress energy tensor which was considered in addition to the Einstein equation was

*benidav@post.bgu.ac.il
†guendel@bgu.ac.il

$$T^{\mu\nu} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}, \tag{1}$$

where ϕ is a scalar field and $U(\phi)$ is the potential for that scalar. Assuming homogeneous and isotropic behavior, the scalar field should be only time dependent $\phi = \phi(t)$. Then the kinetic term $-\frac{1}{2}\phi^{,\mu}\phi^{,\nu}$ is parameterizing the dark matter, because it contains only energy density with no pressure, and $U(\phi)g^{\mu\nu}$ is parameterizing the dark energy. The basic requirement for this stress energy tensor is its conservation law $\nabla_\mu T^{\mu\nu} = 0$. By assuming a constant potential $U(\phi) = \text{const}$, the model provides from the potential the traditional cosmological constant and the kinetic term of the scalar field is shown to provide, from the conservation law of the energy momentum tensor, that the kinetic term dependence has a dustlike behavior:

$$-\frac{1}{2}\nabla_\mu(\phi^{,\mu}\phi^{,\nu}) = 0 \Rightarrow \dot{\phi}^2 \sim \frac{1}{a^3}. \tag{2}$$

This simple case refers to the classical ΛCDM model. The special advantage of this model is a unification of dark energy and dark matter from one scalar field and has an interesting possibility for exploring the coincidence problem.

The lack of an action principle for this model brought us to a reformulation of the unification between dark energy and dark matter idea put forward by Gao, Kunz, Liddle, and Parkinson [22] in the framework of a dynamical spacetime theory [23,24], which forces conservation of the energy momentum tensor in addition to the covariant conservation of the stress energy momentum tensor that appears in the Einstein equation. In the next section, we explore the equations of motion for these theories. In the third section, we solve analytically the theory for constant potentials which reproduce the ΛCDM model with a bounce, which gives a possibility to solve the initial big bang singularity. In the last section, we solve the theory for an exponential potential which gives a good possibility for solving the coincidence problem.

II. DYNAMICAL SPACETIME THEORY

A. A basic formulation

One of the basic features in the standard approach to theories of gravity is the local conservation of an energy momentum tensor. In the field theory case, it is derived as a result rather than a starting point. For example, the conservation of energy can be derived from the time translation invariance principle. The local conservation of an energy momentum tensor can be a starting point rather than a derived result. Let us consider a four-dimensional case where conservation of a symmetric energy momentum tensor $T^{\mu\nu}_{(\chi)}$ is imposed by introducing the term in the action

$$\mathcal{S}_{(\chi)} = \int d^4x\sqrt{-g}\chi_{\mu;\nu}T^{\mu\nu}_{(\chi)}, \tag{3}$$

where $\chi_{\mu;\nu} = \partial_\nu\chi_\mu - \Gamma^\lambda_{\mu\nu}\chi_\lambda$. The vector field χ_μ called a dynamical spacetime vector, because of the energy density of $T^{\mu\nu}_{(\chi)}$, is a canonically conjugated variable to χ_0, which is what we expected from a dynamical time:

$$\pi_{\chi_0} = \frac{\partial\mathcal{L}}{\partial\dot{\chi}^0} = T^0_0(\chi). \tag{4}$$

If $T^{\mu\nu}_{(\chi)}$ is independent of χ_μ and having $\Gamma^\lambda_{\mu\nu}$ being defined as the Christoffel connection coefficients (the second-order formalism), then the variation with respect to χ_μ gives a covariant conservation law:

$$\nabla_\mu T^{\mu\nu}_{(\chi)} = 0. \tag{5}$$

From the variation of the action with respect to the metric, we get a conserved stress energy tensor $G^{\mu\nu}$ (in appropriate units), which is well known from the Einstein equation:

$$G^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}[\mathcal{L}_\chi + \mathcal{L}_m], \qquad \nabla_\mu G^{\mu\nu} = 0, \tag{6}$$

where $G^{\mu\nu}$ is the Einstein tensor, $\mathcal{L}_\chi$ is the Lagrangian in (3), and $\mathcal{L}_m$ is an optional action that involves other contributions.

Some basic symmetries that hold for the dynamical spacetime theory are two independent shift symmetries:

$$\chi_\mu \to \chi_\mu + k_\mu, \qquad T^{\mu\nu}_{(\chi)} \to T^{\mu\nu}_{(\chi)} + \Lambda g^{\mu\nu}, \tag{7}$$

where Λ is some arbitrary constant and k_μ is a Killing vector of the solution. This transformation will not change the equations of motion, which means also that the process of redefinition of the energy momentum tensor in the action (3) will not change the equations of motion. Of course, such a type of redefinition of the energy momentum tensor is exactly what is done in the process of normal ordering in the quantum field theory, for instance.

B. A connection to modified measures

A particular case of the stress energy tensor with the form $T^{\mu\nu}_{(\chi)} = \mathcal{L}_1 g^{\mu\nu}$ corresponds to a modified measure theory. By substituting this stress energy tensor into the action itself, the determinant of the metric is canceled:

$$\sqrt{-g}\chi^\mu_{;\mu}\mathcal{L}_1 = \partial_\mu(\sqrt{-g}\chi^\mu)\mathcal{L}_1 = \Phi\mathcal{L}_1, \tag{8}$$

where $\Phi = \partial_\mu(\sqrt{-g}\chi^\mu)$ is like a "modified measure." A variation with respect to the dynamical time vector field will give a constraint on $\mathcal{L}_1$ to be a constant:

$$\partial_\alpha \mathcal{L}_1 = 0 \Rightarrow \mathcal{L}_1 = M = \text{const.} \tag{9}$$

This situation corresponds to the two-measures theory [25–27], where, in addition to the regular measure of integration in the action, $\sqrt{-g}$ includes another measure of integration which is also a density and a total derivative. Notable effects that can be obtained in this way are the spontaneous breaking of the scale invariance, the seesaw cosmological effects [25], the resolution of the 5th force problem in quintessential cosmology [28], and a unified picture of both inflation and slowly accelerated expansion of the present Universe [29,30]. As we mentioned before in the introduction, the two-measures theory can serve to build unified models of dark energy and dark matter.

Usually, the construction of this measure is from four scalar fields φ_a, where $a = 1, 2, 3, 4$:

$$\Phi = \frac{1}{4!}\varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{abcd}\partial_\alpha\varphi^{(a)}\partial_\beta\varphi^{(b)}\partial_\gamma\varphi^{(c)}\partial_\delta\varphi^{(d)}, \tag{10}$$

and then we can rewrite an action that uses both of these densities:

$$S = \int d^4x\Phi\mathcal{L}_1 + \int d^4x\sqrt{-g}\mathcal{L}_2. \tag{11}$$

As a consequence of the variation with respect to the scalar fields φ_a, assuming that $\mathcal{L}_1$ and $\mathcal{L}_2$ are independent of the scalar fields φ_a, we obtain that for $\Phi \neq 0$ it implies that $\mathcal{L}_1 = M = \text{const}$ as in the dynamical time theory with the case of (9).

III. DE-DM UNIFIED THEORY FROM DYNAMICAL SPACETIME

A suggestion of an action which can produce DE-DM unification takes the form

$$\mathcal{L} = -\frac{1}{2}R + \chi_{\mu;\nu}T^{\mu\nu}_{(\chi)} - \frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi). \tag{12}$$

Consisting of an Einstein Hilbert action ($8\pi G = 1$), quintessence, and dynamical spacetime action, when the original stress energy tensor $T^{\mu\nu}_{(\chi)}$ is the same as the stress energy tensor (1), Gao and colleagues used

$$T^{\mu\nu}_{(\chi)} = -\frac{1}{2}\phi^{,\mu}\phi^{,\nu} + U(\phi)g^{\mu\nu}. \tag{13}$$

The action depends on three different variables: the scalar field ϕ, the dynamical spacetime vector χ_μ, and the metric $g_{\mu\nu}$. Therefore, there are three sets for the equation of motions. For the solution, we assume homogeneity and isotropy; therefore, we solve our theory with a Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a(t)^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right). \tag{14}$$

According to this ansatz, the scalar field is just a function of time $\phi(t)$ and the dynamical vector field will be taken only with a time component $\chi_\mu = (\chi_0, 0, 0, 0)$, where χ_0 is also just a function of time. A variation with respect to the dynamical spacetime vector field χ_μ will force conservation of the original stress energy tensor, which in Friedmann-Lemaître-Robertson-Walker metric (FRWM) gives the relation

$$\ddot{\phi} + \frac{3}{2}\mathcal{H}\dot{\phi} + U'(\phi) = 0. \tag{15}$$

Compared with the equivalent equation which comes from the quintessence model, this model gives a different and smaller friction term, as compared to the canonical scalar field. Therefore, for an increasing redshift, the densities for the scalar field will increase slower than in the standard quintessence.

The second variation with respect to the scalar field ϕ gives a nonconserved current:

$$\chi^\lambda_{;\lambda}U'(\phi) - V'(\phi) = \nabla_\mu j^\mu, \tag{16a}$$

$$j^\mu = \frac{1}{2}\phi_{,\nu}(\chi^{\mu;\nu} + \chi^{\nu;\mu}) + \phi^{,\mu}, \tag{16b}$$

and the derivatives of the potentials are the source of this current. For constant potentials the source becomes zero, and we get a covariant conservation of this current. In a FLRW metric, this equation of motion takes the form

$$\ddot{\phi}(\dot{\chi}_0 - 1) + \dot{\phi}[\ddot{\chi}_0 + 3\mathcal{H}(\dot{\chi}_0 - 1)] = U'(\phi)(\dot{\chi}_0 + 3\mathcal{H}\chi_0) - V'(\phi). \tag{17}$$

Substituting the term of the potential derivative $U'(\phi)$ from Eq. (15):

$$[1 - 2\dot{\chi}_0 - 3\mathcal{H}\chi_0]\ddot{\phi} - \left[\ddot{\chi}_0 - 3\mathcal{H} + \frac{9}{2}\mathcal{H}(\dot{\chi}_0 + \chi_0\mathcal{H})\right]\dot{\phi} + V'(\phi) = 0. \tag{18}$$

The last variation, with respect to the metric, gives the stress energy tensor that is defined by the value of the Einstein tensor:

$$G^{\mu\nu} = g^{\mu\nu}\left(\frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + V(\phi) + \frac{1}{2}\chi^{\alpha;\beta}\phi_{,\alpha}\phi_{,\beta} + \chi^\lambda\phi_{,\lambda}U'(\phi)\right) - \frac{1}{2}\phi^{,\mu}((\chi^\lambda_{;\lambda} + 2)\phi^{,\nu} + \chi^{\lambda;\nu}\phi_{,\lambda} + \chi^\lambda\phi^{;\nu}_{;\lambda}) - \frac{1}{2}(\chi^\lambda\phi^{;\mu}_{;\lambda}\phi^{,\nu} + \chi^{\lambda;\mu}\phi_{,\lambda}\phi^{,\nu}). \tag{19}$$

For the spatially homogeneous, cosmological case, the energy density and the pressure of the scalar field are, respectively,

$$\rho = \dot{\phi}^2\left(\dot{\chi}_0\left(1-\frac{3}{2}\mathcal{H}\right)-\frac{1}{2}\right)+V(\phi)-\dot{\phi}\dot{\chi}_0(U'(\phi)+\ddot{\phi}), \tag{20a}$$

$$p = \frac{1}{2}\dot{\phi}^2(\dot{\chi}_0-1)-V(\phi)-\chi_0\dot{\phi}U'(\phi). \tag{20b}$$

Substituting the potential derivative $U'(\phi)$ from Eq. (15) into the energy density term, makes the equation simpler:

$$\rho = \left(\dot{\chi}_0-\frac{1}{2}\right)\dot{\phi}^2+V(\phi), \tag{21}$$

which no longer has dependence on the potential $U(\phi)$ or its derivatives. Those three variations are sufficient for building a complete solution for the theory. Let us see a few simple cases.

IV. THE EVOLUTION OF THE HOMOGENEOUS SOLUTIONS

A. A bouncing ΛCDM solution

In order to compute the evolution of the scalar field and to check whether it is compatible with the observable Universe, we have to specify a form for the potentials. Let us take a simplified case of constant potentials:

$$U(\phi) = C, \qquad V(\phi) = \Omega_\Lambda. \tag{22}$$

Overall, in the equations of motion, only the derivative the potential $U(\phi)$ appears, not the potential itself. Therefore, a constant part of the potential $U(\phi)$ does not contribute to the solution. However, $V(\phi)$, as we shall see below, gives the cosmological constant. The conservation of the stress energy tensor from Eq. (15) gives

$$\dot{\phi}^2 = \frac{2\Omega_m}{a^3}, \tag{23}$$

where Ω_m is an integration constant which appears from the solution. From the second variation, with respect to the scalar field ϕ, a conserved current is obtained, which from Eq. (18) gives the exact solution of the dynamical time vector field:

$$\dot{\chi}_0 = 1-\kappa a^{-1.5}, \tag{24}$$

where κ is another integration constant. Eventually, the densities and the pressure for this potentials are given by (21). By substituting the solutions for the scalar $\dot{\phi}$ and the vector $\dot{\chi}_0$ (in units with $\rho_c = \frac{8\pi G}{3H_0^2} = 1$), we get

$$\rho = \Omega_\Lambda - \frac{\Omega_\kappa}{a^{4.5}}+\frac{\Omega_m}{a^3}, \tag{25a}$$

$$p = -\Omega_\Lambda - \frac{1}{2}\frac{\Omega_\kappa}{a^{4.5}}, \tag{25b}$$

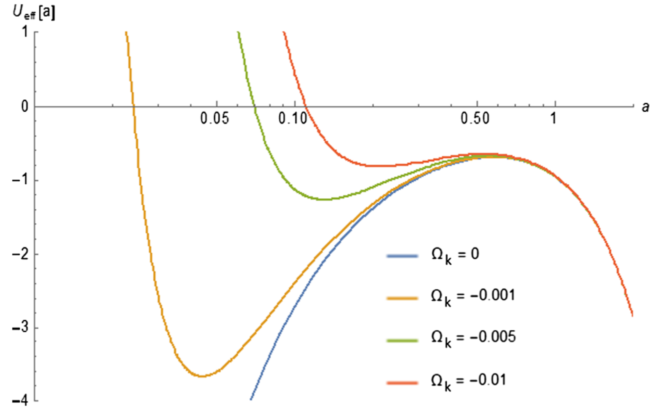

FIG. 1. Plot of the effective potential. For $\Omega_\kappa \neq 0$, there is a bouncing universe with dynamical dark energy.

where $\Omega_\kappa = \kappa\Omega_m$. Notice that Ω_m and Ω_κ are integration constants the solution contains and Ω_Λ is a parameter from the action of the theory. We can separate the result into three different "dark fluids": dark energy ($\omega = -1$), dark matter ($\omega = 0$), and an exotic part ($\omega = \frac{1}{2}$), which is responsible for the bounce (for $\kappa > 0$). From Eq. (23), the solution produces a positive Ω_m, since it is proportional to $\dot{\phi}^2$. For Ω_Λ the measurements for the late Universe forces the choice of this parameter to be positive. However, for other solutions (in the context of anti–de Sitter space, for instance), this parameter could be negative from the beginning.

In Fig. 1, we can see the effective potential for different values of Ω_κ. For $\Omega_\kappa = 0$, the solution returns to the known ΛCDM model. However, for $\Omega_\kappa < 0$, we obtain a bouncing solution which also returns to the ΛCDM for late time expansion.

In addition to those solutions, there is a strong correspondence between the zero component of the dynamical spacetime vector field and the cosmic time. For ΛCDM, there is no bouncing solution $\kappa = 0$, and therefore, from Eq. (24), we get $\chi_0 = t$ that implies that the dynamical time is exactly the cosmic time. For bouncing ΛCDM (see Fig. 2), we obtain a relation between the dynamical and the

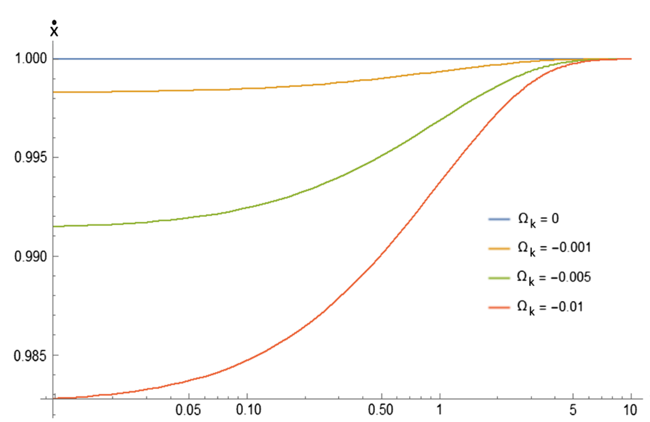

FIG. 2. Plot of $\dot{\chi}_0$ vs the cosmic time.

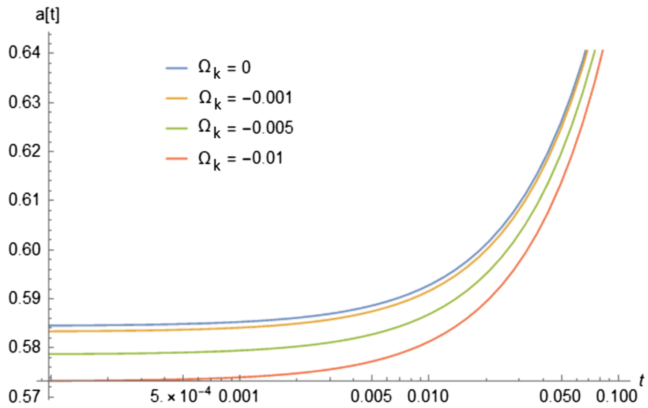

FIG. 3. Plot of the scale parameter vs the cosmic time. In any case, $\dot{\chi}_0 \approx 1$.

cosmic time with some delay between the dynamical time and the cosmic time for the early Universe (in the bouncing region). For the late Universe, the dynamical time returns back to run as fast as the cosmic time again. This relation between the dynamical and the cosmic time may have an interesting application in the solution to "the problem of time" in quantum cosmology which will discussed elsewhere. Notice that the dynamical time is a field variable while the cosmic time is a coordinate. The scale parameter evolution depicted in Fig. 3 can show us the initial conditions where $\dot{a}(t) = 0$, because at that point $a(t)$ is a minimum. In addition, for all cases the initial condition for the scale parameter is not zero $a(0) \neq 0$. These features imply a bouncing universe solution.

B. Interacting DE-DM

1. Autonomous system method

For studying the evolution of the scalar field in the case of interacting DE-DM, we address more generic potentials. For instance,

$$U(\phi) = C, \qquad V(\phi) = \Omega_\Lambda e^{-\beta\phi}, \tag{26}$$

where $\beta > 0$ (if not, we can perform the transformation $\phi \to -\phi$). In the limit $\beta \to 0$, the solution returns to the constant potentials case, and therefore the model is continuously connected to ΛCDM, at least as far as the background evolution is concerned. The first equation of motion (15) gives us the last case (23) or in this form

$$\ddot{\phi} = -\frac{3}{2}\mathcal{H}\dot{\phi}. \tag{27}$$

The equation of motion with respect to the scalar field ϕ can be expressed with a new dimensionless parameter:

$$\delta = \dot{\chi}_0 - 1, \tag{28}$$

which represents the difference of the rates of change between the zero component of the dynamical spacetime vector and the actual cosmic time. The equation of motion (17) in terms of this variable gets the form

$$\dot{\phi}\left(\dot{\delta} + \frac{3}{2}\mathcal{H}\delta\right) = \beta V(\phi). \tag{29}$$

Notice that for $\beta = 0$ the relation for $\delta = 2\kappa a^{-1.5}$ is Eq. (24). The main equations of the dynamical system are given by the following dimensionless quantities:

$$x = \frac{\dot{\phi}}{\sqrt{6}H}, \qquad y = \frac{\sqrt{V(\phi)}}{\sqrt{3}H}, \tag{30}$$

where x and y are represent the density parameters of the kinetic (dark-matter-like) and potential (dark-energy-like) terms, respectively. With those three new parameters (x, y, δ), the equation of motion with respect to the metric is written as

$$(1 + 2\delta)x^2 + y^2 = 1. \tag{31}$$

Assuming low values of β, the dynamical time and the cosmic time approximately coincide (see Fig. 3), and therefore $\delta \approx 0$. The phase portrait in that case should not deviate too much from a closed circle. Hence, Eq. (30) can be written by the following autonomous system equations:

$$\frac{dx}{d\tau} = -\frac{3x}{4}(x^2 - 1 + 3y^2), \tag{32a}$$

$$\frac{dy}{d\tau} = -\frac{y}{4}(-9 + 3x^2 + 9y^2 + 2\sqrt{6}x\beta), \tag{32b}$$

where $\tau = \ln a$. The equation of state ω also can be written as

$$\omega_\chi = \frac{1}{2}(1 - x^2 - 3y^2). \tag{33}$$

The properties of a few fixed points for the exponential potential are presented in Table I. In addition, the phase plane of the autonomous system shown in Fig. 4, with the points that are mentioned in the table. The features of the fixed points can separate to two cases.

TABLE I. The properties of the critical points for the exponential potential.

Name	Existence	Stability	Universe
A	All β	Unstable	…
B	All β	Stable for $\beta > \sqrt{\frac{3}{2}}$	Dark matter
C	All β	Asymptotically stable	Dark energy
D	$\beta > \sqrt{\frac{3}{2}}$	Unstable saddle point	Unified DE-DM

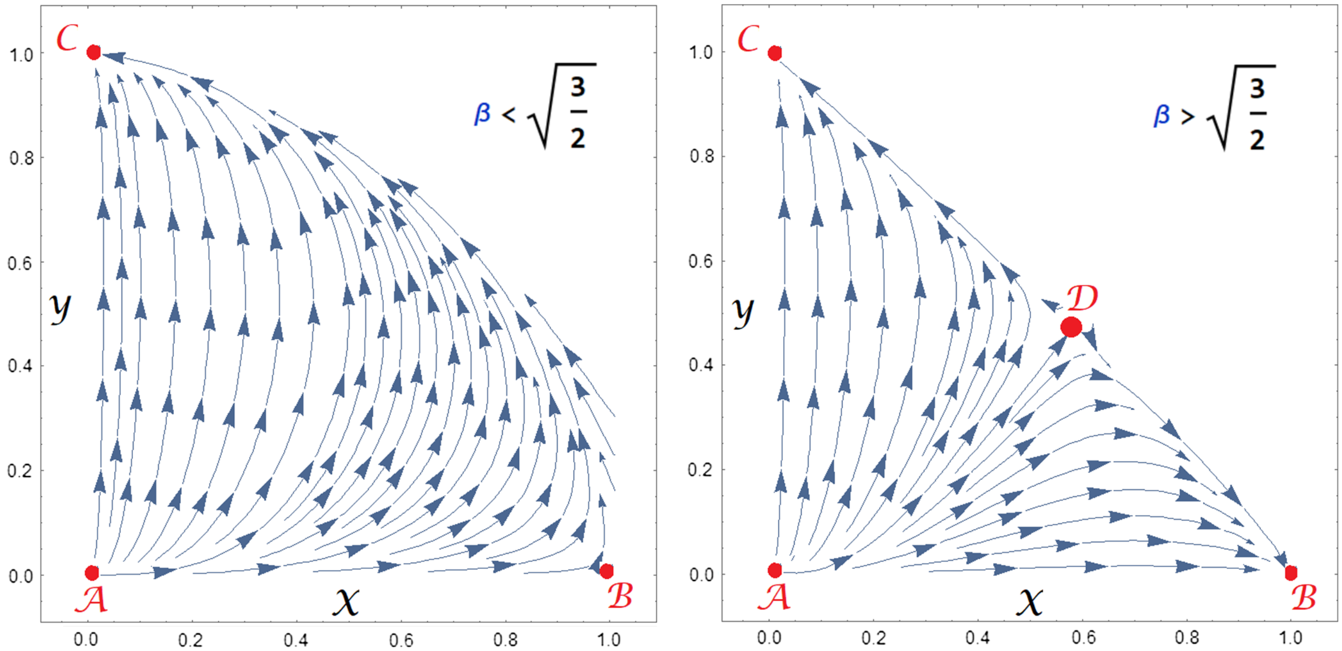

FIG. 4. The phase plane for different values of β.

One case is when $\beta < \sqrt{\frac{3}{2}}$ and all of the solutions are flowing into a dark energy dominated universe [point C $(x = 0, y = 1)$]. The dark matter dominated universe is an unstable point [point B $(x = 1, y = 0)$] that the universe goes though which corresponds to the dark matter epoch. In any case, point A [$(x = 0, y = 0)$, which represents no dark matter and no dark energy, does not really exist because of the contradiction to Eq. (31). However, if the initial condition starts close to this point, it is driven into dark energy dominance eventually, as you can see in Fig. 5. Also, for this case the shape of the phase portrait looks as a circle, which ensures our assumption about the identification between the dynamical spacetime and the cosmic time.

In the second case, $\beta > \sqrt{\frac{3}{2}}$ and there are two stable fixed points. One for dark energy (C) and one for dark matter (B). If the initial conditions are close enough to those points, it will be attracted into them. In addition, a saddle point D

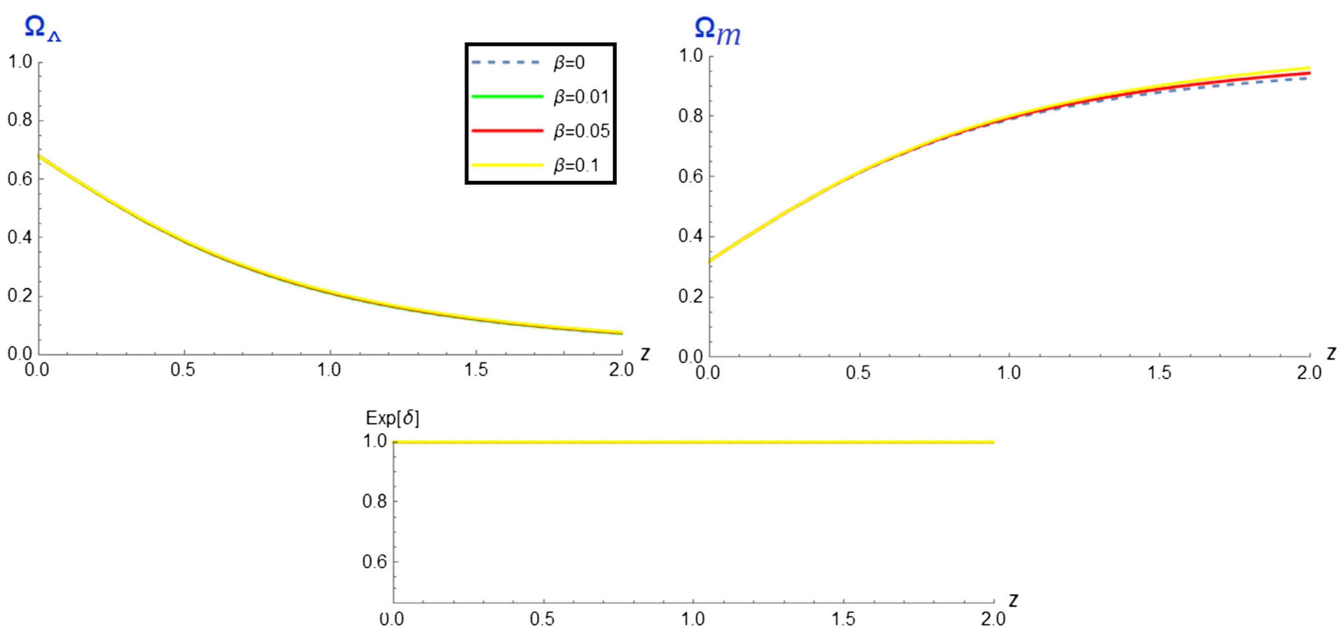

FIG. 5. The evolution of DE-DM ratios and $e^{\delta} \sim 1$ for small values of β.

$\left(x = \sqrt{\frac{3}{2}}\frac{1}{\beta}, y = \frac{\sqrt{2\beta^2-3}}{\sqrt{6}\beta}\right)$ is obtained. For this point, the ratio between the pressure and the density is $\omega = -\frac{2}{3} + \frac{1}{\beta^2}$. Some solutions are attracted to this point, but eventually they are repelled to the closer point. However, the case of $\beta > \sqrt{\frac{3}{2}}$ contradicts the assumption that β is small enough in order not to deviate from ΛCDM. Also, for this case the shape of the phase portrait deviates from a circle, which implies a big deviation between the dynamical spacetime and the cosmic time.

This modification, which adds one exponential potential, is not the most general case, since we could also add an additional potential which would enter into the $T^{\mu\nu}_{(\chi)}$. We suspect that some form of the potentials which are more general could cause point D to become stable and will lead us to a more comprehensive understanding of the cosmic coincidence problem, which we will investigate in the future.

2. Evolution of physical quantities

In order to assess the viability of the model, let us see how some physical quantities change vs the redshift (z). The connection between the cosmic time derivative and a redshift derivative is

$$\frac{d}{dt} = -\mathcal{H}(z)(z+1)\frac{d}{dz}, \tag{34}$$

which has been obtained from the scale factor dependence on z, $a(z) = \frac{a_0}{z+1}$. Figure 5 describes the cosmological energies densities Ω_m and Ω_Λ vs the redshift. For the $\beta = 0$ case, which refers to ΛCDM model (any time we can set Ω_κ to be zero or small), we can see that in earlier times Ω_m becomes dominant; for earlier times, that is, for the very early Universe, Ω_κ (which we have taken to be very small except for the very early Universe) dominates. For different values of β, we can see a slight shift from ΛCDM, which should be more dominant in the early Universe. The variable δ, that measures the difference in the evolution of the dynamical time and the cosmic time, which in the case of $\beta = 0$ gives a contribution that can be parametrized by Ω_κ, has been taken to be very close to zero in all cases except for the very early Universe, because there a strong impact exists, close to the bounce that replaces now the traditional big bang.

In Fig. 6, we can see the evolution of the equation of state of the whole Universe as a function of the redshift. It behaves as cold dark matter dominated at higher redshifts and dark energy for the lower redshifts. The behavior does not tremendously change for those values of the redshift, but the deviations are measurable.

The set of potentials that were suggested in this section have a nice feature which reduces the dependence on the number of quantities. In this way, a suggestive and

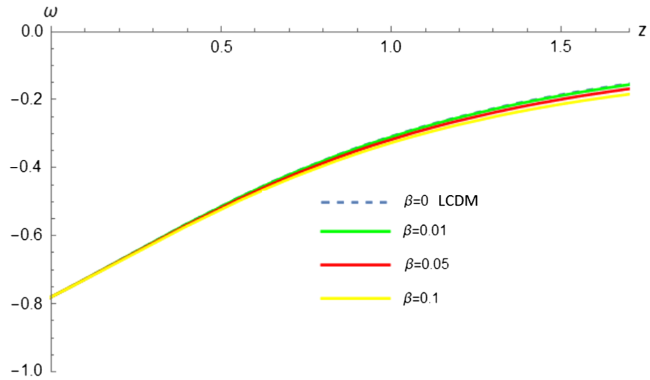

FIG. 6. The equation of state of the Universe for different values of β.

convenient parametrization of the solution uses the variable $\delta = \dot{\chi}_0 - 1$, which contains all the dependence on χ_0. In the future, it would be interesting to investigate how different potentials would affect the physical quantities of the Universe. However, unlike other models of dark energy and dark matter, even a trivial assumption of constant potentials leads directly to a unification of dark energy and dark matter. In any case, any generalization should assume a constant potential asymptotically.

V. DISCUSSION AND FUTURE WORK

In this paper, the unified dark energy and dark matter from a scalar field different from quintessence is formulated through an action principle. Introducing the coupling of a dynamical spacetime vector field to an energy momentum tensor that appears in the action determines the equation of motion of the scalar field from the variation of the dynamical spacetime vector field or effectively from the conservation law of an energy momentum tensor, as in Ref. [22]. The energy momentum tensor that is introduced in the action is related but not, in general, the same as the one that appears in the right-hand side of the gravitational equations, as opposed to the non-Lagrangian approach of Ref. [22], so our approach and that of Ref. [22] are not equivalent. However, in many situations the solutions studied in Ref. [22] can be also obtained here, but there are other solutions, in special nonsingular bounce solutions which are not present in Ref. [22].

In those simple solutions, the dynamical time behaves very close to the cosmic time. In particular, in solutions which are exactly ΛCDM, the cosmic time and the dynamical time exactly coincide with each other. If there is a bounce, the deviation of the dynamical time with respect to the cosmic time takes place only very close to the bounce region. The use of this dynamical time as the time in the Wheeler–de Witt equation should also be a subject of interest.

In principle, we can introduce two different scalar potentials: one coupled directly to $\sqrt{-g}$ and the other appearing in the original stress energy tensor $T^{\mu\nu}_{(\chi)}$. So far, for the purposes of starting the study of the theory, we have introduced only a scalar potential coupled directly to $\sqrt{-g}$ and shown that this already leads to an interacting dark energy–dark matter model, although the full possibilities of the theory will be revealed when the two independent potentials are introduced.

Possible signatures for this model or for more generalized forms could be identified from the cosmological perturbation theory. For instance, the perturbation for the scalar field is clear. However, the perturbation for the vector field could be represented with more degrees of freedom which can reproduce a different power spectrum for the cosmic microwave background (CMB) anisotropies, for instance. But, more than this, the model that was suggested in the last part was only with an exponential potential. However, many combinations of potentials are applicable for testing the evolution for the energy densities and using data fitting for those models. The benefits for these models are that they still preserve the speed of gravity equal to the speed of light and also that arise from an action principle. Researching those families of solutions with more general potentials could help solve the coincidence problem.

The effects studied in the context of the bouncing solution, which can prevent the initial big bang singularity, could have consequences for the radially falling solutions, since as we have seen the kappa term can introduce a repulsive force that prevents the big bang singularity; there will very likely be a corresponding effect when we study radial collapse of matter, and then the analogous term, that in the homogeneous cosmology solutions prevents the big bang singularity, will in this case prevent the collapse to very high densities. This will, in turn, suppress the structure formation at low redshifts as compared to the expectations from the perturbations observed in the CMB, thus maybe explaining the σ_8 [31–33] -Ω_m tension. Notice that this effect on perturbations can take place even for constant potentials, that is, without modifying the standard ΛCDM homogeneous background, since in the homogeneous background the κ term acts only in the very early Universe.

Finally, another direction for research has been started by studying models of this type in the context of higher-dimensional theories, where they can provide a useful framework to study the "inflation-compactification" epoch and an exit from this era to the present ΛCDM epoch could be further explored [34].

ACKNOWLEDGMENTS

This article is supported by COST Action No. CA15117 "Cosmology and Astrophysics Network for Theoretical Advances and Training Action" (CANTATA) of the COST (European Cooperation in Science and Technology). In addition, we thank the Foundational Questions Institute FQXi for support, in particular, support for our conference BASIC2018 at Stella Maris Bahamas, where part of this research was carried out.

[1] V. F. Cardone, A. Troisi, and S. Capozziello, Phys. Rev. D **69**, 083517 (2004).
[2] G. Dvali and M. S. Turner, arXiv:astro-ph/0301510.
[3] S. Basilakos, Mon. Not. R. Astron. Soc. **395**, 2347 (2009).
[4] R. J. Scherrer, Phys. Rev. Lett. **93**, 011301 (2004).
[5] A. Arbey, Phys. Rev. D **74**, 043516 (2006).
[6] E. Guendelman, E. Nissimov, and S. Pacheva, Eur. Phys. J. C **76**, 90 (2016).
[7] E. Guendelman, D. Singleton, and N. Yongram, J. Cosmol. Astropart. Phys. 11 (2012) 044.
[8] E. Guendelman, E. Nissimov, and S. Pacheva, Eur. Phys. J. C **75**, 472 (2015).
[9] S. Ansoldi and E. I. Guendelman, J. Cosmol. Astropart. Phys. 05 (2013) 036.
[10] E. Guendelman, E. Nissimov, and S. Pacheva, Bulg. J. Phys. **44**, 15 (2017).
[11] G. Koutsoumbas, K. Ntrekis, E. Papantonopoulos, and E. N. Saridakis, J. Cosmol. Astropart. Phys. 02 (2018) 003.
[12] Z. Haba, A. Stachowski, and M. Szydłowski, J. Cosmol. Astropart. Phys. 07 (2016) 024.
[13] D. Benisty and E. I. Guendelman, Eur. Phys. J. C **77**, 396 (2017).
[14] D. Benisty and E. I. Guendelman, Int. J. Mod. Phys. D **26**, 1743021 (2017); Eur. Phys. J. C **77**, 396 (2017).
[15] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. **119**, 161101 (2017).
[16] P. Creminelli and F. Vernizzi, Phys. Rev. Lett. **119**, 251302 (2017).
[17] J. M. Ezquiaga and M. Zumalacárregui, Phys. Rev. Lett. **119**, 251304 (2017).
[18] T. Baker, E. Bellini, P. G. Ferreira, M. Lagos, J. Noller, and I. Sawicki, Phys. Rev. Lett. **119**, 251301 (2017).
[19] J. Sakstein and B. Jain, Phys. Rev. Lett. **119**, 251303 (2017).
[20] L. Lombriser and A. Taylor, J. Cosmol. Astropart. Phys. 03 (2016) 031.
[21] L. Lombriser and N. A. Lima, Phys. Lett. B **765**, 382 (2017).
[22] C. Gao, M. Kunz, A. R. Liddle, and D. Parkinson, Phys. Rev. D **81**, 043520 (2010).
[23] E. I. Guendelman, Int. J. Mod. Phys. A **25**, 4081 (2010).
[24] D. Benisty and E. I. Guendelman, Mod. Phys. Lett. A **31**, 1650188 (2016).
[25] E. I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999).
[26] E. I. Guendelman and A. B. Kaganovich, Phys. Rev. D **53**, 7020 (1996).

[27] E. I. Guendelman and A. B. Kaganovich, Classical Quantum Gravity **25**, 235015 (2008).
[28] E. I. Guendelman and A. B. Kaganovich, Ann. Phys. (Amsterdam) **323**, 866 (2008).
[29] E. I. Guendelman and O. Katz, Classical Quantum Gravity **20**, 1715 (2003).
[30] E. Guendelman, R. Herrera, P. Labrana, E. Nissimov, and S. Pacheva, Gen. Relativ. Gravit. **47**, 10 (2015).
[31] G. Lambiase, S. Mohanty, A. Narang, and P. Parashari, arXiv:1804.07154.
[32] B. J. Barros, L. Amendola, T. Barreiro, and N. J. Nunes, arXiv:1802.09216.
[33] L. Kazantzidis and L. Perivolaropoulos, Phys. Rev. D **97**, 103503 (2018).
[34] D. Benisty and E. I. Guendelman, arXiv:1805.09314.