

Interpolating formula for the $0\nu\beta\beta$ -decay half-life in the case of light and heavy neutrino mass mechanisms

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We revisit the “interpolating formula” proposed in our previous publication. It allows one to calculate the $0\nu\beta\beta$ -decay half-life for arbitrary neutrino mass without the involvement of the complicated results for nuclear matrix elements (NMEs) obtained within specific nuclear structure models. The formula derives from the finding that the value of a properly normalized ratio of the NMEs for the light and heavy neutrino mass mechanisms weakly depends on the isotope. From this fact, it follows, in particular, that the light and heavy neutrino mass mechanisms can hardly be distinguished in a model-independent way searching for $0\nu\beta\beta$ decay of different nuclei. Here, we show that this formula holds for all the known nuclear structure approaches. We give a mathematical justification of our results examining analytical properties of the NMEs. We also consider several simplified benchmark scenarios within left-right symmetric models and analyze the conditions for the dominance of the light or heavy neutrino mass mechanisms in $0\nu\beta\beta$ decay.

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I. INTRODUCTION

Neutrinoless double-beta decay ($0\nu\beta\beta$) is a lepton number–violating (LNV) process changing the lepton number by two units $\Delta L = 2$. It is forbidden in the Standard Model (SM), in which the lepton number is conserving. Basically, there are two sources of lepton-number violation: Majorana neutrino mass and LNV vertices. The latter may emerge from numerous high-scale models giving rise to the corresponding mechanisms of $0\nu\beta\beta$ decay. Once this process is observed, the question of distinguishing between the dominant mechanisms will arise. Certainly, this task is highly nontrivial. One may hope that measurements of the $0\nu\beta\beta$ half-life with different isotopes would facilitate its solution due to the variability of nuclear matrix elements (NMEs) of particular mechanisms from one isotope to the other. In the present paper, we show

that at least the light and heavy Majorana neutrino mass mechanisms are indistinguishable in this way without additional hypothesis. This fact becomes especially comprehensible in terms of the so-called interpolating formula (IntF) [1] merging the light and heavy neutrino mass ranges in the NMEs and allowing a transparent physical interpretation of the above fact. The IntF is a simple analytical formula representing with an accuracy of 30% or better the NME as a function of the Majorana neutrino mass. This accuracy is sufficient for practical purposes, taking into account the limited accuracy of the available nuclear structure approaches to the NME calculations. In what follows, we will show that the IntF is valid for all these nuclear structure approaches with the above-indicated accuracy and elucidate some of its other useful properties. On the particle physics side, we adopt a generic scenario with Majorana neutrinos of arbitrary value masses and consider their contribution to $0\nu\beta\beta$ decay via mass mechanism mediated by both left- and right-handed weak currents. Then, for the sake of concreteness, we consider the neutrino mass mechanism within the left-right symmetric models (LRSMs) [2,3] and extend our analysis toward some more particular scenarios.

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II. NEUTRINO MASS MECHANISM OF $0\nu\beta\beta$ DECAY

We start with a generic Majorana neutrino mass mechanism of $0\nu\beta\beta$ decay induced by the low-energy effective Lagrangian

$$\mathcal{L}^\beta = \frac{G_\beta}{\sqrt{2}} [j_L^\rho J_{L\rho}^\dagger + \lambda j_R^\rho J_{R\rho}^\dagger + \text{H.c.}] \quad (1)$$

with the left-/right-handed hadronic $J_{L/R}$ and leptonic $j_{L/R}$ currents. As usual, $G_\beta = G_F \cos \theta_C$, where G_F and θ_C are Fermi constant and the Cabbibo angle, respectively. The dimensionless parameter λ depends on the underlying high-scale model. In the particular case of the left-right symmetric (LRS) models, based on the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group [2,3], the Lagrangian (1) appears at low energies after integrating out $W_{L,R}^\pm$ massive gauge bosons. In this model,

$$\lambda = (M_{W_L}/M_{W_R})^2, \quad (2)$$

where M_{W_L} and M_{W_R} ($M_{W_L} < M_{W_R}$) are masses of W_L and W_R gauge bosons, respectively. The current constraint on the mass of W_R $M_{W_R} \geq 2.9$ TeV [4] sets the limit

$$\lambda \leq 7.7 \times 10^{-4}. \quad (3)$$

The upper limit $\lambda = 7.7 \times 10^{-4}$ we use everywhere in the present paper as a reference value for this parameter. Since we focus on the mass mechanism, we discard in Eq. (1) the $j_{L,R} J_{R,L}$ terms irrelevant in this case (for a review see, for instance, Ref. [5]). In Eq. (1), the explicit form of the left- and right-handed hadronic currents $J_{L,R}^\dagger$ in nuclei can be found, e.g., in Ref. [6]. The leptonic currents are given by

$$j_L^\rho = \bar{e}_L \gamma^\rho \nu'_{eL}, \quad j_R^\rho = \bar{e}_R \gamma^\rho \nu'_{eR}.$$

The ν'_{eL} and ν'_{eR} are the weak eigenstate electron neutrinos, which are expressed as superpositions of the light and heavy Majorana mass eigenstate neutrinos ν_j and N_k as

$$\begin{aligned} \nu'_{eL} &= \sum_{j=1}^3 U_{ej} \nu_j + \sum_{k=1}^n S_{ek} N_k^C, \\ \nu'_{eR} &= \sum_{j=1}^3 T_{ej}^* \nu_j^C + \sum_{k=1}^n V_{ek}^* N_k, \end{aligned} \quad (4)$$

where the unitary matrix

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \quad (5)$$

is the generalization of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which diagonalizes the general $(3+n) \times (3+n)$ neutrino mass matrix

$$\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \quad (6)$$

in the basis $(\nu'_{eL}, \nu'_{\mu L}, \nu'_{\tau L}, N_{1R}^C, \dots, N_{nR}^C)$. Here, $M_{L,R}$ and M_D are Majorana and Dirac mass terms, respectively. After diagonalization, one should end up with three light ν_i ($i = 1, 2, 3$) and n heavy N_k ($k = 1, \dots, n$) Majorana neutrino mass eigenstates with the masses m_i and M_k , respectively. In the LRS models, $n = 3$. The smallness of m_i can be guaranteed by the seesaw-I condition $M_R \gg M_D$. As is well known, this leads to very heavy states N_k with masses $M_k \gg 1$ TeV being beyond the experimental reach. In the scenarios with $n > 3$, the inverse seesaw mechanism can be implemented. In this case among N_k , accompanying the light ν_i states, there can appear moderately heavy or even light Majorana states. Actually, their masses can be of arbitrary value. This is the case of our particular interest, for which we designed the above-mentioned interpolating formula.

Assuming the dominance of the mass mechanism, we write down the $0\nu\beta\beta$ -decay half-life

$$\begin{aligned} [T_{1/2}^{0\nu}]^{-1} &= G^{0\nu} g_A^4 m_p^2 \left(\left| \sum_{j=1}^3 U_{ej}^2 m_j M_{LL}^{0\nu}(m_j) + \sum_{k=1}^n S_{ek}^2 M_k M_{LL}^{0\nu}(M_k) \right|^2 + \lambda^2 \right. \\ &\quad \left. \times \left| \sum_{j=1}^3 T_{ej}^2 m_j M_{RR}^{0\nu}(m_j) + \sum_{k=1}^n V_{ek}^2 M_k M_{RR}^{0\nu}(M_k) \right|^2 \right). \end{aligned} \quad (7)$$

The proton mass is denoted by m_p , and g_A is the unquenched value of axial-vector coupling constant ($g_A = 1.269$). The phase-space factor $G^{0\nu}$ is tabulated for various $0\nu\beta\beta$ -decaying nuclei in Ref. [7]. The NMEs $M^{0\nu}$ as functions of neutrino mass m_ν ($m_\nu = m_i$ or M_k) are given by [1]

$$M_{LL,RR}^{0\nu}(m_\nu) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p \times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+ | J_{L,R}^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_{L,R,\mu}^\dagger(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_\nu^2} (\sqrt{p^2 + m_\nu^2} + E_n - \frac{E_I - E_F}{2})}. \quad (8)$$

Here, R and m_e are the nuclear radius and the mass of electron, respectively. We use as usual $R = r_0 A^{1/3}$ with $r_0 = 1.2$ fm. Initial and final nuclear ground states with energies E_I and E_F are denoted by $|0_I^+\rangle$ and $|0_F^+\rangle$, respectively. The summation runs over intermediate nuclear states $|n\rangle$ with energies E_n . The weak one-body nuclear charged current $J_{L,R}$ [1,6] depends on the effective value of axial-vector coupling constant g_A^{eff} of the nucleon, which is renormalized to a smaller, the so-called quenched value, g_A^{eff} [8].

III. INTERPOLATING FORMULA FOR THE $0\nu\beta\beta$ -DECAY HALF-LIFE

For the Majorana neutrino exchange mechanism in the literature, there are usually two limiting cases considered: light $m_i \ll p_F$ and heavy $M_i \gg p_F$ neutrinos, where $p_F \sim 200$ MeV is the Fermi momentum. For these limiting cases, the half-life formula (8) is reduced to

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \times \begin{cases} |\eta_\nu|^2 |M_\nu^{0\nu}|^2, & \text{for } m_i \ll p_F, \\ |\eta_N|^2 |M_N^{0\nu}|^2, & \text{for } M_k \gg p_F, \end{cases} \quad (9)$$

with

$$\begin{aligned} |\eta_\nu|^2 m_e^2 &= \left| \sum_{j=1}^3 U_{ej}^2 m_j \right|^2 + \lambda^2 \left| \sum_{j=1}^3 T_{ej}^2 m_j \right|^2 \\ &\simeq \left| \sum_{j=1}^3 U_{ej}^2 m_j \right|^2 \\ |\eta_N|^2 \frac{1}{m_p^2} &= (|\eta_N^L|^2 + |\eta_N^R|^2) \frac{1}{m_p^2} \\ &= \left| \sum_{k=1}^n S_{ek}^2 \frac{1}{M_k} \right|^2 + \lambda^2 \left| \sum_{k=1}^n V_{ek}^2 \frac{1}{M_k} \right|^2. \end{aligned} \quad (10)$$

Here, the NMEs $M_\nu^{0\nu}$ and $M_N^{0\nu}$ are derived from the NME $M^{0\nu}$ in Eq. (8) in the following way:

$$M^{0\nu}(m_i \rightarrow 0) = \frac{1}{m_p m_e} M_\nu^{0\nu}, \quad (11)$$

$$M^{0\nu}(M_i \rightarrow \infty) = \frac{1}{M_i^2} M_N^{0\nu}. \quad (12)$$

In the case of a neutrino spectrum with mass states N_k of an arbitrary mass value, one has to apply Eq. (8) for the NME calculations, resulting in a complicated function of the neutrino mass. This is a real hassle for use in practice. Fortunately, there is a very good approximate analytical representation for Eq. (8) proposed in Ref. [1] (and references therein) and having a remarkably simple form,

$$M_{LL,RR}^{0\nu}(m_\nu) \simeq M_N^{0\nu} \frac{1}{\langle p^2 \rangle + m_\nu^2}. \quad (13)$$

This is what we call the ‘‘interpolating formula’’ since it interpolates two limiting cases (11) and (12) and is valid to a good accuracy for an arbitrary value of m_ν . Equation (13) contains the parameter

$$\langle p^2 \rangle = m_p m_e \frac{M_N^{0\nu}}{M_\nu^{0\nu}} \quad (14)$$

with the dimension of (mass)². The form of Eq. (13) suggests the interpretation of $\langle p^2 \rangle$ as the mean square momentum of the virtual neutrino propagating between two β -decaying nucleons. Therefore, it is expected to be of the order of $p_F^2 \sim (200 \text{ MeV})^2$. The current values of the matrix elements $M_\nu^{0\nu}$ and $M_N^{0\nu}$ calculated within different nuclear structure approaches can be found in Tables 6 and 7 of Ref. [8]. The value of corresponding parameter $\sqrt{\langle p^2 \rangle}$ is given for various isotopes together with its averaged value $\sqrt{\langle p^2 \rangle_a}$ with variance σ in Table I. The unquenched value of axial-vector coupling constant is assumed: $g_A^{\text{eff}} = g_A = 1.25$ – 1.27 . We see that the value of $\sqrt{\langle p^2 \rangle}$ depends noticeably on the chosen nuclear structure method and considered choice of the two-nucleon short-range correlation function. The values of $\sqrt{\langle p^2 \rangle_a}$ are displayed for different nuclear structure approaches and types of two-nucleon short-range correlations (src) in Fig. 1. The largest value of the parameter $\sqrt{\langle p^2 \rangle_a} \simeq 200$ MeV is found for the quasiparticle random phase approximation (QRPA) with isospin restoration and CD-Bonn two-nucleon src. Surprisingly, within all the considered nuclear structure approaches, the variance σ is very small, being of the order of 3%–10%; i.e., the value of $\langle p^2 \rangle$ is practically the same for all isotopes of experimental interest and can be replaced with averaged value $\langle p^2 \rangle_a$. In the Appendix, we discuss this finding from the view point of the analytical properties of the NME in Eq. (8) as a function in the complex plane of m_ν . The above conclusion is also supported by the statistical treatment of $M_\nu^{0\nu}$ and $M_N^{0\nu}$ NMEs performed in Ref. [9].

Using the parameter $\langle p^2 \rangle_a$ in the interpolating formula (13), we can write to a good accuracy the $0\nu\beta\beta$ -decay half-life for the Majorana neutrino exchange mechanism as

$$[T_{1/2}^{0\nu}]^{-1} = \eta_{\nu N}^2 C_{\nu N}, \quad (15)$$

where

$$C_{\nu N} = g_A^4 |M_\nu^{0\nu}|^2 G^{0\nu} \quad (16)$$

and

TABLE I. The values of the parameter $\sqrt{\langle p^2 \rangle}$ of the interpolating formula (13), (14) for a given isotope and their average value $\sqrt{\langle p^2 \rangle}_a$ used in Eq. (17) with the variance σ (in parentheses) calculated within different nuclear structure approaches: interacting shell model (ISM) [Strasbourg-Madrid (StMa) [10] and Central Michigan University (CMU) [11] groups], interacting boson model (IBM) [12], quasiparticle random phase approximation [Tuebingen-Bratislava-Caltech (TBC) [13,14] and Jyväskylä (Jy) [15] groups], projected Hartree-Fock Bogoliubov approach (PHFB) [16], and covariant density functional theory (CDFT) [17]. The Argonne, CD-Bonn, and UCOM two-nucleon short-range correlations (src) are taken into account. The nonquenched value of weak axial-vector coupling g_A is assumed.

Method	g_A	src	$\sqrt{\langle p^2 \rangle}$ (MeV)												$\sqrt{\langle p^2 \rangle}_a$ (σ) (MeV)
			⁴⁸ Ca	⁷⁶ Ge	⁸² Se	⁹⁶ Zr	¹⁰⁰ Mo	¹¹⁰ Pd	¹¹⁶ Cd	¹²⁴ Sn	¹²⁸ Te	¹³⁰ Te	¹³⁶ Xe	¹⁵⁰ Nd	
ISM-StMa	1.25	UCOM	178	150	149					160		161	159		160(10)
ISM-CMU	1.27	Argonne	178	134	138					153		159	170		155(17)
		CD-Bonn	203	165	162					177		184	197		181(17)
IBM	1.27	Argonne	113	103	103	129	136	135	130	109	109	109	107	155	120(17)
QRPA-TBC	1.27	Argonne	189	163	164	180	174	166	157	186	178	180	183		175(11)
		CD-Bonn	231	193	194	211	204	194	182	214	207	209	211		205(13)
QRPA-Jy	1.26	CD-Bonn		191	192	217	207	187	177	202	196	201	175		194(13)
PHFB	1.25	Argonne				130	127	124			131	132		121	128(4)
		CD-Bonn				150	145	143			150	150		139	146(5)
CDFT	1.25	Argonne	122	129	131	129	131		133	138		138	137	138	132(5)

$$\eta_{\nu N}^2 = \left| \sum_{j=1}^3 U_{ej}^2 \frac{m_j}{m_e} + \sum_{k=1}^n S_{ek}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_k^2} \frac{M_k}{m_e} \right|^2 + \lambda^2 \left| \sum_{j=1}^3 T_{ej}^2 \frac{m_j}{m_e} + \sum_{k=1}^n V_{ek}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_k^2} \frac{M_k}{m_e} \right|^2 \quad (17)$$

for arbitrary mass M_k . The sum runs over $j = 1, 2, 3$ and $k = 1, \dots, n$. The values of parameter $C_{\nu N}$ are given for various isotopes in Table II. The interpolating formula in Eq. (15) reproduces the “exact” QRPA result with rather good accuracy except for the transition region where its deviation, as seen from Fig. 8, amounts to 20%–25%. The parameter $\eta_{\nu N}$ is a general LNV parameter for the light and

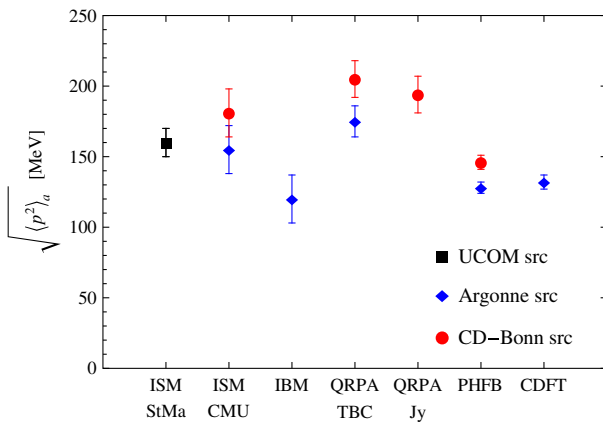


FIG. 1. The average value $\sqrt{\langle p^2 \rangle}_a$ over the set of the considered isotopes with variance σ calculated within different nuclear structure approaches. The notations are the same as in Table I.

heavy neutrino mass mechanisms, which is practically independent of the isotope under consideration.

IV. LIGHT VS HEAVY NEUTRINO MASS MECHANISMS

From the conclusion of the previous section and Eq. (17), it follows that, contrary to the previous expectations in the literature (see, for instance, Refs. [18,19]), the dominance of light or heavy neutrino mechanisms of $0\nu\beta\beta$ decay cannot be recognized just by observation of this process with different isotopes. An additional theoretical or experimental input about neutrino masses and mixing is needed to shed light on the particular role of each of these mechanisms.

Let us give a couple of examples of model inputs allowing us to distinguish the two above-mentioned mechanisms.

For a scenario with three SM singlet neutrinos $\nu_{e,\mu,\tau R}$, the 6×6 mixing matrix \mathcal{U} in Eq. (5) is completely parametrized with 15 angles, 10 Dirac and 5 Majorana CP -violating phases. Let consider some viable structures of this mixing matrix.

A. Uncoupled light and heavy neutrino sectors

In the particular case of

$$\mathcal{U} = \begin{pmatrix} U_0 & \mathbf{0} \\ \mathbf{0} & V_0 \end{pmatrix}, \quad (18)$$

there is no mixing between heavy and light neutrino sectors. Then, we have

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} (m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2) \quad (19)$$

TABLE II. The value of the parameter $C_{\nu N}$ in Eq. (16) for the isotopes of experimental interest. The calculated light neutrino exchange NME $M_{\nu}^{0\nu}$ within the interacting shell model (ISM) [Strasbourg-Madrid (StMa) [10] and Central Michigan University (CMU) [11] groups], interacting boson model (IBM) [12], QRPA [Tuebingen-Bratislava-Caltech (TBC) [13,14] and Jyvaskyla (Jy) [15] groups], projected Hartree-Fock Bogoliubov approach (PHFB) [16], and covariant density functional theory (CDFT) [17] are considered. The Argonne, CD-Bonn, and UCOM two-nucleon short-range correlations (src) are taken into account. The nonquenched value of the weak axial-vector coupling g_A is assumed.

Method	g_A	src	$C_{\nu N}$ (10^{-14} yrs $^{-1}$)											
			^{48}Ca	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{110}Pd	^{116}Cd	^{124}Sn	^{128}Te	^{130}Te	^{136}Xe	^{150}Nd
ISM-StMa	1.25	UCOM	4.38	4.56	17.3						15.1	24.4	17.1	
ISM-CMU	1.27	Argonne	4.12	6.96	26.8						9.38		11.8	10.0
		CD-Bonn	4.98	7.81	30.3						10.8		13.7	11.7
IBM	1.27	Argonne	19.7	13.4	36.7	42.7	73.5	20.5	41.6	23.9	2.56	50.5	35.2	27.0
QRPA-TBC	1.27	Argonne	1.88	16.3	56.7	39.5	120.	41.4	70.7	15.4	3.17	55.8	18.0	
		CD-Bonn	2.24	19.0	66.4	46.8	141.	48.9	81.6	19.9	3.93	70.4	22.9	
QRPA-Jy	1.26	CD-Bonn		16.5	35.6	51.1	61.0	51.6	76.4	64.0	3.59	57.3	31.1	
PHFB	1.25	Argonne				40.5	132.	59.6			2.18	50.4		23.7
		CD-Bonn				44.6	143.	64.7			2.39	55.0		25.6
CDFT	1.25	Argonne	47.3	22.4	74.0	216.	173.		128.	42.3		88.2	68.0	113.

$$m_{\beta\beta} = \left| \sum_{j=1}^3 U_{ej}^2 \frac{m_j}{1 + m_j^2 / \langle p^2 \rangle_a} \right| \simeq \left| \sum_{j=1}^3 U_{ej}^2 m_j \right|, \quad (20)$$

$$M_{\beta\beta}^R = \lambda \left| \sum_{k=1}^3 U_{ek}^2 \frac{M_k}{1 + M_k^2 / \langle p^2 \rangle_a} \right| \simeq \lambda \langle p^2 \rangle_a \left| \sum_{k=1}^3 U_{ek}^2 \frac{1}{M_k} \right|. \quad (21)$$

In this scenario, U_0 can be identified with the PMNS mixing matrix U . Thus, we assume $U_0 = U$. The mixing matrix V_0 for the heavy neutrinos is unknown, but it is similar to U_0 in the light neutrino sector; then, $V_0 = U$ is frequently assumed. For sake of simplicity, we consider two different cases for the heavy neutrino masses:

$$M_i = \begin{cases} m_i / \zeta_r & \text{constant ratios} \\ \zeta_p / m_i & \text{constant products.} \end{cases} \quad (22)$$

In the case of the constant products $\zeta_p = m_i M_i$, we have for the LNV parameter in Eq. (19)

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} \left(1 + \lambda^2 \left(\frac{\langle p^2 \rangle_a}{\zeta_p} \right)^2 \right) m_{\beta\beta}^2 \equiv \kappa^2 m_{\beta\beta}^2. \quad (23)$$

Thus, in this scenario, the presence of heavy neutrinos leads to a vertical shift of the standard plot in Fig. 2 by a constant factor κ . As a result, the $0\nu\beta\beta$ -decay experimental upper bound on $m_{\beta\beta}$ is significantly less stringent, if $\zeta_p \ll \lambda \langle p^2 \rangle_a \simeq 24 \text{ MeV}^2$. In our estimation, we used the upper-bound value in (3), i.e., $\lambda = 7.7 \times 10^{-4}$, and $\sqrt{\langle p^2 \rangle_a} = 175 \text{ MeV}$ calculated within the QRPA by assuming Argonne potential and $g_A = 1.27$ (see Table II).

In the case of the constant ratios $\zeta_r = m_i / M_i$ in Eq. (22), the effective Majorana neutrino mass $M_{\beta\beta}$ is shown in Fig. 3. Contribution of $M_{\beta\beta}$ becomes comparable to $m_{\beta\beta}$ as soon as $\zeta_r = 10^{-17}$, which corresponds to $M_i \sim 10^{16} \text{ eV} = 10^4 \text{ TeV}$, with $\lambda = 7.7 \times 10^{-4}$ being assumed again. Notice the reverse behavior of the mass hierarchies: normal hierarchy no longer exhibits a region unbounded from below, while inverted hierarchy does.

B. Seesaw-mixed light and heavy neutrino sectors

Assuming for simplicity the flavor universal mixing between the active and sterile neutrino sectors, the seesaw mixing matrix \mathcal{U} takes the form

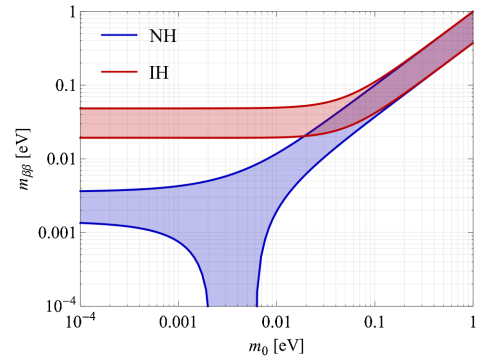


FIG. 2. The effective Majorana neutrino mass $m_{\beta\beta}$ as a function of the lightest neutrino mass for the normal (blue) and inverted (red) hierarchy of neutrino masses. The best-fit values of neutrino oscillation parameters from the global analysis of neutrino oscillation data [20] are considered. NH and IH stand for the normal and inverted hierarchy of neutrino masses, respectively.

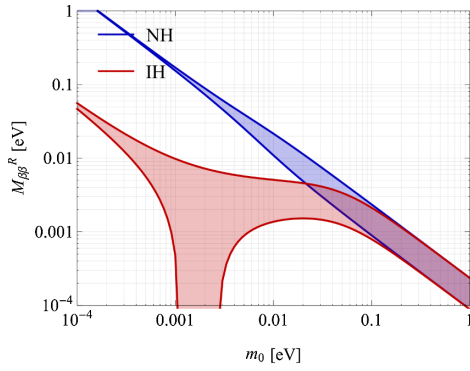


FIG. 3. Scenario IV A with $m_i/M_i = \text{const}$. The effective Majorana neutrino mass $M_{\beta\beta}^R$ as a function of the lightest neutrino mass m_0 for the normal (blue) and inverted (red) hierarchy of neutrino masses. The same notation as in Fig. 2 is used.

$$U = \begin{pmatrix} U_0 & \zeta \mathbf{1} \\ -\zeta \mathbf{1} & V_0 \end{pmatrix}. \quad (24)$$

Here, $\zeta = \frac{m_D}{m_{\text{LNV}}}$, where m_D is the typical scale of the charged leptons masses and m_{LNV} is the LNV scale of the order of the Majorana masses M_i of the heavy neutrinos. As in the previous scenario, U_0 can be identified with the PMNS U matrix. Thus, we assume $U_0 = U$. For V_0 , the analog of U_0 in the heavy neutrino sector, we find from the unitarity conditions

$$V_0 = U^\dagger = \begin{pmatrix} c_{12}c_{13}e^{-i\alpha_1} & (-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta})e^{-i\alpha_1} & (s_{12}s_{23} - c_{12}s_{13}c_{23}e^{-i\delta})e^{-i\alpha_1} \\ s_{12}c_{13}e^{-i\alpha_2} & (c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta})e^{-i\alpha_2} & (-c_{12}s_{23} - s_{12}s_{13}c_{23}e^{-i\delta})e^{-i\alpha_2} \\ s_{13}e^{i\delta} & c_{13}s_{23} & c_{13}c_{23} \end{pmatrix}. \quad (27)$$

We note that each element of the first row is multiplied by the same phase factor $e^{-i\alpha_1}$. Analogously, the second row is multiplied by $e^{-i\alpha_2}$. Therefore, the Majorana phases $\alpha_{1,2}$ do not affect the heavy neutrino LNV parameter $M_{\beta\beta}^R$ in this case. On the contrary, the Dirac phase δ , which does not affect the light neutrino LNV parameter $m_{\beta\beta}$, will impact the value of $M_{\beta\beta}^R$. The seesaw structure of (24) implies $m_i \simeq m_D^2/m_{\text{LNV}}$ and $M_i \simeq m_{\text{LNV}}$. For a product of light and heavy neutrino masses, let us assume $m_i M_i \simeq m_D^2$. If the LNV scale is significantly larger than $\langle p^2 \rangle_a$, we find

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} (m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2) \quad (28)$$

with

$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|. \quad (29)$$

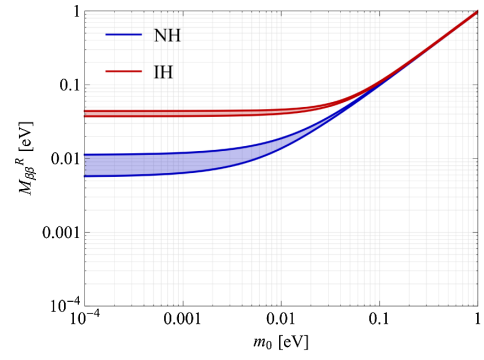


FIG. 4. Scenario IV B. The effective Majorana neutrino mass $M_{\beta\beta}^R$ in Eq. (29) with $m_D = 5$ MeV as a function of the lightest neutrino mass m_0 for the normal (blue) and inverted (red) hierarchy of neutrino masses. The same notation as in Fig. 2 is used.

$$V_0 = U_0^\dagger \quad (25)$$

and

$$\begin{aligned} U_0 U_0^\dagger &= (1 - \zeta^2) \mathbf{1}, \\ V_0 V_0^\dagger &= (1 - \zeta^2) \mathbf{1}. \end{aligned} \quad (26)$$

It is assumed that a small violation of the unitarity of U_0 and V_0 matrices is beyond the current accuracy of phenomenological determination of elements of the PMNS matrix. The matrix V_0 takes the form

We note that for $m_D \simeq 5$ MeV the coefficient $\lambda \langle p^2 \rangle_a / m_D^2$ entering $M_{\beta\beta}^R$ in Eq. (29) is close to unity and it might be that contributions from the light and heavy neutrinos to $\eta_{\nu N}$ are comparable. However, $M_{\beta\beta}^R$ is not proportional to $m_{\beta\beta}$ as off-diagonal elements of matrices U_0 and (U^\dagger) are different. Therefore, a detailed analysis is needed to establish a useful constraint on the Yukawa potential associated with neutrinos. In Fig. 4, we show $M_{\beta\beta}^R$ as function of the lightest neutrino mass both for normal and inverted hierarchy by assuming $m_D \simeq 5$ MeV (and $\lambda = 7.7 \times 10^{-4}$).

Within the seesaw structure, one can also assume $m_i \simeq \zeta^2 M_i$. Then, we find

$$M_{\beta\beta}^R = \lambda \zeta^2 \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 \frac{\langle p^2 \rangle_a}{m_j} \right|. \quad (30)$$

For $\zeta^2 = 10^{-17}$ and $\lambda = 7.7 \times 10^{-4}$, the effective mass $M_{\beta\beta}^R$ in Eq. (30) is plotted in Fig. 5. We see again that for a

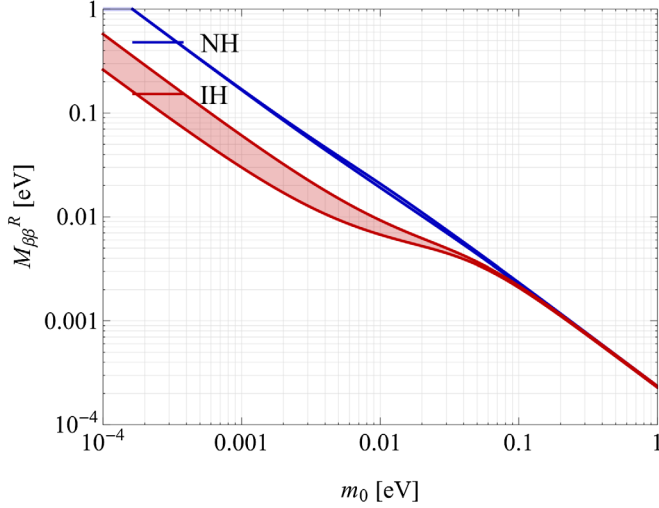


FIG. 5. The same as in Fig. 4, but for $M_{\beta\beta}^R$ defined according to Eq. (30) with $\zeta^2 = 10^{-17}$. The same notation as in Fig. 2 is used.

chosen set of parameters the value of $M_{\beta\beta}^R$ can be comparable with $m_{\beta\beta}$ (see Fig. 2).

In Table III, we show upper bounds on $\eta_{\nu N}$ in Eq. (28) derived from the current $0\nu\beta\beta$ -decay experiments. As seen, the most stringent bound comes out from the ^{136}Xe $0\nu\beta\beta$ -decay experiment, Ref. [21]. For this bound, we analyzed the separate contributions of the light and heavy neutrinos to $0\nu\beta\beta$ decay. Figure 6 displays the corresponding results in the plane of the parameters m_D and m_0 ($m_i \simeq m_D^2/M_i$ is assumed) for the cases of the normal (upper panel) and inverted hierarchy (lower panel) of neutrino masses. We see that in the considered scenario for normal (inverted) hierarchy the values $m_D \leq 1.4$ MeV ($m_D \leq 2.9$ MeV) are already excluded by the existing experimental data on $0\nu\beta\beta$ decay. We also see that in the case of normal (inverted) neutrino mass hierarchy the heavy neutrino exchange mechanism cannot dominate over the light one in the region $m_0 \geq 0.08$ eV ($m_0 \geq 0.065$ eV). The constraint from the $0\nu\beta\beta$ -decay experiment implies that the limit on the mass of lightest heavy neutrino is $M_3 > 38$ TeV and $M_2 > 171$ TeV in the cases of normal and inverted hierarchy, respectively.

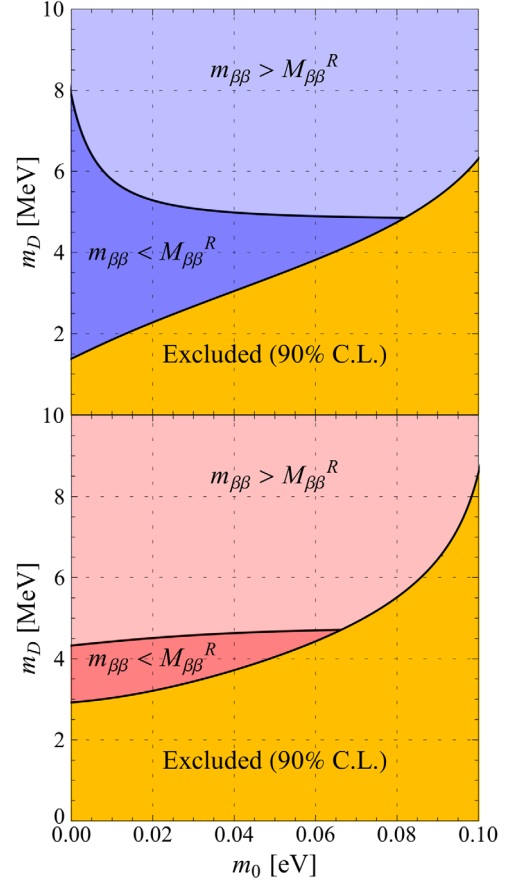


FIG. 6. Scenario IV B with the mass relation $m_i \simeq m_D^2/M_i$. The comparison of the light $m_{\beta\beta}$ and heavy $M_{\beta\beta}^R$ neutrino contributions to $0\nu\beta\beta$ decay for the normal (inverted) hierarchy is shown in the upper (lower) panel.

Figure 7 shows results of an analysis similar to the above-discussed one, but for the $\zeta^2 = m_i/M_i$ scenario. In this case, the heavy neutrino mechanism cannot dominate in practically the same domain of m_0 as previously. It is concluded that in the case of normal (inverted) hierarchy $\zeta \leq 1.75 \times 10^{-8}$ ($\zeta \leq 1.65 \times 10^{-8}$). We note that within the considered seesaw scenario within the LRSM the effective Majorana neutrino mass $m_{\beta\beta}$ cannot be identified with the first element of the Dirac-Majorana mass (see Appendix B) $(M_L)_{ee}$, which contains the additional term $\zeta^2 M_1$ in magnitude

TABLE III. Upper bounds on the effective lepton number-violating parameter $\eta_{\nu N}$ imposed by the current constraints on the $0\nu\beta\beta$ -decay half-life $T_{1/2}^{0\nu\text{-exp}}$ (the first row). The values in the second and the third rows were obtained using the largest and lowest values of $C_{\nu N}$ for a given isotope from Table II, respectively.

	^{48}Ca	^{76}Ge	^{82}Se	^{100}Mo	^{116}Cd	^{130}Te	^{136}Xe
$T_{1/2}^{0\nu\text{-exp}}$ (yrs)	2.0×10^{22} [22]	5.3×10^{25} [23]	2.5×10^{23} [24]	1.1×10^{24} [25]	1.7×10^{23} [26]	4.0×10^{24} [27]	1.07×10^{26} [21]
$\eta_{\nu N} \times 10^6$	10.3	0.290	2.32	0.724	2.14	0.532	0.117
	33.8	0.643	4.81	1.22	3.76	1.455	0.306

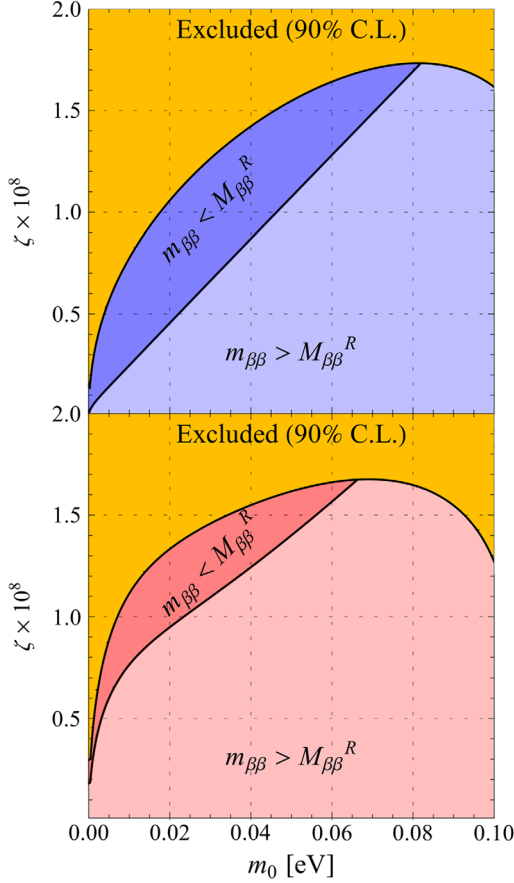


FIG. 7. The same as in Fig. 6, but with the mass relation $\zeta^2 = m_i/M_i$.

comparable with m_1 . The corresponding term in $m_{\beta\beta}$ has been neglected as it is suppressed by properties of the neutrino propagator for large neutrino mass. For the same reason, $M_{\beta\beta}$ cannot be identified with $(M_R)_{ee}$.

V. CONCLUSIONS

In summary, we have shown that the ratio of nuclear matrix elements for the light and heavy neutrino mass mechanisms exhibits practically no dependence on the isotope for all favored nuclear structure methods. This quantity, when properly scaled, can be identified with the squared average neutrino momentum $\langle p^2 \rangle$ of the interpolating formula including light and heavy neutrino exchange mechanisms. The universality of the averaged value of $\langle p^2 \rangle$ for a set of isotopes allows the determination of a new LNV parameter $\eta_{\nu N}$, which is a sum of squared LNV parameters $m_{\beta\beta}$ and $M_{\beta\beta}^R$ characterizing the light and heavy neutrino exchange mechanisms, respectively. Thus, the observation of $0\nu\beta\beta$ decay on two and more nuclear isotopes will allow one to deduce information about the size of $\eta_{\nu N}$ but not about the relative contribution of the light or heavy neutrino-exchange mechanism to the decay rate. An additional theoretical or experimental input about neutrino

masses and mixing is needed to shed light on the particular role of each of these mechanisms. For an example, we considered a simplified seesaw-type 6×6 neutrino mixing matrix (24), which implies that the 3×3 mixing matrix of heavy neutrinos is the Hermitian conjugate of the 3×3 PMNS mixing matrix of light neutrinos. Assuming several viable seesaw relations among the light m_i and heavy M_i neutrino masses ($i = 1, 2$, and 3), useful constraints on the parameters, in particular Dirac neutrino mass m_D , entering these relations have been obtained from the experimental lower bounds on the $0\nu\beta\beta$ -decay half-life. The region of dominance of heavy over light neutrino exchange mechanisms for the considered scenarios has been identified.

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APPENDIX A: ANALYTICAL PROPERTIES OF THE NMES AND THE INTERPOLATING FORMULA

Here, we give some comments on the possible improvement of our interpolating formula in Eq. (13), which we call the “monopole” approximation. Numerically, the latter is already a very good approximation to the exact NMEs given by Eq. (8) and calculated in the framework of any specific nuclear structure approach. However, in certain cases, one may need an approximate formula having not only a good numerical precision but also the analytical properties in the complex plane of m_ν the same or maximally close to the exact NME defined in expression (8).

Obviously, the monopole approximation (13) has two imaginary poles in the complex plane of m_ν , while they are absent in the exact expression (8). Below, we describe a class of approximations with the analytic properties of the exact NME (8).

Let us rewrite Eq. (8) in the form

$$M_{LL,RR}^{0\nu}(m_\nu) = \frac{4\pi}{(2\pi)^3} \int_0^\infty p^2 dp \frac{\varphi(p)}{E_p(E_p + \Delta)}, \quad (\text{A1})$$

where $\Delta = E_n - (E_l - E_f)/2 > 0$, $E_p = \sqrt{p^2 + m_\nu^2}$,

$$\varphi(p) = \int d\mathbf{x} d\mathbf{y} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \varphi(\mathbf{x}, \mathbf{y}), \quad (\text{A2})$$

and

$$\varphi(\mathbf{x}, \mathbf{y}) = \frac{1}{m_p m_e} \frac{4\pi R}{g_A^2} \sum_n \langle 0_F^+ | J_{LR}^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_{\mu LR}^\dagger(\mathbf{y}) | 0_i^+ \rangle. \quad (\text{A3})$$

The function $\varphi(\mathbf{x}, \mathbf{y})$ describes a distribution of currents inside the nucleus. In Eq. (A1), the neutrino mass enters the denominator of the integrand.

Analytic properties of functions defined in terms of a contour integral are fixed by the Landau rules [28,29].

The singular points of the first kind are associated with singular behavior of the integrand at the end points of the integration contour. In the case of Eq. (A1), these singularities could occur provided that $\chi(p) \equiv E_p(E_p + \Delta) = 0$ for $p = 0$ or ∞ . This equation can be fulfilled for $p = 0$ only to give $m = 0$ and $m = \pm\Delta$. The points $m = \pm\Delta$ are located on the different sheets of the Riemann surface of $M_{LL,RR}^{0\nu}(m_\nu)$. It is clear that model-dependent features of the nuclear structure entering $\varphi(\mathbf{x}, \mathbf{y})$ do not affect the end point singularities.

Singular points of the second kind are associated with the pinch singularities of the integrand. To find them, the equations $\chi(p)/\varphi(p) = 0$ and $(\chi(p)/\varphi(p))' = 0$, which localize high-order poles of the integrand in the complex p plane, are to be solved. These singularities depend on $\varphi(\mathbf{x}, \mathbf{y})$ and thereby on the nuclear structure model.

Analytic properties of $M_{LL,RR}^{0\nu}(m_\nu)$ as a function of Δ are particularly simple. Changing the variable in Eq. (A4) to $p = m \sinh \theta$, we arrive at the dispersion integral

$$M_{LL,RR}^{0\nu}(m_\nu) = \frac{4\pi m}{(2\pi)^3} \int_0^\infty \sinh^2 \theta d\theta \frac{\varphi(m \sinh \theta)}{\cosh \theta - \xi}, \quad (\text{A4})$$

where $\xi = -\Delta/m_\nu$. This equation shows that $M_{LL,RR}^{0\nu}$ is an analytic function in the complex ξ plane with the cut $(1, +\infty)$ corresponding to the cut $(-\Delta, 0)$ in m_ν . Provided $\varphi(p)$ is an analytic function for $|p| < \infty$ and the integral (A4) converges, $M_{LL,RR}^{0\nu}(m)$ turns out to be an analytic function in the complex m_ν plane with the cut $(-\Delta, 0)$. On the second sheet of the Riemann surface, one finds a branch point $m = +\Delta$.

As we discussed before, the monopole parametrization (13) is numerically very accurate. This parametrization corresponds to an approximation of the spectral function with the delta function: $\phi_m(p) \sim \delta(p^2 - \langle p^2 \rangle)$. Then, for the formula with the correct analytical properties, which we are going to construct here, we chose the spectral function in a form close to the $\phi_m(p)$ to guarantee its numerical accuracy comparable with the monopole parametrization. We may choose

$$\phi(p) = \exp\left(-\frac{\rho^2 p^2}{2}\right) \frac{\sinh(\rho^2 p p_0)}{\rho^2 p p_0} \quad (\text{A5})$$

with the free parameters, ρ and $p_0 \sim \langle p^2 \rangle^{1/2}$, which can be fixed by normalization to the exact values at zero and at

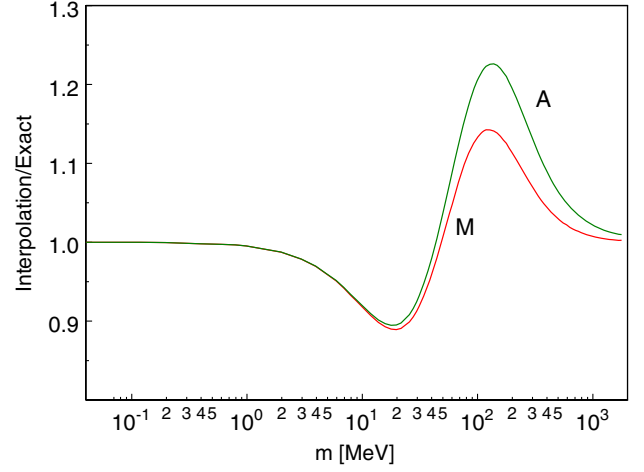


FIG. 8. Ratio between the interpolation formulas and the exact calculation for ^{76}Ge vs neutrino mass. The curve M is for the monopole interpolation (13), and the curve A shows the ratio for the analytic interpolation formula.

infinity. The function $\phi(p)$ for $p = p_0$ is close to the maximum, and the value of $1/\rho$ determines the width of momentum distribution. This spectral function is analytic for $|p| < \infty$, and it generates model-independent end point singularities only. The corresponding interpolating formula appears to be an analytic function in the complex m_ν plane with the cut $(-\Delta, 0)$. The cut position is model independent. The discontinuity depends on $\phi(p)$ and is model dependent. A particularly strong effect on the behavior of analytic functions in a fixed domain comes from nearest singularities. Taking into account that $\Delta \sim 10$ MeV, an improved description of the neutrino mass dependence can be expected around zero neutrino mass in the circle with a radius of a few tens of MeV. This scale is smaller than the characteristic momentum transfer $p_0 \sim 200$ MeV. Reasonable accuracy is also expected for the large m_ν domain, provided the spectral function (A5) approximates closely the monopole spectral function found to be successful phenomenologically.

The ratio between the interpolating formula of Eq. (13) and the exact calculation for ^{76}Ge is shown in Fig. 8. The result is compared to the interpolating formula of the spectral function (A5) with $\rho = 5$ fm and $p_0 = 0.84/\text{fm}$. For low neutrino masses up to about 40 MeV, the analytic interpolation formula approximates the exact result with better accuracy. For higher masses, the nuclear structure at about 200 MeV becomes important, which could reflect a contribution of the model-dependent pinch singularities that we do not consider.

APPENDIX B: DIRAC-MAJORANA NEUTRINO MASS TERM WITHIN SEESAW IN LRSM

In this Appendix, the Dirac-Majorana neutrino mass term associated with the seesaw mass mechanism within the LRSM and particular case of neutrino mixing given in Eq. (24) is presented. We have

$$\begin{aligned}\mathcal{L}_{D+M} &= -\frac{1}{2} \begin{pmatrix} \overline{\nu'_L} & \overline{\nu'_R} \end{pmatrix} \begin{pmatrix} M_L & M_D \\ M_D^\dagger & M_R \end{pmatrix} \begin{pmatrix} \nu'_L \\ \nu'_R \end{pmatrix} + \text{H.c.} \\ &= -\frac{1}{2} \sum_{i=1}^3 (m_i \bar{\nu}_i \nu_i + M_i \bar{N}_i N_i). \end{aligned} \quad (\text{B1})$$

Here, $\nu'_L = (\nu'_{eL}, \nu'_{\mu L}, \nu'_{\tau L})^T$ and $\nu'_R = (\nu'_{eR}, \nu'_{\mu R}, \nu'_{\tau R})^T$ are three-component columns of the active left-handed $\nu'_{\alpha L}$ and sterile right-handed $\nu'_{\alpha R}$ ($\alpha = e, \mu, \tau$) flavor-neutrino fields, respectively. The elements of the 6×6 Dirac-Majorana mass matrix \mathcal{M} are calculated as follows:

$$\begin{aligned}\mathcal{M} &= \begin{pmatrix} M_L & M_D \\ M_D^\dagger & M_R \end{pmatrix} \\ &= \begin{pmatrix} U & \zeta \mathbb{1} \\ -\zeta \mathbb{1} & U^\dagger \end{pmatrix}^* \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} \begin{pmatrix} U & \zeta \mathbb{1} \\ -\zeta \mathbb{1} & U^\dagger \end{pmatrix}^\dagger. \end{aligned} \quad (\text{B2})$$

Here, m and M stand for diagonal 3×3 mass matrices $m = \text{diag}(m_1, m_2, m_3)$ and $M = \text{diag}(M_1, M_2, M_3)$, respectively. By assuming the seesaw relation $m_i \sim \zeta^2 M_i$ ($i = 1, 2, 3$) between light and heavy neutrino masses, the elements of \mathcal{M} expressed in terms of three mixing angles θ_{12} , θ_{13} , and θ_{23} ; three CP phases α_1 , α_2 , and δ ; six neutrino masses; and the seesaw parameter ζ are given by

$$\begin{aligned}(M_L)_{ee} &= \zeta^2 M_1 + c_{12}^2 c_{13}^2 e^{-i2\alpha_1} m_1 \\ &\quad + s_{12}^2 c_{13}^2 e^{-i2\alpha_2} m_2 + s_{13}^2 e^{i2\delta} m_3, \\ (M_L)_{e\mu} &= -c_{12} c_{13} (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{-i\delta}) e^{-i2\alpha_1} m_1 \\ &\quad + s_{12} c_{13} (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta}) e^{-i2\alpha_2} m_2 \\ &\quad + s_{13} c_{13} s_{23} e^{i\delta} m_3, \\ (M_L)_{e\tau} &= c_{12} c_{13} (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta}) e^{-i2\alpha_1} m_1 \\ &\quad - s_{12} c_{13} (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{-i\delta}) e^{-i2\alpha_2} m_2 \\ &\quad + s_{13} c_{13} c_{23} e^{i\delta} m_3, \\ (M_L)_{\mu\mu} &= (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{-i\delta})^2 e^{-i2\alpha_1} m_1 \\ &\quad + (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta})^2 e^{-i2\alpha_2} m_2 \\ &\quad + c_{13}^2 s_{23}^2 m_3 + \zeta^2 M_2, \\ (M_L)_{\mu\tau} &= (-s_{12} s_{23} + c_{12} s_{13} c_{23} e^{-i\delta}) \\ &\quad \times (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{-i\delta}) e^{-i2\alpha_1} m_1 \\ &\quad - (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{-i\delta}) \\ &\quad \times (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta}) e^{-i2\alpha_2} m_2 \\ &\quad + c_{13}^2 s_{23} c_{23} m_3, \\ (M_L)_{\tau\tau} &= (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta})^2 e^{-i2\alpha_1} m_1 \\ &\quad + (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{-i\delta})^2 e^{-i2\alpha_2} m_2 \\ &\quad + c_{13}^2 c_{23}^2 m_3 + \zeta^2 M_3, \end{aligned} \quad (\text{B3})$$

$$\begin{aligned}(M_D)_{ee} &= \zeta [-c_{12} c_{13} e^{-i\alpha_1} m_1 + c_{12} c_{13} e^{i\alpha_1} M_1], \\ (M_D)_{e\mu} &= \zeta [-s_{12} c_{13} e^{-i\alpha_2} m_2 + s_{12} c_{13} e^{i\alpha_2} M_1], \\ (M_D)_{e\tau} &= \zeta [-s_{13} e^{i\delta} m_3 + s_{13} e^{-i\delta} M_1], \\ (M_D)_{\mu e} &= \zeta [(s_{12} c_{23} + c_{12} s_{13} s_{23} e^{-i\delta}) e^{-i\alpha_1} m_1 \\ &\quad - (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta}) e^{i\alpha_1} M_2], \\ (M_D)_{\mu\mu} &= \zeta [-(c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta}) e^{-i\alpha_2} m_2 \\ &\quad + (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}) e^{i\alpha_2} M_2], \\ (M_D)_{\mu\tau} &= \zeta [-c_{13} s_{23} m_3 + c_{13} s_{23} M_2], \\ (M_D)_{\tau e} &= \zeta [-(s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta}) e^{-i\alpha_1} m_1 \\ &\quad + (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta}) e^{i\alpha_1} M_3], \\ (M_D)_{\tau\mu} &= \zeta [(c_{12} s_{23} + s_{12} s_{13} c_{23} e^{-i\delta}) e^{-i\alpha_2} m_2 \\ &\quad - (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta}) e^{i\alpha_2} M_3], \\ (M_D)_{\tau\tau} &= \zeta [-c_{13} c_{23} m_3 + c_{13} c_{23} M_3], \end{aligned} \quad (\text{B4})$$

$$\begin{aligned}(M_R)_{ee} &= \zeta^2 m_1 + c_{12}^2 c_{13}^2 e^{i2\alpha_1} M_1 \\ &\quad + (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta})^2 e^{i2\alpha_1} M_2 \\ &\quad + (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta})^2 e^{i2\alpha_1} M_3, \\ (M_R)_{e\mu} &= s_{12} c_{12} c_{13}^2 e^{i(\alpha_1+\alpha_2)} M_1 \\ &\quad - (s_{12} c_{23} + c_{12} s_{13} s_{23} e^{i\delta}) \\ &\quad \times (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta}) e^{i(\alpha_1+\alpha_2)} M_2 \\ &\quad - (s_{12} s_{23} - c_{12} s_{13} c_{23} e^{i\delta}) \\ &\quad \times (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta}) e^{i(\alpha_1+\alpha_2)} M_3, \\ (M_R)_{e\tau} &= c_{12} s_{13} c_{13} e^{-i\delta} e^{i\alpha_1} M_1 \\ &\quad - c_{13} (s_{12} s_{23} c_{23} + c_{12} s_{13} s_{23}^2 e^{i\delta}) e^{i\alpha_1} M_2 \\ &\quad + c_{13} (s_{12} s_{23} c_{23} - c_{12} s_{13} c_{23}^2 e^{i\delta}) e^{i\alpha_1} M_3, \\ (M_R)_{\mu\mu} &= \zeta^2 m_2 + s_{12}^2 c_{13}^2 e^{i2\alpha_2} M_1 \\ &\quad + (c_{12} c_{23} - s_{12} s_{13} s_{23} e^{i\delta})^2 e^{i2\alpha_2} M_2 \\ &\quad + (c_{12} s_{23} + s_{12} s_{13} c_{23} e^{i\delta})^2 e^{i2\alpha_2} M_3, \\ (M_R)_{\mu\tau} &= s_{12} s_{13} c_{13} e^{-i\delta} e^{i\alpha_2} M_1 \\ &\quad + c_{13} (c_{12} s_{23} c_{23} - s_{12} s_{13} s_{23}^2 e^{i\delta}) e^{i\alpha_2} M_2 \\ &\quad - c_{13} (c_{12} s_{23} c_{23} + s_{12} s_{13} c_{23}^2 e^{i\delta}) e^{i\alpha_2} M_3, \\ (M_R)_{\tau\tau} &= s_{13}^2 e^{-i2\delta} M_1 + c_{13}^2 s_{23}^2 M_2 \\ &\quad + c_{13}^2 c_{23}^2 M_3 + \zeta^2 m_3. \end{aligned} \quad (\text{B5})$$

We note that, due to the seesaw relation $m_i \sim \zeta^2 M_i$, terms ζm_i and $\zeta^2 m_i$ entering elements of matrices M_D and M_R , respectively, can be safely neglected, unlike terms $\zeta^2 M_i$ appearing in the diagonal elements of the M_L matrix.

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