Three-body $J^P = 0^+, 1^+, 2^+$ $B^*B^*\bar{K}$ bound states

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Three-body systems with short-range interactions display universal features that have been extensively explored in atomic physics, but apply to hadron physics as well. Systems composed of two noninteracting identical particles (species H) of mass M and a third particle (species P) of mass m that interacts attractively with the other two have the property that they are more likely to bind for larger values of the mass ratio M/m. This is particularly striking if the HHP system is in P-wave (while the interacting pair is in S-wave), in which case one would not normally expect the formation of a three-body state. If we assume that the $B^*\bar{K}$ binds to form the B^*_{s1} heavy meson and notice that the mass ratio of the B^* to $\bar{K}$ is $M/m = 10.8$, concrete calculations indicate that there should be a three-body $B^*B^*\bar{K}$ bound state between 30–40 MeV below the $B^*_{s1}B^*$ threshold. For the $\Xi_{bb}\Xi_{bb}\bar{K}$ system the mass imbalance is about $M/m = 20.5$ and two bound states are expected to appear, a fundamental and an excited one located at 50–90 and 5–15 MeV below the $\Xi_{bb}\Omega^*_{bb\frac{1}{2}}$ threshold (where $\Omega^*_{bb\frac{1}{2}}$ denotes the $\Xi_{bb}\bar{K}$ bound state). We indicate the possibility of analogous P-wave three-body bound states composed of two heavy baryons and a kaon or antikaon and investigate the conditions under which the Efimov effect could appear in these systems.

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I. INTRODUCTION

The three boson system in the unitary limit shows a geometric spectrum of shallow bound states, the Efimov effect [1]. In this limit, there is a geometric tower of three-body bound states for which the ratio of the binding energies of the nth and $(n+1)$-th excited state is given by $E_n/E_{n+1} \simeq 512$, a prediction that has been experimentally confirmed with cesium atoms [2]. The existence of a geometric spectrum extends to other three-body systems where only two of the three particles interact resonantly [1,3,4]. Recently it has been found that a similar geometric spectrum might also arise in specific two-body hadronic systems, for instance $\Sigma_c\bar{D}^* - \Lambda_{c1}\bar{D}$ and $\Sigma_c\Xi'_b - \Lambda_{c1}\Xi_b$ [5]. Particularly interesting are three-body systems with a mass imbalance in which we have two-identical particles of the species H with mass M and a third particle of the species P with mass m. When the HH subsystem is noninteracting and the HP subsystem is resonant, the three-body system will eventually display a geometrical Efimov-like spectrum if the ratio M/m is big enough [3,4], as observed in experiments with lithium and cesium atoms [6]. This is not such a surprise if the system is in S-wave, where there will always be a geometric spectrum.[1] But the cases in which the system is in P-wave or higher is much more interesting, as they are less trivial. For P-wave this happens for $M/m \geq 13.6$ while for D-wave the threshold is $M/m \geq 38.6$ [7]. Kartavtsev and Malykh also made the remarkable discovery [8] that in the P-wave case there is a universal three-body state for $M/m \geq 8.176$ and a second one for $M/m \geq 12.917$. By universal it is meant that the binding energies of these three-body bound states depend only on the two-body binding energy.

The bottom-line is that three-body systems with large mass imbalances are more likely to bind. This is particularly interesting in view of the recent renaissance of heavy hadron spectroscopy triggered by the discovery of the $X(3872)$ [9] (which has been theorized to be a shallow two-body bound state [10–12]). The DK and D^*K systems display a strong s-wave attraction that generates a bound state at about 45 MeV below threshold [13–17]. These bounds states are suspected to be the $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ charmed mesons, partly because the DK and D^*K bind at the right location partly because of other

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[1] In the absence of spin, isospin, or other quantum numbers that might translate into numerical factors diminishing attraction.

reasons, like the fact that the masses of the D_{s0}^*/D_{s1}^* are similar to (instead of markedly heavier than) those of the D_0/D_1 charmed mesons or the analysis of the D_{s0}^*/D_{s1}^* wave function from lattice data [18,19]. Owing to heavy flavor symmetry this idea extends to the $B\bar{K}$ and $B^*\bar{K}$ cases, which form the B_{s0}^* and B_{s1}^* mesons. Last, if we consider heavy antiquark-diquark symmetry (HADS) [20–22] then a new set of bound states involving $\Xi_{cc}\bar{K}$, $\Xi_{cc}^*\bar{K}$, $\Xi_{bb}\bar{K}$ and $\Xi_{bb}^*\bar{K}$ should appear, which we will call the $\Omega_{cc\frac{1}{2}}^*$, $\Omega_{cc\frac{3}{2}}^*$, $\Omega_{bb\frac{1}{2}}^*$ and $\Omega_{bb\frac{3}{2}}^*$ in analogy with the D_{s0}^*, D_{s1}^* notation. Owing to the slightly larger reduced masses the binding energies are also a bit bigger than in the DK and D^*K cases, of the order of 60–70 MeV [23]. In particular, if we consider the $B^*B^*\bar{K}$ and $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi_{bb}^*\Xi_{bb}^*\bar{K}$, the masses' imbalances are remarkable, 10.8 and 20.5, respectively (where for the mass of the doubly bottom baryons we have used the lattice QCD determination of Ref. [24]). This points out to the possibility of P-wave three-body bound states. Concrete calculations show that this is, indeed, the case for the bottom hadrons, with $B^*B^*\bar{K}$ binding about 30–40 MeV below the $B_{s1}^*B^*$ threshold, where it is interesting to notice that this state can also be predicted in a two-body description involving the $B_{s1}^*B^*$ mesons interacting by means of a one antikaon exchange potential [25]. For the $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi_{bb}^*\Xi_{bb}^*\bar{K}$ two bound states appear, a shallow one with a binding of 5–15 MeV below the $\Omega_{bb\frac{1}{2}}^*\Xi_{bb}/\Omega_{bb\frac{3}{2}}^*\Xi_{bb}^*$ threshold and a second one at 50–90 MeV. Meanwhile the charmed mesons and doubly charmed baryons are unlikely to bind in P-wave as a consequence of the insufficient mass imbalance. Yet they will likely bind in S-wave [25], as happen in other S-wave, mass-imbalanced three hadron systems like the $\rho D^*\bar{D}^*$ [26], $\rho B^*\bar{B}^*$ [27] and $KD^*\bar{D}^*$ [28] systems.

This idea also applies to other P-wave HHP hadron systems with large masses imbalances. If we consider the H hadron to be a bottom baryon and the P hadron to be a kaon or antikaon, the HP interaction is of a Weinberg-Tomozawa type and in a few cases might be strong enough as to bind the HP subsystem [29]. If this is the case, this will likely imply the existence of HHP bound states. At this point the natural question arises of whether the P-wave Efimov effect will be present in these systems if the HP interaction is resonant. The answer is negative for two bottom baryon plus a kaon/antikaon because the mass imbalance is not large enough. However, from HADS [20], we expect the existence of doubly heavy tetraquark partners of the heavy baryons. If these doubly heavy tetraquarks are stable they will be the perfect candidates. In this regard, we notice that the recent discovery of a doubly charmed baryon by the LHCb [30] strongly points towards the stability of doubly heavy tetraquarks in the bottom sector [31,32].

The manuscript is structured as follows: after the introduction, we explain the Faddeev equations for the $B^*B^*\bar{K}$ system in Sec. II. We discuss the conditions for the appearance of the P-wave Efimov effect in Sec. III. Then we show the predictions for $B^*B^*\bar{K}$ and $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi_{bb}^*\Xi_{bb}^*\bar{K}$ P-wave three-body states in Sec. IV. Finally, we present our conclusions at the end.

II. FADDEEV EQUATIONS FOR THE HHP SYSTEM IN P-WAVE

Here we present the Faddeev equations for solving the HHP bound state problem for the P-wave case. This is done for the particular case of contact interactions. If the HP system is $B^*\bar{K}$, $\Xi_{bb}\bar{K}$ or $\Xi_{bb}^*\bar{K}$, the binding momentum of the antikaon lies on the vicinity of 200 MeV. This is comparable with the mass of the antikaon, $m_K = 495$ MeV, which means that relativistic kinematics might have a moderate impact on the calculations. For this reason we will present first the standard nonrelativistic Faddeev equations and then we will explain how to include corrections coming from the relativistic antikaon kinematics. Concrete calculations show that though relativistic corrections are not negligible, they are not required at the level of accuracy at which the HHP bound states can be computed now.

A. The Equations

We begin with the Faddeev decomposition of the HHP wave functions

$$\Psi_{3B} = [\phi(\vec{k}_{23}, \vec{p}_1) - \phi(\vec{k}_{31}, \vec{p}_2)] \left| 1 \otimes \frac{1}{2} \right\rangle_{1/2}, \qquad (1)$$

with particles 1, 2 and 3 corresponding to species H, H and P (particles 1 and 2 are identical). This decomposition indicates that the HH subsystem is antisymmetric in the spatial coordinates, which implies it has odd orbital angular momentum $L_{12} = 1, 3, 5, \ldots$ with the $L_{12} = 1$ component dominant at low momenta. It also assumes that there is no interaction in the HH subsystem, a hypothesis that we will review in a few lines. The Jacobi momenta $\vec{k}_{ij}$ and $\vec{p}_k$ are defined as usual,

$$\vec{k}_{ij} = \frac{m_j\vec{k}_i - m_i\vec{k}_j}{m_i + m_j}, \qquad (2)$$

$$\vec{p}_k = \frac{1}{M_T}[(m_i + m_j)\vec{k}_k - m_k(\vec{k}_i + \vec{k}_j)], \qquad (3)$$

with m_1, m_2, m_3 the masses of particles 1, 2, 3 (we take $m_1 = m_2 = M$ and $m_3 = m$), $M_T = m_1 + m_2 + m_3$ the total mass and ijk an even permutation of 123. The ket refers to the isospin wave function of the system in the notation

$$|I_{12} \otimes I_3\rangle_{I_T}, \qquad (4)$$

where I_{12} is the isospin of particles 1 and 2, I_3 the isospin of particle 3 and I_T the total isospin. The choice $I_{12} = 1$, $I_T = \frac{1}{2}$ is the combination with the biggest overlap into the $I = 0$ channel of the HP subsystem, where the B_{s1}^* bound state is expected to happen. The spin wave function is not explicitly indicated: the B^*'s are bosons, their isospin wave function is symmetric and the spatial wave function is antisymmetric, from which we deduce that $S_{12} = 1$. The coupling of the spin of the B^* mesons with their orbital angular momentum $L_{12} = 1$ leads to the conclusion that the quantum numbers of the three-body bound states are $J^P = 0^+$, 1^+ and 2^+. The same logic applies if we consider the Ξ_{bb} and Ξ_{bb}^* baryons, though in this case we have spin $\frac{1}{2}$ and $\frac{3}{2}$ fermions: we end up with $S_{12} = 1$ or $S_{12} = 1, 3$ for the spin wave function, where the quantum numbers of the states are $J^P = 0^+$, 1^+ and 2^+ for $\Xi_{bb}\Xi_{bb}\bar{K}$ and $J^P = 0^+$, 1^+, 2^+, 3^+ and 4^+ for $\Xi_{bb}^*\Xi_{bb}^*\bar{K}$.

The interaction in the HH subsystem is not zero but it is expected to be small. The orbital angular momentum is $L_{12} \geq 1$, which effectively suppresses the short-range components of the interaction (rho- and omega-exchange, for instance). If we consider the long-range interaction instead, which is given by the one pion exchange (OPE) potential, we notice that the isospin of the HH subsystem is $I_{12} = 1$. The OPE potential is known to be weak in the isovector configurations of the B^*B^* and $B^*\bar{B}^*$ systems, as already pointed out in the seminal work of Törnqvist [33]. For the $\Xi_{bb}\Xi_{bb}$ system the strength of OPE is $1/9$ of that of the B^*B^* system, a result which can derived from HADS (see Ref. [34] for instance). That is, OPE is suppressed for the HH configurations we are considering here. This in turn implies that the HH interaction is likely to be a perturbative effect that we can neglect owing to the exploratory character of the current calculations.

The HP interaction is of a short-range type. We can write it as

$$V_{23} = C g(k) g(k'), \tag{5}$$

where k, k' are the initial and final relative momenta of particles 2 and 3, while $g(k)$ is the regulator function we are using. From this potential the T-matrix is given by the ansatz

$$T_{23}(Z) = \tau_{23}(Z) g(k) g(k'), \tag{6}$$

where Z refers to the energy. The coupling C is determined from the condition that $\tau_{23}(Z)$ has a pole at the location of the B_{s1}^* strange-bottom meson. For the Faddeev component of the wave function there is the well-known ansatz

$$\phi(\vec{k}, \vec{p}) = \frac{g(k)}{Z - \frac{k^2}{2\mu_{23}} - \frac{p^2}{2\mu_1}} a_1(p) Y_{1m}(\hat{p}), \tag{7}$$

where Y_{1m} is a spherical harmonic and μ_{ij} and μ_k are reduced masses defined as

$$\frac{1}{\mu_{ij}} = \frac{1}{m_i} + \frac{1}{m_j}, \tag{8}$$

$$\frac{1}{\mu_k} = \frac{1}{m_k} + \frac{1}{m_i + m_j}. \tag{9}$$

The wave function is fully determined by $a_1(p)$, for which the Faddeev equations can be reduced to[2]

$$a_1(p_1) = -\frac{3}{4} \tau_{23}(Z_{23}) \int \frac{d^3\vec{p}_2}{(2\pi)^3} B_{12}^1(\vec{p}_1, \vec{p}_2) a_1(p_2), \tag{10}$$

where $Z_{23} = Z - \frac{p_1^2}{2m_1} \frac{M_T}{m_2 + m_3}$ and B_{12}^1 is given by

$$B_{12}^1(\vec{p}_1, \vec{p}_2) = \frac{g(q_1) g(q_2)}{Z - \frac{p_1^2}{2m_1} - \frac{p_2^2}{2m_2} - \frac{p_3^2}{2m_3}} P_1(\hat{p}_1 \cdot \hat{p}_2), \tag{11}$$

where $P_1(x)$ is a Legendre polynomial. We have that $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$ and

$$\vec{q}_i = \frac{m_j \vec{p}_k - m_k \vec{p}_j}{m_j + m_k}, \tag{12}$$

with ijk an even permutation of 123. Once we have all the pieces we can solve the eigenvalue equation by discretization and obtain the energy of the bound states.

B. Inclusion of relativistic effects

Previously we have considered the kaons to behave nonrelativistically. The binding momentum of the typical HP bound states is close to 200 MeV. This indicates that relativistic corrections to the kaon kinematics might have a moderate impact on the three-body binding. The derivation of relativistic Faddeev equations for systems with contact-range interactions is not unique, a situation which is analogous to what happens in the two-body system [35–39]. Here we choose to follow the prescription of Garcilazo and Mathelisch [40,41], which reproduces the Kadyshevsky equation [35] for the two-body sector. We adapt this prescription to the problem at hand, where the only nonrelativistic particle is the kaon and the mass of the heavy hadrons is considerably larger than the kaon energy. This amounts to the following change in the two-body propagator for the calculation of the two-body T-matrix,

[2] Notice that the integral equation for $a_1(p_1)$ does not include the integration on the $\hat{p}_1$ angular variable. The reason is that it is not required: the integrand on the right hand side of Eq. (10) depends only on the angle between $\vec{p}_1$ and $\vec{p}_2$, which is already taken care of by the integration on the $\hat{p}_2$ angular variable.

$$\frac{1}{Z - \frac{p_2^2}{2m_2} - \frac{p_3^2}{2m_3}} \to \frac{m_3}{\omega_3(p_3)} \frac{1}{Z - \frac{p_2^2}{2m_2} - \epsilon_3(p_3)}, \tag{13}$$

plus the analogous modification for B_{12}^1,

$$B_{12}^1(\vec{p}_1, \vec{p}_2) \to \frac{m_3}{\omega_3(p_3)} \frac{g(q_1)g(q_2)P_1(\hat{p}_1 \cdot \hat{p}_2)}{Z - \frac{p_1^2}{2m_1} - \frac{p_2^2}{2m_2} - \epsilon_3(p_3)}, \tag{14}$$

with $\omega_3(q) = \sqrt{m_3^2 + q^2}$, where $m_3 = m_K$ is the kaon energy and $\epsilon_3(q) = \omega_3(q) - m_3$. The advantage of this prescription is that the changes are easy to implement from the computational point of view.

Besides kinematics, another relativistic effect is that the HP interaction is of a Weinberg-Tomozawa type, which is not momentum independent as previously assumed. The correct momentum dependence is indeed

$$V_{23} = C\left[\frac{\omega_3(k) + \omega_3(k')}{2m_3}\right] g(k)g(k'). \tag{15}$$

This potential can also be rewritten as

$$V_{23} = C[1 + f(k) + f(k')]g(k)g(k'), \tag{16}$$

where $f(k) = (\omega_3(k) - m_3)/2m_3$. The T-matrix for this potential admits a well-known ansatz

$$T_{23}(Z) = g(k)g(k')[\tau_{23}^A(Z) + \tau_{23}^B(Z)(f(k) + f(k')) + \tau_{23}^C(Z)f(k)f(k')], \tag{17}$$

plus the following ansatz for the Faddeev component,

$$\phi(\vec{k}, \vec{p}) = \frac{g(k)[a_1(p) + b_1(p)f(k)]}{Z - \frac{p_1^2}{2m_1} - \frac{p_2^2}{2m_2} - \epsilon(p_3)} Y_{1m}(\hat{p}), \tag{18}$$

where $p_1 = p$, $p_2 = k - m_2 p/(m_2 + m_3)$ and $p_3 = -k - m_3 p/(m_2 + m_3)$. This leads to a different set of Faddeev equations:

$$a_1(p_1) = -\frac{3}{4}\int \frac{d^3\vec{p}_2}{(2\pi)^3}[\tau_{23}^A + \tau_{23}^B f(q_1)]B_{12}^1(\vec{p}_1, \vec{p}_2) \times (a_1(p_2) + b_1(p_2)f(q_2)), \tag{19}$$

$$b_1(p_1) = -\frac{3}{4}\int \frac{d^3\vec{p}_2}{(2\pi)^3}[\tau_{23}^B + \tau_{23}^C f(q_1)]B_{12}^1(\vec{p}_1, \vec{p}_2) \times (a_1(p_2) + b_1(p_2)f(q_2)), \tag{20}$$

where the $\tau_{23}^{(A,B,C)}$ components of the T-matrix are evaluated at $Z = Z_{23}$.

III. THE EFIMOV EFFECT IN THE HHP SYSTEM

Now we consider the Faddeev equations in the unitary limit, i.e., when the binding energy of the HP state approaches zero. For $Z \to 0$ and momenta p_1, p_2 well below the cutoff we have the simplifications

$$\tau_{23}(Z_{23}) \to -\frac{2\pi}{\mu_{23}}\sqrt{\frac{\mu_{23}}{\mu_1}}\frac{1}{p_1}, \tag{21}$$

$$\int \frac{d^2\hat{p}_2}{4\pi} B_{12}^1 \to +\frac{m}{p_1 p_2} Q_1\left(\frac{M+m}{2M}\frac{p_1^2 + p_2^2}{p_1 p_2}\right), \tag{22}$$

where $Q_1(z)$ is a Legendre function of the second kind,

$$Q_1(z) = \frac{z}{2}\log\frac{z+1}{z-1} - 1. \tag{23}$$

If we ignore the purely polynomial terms in p_1 and p_2, we end up with the equation

$$p_1^3 a(p_1) = \frac{3}{4}\frac{1}{\pi}\sqrt{\frac{\mu_1}{\mu_{23}}}\left(\frac{M+m}{2M}\right)^2 \int_0^\infty dp_2 a(p_2) \times (p_1^2 + p_2^2)\log\left(\frac{p_1^2 + p_2^2 + \frac{2M}{M+m}p_1 p_2}{p_1^2 + p_2^2 - \frac{2M}{M+m}p_1 p_2}\right). \tag{24}$$

If we assume a solution of the type $a(p) = b(p)/p^3$ with $b(p) = p^s$, we find the eigenvalue equation

$$1 = \frac{3}{4}\frac{1}{\pi}\sqrt{\frac{\mu_1}{\mu_{23}}}\left(\frac{M+m}{2M}\right)^2 \times \int_0^\infty dx x^{s-3}(1 + x^2)\log\left(\frac{1 + x^2 + \frac{2M}{M+m}x}{1 + x^2 - \frac{2M}{M+m}x}\right) = \frac{3}{4}I_E^1(s). \tag{25}$$

The integral $I_E^1(s)$ is analytically solvable [7],

$$I_E^1(s) = \frac{1}{2\sin^2\alpha\cos\alpha}\left[\frac{1}{is-1}\frac{\sin[(is-1)\alpha]}{\cos[(is-1)\frac{\pi}{2}]} + \frac{1}{is+1}\frac{\sin[(is+1)\alpha]}{\cos[(is+1)\frac{\pi}{2}]}\right], \tag{26}$$

where α is

$$\alpha = \text{asin}\left(\frac{1}{1+\delta}\right), \tag{27}$$

with $\delta = m/M$ the inverse of the mass imbalance. For $M/m \geq 20.587$ the eigenvalue equation admits complex solutions of the type $s = \pm i s_1$, indicating the existence of an Efimov geometric spectrum.

If we consider the $B\bar{K}$ and $\Xi_{bb}\bar{K}$ cases, the existence of a geometrical spectrum is a theoretical possibility rather than a practical one: these systems are too tightly bound to show this type of universality. Yet there are many hadrons in with the Weinberg-Tomozawa interaction with a kaon or antikaon might result in a bound state [42–46]. The isospin structure can be different, leading to the eigenvalue equation

$$1 = c_I I_E^1(s), \tag{28}$$

where c_I is an isospin factor that depends on the particular case under consideration. A few examples with a strongly attractive Weinberg-Tomozawa term include the $\Xi'_Q K$, $\Omega_Q K$ and $\Sigma_Q \bar{K}$ [29]. The isospin factors and masses imbalances required for the P-wave Efimov effect are listed in Table I, where the relative strength of the Weinberg-Tomozawa term has also been included. For the $\Xi'_b\Xi'_b K$ system the isospin factor is identical to that of BBK and $\Xi_{bb}\Xi_{bb}\bar{K}$, i.e., $c_I = \frac{3}{4}$. For the $\Omega_Q\Omega_Q K$ system the isospin factor is $c_I = 1$ and the mass imbalance required for a geometrical spectrum is the standard 13.6. This is to be compared with a mass imbalance of 12.1 for the $\Omega_b K$ case. For the $\Sigma_Q\Sigma_Q\bar{K}$ system the isospin factors are $\frac{2}{3}$ and $\frac{5}{6}$ for total isospin $I = \frac{1}{2}$ and $I = \frac{3}{2}$, respectively, which require mass imbalances of 24.5 and 17.7. This limit is, however, not reached for $Q = b$, in which case the mass imbalance is 11.7 for both the Σ_b and Σ_b^*. Notice that the previous HP molecules are expected to have a finite width: the $\Xi'_Q K$ system can decay into $\Sigma_Q\pi$, while the $\Sigma_Q\bar{K}$ and $\Omega_Q K$ can both decay into $\Xi'_Q\pi$. Their corresponding HHP bound states will also have a finite width.

TABLE I. P-wave HHP three-body systems with large mass imbalances where the Weinberg-Tomozawa interaction of the HP subsystem might be able to produce binding. The relative strength of the Weinberg-Tomozawa is denoted by $C_{\rm WT}$. This leads to the coupling $C = C_{\rm WT}/2f_\pi^2$ in the potentials of Eqs. (5) and (15). The approximate binding energy—if known—of the HP system is shown in the column $B_{\rm HP}$: the $N\bar{K}$ value is taken from Ref. [42], the $B^*\bar{K}$ from Ref. [23] and the $\Xi_{bb}\bar{K}$ value is deduced from HADS. For the masses of the experimentally observed heavy hadrons we use the isospin average of the values listed in the PDG [47], i.e., $M(B^*) = 5325$ MeV, $M(\Xi'_b) = 5935$ MeV, $M(\Sigma_b) = 5813$ MeV and $M(\Omega_b) = 6046$ MeV. For the mass of the doubly bottom baryon Ξ_{bb} we use the central value of the lattice calculation of Ref. [24], i.e., $M(\Xi_{bb}) = 10127$ MeV. We consider the HHP and HP systems to have isospin I_T and I_{12}, resulting in the isospin factor c_I. If the HP system happens to bind near the threshold and the mass imbalance (M/m) of the H hadron and the P pseudo Nambu-Goldstone boson is larger than the critical value $(M/m)_{\rm crit}$, the HHP system might display the P-wave Efimov effect. Even though for the heavy hadrons listed above the mass imbalance does not reach the critical value, it is probable for these systems to have three-body bound states as the ones we have computed for the B^*B^*K and $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi^*_{bb}\Xi^*_{bb}\bar{K}$ systems. From HADS, we expect the Ξ'_Q, Σ_Q and Ω_Q heavy baryons to have doubly heavy tetraquark partners T_{QQ}, leading to mass imbalances twice as big as the ones listed in this table.

HHP	$C_{\rm WT}$	$B_{\rm HP}$ (MeV)	I_T	I_{12}	c_I	(M/m)	$(M/m)_{\rm crit}$
$NN\bar{K}$	−3	8	$\frac{1}{2}$	1	$\frac{3}{4}$	1.9	20.6
B^*B^*K	−2	60–70	$\frac{1}{2}$	1	$\frac{3}{4}$	10.8	20.6
$\Xi_{bb}\Xi_{bb}\bar{K}$	−2	60–70	$\frac{1}{2}$	1	$\frac{3}{4}$	20.5	20.6
$\Xi'_b\Xi'_b K$	−2	N/A	$\frac{1}{2}$	1	$\frac{3}{4}$	12.0	20.6
$\Sigma_b\Sigma_b\bar{K}$	−3	N/A	$\frac{1}{2}$	1	$\frac{2}{3}$	11.7	24.5
			$\frac{3}{2}$	2	$\frac{5}{6}$	11.7	17.7
$\Omega_b\Omega_b K$	−2	N/A	$\frac{1}{2}$	0	1	12.1	13.6

From HADS [20], we naively expect the existence of doubly heavy tetraquark T_{QQ} partners of the $T_Q = \Lambda_Q/\Xi_Q$ and $S_Q = \Sigma_Q/\Xi'_Q/\Omega_Q$ heavy baryons. The Weinberg-Tomozawa interactions for the doubly heavy tetraquarks will be identical to those of the heavy baryons, but their mass imbalances will be about twice as high as the ones listed in Table I. This means that the $T_{QQ}T_{QQ}P$ system is a possible candidate for the P-wave Efimov effect in hadronic physics. However, the existence of strongly and electromagnetically stable tetraquarks is not guaranteed, as it depends on their locations being below the relevant open charm/bottom thresholds [48]. In this regard, it has been pointed out that the actual location of the doubly charmed Ξ_{cc}^{++} baryon, recently observed by the LHCb Collaboration [30], suggests the stability of the doubly bottomed tetraquarks [31,32].

IV. THREE-BODY $B^*B^*\bar{K}$ STATES

Now we calculate the location of the HHP bound states for $H = B/\Xi_{bb}/\Xi^*_{bb}$ and $P = \bar{K}$. For that we need to know the location of the HP bound states, which is not available experimentally except for the DK and D^*K cases (unfortunately these two systems do not have a large enough mass imbalance to form a P-wave bound state). From heavy flavor symmetry, we expect, however, the $B\bar{K}$ and $B^*\bar{K}$ potential to be identical to that of the DK and D^*K. The same is true for $\Xi_{bb}\bar{K}$ if we consider HADS. Besides, the strength of the Weinberg-Tomozawa terms should also be identical in all these HP systems, thus cementing the previous conclusions obtained from heavy quark symmetry. The only difference with the DK and D^*K systems is that the reduced mass is a bit larger, approaching the kaon mass in the $m_Q \to \infty$ limit. There are a few theoretical calculations of the masses of the aforementioned HP systems, which are usually inside the 60–70 MeV window [14,15,23]. Here, for consistency we will simply recalculate the location of the HP partners of the DK and D^*K systems from the assumption that the binding energies of the later are known. We will do two calculations, a nonrelativistic and a relativistic one. For the nonrelativistic one, we use the potential

$$V^{\mathrm{NR}} = C(\Lambda) g_\Lambda(k') g_\Lambda(k), \tag{29}$$

where $C(\Lambda)$ is a running coupling constant and $g_\Lambda(k) = e^{-(k^2/\Lambda^2)^n}$ is a gaussian regulator with $n = 2$. For the relativistic calculation we will include the correct Weinberg-Tomozawa energy dependence

$$V^{\mathrm{R}} = C(\Lambda) \frac{\omega_K(k) + \omega_K(k')}{2m_K} g_\Lambda(k') g_\Lambda(k), \tag{30}$$

with $\omega_K(q) = \sqrt{m_K^2 + q^2}$, where we also modify the two-body propagator in line with Eq. (13). We choose the cutoff to float in the $\Lambda = 0.5$–1.0 GeV window, i.e., a cutoff around the breakdown scale of the previous description (which is set by the vector meson mass $m_\rho = 0.77$ GeV). Now if we fix the DK and D^*K binding to 45 MeV, in the nonrelativistic case we obtain a binding energy of 57–74 MeV and 60–83 MeV for the $B^*\bar{K}$ and $\Xi_{bb}\bar{K}$ molecules. In the relativistic case, these numbers increase a bit to 59–81 MeV and 66–93 MeV, respectively. For the three-body system, we define the binding energy with respect to the particle-dimer threshold, that is, with respect to $2M + m - B_2$. This means that the location of the three-body bound states is

$$M(HHP) = 2M + m - B_2 - B_3. \tag{31}$$

With this definition the $B^*B^*\bar{K}$ binding energy lies in the range of 32–42 MeV and 32–33 MeV for the nonrelativistic and relativistic cases, respectively. For the $\Xi_{bb}\Xi_{bb}\bar{K}$ molecules we find a fundamental and excited state at 50–90 and 8–14 MeV for nonrelativistic antikaons and 52–83 and 2–14 MeV for relativistic antikaons. These results are summarized in Table II.

TABLE II. Two- and three-body binding energies in MeV for the $B^*B^*\bar{K}$ and $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi_{bb}^*\Xi_{bb}^*\bar{K}$ for different values of the cutoff and depending on the kinematics (nonrelativistic and relativistic, indicated by the superscripts NR and R). The two-body binding energy B_2 refers to the hadron-antikaon system, while the three-body binding energy B_3 is computed with respect to the two-body binding threshold, i.e., with respect to $(2M + m - B_2)$ with M the mass of the hadron and m the mass of the antikaon. We make no difference between the $\Xi_{bb}\Xi_{bb}\bar{K}/\Xi_{bb}^*\Xi_{bb}^*\bar{K}$ systems as there is no noticeable change in the predicted binding energies owing to the similar masses of the Ξ_{bb} and Ξ_{bb}^* baryons, $M(\Xi_{bb}) = 10127$ MeV and $M(\Xi_{bb}^*) = 10151$ MeV according to Ref. [24].

HHP	B_2^{NR}	B_3^{NR}	B_2^{R}	B_3^{R}
$B^*B^*\bar{K}$	57–74	32–42	59–81	32–33
$\Xi_{bb}\Xi_{bb}\bar{K}$	60–83	8–14	66–93	2–14
		50–90		52–83

In the previous calculations, we have treated $C(\Lambda)$ as a running coupling constant. Yet its strength is expected to be given by

$$C = \frac{C_{\mathrm{WT}}}{2f_\pi^2}, \tag{32}$$

with $C_{\mathrm{WT}} = -2$, where we take the $f_\pi = 132$ MeV normalization. This suggest a different approach: to treat the coupling C as known and to choose a cutoff that reproduces the location of the DK and D^*K poles. In this case, we obtain $\Lambda_{\mathrm{WT}} = 0.892$ and 0.823 GeV for the relativistic and nonrelativistic cases. If we redo the calculations for this "privileged" cutoff, the $B^*\bar{K}$ and $B^*B^*\bar{K}$ lie now at

$$B_2^{\mathrm{NR}} = 71 \text{ MeV} \quad \text{and} \quad B_3^{\mathrm{NR}} = 40 \text{ MeV}, \tag{33}$$

$$B_2^{\mathrm{R}} = 72 \text{ MeV} \quad \text{and} \quad B_3^{\mathrm{R}} = 30 \text{ MeV}, \tag{34}$$

depending on whether we are using relativistic or nonrelativistic kinematics. Meanwhile, for the $\Xi_{bb}\bar{K}$ and $\Xi_{bb}\Xi_{bb}\bar{K}$ systems, we have

$$B_2^{\mathrm{NR}} = 78 \text{ MeV} \quad \text{and} \quad B_3^{\mathrm{NR}} = 9/81 \text{ MeV}, \tag{35}$$

$$B_2^{\mathrm{R}} = 79 \text{ MeV} \quad \text{and} \quad B_3^{\mathrm{R}} = 4/67 \text{ MeV}, \tag{36}$$

where we remind that there is an excited and a fundamental $\Xi_{bb}\Xi_{bb}\bar{K}$ state.

For comparison purposes, we can consider the case of the $\Lambda(1405)$, which is traditionally considered to be a $N\bar{K}$ bound state. The strength of the WT term is $C_{\mathrm{WT}} = -3$ for this system. The $\Lambda(1405)$ is known to have a double pole structure [43,44], which comes from the fact that the $N\bar{K}$ channel mixes with the $\Sigma\pi$ channel and where the two channels are attractive enough to generate a pole with the quantum numbers of the $\Lambda(1405)$. One of the poles is mostly an $N\bar{K}$ bound state. If we ignore the $\Sigma\pi$ channel, we end up with a standard bound state which is estimated to be located at 1427 MeV [42], i.e., a binding energy of 8 MeV. The cutoffs for which this $\Lambda(1405)$ pole is reproduced with the formalism presented here are $\Lambda_{\mathrm{WT}} = 0.596$ and 0.571 GeV for nonrelativistic and relativistic antikaon kinematics, which are markedly lower than in the DK and D^*K systems. The conclusion is that we are not really sure about what is the exact cutoff to use in the $\Xi_Q'K$, $\Sigma_Q\bar{K}$ and $\Omega_b K$ systems, but we can expect it to be somewhere in between the two values that we have deduced from the $N\bar{K}$ and DK/D^*K systems. That is, we expect the cutoff to be somewhere in the $\Lambda = 0.6$–0.9 GeV window. As a matter of fact, for $\Lambda = 0.6$ MeV all the HP two-body system of Table I ($\Xi_b'K$, $\Sigma_b\bar{K}$ and ΩK) bind and the same is true for the HHP P-wave three-body systems. For $\Lambda = 0.9$ GeV the binding energies can in a few cases—in particular the $\Sigma_b\bar{K}$ system—be of the order of a few hundred MeV, clearly outside the expected range of validity of the type of description we are using. The conclusion is that the spread generated by the cutoff variation is excessively large: reliable predictions cannot be done until we find a suitable heavy baryon and kaon/antikaon bound state from which to fix the contact interaction or the cutoff. For this reason we will refrain to do concrete predictions about these systems in this work, except noting their probable existence.

V. CONCLUSIONS

In this work, we have considered the P-wave three-body $B^*B^*\bar{K}$ system. In this system, the $B^*\bar{K}$ interaction is strong enough as to generate a bound state, the B^*_{s1}. In addition, the mass imbalance between the B^* and the $\bar{K}$ is remarkable, a feature that points out to the possibility of P-wave three-body bound states. Concrete calculations indicate that there are indeed P-wave $B^*B^*\bar{K}$ bound states with quantum numbers $J^P = 0^+$, 1^+ and 2^+ located at 30–40 MeV below the B^*B_{s1} threshold. Owing to heavy antiquark-diquark symmetry [20–22] this idea can be easily extended to the $\Xi_{bb}\Xi_{bb}\bar{K}$ system, where there are two bound states as a consequence of the larger mass imbalance. In this latter case, the excited and fundamental states are located about 5–15 and 50–90 MeV below the $\Xi_{bb}\Omega^*_{bb\frac{1}{2}}$ threshold, where $\Omega^*_{bb\frac{1}{2}}$ refers to the theorized $\Xi_{bb}\bar{K}$ bound state. In general, the antikaon can be treated nonrelativistically in these three-body systems, with relativistic corrections playing a minor role, as we have explicitly checked with calculations. As a consequence of the isospin and angular momentum of the B^*B^*, $\Xi_{bb}\Xi_{bb}$ and $\Xi^*_{bb}\Xi^*_{bb}$ subsystems, the possible interaction between the heavy hadrons is expected to have a very limited impact on the location of the three-body states. It is interesting to notice that the $B^*B^*\bar{K}$ state can also be predicted in a complimentary two-body description, in which case we consider a B^*B_{s1} pair interacting by means of a one antikaon exchange potential [25]. In this interpretation, the location of the bound states is a bit more shallow, about half the binding energy computed here. Nonetheless these figures are still compatible with the calculations presented here.

This idea could also apply to other HHP systems, particularly if we consider that the Weinberg-Tomozawa interaction between a hadron and a pseudo Nambu-Goldstone boson can be strong in some cases. A few candidate HP systems include the $\Xi'_Q K$, $\Omega_Q K$ and the $\Sigma_Q \bar{K}$. If we consider heavy antiquark-diquark symmetry and the observation that the recent discovery of the Ξ^{++}_{cc} doubly charmed baryon [30] probably implies the existence of doubly heavy tetraquarks in the bottom sector [31,32], there is the possibility of a tetraquark-tetraquark-kaon/antikaon three-body system capable of fulfilling the conditions for the P-wave Efimov effect.

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