$B \to f_2(1270)$ form factors with light-cone sum rules

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We construct the quark-antiquark chiral odd distribution amplitudes including twist-four mass contributions for tensor mesons. We also give quark-antiquark-gluon distribution amplitudes, where we calculate the input parameters with QCD sum rules. With the help of equations of motion, we determine the twist-three and twist-four distribution amplitudes including SU(3) breaking terms. We use QCD light-cone sum rules to derive the form factors for the decay $B \to f_2(1270)$ with vector, axial-vector and tensor currents. We also give the q^2 dependence of the form factors.

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I. INTRODUCTION

Semileptonic B decays involving light mesons are especially promising in the search for new physics. This can be illustrated, e.g., by the recent discussion about angular distributions in $B \to K^{*0}\mu^+\mu^-$ [1–3]. Decays into tensor mesons have the advantage that three different polarizations of the final tensor meson are possible and therefore provide additional sensitivity to search for deviations from the helicity structure of the electroweak interaction. (For a general introduction, see the minireviews by A. Gritsan (pp. 1252–1255 in the 2017 online update) and P. Eerola, M. Kreps and Y. Kwon (pp. 1137–1149) in [4] and references given there.) In fact, it was demonstrated by BELLE in a recent measurement of the transition form factor $\gamma^*\gamma \to f_2(1270)$ at large momentum transfers that already, with the existing detectors, relevant polarization sensitive data can be obtained [5]. The uncertainty of the standard model predictions is dominated by QCD uncertainties. A precise calculation of the $B \to f_2(1270)$ decay form factors, which is the topic of this contribution, can reduce these theoretical uncertainties.

Tensor mesons have already been the topic of earlier work. In Ref. [6], the chiral-even and -odd distribution amplitudes (DAs) were constructed and the decay constants were calculated, while in Ref. [7] the chiral-even DAs including meson mass corrections and three-particle twist-three DAs were studied. The present contribution is largely based on that work. The definitions of the B to tensor meson form factors can be found in [8–10]. There are a few studies of the B to $f_2(1270)$ decay, for example using a perturbative QCD approach [10] or using light-cone sum rules [11,12].

In this paper, we calculate the form factors for the B meson decaying into the tensor meson $f_2(1270)$ by using the framework of light-cone sum rules (LCSR) [13–15]. We give for the first time the chiral odd quark-antiquark DAs, including higher-twist contributions and meson mass corrections. We also construct new three-particle quark-antiquark-gluon DAs with tensor structure. With the help of equations of motion (EOM) we can represent the higher-twist DAs in terms of lower-twist DAs including SU(3) breaking terms for the first time. We determine quark-gluon coupling constants appearing in the three particle DAs using QCD sum rules. In doing so, we assume that $f_2(1270)$ is a pure nonstrange isospin singlet state $1/\sqrt{2}(\bar{u}u + \bar{d}d)$ and $f_2'(1525)$ is a pure strange state $\bar{s}s$ which is equivalent to assuming a vanishing mixing angle [16,17].

The paper is organized as follows. In Sec. II, we give the form factors and the related LCSR expressions. Section III contains the numerical analysis of the sum rules and our results. In the Appendix, we define the leading and higher-twist DAs of the tensor mesons.

II. FORM FACTORS AND LIGHT-CONE SUM RULES

We define the semileptonic $B \to f_2(1270)$ form factors by [8–10]

$$\langle f_2^\lambda(P)|\bar{u}(0)\gamma_\mu b(0)|B(P')\rangle = \frac{2}{m_B + m_{f_2}}\epsilon_{\mu\nu\alpha\beta}e^{\nu}_{(\lambda)*}P'^{\alpha}P^{\beta}\tilde{V}(q^2), \quad (1)$$

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$$\begin{aligned}\langle f_2^\lambda(P)|\bar{u}(0)\gamma_\mu\gamma_5 b(0)|B(P')\rangle &= i(m_B+m_{f_2})e_\mu^{(\lambda)*}\tilde{A}_1(q^2)\\ &\quad - i\frac{e^{(\lambda)*}\cdot P'}{m_B+m_{f_2}}(P'+P)_\mu\tilde{A}_2(q^2)\\ &\quad - 2im_{f_2}\frac{e^{(\lambda)*}\cdot P'}{q^2}q_\mu[\tilde{A}_3(q^2)-\tilde{A}_0(q^2)], \end{aligned} \tag{2}$$

$$\begin{aligned}\langle f_2^\lambda(P)|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 b(0)|B(P')\rangle &= \tilde{A}(q^2)[e_\mu^{(\lambda)*}(P+P')_\nu - e_\nu^{(\lambda)*}(P+P')_\mu]\\ &\quad - \tilde{B}(q^2)[e_\mu^{(\lambda)*}q_\nu - e_\nu^{(\lambda)*}q_\mu]\\ &\quad - 2\tilde{C}(q^2)\frac{e^{(\lambda)*}\cdot q}{m_B^2-m_{f_2}^2}[P_\mu q_\nu - P_\nu q_\mu], \end{aligned} \tag{3}$$

where $q_\alpha = P'_\alpha - P_\alpha$, $e_\alpha^{(\lambda)*} = \frac{e_{\alpha\beta}^{(\lambda)*}q^\beta}{m_B}$ and

$$\tilde{A}_3(q^2) = \frac{m_B+m_{f_2}}{2m_{f_2}}\tilde{A}_1(q^2) - \frac{m_B-m_{f_2}}{2m_{f_2}}\tilde{A}_2(q^2).$$

The tensor form factors can also be defined by the two following matrix elements,

$$\begin{aligned}&\langle f_2^\lambda(P)|\bar{u}(0)\sigma^{\mu\nu}q_\nu b(0)|B(P')\rangle\\ &\quad = -2i\epsilon^{\mu\nu\alpha\beta}P'_\nu P_\alpha e_\beta^{(\lambda)*}\tilde{T}_1(q^2),\\ &\langle f_2^\lambda(P)|\bar{u}(0)\sigma^{\mu\nu}\gamma_5 q_\nu b(0)|B(P')\rangle\\ &\quad = \tilde{T}_2(q^2)[(m_B^2-m_{f_2}^2)e^{(\lambda)*\mu} - e^{(\lambda)*}\cdot P'(P'+P)^\mu]\\ &\qquad + \tilde{T}_3(q^2)e^{(\lambda)*}\cdot P'\left[q^\mu - \frac{q^2}{m_B^2-m_{f_2}^2}(P'+P)^\mu\right],\end{aligned}$$

which then leads to

$$\begin{aligned}\tilde{T}_1(q^2) &= \tilde{A}(q^2),\\ \tilde{T}_2(q^2) &= \tilde{A}(q^2) - \frac{q^2}{m_B^2-m_{f_2}^2}\tilde{B}(q^2),\\ \tilde{T}_3(q^2) &= \tilde{B}(q^2)+\tilde{C}(q^2).\end{aligned}$$

To get access to these form factors, we use the two-point correlation function

$$\Pi_a(q,P) = i\int d^4x e^{iqx}\langle f_2^\lambda(P)|T\{\bar{q}_1(x)\Gamma_a b(x) j_B(0)\}|0\rangle, \tag{4}$$

with the Lorentz structures

$$\Gamma_\mu = \gamma_\mu, \qquad \Gamma_{\mu5} = \gamma_\mu\gamma_5, \qquad \Gamma_{\mu\nu5} = \sigma_{\mu\nu}\gamma_5.$$

Here,

$$j_B(0) = \bar{b}(0)i\gamma_5 q_2(0)$$

is the interpolating current for the B-meson.

The decay constant f_B of the B-meson is defined by

$$\langle B(P')|\bar{b}(0)i\gamma_5 q_2(0)|0\rangle = \frac{f_B m_B^2}{m_b}. \tag{5}$$

The standard procedure of light-cone sum rules is to calculate the correlation function (4) in two different ways. On the one hand, for large virtualities, we use operator product expansion (OPE) around the light cone so that we can represent the correlation function in terms of the light-cone DAs, which are given in Appendix. On the other hand we can insert a complete set of eigenstates with the quantum numbers of the B-meson and isolate the ground state. These two different representations can be matched using dispersion relations and quark-hadron duality. Using Borel-transformation to eliminate subtraction terms and to suppress higher states leads to the final sum rules. For the hadronic representation after inserting a complete set of eigenstates and isolating the ground state, we get, e.g., for the vector current

$$\Pi_\mu(q,P) = \frac{\langle 0|\bar{q}_1\gamma_\mu b|B\rangle\langle B|\bar{b}i\gamma_5 q_2|0\rangle}{m_B^2-q^2} + \sum_h \frac{\langle 0|\bar{q}_1\gamma_\mu b|h\rangle\langle h|\bar{b}i\gamma_5 q_2|0\rangle}{m_h^2-q^2}.$$

Inserting Eqs. (1), (5) and rewriting the higher states into a dispersion integral over a spectral density, describing the excited and continuum states, we get

$$\Pi_\mu(q,P) = \frac{2f_B m_B^2\epsilon_{\mu\nu\alpha\beta}e^\nu_{(\lambda)*}P'^\alpha P^\beta\tilde{V}(q^2)}{m_b(m_B+m_{f_2})(m_B^2-q^2)} + \int_{s_0^h}^\infty \frac{ds}{s-q^2}\rho_\mu(s).$$

Here s_0^h is the threshold of the lowest continuum state. Applying a Borel-transformation,

$$\frac{1}{s-P'^2} \to e^{-s/M^2}, \qquad (P'^2)^n \to 0,$$

we get for the vector case and the other two cases after the same procedure

$$\Pi_\mu(q,P) = 2f_B m_B^2 e^{-m_B^2/M^2}\frac{\tilde{V}(q^2)}{(m_B+m_{f_2})m_b}\epsilon_{\mu\nu\alpha\beta}e^{\nu}_{(\lambda)*}q^\alpha P^\beta,$$

$$\Pi_{\mu 5}(q,P) = \frac{if_B m_B^2}{m_b}e^{-m_B^2/M^2}\bigg[(m_B+m_{f_2})e_\mu^{(\lambda)*}\tilde{A}_1(q^2) - (e^{(\lambda)*}\cdot q)(2P_\mu+q_\mu)\frac{\tilde{A}_2(q^2)}{m_B+m_{f_2}} - 2m_{f_2}\frac{e^{(\lambda)*}\cdot q}{q^2}q_\mu(\tilde{A}_3(q^2)-\tilde{A}_0(q^2))\bigg],$$

$$\Pi_{\mu\nu 5}(q,P) = \frac{f_B m_B^2}{m_b}e^{-m_B^2/M^2}\bigg[-\tilde{A}(q^2)((2P_\mu+q_\mu)e_\nu^{(\lambda)*} - (2P_\nu+q_\nu)e_\mu^{(\lambda)*}) - \tilde{B}(q^2)(e_\mu^{(\lambda)*}q_\nu - e_\nu^{(\lambda)*}q_\mu) - 2\tilde{C}(q^2)\frac{e^{(\lambda)*}\cdot q}{m_B^2-m_{f_2}^2}(P_\mu q_\nu - P_\nu q_\mu)\bigg],$$

with M^2 being the Borel parameter. For simplicity, we do not write down the spectral densities. Later we will use quark-hadron duality to subtract these contributions from our OPE result.

For the OPE, we contract the two b-quarks in (4) using the quark propagator in a background field [18,19]

$$\begin{aligned}\langle 0|T\{b^i(x)\bar{b}^j(0)\}|0\rangle &= -i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{\not{k}+m_b}{m_b^2-k^2}\delta_{ij} \\ &- ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dvG^{\mu\nu a}(vx)\left(\frac{\lambda^a}{2}\right)^{ij} \\ &\times\left(\frac{\not{k}+m_b}{2(m_b^2-k^2)^2}\sigma_{\mu\nu} + \frac{1}{m_b^2-k^2}vx_\mu\gamma_\nu\right).\end{aligned}$$

So we get, e.g., for the vector current

$$\begin{aligned}\Pi_\mu(q,P) = i\int\frac{d^4xd^4k}{(2\pi)^4}\frac{e^{ix(q-k)}}{m_b^2-k^2} &\times\Big(m_b\langle f_2^\lambda(P)|\bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0)|0\rangle \\ &+k^\nu\langle f_2^\lambda(P)|\bar{q}_1(x)\gamma_\mu\gamma_\nu\gamma_5 q_2(0)|0\rangle \\ &+\int_0^1 dv\langle f_2^\lambda(P)|\bar{q}_1(x)\gamma_\mu G^{\alpha\beta}(vx) \\ &\times\left(\frac{\not{k}+m_b}{2(m_b^2-k^2)}\sigma_{\alpha\beta} + \frac{vx_\alpha\gamma_\beta}{m_b^2-k^2}\right)\gamma_5 q_2(0)|0\rangle\Big).\end{aligned}$$

After rewriting the Lorentz structures, if necessary, the resulting matrix elements are expressed in terms of the light-cone DAs from Appendix. After performing the x and k integration, the general structure, shown in simplified form looks like

$$\Pi_\mu(q,P) \sim \sum_{n=1}^{5}\int_0^1 du\frac{\mathcal{A}(u)}{D^n}, \qquad (6)$$

where $\mathcal{A}(u)$ is one of the DAs from Appendix and the denominator is

$$D = m_b^2 - (q+uP)^2.$$

We have to write (6) as a dispersion integral in P'^2,

$$\Pi_\mu(q,P) \sim \frac{1}{\pi}\int_{m_b^2}^{\infty}\frac{ds}{s-P'^2}\mathrm{Im}_s\mathcal{A}(s),$$

which we can achieve by substituting

$$\begin{aligned}s(u) &= \frac{1}{u}(m_b^2 - \bar{u}q^2 + u\bar{u}m_{f_2}^2), \\ u(s) &= \frac{1}{2m_{f_2}^2}\Big[m_{f_2}^2 + q^2 - s + \sqrt{(s-q^2-m_{f_2}^2)^2 + 4m_{f_2}^2(m_b^2-q^2)}\Big],\end{aligned}$$

in the denominator (6), with $\bar{u} = 1-u$ and perform a partial integration whenever the power of the denominator is larger than one. Now the contributions of the excited and continuum states can be approximated using quark-hadron duality,

$$\int_{s_0^h}^{\infty}\frac{ds}{s-P'^2}\rho(s) \approx \frac{1}{\pi}\int_{s_0}^{\infty}\frac{ds}{s-P'^2}\mathrm{Im}_s\mathcal{A}(s),$$

where s_0 is the duality threshold. For the final sum rules, we use following shorthand notation for the DAs,

$$\begin{aligned}\frac{d}{du}\hat{\mathcal{A}}(u) &= -\mathcal{A}(u), \\ \frac{d}{d\alpha_3}\tilde{\mathcal{T}}(\underline{\alpha}) &= -\mathcal{T}(\underline{\alpha}),\end{aligned}$$

with $\hat{\mathcal{A}}(0) = \hat{\mathcal{A}}(1) = \tilde{\mathcal{T}}(\alpha_3=0) = \tilde{\mathcal{T}}(\alpha_3=1) = 0$. For $\hat{\hat{\mathcal{A}}}(u)$, $\tilde{\tilde{\mathcal{T}}}(\underline{\alpha})$, we have two derivatives, etc. Performing a Borel-transformation, one obtains the final sum rules for the form factors

$$\tilde{V}(q^2) = \frac{(m_B + m_{f_2})m_b}{2f_B m_B} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} [8(1-\delta_+) m_b m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{g}}_a(s) - 2m_{f_2} f_{f_2}^T \mathrm{Im}_s \hat{A}(s)$$
$$-(3m_b^2+1) m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\mathbb{A}}(s)],$$

$$\tilde{A}_1(q^2) = \frac{m_b e^{m_B^2/M^2}}{f_B m_B (m_B + m_{f_2})} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[2m_b m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{B}_1(s) - 4m_{f_2}^3 m_b^2 f_{f_2}^T \mathrm{Im}_s \hat{\hat{C}}(s) - 4(1-\delta_+^T) m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{h}}_\parallel^{(s)}(s)$$
$$+8 m_b m_{f_2}^4 f_{f_2} \mathrm{Im}_s \hat{\hat{\hat{C}}}_1(s) - 2m_{f_2} f_{f_2}^T (uP^2 + Pq) \mathrm{Im}_s \hat{A}(s) + (3m_b^2+1) m_{f_2}^3 f_{f_2}^T (uP^2+Pq) \mathrm{Im}_s \hat{\mathbb{A}}(s)$$
$$+16 m_{f_2}^3 f_{f_2}^T (uP^2+Pq) \mathrm{Im}_s \hat{\hat{\hat{B}}}(s) + 8 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv f_{f_2}^T m_{f_2}^5 \mathrm{Im}_s \left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) - \frac{m_{f_2}^2}{2} \tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}) \right) \Bigg|_{\substack{\alpha_1 = 1-\alpha_2-\alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg],$$

$$\tilde{A}_2(q^2) = -\frac{(m_b + m_{f_2})m_b}{2f_B m_B} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[8 m_b m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{A}}_1(s) + 24 m_b^3 m_{f_2}^4 f_{f_2} \mathrm{Im}_s \hat{\hat{\phi}}_4(s) + 2m_{f_2} f_{f_2}^T \mathrm{Im}_s \hat{A}(s)$$
$$-(3m_b^2+1) m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\mathbb{A}}(s) - 8(1-\delta_+^T) u m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{h}}_\parallel^{(s)}(s) + 24 u m_b m_{f_2}^4 f_{f_2} \mathrm{Im}_s \hat{\hat{\hat{C}}}_1(s)$$
$$+4 u m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{C}}(s) - m_{f_2}^3 f_{f_2}^T (56 + 48(uPq + q^2)) \mathrm{Im}_s \hat{\hat{\hat{B}}}(s)$$
$$+8 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv f_{f_2}^T m_{f_2}^5 \left(3u \mathrm{Im}_s \left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) - \frac{m_{f_2}^2}{2} \tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}) \right) - \frac{1}{m_{f_2}^2} \mathrm{Im}_s \left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) - \frac{m_{f_2}^2}{2} \tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}) \right) \right) \Bigg|_{\substack{\alpha_1 = 1-\alpha_2-\alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg],$$

$$\tilde{A}_0(q^2) = \frac{q^2}{2m_{f_2}} \Bigg(\frac{m_b}{f_B m_b} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[48 m_{f_2}^3 f_{f_2}^T (uP^2 + Pq) \mathrm{Im}_s \hat{\hat{\hat{B}}}(s) + 24 m_b m_{f_2}^4 f_{f_2} \mathrm{Im}_s \hat{\hat{\hat{C}}}(s)$$
$$-8(1-\delta_+^T) m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{h}}_\parallel^{(s)}(s) + 4 m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{C}}(s)$$
$$+24 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv m_{f_2}^5 f_{f_2}^T \mathrm{Im}_s \left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) - \frac{m_{f_2}^2}{2} \tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}) \right) \Bigg|_{\substack{\alpha_1 = 1-\alpha_2-\alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg] + \frac{\tilde{A}_2(q^2)}{m_B + m_{f_2}} \Bigg)$$
$$+ \frac{m_b + m_{f_2}}{2m_{f_2}} \tilde{A}_1(q^2) - \frac{m_b - m_{f_2}}{2m_{f_2}} \tilde{A}_2(q^2),$$

$$\tilde{A}(q^2) = -\frac{m_b}{2f_B m_B} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[-2 m_b m_{f_2} f_{f_2}^T \mathrm{Im}_s \hat{A}(s) + 3 m_b^3 m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\mathbb{A}}(s) + 16 m_b m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{\hat{B}}}(u)$$
$$+8(1-\delta_+) m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{g}}_a(s) - 4 m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{A}}_1(s) + 2u m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{B}_1(s) + 2(3m_b^2+1) m_{f_2}^4 f_{f_2} \mathrm{Im}_s \hat{\hat{\phi}}_4(s)$$
$$+2 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv m_{f_2}^2 f_{f_2} (\mathrm{Im}_s(\mathcal{V}(\underline{\alpha}) - \mathcal{A}(\underline{\alpha})) - 4 m_{f_2}^2 \mathrm{Im}_s(\tilde{\tilde{\mathcal{V}}}(\underline{\alpha}) - \tilde{\tilde{\mathcal{A}}}(\underline{\alpha}))$$
$$+2u m_{f_2}^2 \mathrm{Im}_s(\tilde{\mathcal{V}}(\underline{\alpha}) - \tilde{\mathcal{A}}(\underline{\alpha}))) |_{\substack{\alpha_1 = 1 - \frac{u-\alpha_3}{v} - \alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg],$$

$$\tilde{B}(q^2) = \frac{m_b}{f_B m_B} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[-8(1-\delta_+) m_{f_2}^2 f_{f_2} (uP^2 + Pq) \mathrm{Im}_s \hat{\hat{g}}_a(s) + 2 m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{B}_1(s)$$
$$+4 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv m_{f_2}^4 f_{f_2} \mathrm{Im}_s(\tilde{\mathcal{V}}(\underline{\alpha}) - \tilde{\mathcal{A}}(\underline{\alpha})) |_{\substack{\alpha_1 = 1 - \frac{u-\alpha_3}{v} - \alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg] + \tilde{A}(q^2),$$

$$\tilde{C}(q^2) = -\frac{(m_B^2 - m_{f_2}^2) m_b}{2f_B m_B} e^{m_B^2/M^2} \frac{1}{\pi} \int_{m_b^2}^{s_0} ds e^{-s/M^2} \Bigg[-48 m_b m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s \hat{\hat{\hat{B}}}(s) + 8(1-\delta_+) m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{g}}_a(s)$$
$$+8 m_{f_2}^2 f_{f_2} \mathrm{Im}_s \hat{\hat{A}}_1(s) - 6(4m_b^2+1) f_{f_2} \mathrm{Im}_s \hat{\hat{\phi}}_4(s) + 3 \int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv m_{f_2}^2 f_{f_2} \mathrm{Im}_s(\tilde{\tilde{\mathcal{V}}}(\underline{\alpha}) - \tilde{\tilde{\mathcal{A}}}(\underline{\alpha})) |_{\substack{\alpha_1 = 1-\alpha_2-\alpha_3 \\ \alpha_2 = \frac{u-\alpha_3}{v}}} \Bigg].$$

III. NUMERICAL ANALYSIS AND DISCUSSION

For the numerical analysis, we use the following input values for the masses [20]

$$m_{f_2} = 1.275 \text{ GeV}, \qquad m_B = 5.279 \text{ GeV},$$

and for the decay constants at a scale of $\mu = 1$ GeV, we use [6,7]

$$f_{f_2} = 0.101(10) \text{ GeV}, \qquad f_{f_2}^T = 0.117(25) \text{ GeV}.$$

We use the pole b-quark mass, as always for LCSR, given by $m_b = 4.8(1)$ GeV and for the B-meson decay constant f_B, we use the tree-level sum rule from [21]. All the scale-dependent parameters are evaluated at the factorization scale $\mu_f = \sqrt{m_B^2 - m_b^2}$. We choose the Borel parameter window to be $M^2 = 4$–8 GeV^2 and the duality threshold $s_0 = 35.5 \pm 2$ GeV^2, which is consistent with other studies of the B-meson [22]. All the other input values are given in Appendix.

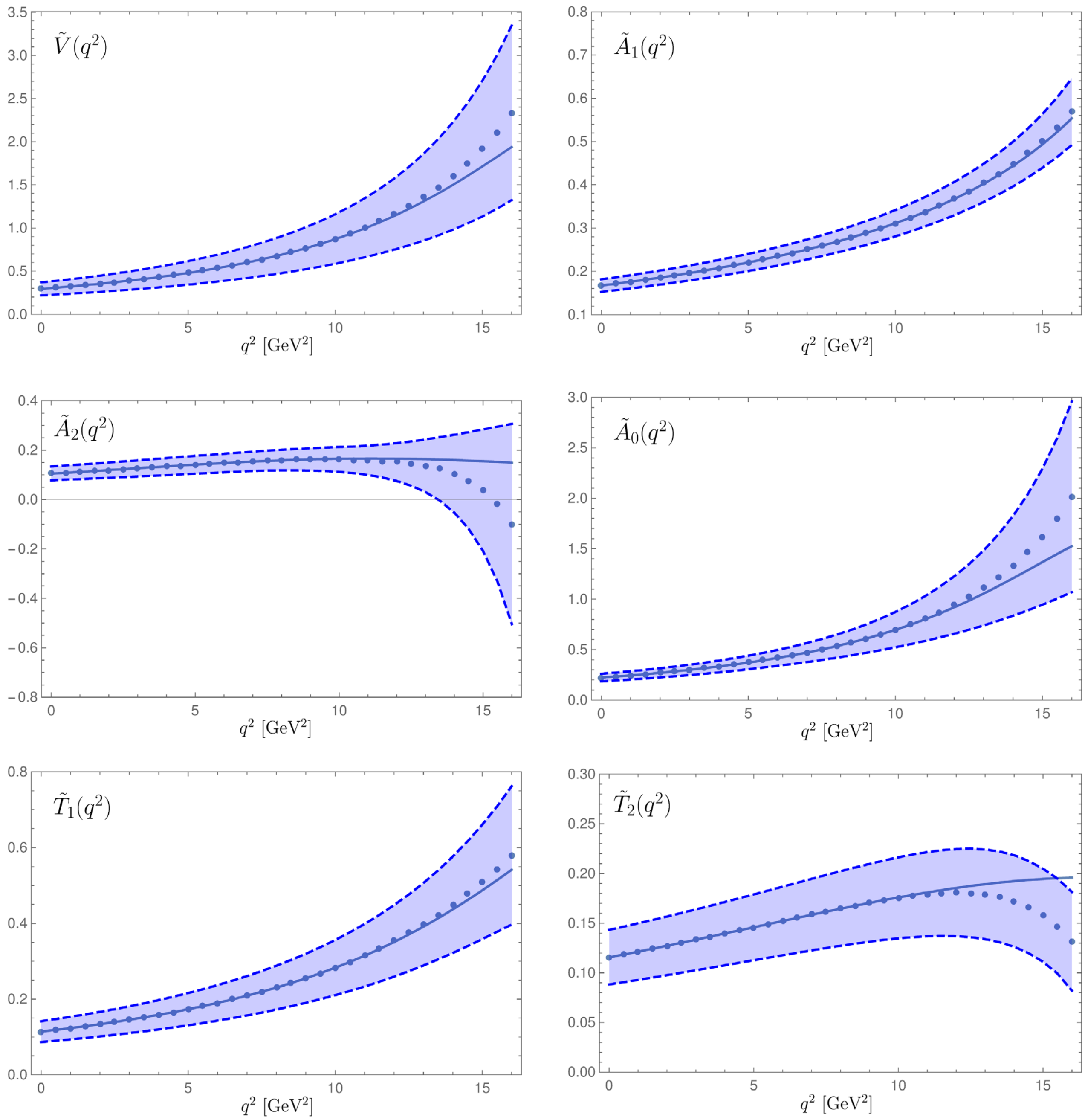

FIG. 1. q^2 dependence of the form factors. The solid line gives the central value of the fit to the sum rule results (dots) and the dashed lines are the uncertainties from varying the input parameters.

TABLE I. Results from fitting the $B \to f_2(1270)$ form factors obtained by LCSR to the three parameter form in (7).

Form factor	$\tilde{F}(0)$	a	b
$\tilde{V}$	0.30 ± 0.03	2.38 ± 0.4	1.50 ± 0.73
$\tilde{A}_1$	0.17 ± 0.01	1.41 ± 0.50	0.35 ± 1.40
$\tilde{A}_2$	0.11 ± 0.02	1.84 ± 1.46	2.30 ± 4.09
$\tilde{A}_0$	0.22 ± 0.02	2.57 ± 0.77	1.89 ± 2.23
$\tilde{T}_1$	0.11 ± 0.02	2.14 ± 1.14	1.34 ± 3.19
$\tilde{T}_2$	0.12 ± 0.01	1.35 ± 1.24	1.11 ± 3.39
$\tilde{T}_3$	-0.02 ± 0.04	1.94 ± 17.51	0.71 ± 49.40

The LCSR are assumed to give a reasonable approximation up to $q^2 \leq q^2_{\max} = (m_B - m_{f_2})^2 = 16.07\ \mathrm{GeV}^2$. To avoid fitting artifacts, we limit the actual fit range to $0 \leq q^2 \leq 10\ \mathrm{GeV}^2$. The deviations from the fit curves for large q^2 in fact indicate the break down of the approximation. We choose a parametrization for the form factors with the three parameters $\tilde{F}(0)$, a and b,

$$F(q^2) = \frac{\tilde{F}(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}. \qquad (7)$$

We perform a weighted fit using as weights the uncertainties from varying the input parameters and add the errors in quadrature. The cited errors indicate an increase of χ^2 by 1. For asymmetric errors we take the mean value and shift the central value by the difference of the asymmetric error and the mean value to get symmetric errors. As one can see from Fig. 1, our $\chi^2/$ d.o.f. is nearly zero for all form factors and $q^2 \leq 10\ \mathrm{GeV}^2$, indicating that the parametrization (7) is a very efficient one. We do not show any q^2 dependence of the form factor $\tilde{T}_3(q^2)$ because this form factor is close to zero in the whole fitting range due to the fact that $\tilde{B}(q^2)$ and $\tilde{C}(q^2)$ have nearly the same magnitude but different signs. Our results can be found in Table I and in Figs. 1, 2.

We observe that the contributions from the mass terms $\mathbb{A}(u)$ and $\phi_4(u)$ to the form factors are not negligible as can already be seen in Fig. 2. More precisely, the effect of these mass terms is for all the form factors less than 30% for $q^2 = 0$. For $q^2 \neq 0$ the contributions of the meson mass terms to the form factors $\tilde{V}(q^2)$, $\tilde{A}_1(q^2)$ and $\tilde{A}_0(q^2)$ stays under 30%. For the form factors $\tilde{A}_2(q^2)$, $\tilde{T}_1(q^2)$, $\tilde{T}_2(q^2)$ and $\tilde{T}_3(q^2)$ the effect of the meson mass terms increases for higher values of q^2. Worth mentioning is the form factor $\tilde{A}_0(q^2)$, which depends on $\tilde{A}_1(q^2)$ and $\tilde{A}_2(q^2)$ but, due to cancellations the effect of the meson mass terms is less than 13% for the whole range of q^2. The comparison with other theoretical approaches, which is illustrated in Fig. 2 by the $q^2 = 0$ values of the form factors, illustrates the improved precision we achieved. This comparison requires, however, some explanations. The method used in [10] is a calculation within a specific "pQCD" approach based on $k_\perp$ factorization. The discrepancies between their results and ours (black bullets) is substantially larger than the systematic uncertainty we expect for our LCSR calculation. Therefore, we conclude that we disagree with the findings of [10]. In contrast, [11,12] are also LCSR calculations which allows to trace back the discrepancies to the fact that we have calculated higher contributions. In all cases, the error bars represent the variation observed when the LCSR input parameters are varied in reasonable bounds. They do not include any estimate of neglected higher order terms. Therefore, [11] should be compared to our grey bullets which do not contain meson mass corrections as these where also not taken into account in [11]. The difference between our grey bullets and the green squares shows that the higher-twist contributions and three-particle DAs we take into account make a significant difference, especially for $\tilde{V}(0)$, though not a very large one. The same can be said of the meson mass terms, comparing our grey and black bullets. Thus, one can conclude that to reach high precision all these effects have to be included and that our results are

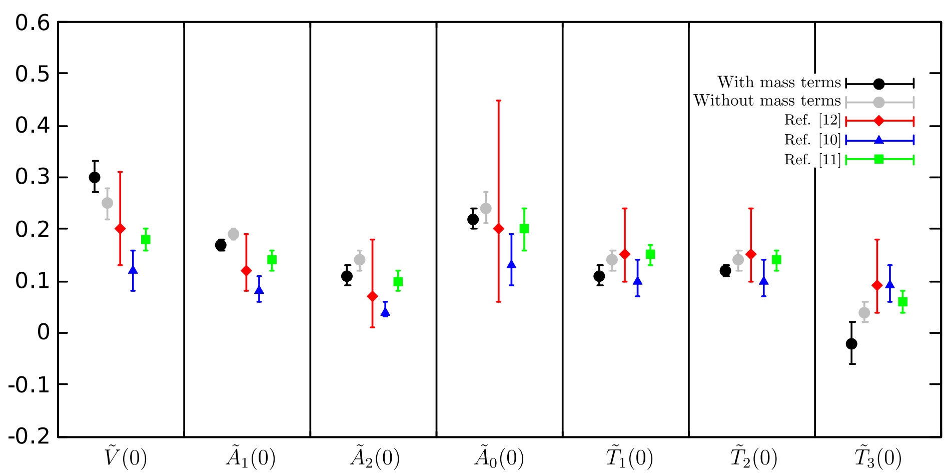

FIG. 2. The values of the form factors for $q^2 = 0$ from different theoretical approaches.

in fact far more precise than earlier calculations even though this does not show up in all cases in the cited error bars.

To summarize, we calculated the $B \to f_2(1270)$ form factors with LCSR using chiral-even and chiral-odd tensor meson DAs, including for the first time twist-four meson mass terms. We observe that these mass terms have a noticeable impact on the sum rules and should be taken into account in future studies. Especially for the region of $q^2 \neq 0$ these mass terms can play an important role. The effects of still higher-twist terms are probably smaller than the uncertainties arising from the choice of the Borel parameter, which is illustrated by the cited error bars. However, this can only be checked by future calculations. In such future investigations, we would, e.g., also consider additional SU(3) breaking terms. Especially for decays involving a strange quark, such SU(3) breaking terms can probably yield important contributions.

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APPENDIX: DISTRIBUTION AMPLITUDES

In previous studies, the chiral-even quark-antiquark light-cone DAs for the f_2-meson were defined as matrix elements of nonlocal light-ray operators [6,7,23]

$$\begin{aligned}\langle f_2(P,\lambda)|\bar{q}(z_2n)\gamma_\mu q(z_1n)|0\rangle &= f_{f_2}m_{f_2}^2\left[\frac{e_{nn}^{(\lambda)*}}{(pn)^2}p_\mu\int_0^1 du e^{iz_{12}(pn)}\phi_2(u,\mu)\right.\\ &\quad+\frac{e_{\perp\mu n}^{(\lambda)*}}{pn}\int_0^1 du e^{iz_{12}(pn)}g_v(u,\mu)\\ &\quad\left.-\frac{1}{2}n_\mu m_{f_2}^2\frac{e_{nn}^{(\lambda)*}}{(pn)^3}\int_0^1 du e^{iz_{12}(pn)}g_4(u,\mu)\right], \qquad (A1)\end{aligned}$$

$$\begin{aligned}\langle f_2(P,\lambda)|\bar{q}(z_2n)\gamma_\mu\gamma_5 q(z_1n)|0\rangle &= -if_{f_2}m_{f_2}^2(1-\delta_+)\epsilon_{\mu\nu\alpha\beta}\frac{n^\nu p^\alpha}{pn}\frac{e_{\beta n}^{(\lambda)*}}{pn}\\ &\quad\times\int_0^1 du e^{iz_{12}(pn)}g_a(u,\mu). \qquad (A2)\end{aligned}$$

In the same manner, we can define the chiral odd DAs[1]

$$\begin{aligned}\langle f_2(P,\lambda)|\bar{q}(z_2n)q(z_1n)|0\rangle &= f_{f_2}^T\frac{e_{nn}^{(\lambda)*}}{(pn)^2}m_{f_2}^3(1-\delta_+^T)\int_0^1 du e^{iz_{12}(pn)}h_\parallel^{(s)}(u,\mu), \qquad (A3)\end{aligned}$$

$$\begin{aligned}\langle f_2(P,\lambda)|\bar{q}(z_2n)\sigma_{\mu\nu}q(z_1n)|0\rangle &= if_{f_2}^T\left[m_{f_2}\frac{(e_{\perp n\mu}^{(\lambda)*}p_\nu - e_{\perp n\nu}^{(\lambda)*}p_\mu)}{pn}\int_0^1 du e^{iz_{12}(pn)}\phi_\perp(u,\mu)\right.\\ &\quad+m_{f_2}^3(p_\mu n_\nu - p_\nu n_\mu)\frac{e_{nn}^{(\lambda)*}}{(pn)^3}\int_0^1 du e^{iz_{12}(pn)}h_\parallel^{(t)}(u,\mu)\\ &\quad\left.+\frac{1}{2}(e_{\perp n\mu}^{(\lambda)*}n_\nu - e_{\perp n\nu}^{(\lambda)*}n_\mu)\frac{m_{f_2}^3}{(pn)^2}\int_0^1 du e^{iz_{12}(pn)}h_4(u,\mu)\right], \qquad (A4)\end{aligned}$$

with $e_{nn}^{(\lambda)*} = e_{\alpha\beta}^{(\lambda)*}n^\alpha n^\beta$ and we use the shorthand notation $z_{12} = \bar{u}z_1 + uz_2$. The polarization tensor $e_{\alpha\beta}^{(\lambda)}$ is traceless, symmetric and satisfies the condition $e_{\alpha\beta}^{(\lambda)}P^\alpha = 0$. Further, we have

$$\begin{aligned}e_{\perp\mu n}^{(\lambda)*} &\equiv g_{\mu\nu}^\perp e_{\nu n}^{(\lambda)*} = e_{\mu n}^{(\lambda)*} - \frac{e_{nn}^{(\lambda)*}}{(pn)}p_\mu + \frac{1}{2}n_\mu e_{nn}^{(\lambda)*}\frac{m_{f_2}^2}{(pn)^2},\\ g_{\mu\nu}^\perp &= g_{\mu\nu} - \frac{1}{pn}(n_\mu p_\nu + n_\nu p_\mu),\end{aligned}$$

where the vectors n_μ and $p_\mu = P_\mu - \frac{n_\mu}{2}\frac{m_{f_2}^2}{pn}$ are light-like, $n^2 = p^2 = 0$. The SU(3) breaking terms are parametrized by

$$\delta_\pm = \frac{f_{f_2}^T}{f_{f_2}}\frac{m_{\bar{q}} \pm m_q}{m_{f_2}}, \qquad \delta_\pm^T = \frac{f_{f_2}}{f_q^T}\frac{m_{\bar{q}} \pm m_q}{m_{f_2}}.$$

Close to the light cone $x^2 \to 0$, the operator product expansion (OPE) of the chiral odd DAs takes the form

$$\begin{aligned}\langle f_2(P,\lambda)|\bar{q}(x)\sigma_{\mu\nu}q(-x)|0\rangle &= if_q^T\left[m_{f_2}\frac{(e_{x\mu}^{(\lambda)*}P_\nu - e_{x\nu}^{(\lambda)*}P_\mu)}{(Px)}\int_0^1 du e^{i\xi(Px)}\left[A(u)\right.\right.\\ &\quad\left.+\frac{1}{4}x^2m_{f_2}^2\mathbb{A}(u)\right]\\ &\quad+m_{f_2}^3(P_\mu x_\nu - P_\nu x_\mu)\frac{e_{xx}^{(\lambda)*}}{(Px)^3}\int_0^1 du e^{i\xi(Px)}B(u)\end{aligned}$$

$$\langle f_2(P,\lambda)|\bar{q}(x)q(-x)|0\rangle = f_q^T\frac{e_{xx}^{(\lambda)*}}{(Px)^2}m_{f_2}^3(1-\delta_+^T)\int_0^1 du e^{i\xi(Px)}h_\parallel^{(s)}(u),$$

[1] In Ref. [6], they already defined the chiral odd DAs but without the mass terms and SU(3) breaking terms.

with the new two-particle twist-four DA $\mathbb{A}(u)$ that can be expressed in terms of the other DAs using QCD EOM, see below and $\xi = 2u - 1$. By comparing to Eqs. (A4) and (A3), we find

$$
\begin{aligned}
A(u) &= \phi_\perp(u), \\
B(u) &= h_\parallel^{(t)}(u) - \frac{\phi_\perp(u)}{2} - \frac{h_4(u)}{2}, \\
C(u) &= h_4(u) - \phi_\perp(u).
\end{aligned}
$$

The OPE for the chiral-even DAs can be found in [7].

We take the three-particle quark-antiquark-gluon DAs from Ref. [7]

$$
\langle f_2(P,\lambda)|\bar{q}(z_3 n) i g G_{\mu\nu}(z_2 n)\gamma_\alpha q(z_1 n)|0\rangle = -f_{f_2} m_{f_2}^2 \frac{p_\alpha}{pn}[p_\mu e_{\perp n\nu}^{(\lambda)} - p_\nu e_{\perp n\mu}^{(\lambda)}] \int \mathcal{D}\alpha e^{ipn\sum \alpha_k z_k} \mathcal{V}(\underline{\alpha}) + \cdots,
$$
$$
\langle f_2(P,\lambda)|\bar{q}(z_3 n) g \tilde{G}_{\mu\nu}(z_2 n)\gamma_\alpha\gamma_5 q(z_1 n)|0\rangle = -f_{f_2} m_{f_2}^2 \frac{p_\alpha}{pn}[p_\mu e_{\perp n\nu}^{(\lambda)} - p_\nu e_{\perp n\mu}^{(\lambda)}] \int \mathcal{D}\alpha e^{ipn\sum \alpha_k z_k} \mathcal{A}(\underline{\alpha}) + \cdots,
$$

and we define a new one for tensor structures

$$
\begin{aligned}
&\langle f_2(P,\lambda)|\bar{q}(z_3 n)\sigma_{\alpha\beta} g G_{\mu\nu}(z_2 n) q(z_1 n)|0\rangle \\
&\quad = -f_{f_2}^T \frac{e_{nn}^{(\lambda)*}}{2(pn)^2} m_{f_2}^3 [p_\alpha p_\mu g_{\beta\nu}^\perp - p_\beta p_\mu g_{\alpha\nu}^\perp - p_\alpha p_\nu g_{\beta\mu}^\perp + p_\beta p_\nu g_{\alpha\mu}^\perp] \int \mathcal{D}\alpha e^{ipn\sum \alpha_k z_k} \mathcal{T}_1(\underline{\alpha}) \\
&\qquad + f_{f_2}^T \frac{m_{f_2}^3}{2} [p_\alpha p_\mu e_{\nu\beta}^{\perp(\lambda)*} - p_\beta p_\mu e_{\nu\alpha}^{\perp(\lambda)*} - p_\alpha p_\nu e_{\mu\beta}^{\perp(\lambda)*} + p_\beta p_\nu e_{\mu\alpha}^{\perp(\lambda)*}] \int \mathcal{D}\alpha e^{ipn\sum \alpha_k z_k} \mathcal{T}_2(\underline{\alpha}) + \cdots,
\end{aligned}
$$

with $e_{\mu\nu}^{\perp(\lambda)*} = e_{\mu'\nu'}^{(\lambda)*} g_{\mu'\mu}^\perp g_{\nu'\nu}^\perp$. For the asymptotic form of the three-particle DAs we take [24,25]

$$
\begin{aligned}
\mathcal{V}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\xi_3 + \frac{1}{2}\omega_3(7\alpha_2 - 3)\right], \\
\mathcal{A}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\frac{1}{2}\tilde{\omega}_3(\alpha_1 - \alpha_3)\right], \\
\mathcal{T}_{1/2}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\xi_3^{\mathcal{T}_{1/2}} + \frac{1}{2}\omega_3^{\mathcal{T}_{1/2}}(7\alpha_2 - 3)\right].
\end{aligned}
$$

The constants ξ_3, ω_3 and $\tilde{\omega}_3$ have been determined in [7] by using QCD sum rules and are at a scale of 1 GeV

$$
\xi_3 = 0.15(8), \qquad \omega_3 = -0.2(3), \qquad \tilde{\omega}_3 = 0.06(1).
$$

Using QCD sum rules we get

$$
\begin{aligned}
\left(\frac{m_{f_2}^2\xi_3^{\mathcal{T}_2}}{2} - \xi_3^{\mathcal{T}_1}\right) &= 0.16(3), \\
\left(\frac{m_{f_2}^2\omega_3^{\mathcal{T}_2}}{2} - \omega_3^{\mathcal{T}_1}\right) &= -0.33(16).
\end{aligned}
$$

In Eqs. (A1)–(A3), the two-particle DAs $\phi_2(u)$, $\phi_\perp(u)$ are leading twist two, $g_v(u)$, $g_a(u)$, $h_\parallel^{(t)}(u)$, $h_\parallel^{(s)}(u)$ are collinear twist three and $g_4(u)$, $h_4(u)$ are twist four. By using the EOM [24], we can represent the twist-three DAs, including the SU(3) breaking terms in terms of the leading DAs and three-particle DAs

$$
\begin{aligned}
(1-\delta_+) g_a(u) &= \int_0^u dv \frac{\Omega(v)}{\bar{v}} - \int_u^1 dv \frac{\Omega(v)}{v}, \\
g_v(u) &= \int_0^u dv \frac{\Omega(v)}{\bar{v}} + \int_u^1 dv \frac{\Omega(v)}{v} + \delta_+\phi_\perp(u) \\
&\quad - \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\mathcal{V}(\underline{\alpha}) \\
&\quad - \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\frac{d}{d\alpha_1} + \frac{d}{d\alpha_3}\right)\mathcal{A}(\underline{\alpha}),
\end{aligned}
$$

with

$$
\begin{aligned}
\Omega(u) &= \phi_2(u) + (\delta_- + \delta_+(2u-1))\phi_\perp'(u) \\
&\quad - \frac{1}{2}\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1} + \alpha_3\frac{d}{d\alpha_3}\right)\mathcal{V}(\underline{\alpha}) \\
&\quad - \frac{1}{2}\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1} - \alpha_3\frac{d}{d\alpha_3}\right)\mathcal{A}(\underline{\alpha}),
\end{aligned}
$$

and

$$
\begin{aligned}
&(1-\delta_+^T)h_\parallel^{(s)}(u)\\
&\quad=\frac{1}{2}\left(\int_0^u dv\frac{\Phi(v)}{\bar v}-\int_u^1 dv\frac{\Phi(v)}{v}\right),\\
h_\parallel^{(t)}&=\frac{1}{2}(u-\bar u)\left(\int_0^u dv\frac{\Phi(v)}{\bar v}-\int_u^1 dv\frac{\Phi(v)}{v}\right)-\delta_+\phi_2(u)\\
&\quad+\frac{d}{du}\int_0^u d\alpha_1\int_0^{\bar u} d\alpha_3\frac{1}{\alpha_2}\left(\mathcal{T}_1(\underline{\alpha})-\frac{1}{2}m_{f_2}^2\mathcal{T}_2(\underline{\alpha})\right),
\end{aligned}
$$

with

$$
\begin{aligned}
\Phi(u)&=3\phi_\perp(u)+\delta_+^T\left(\phi_2(u)-\xi\frac{\phi_2'(u)}{2}\right)+\frac{\delta_-^T}{2}\phi_2'(u)\\
&\quad+\frac{d}{du}\int_0^u d\alpha_1\int_0^{\bar u} d\alpha_3\frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1}+\alpha_3\frac{d}{d\alpha_3}-1\right)\\
&\quad\times\left(\mathcal{T}_1(\underline{\alpha})-\frac{1}{2}m_{f_2}^2\mathcal{T}_2(\underline{\alpha})\right).
\end{aligned}
$$

For the leading-twist DAs, we will use the asymptotic form

$$
\phi_2(u)=-\phi_\perp(u)=30u\bar u(2u-1),
$$

where we defined $\phi_\perp(u)$ with a minus sign so that we have the same signs in Eq. (A4) as in Ref. [6] from which we take the value for f_{f_2}.

Also using the EOM [26], we can express the twist-four DAs by the asymptotic form of lower twist DAs

$$
\begin{aligned}
g_4(u)&=30u\bar u(2u-1),\\
h_4(u)&=-30u\bar u(2u-1),\\
\phi_4(u)&=100u^2\bar u^2(2u-1),\\
\mathbb{A}(u)&=60u^2\bar u^2(2u-1).
\end{aligned}
$$

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