$J/\psi \to \gamma\eta'\pi^+\pi^-$ and the structure observed around the $\bar{p}p$ threshold

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We analyze the origin of the structure observed in the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$ for $\eta'\pi^+\pi^-$ invariant masses close to the antiproton-proton ($\bar{p}p$) threshold, commonly associated with the $X(1835)$ resonance. Specifically, we explore the effect of a possible contribution from the two-step process $J/\psi \to \gamma\bar{N}N \to \gamma\eta'\pi^+\pi^-$. The calculation is performed in the distorted-wave Born approximation which allows an appropriate inclusion of the $\bar{N}N$ interaction in the transition amplitude. The $\bar{N}N$ amplitude itself is generated from a corresponding potential recently derived within chiral effective field theory. We are able to describe the invariant-mass dependence of the measured spectra for the reactions $J/\psi \to \gamma\bar{p}p$ and $J/\psi \to \gamma\eta'\pi^+\pi^-$ around the $\bar{p}p$ threshold. The structure seen in the $\eta'\pi^+\pi^-$ spectrum emerges as a threshold effect due to the opening of the $\bar{p}p$ channel.

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I. INTRODUCTION

The $X(1835)$ resonance, first discovered by the BES Collaboration in 2005 in the decay $J/\psi \to \gamma\eta'\pi^+\pi^-$ [1] and subsequently seen in other reactions [2–4], but only faintly by other groups [5,6], has a long and winding history. Initially the resonance was associated with the anomalous near-threshold enhancement in the antiproton-proton ($\bar{p}p$) invariant-mass spectrum in the reaction $J/\psi \to \gamma\bar{p}p$ [7,8] which would point to a baryonium-type state (or a $\bar{N}N$ quasibound state) as a possible explanation for its structure. However, with increasing statistics [9] it became clear that the two phenomena are not necessarily connected, not least due to a striking difference in the width of the respective resonances required for describing the invariant-mass spectra of the two reactions in question. Yet another facet was added in the most recent publication of the BESIII Collaboration on the decay $J/\psi \to \gamma\eta'\pi^+\pi^-$ [10]. Now the initial peak around 1835 MeV is practically gone but has reappeared as a structure that is located very close to the $\bar{p}p$ threshold, namely around 1870 MeV.

A more detailed coverage of the historical development regarding the $X(1835)$ resonance can be found in recent summary papers [11,12]. These works provide also an overview of the large amount of theoretical investigations performed in the context of the $X(1835)$. Naturally, in many of them an interpretation of the resonance in terms of a baryonium state is the key element. Indeed, some of these studies attempt to establish a direct and quantitative connection between the resonance and predictions of $\bar{N}N$ potentials that were fitted to $\bar{p}p$ scattering data [13,14].

In the present work we aim at a quantitative analysis of the most recent BESIII data on the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$ [10]. The study is based on the hypothesis that the structure seen in the invariant mass spectrum is indeed linked with the opening of the $\bar{p}p$ channel. The incentive for that comes from past studies of e^+e^- annihilation into multipion states. Also in this case, and specifically in the reactions $e^+e^- \to 3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $\omega\pi^+\pi^-\pi^0$, and $e^+e^- \to 2(\pi^+\pi^-)\pi^0$, structures were observed in the experiments at energies around the $\bar{p}p$ threshold [15–18]. Calculations by our group [19] and others [20] suggested that two-step processes $e^+e^- \to \bar{N}N \to$ multipions could play an important role and their inclusion even allowed one to reproduce the data quantitatively near the $\bar{N}N$ threshold. Accordingly, the structures seen in the experiments found a natural explanation as a threshold effect due to the opening of the $\bar{N}N$ channel, for the majority of the measured channels.

As already indicated above, with the new $J/\psi \to \gamma\eta'\pi^+\pi^-$ data [10] the region of interest is now shifted likewise to energies around the $\bar{p}p$ threshold. Accordingly, we investigate the significance of the $\bar{N}N$ channel for the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$. Since the decay $J/\psi \to \gamma\bar{p}p$ constitutes one segment of the assumed two-step process (the other being $\bar{p}p \to \eta'\pi^+\pi^-$), we reconsider this decay process in the present paper. Indeed, we had already shown in earlier studies that it is possible to describe the large near-threshold enhancement observed in the reaction $J/\psi \to \gamma\bar{p}p$ by the final-state interaction (FSI) provided by the $\bar{N}N$ interaction [21–23]; see also Refs. [13,14,24–26].

A main ingredient of our present calculation is the $\bar{N}N$ interaction. Here we build on our latest $\bar{N}N$ potential, derived in the framework of chiral effective field theory (EFT) up to next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) [27]. That potential reproduces the amplitudes determined in a partial-wave analysis (PWA) of $\bar{p}p$ scattering data [28] from the $\bar{N}N$ threshold up to laboratory energies of $T_{\mathrm{lab}} \approx$ 200–250 MeV [27].

The paper is structured in the following way. In Sec. II an overview of the employed formalism is provided. Section III is devoted to the reaction $J/\psi \to \gamma\bar{p}p$, the first segment of the considered two-step process. In particular, a comparison with the $J/\psi \to \gamma\bar{p}p$ data from the BESIII Collaboration is presented. As in our initial study [23], a refit of the $\bar{N}N$ amplitudes in the $^1S_0$ partial wave with isospin $I = 1$ is required. The second segment of the considered two-step process, the reaction $\bar{p}p \to \eta'\pi^+\pi^-$, is discussed in Sec. IV. However, the main focus of this section is on the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$ and results for the $\eta'\pi^+\pi^-$ invariant-mass spectrum are presented. It turns out that the structure observed in the BESIII experiment at invariant masses near the $\bar{N}N$ threshold is very well reproduced, once effects due to the coupling to the $\bar{N}N$ channel are explicitly taken into account. In view of that observation, and in the light of the conjectured $X(1835)$ resonance, the employed $\bar{N}N$ interactions are examined with regard to possible bound states. The paper ends with concluding remarks.

II. FORMALISM

Our study of the processes $J/\psi \to \gamma\bar{p}p$ and $J/\psi \to \gamma\eta'\pi^+\pi^-$ is based on the distorted-wave Born approximation (DWBA). It amounts to solving the following set of formally coupled equations:

$$\begin{aligned} T_{\bar{N}N\to\bar{N}N} &= V_{\bar{N}N\to\bar{N}N} + V_{\bar{N}N\to\bar{N}N} G_0 T_{\bar{N}N\to\bar{N}N}, \\ T_{\bar{N}N\to\eta'\pi\pi} &= V_{\bar{N}N\to\eta'\pi\pi} + T_{\bar{N}N\to\bar{N}N} G_0 V_{\bar{N}N\to\eta'\pi\pi}, \\ A_{J/\psi\to\gamma\bar{N}N} &= A^0_{J/\psi\to\gamma\bar{N}N} + A^0_{J/\psi\to\gamma\bar{N}N} G_0 T_{\bar{N}N\to\bar{N}N}, \end{aligned} \qquad (1)$$

$$\begin{aligned} A_{J/\psi\to\gamma\eta'\pi\pi} &= A^0_{J/\psi\to\gamma\eta'\pi\pi} + A^0_{J/\psi\to\gamma\bar{N}N} G_0 T_{\bar{N}N\to\eta'\pi\pi} \\ &= A^0_{J/\psi\to\gamma\eta'\pi\pi} + A_{J/\psi\to\gamma\bar{N}N} G_0 V_{\bar{N}N\to\eta'\pi\pi}. \end{aligned} \qquad (2)$$

The first line in Eq. (1) is the Lippmann-Schwinger equation from which the $\bar{N}N$ scattering amplitude ($T_{\bar{N}N}$), is obtained, for a specific $\bar{N}N$ potential $V_{\bar{N}N}$; see Refs. [27,29] for details. The quantity G_0 denotes the free $\bar{N}N$ Green's function. The second equation defines the amplitude for $\bar{N}N$ annihilation into the $\eta'\pi^+\pi^-$ channel while the third equation provides the $J/\psi \to \gamma\bar{N}N$ transition amplitude. Finally, Eq. (2) defines the $J/\psi \to \gamma\eta'\pi^+\pi^-$ amplitude. The quantities A^0_ν denote the elementary (or primary) decay amplitudes for J/ψ to $\gamma\bar{N}N$ or $\gamma\eta'\pi\pi$.

General selection rules [23] but also direct experimental evidence [3] suggest that the specific (and unique) $\bar{N}N$ partial wave that plays a role for energies around the $\bar{p}p$ threshold is the $^1S_0$. For it the equation for the amplitude $A_{J/\psi\to\gamma\bar{N}N}$ reads [23]

$$A = A^0 + \int_0^\infty \frac{dpp^2}{(2\pi)^3} A^0 \frac{1}{2E_k - 2E_p + i0^+} T(p,k;E_k), \qquad (3)$$

where k and E_k are the momentum and energy of the proton (or antiproton) in the center-of-mass system of the $\bar{N}N$ pair, i.e., $E_k = \sqrt{m_p^2 + k^2}$, where m_p is the proton (nucleon) mass. The subscript of A indicating the channel is omitted in Eq. (3) for simplicity.

The $\bar{N}N$ T matrix that enters Eq. (3) fulfils

$$\begin{aligned} T(p', k; E_k) = V(p', k) &+ \int_0^\infty \frac{dpp^2}{(2\pi)^3} V(p', p) \\ &\times \frac{1}{2E_k - 2E_p + i0^+} T(p, k; E_k), \end{aligned} \qquad (4)$$

where V represents the $\bar{N}N$ potential in the $^1S_0$ partial wave.

Following the strategy in Refs. [27,29], the elementary annihilation potential for $\bar{N}N \to \eta'\pi^+\pi^-$ and the transition amplitude $A^0_{J/\psi\to\gamma\bar{N}N}$ are parametrized by

$$V_{\bar{N}N\to\eta'\pi\pi}(q) = \tilde{C}_{\eta'\pi\pi} + C_{\eta'\pi\pi} q^2, \qquad (5)$$

$$A^0_{J/\psi\to\gamma\bar{N}N}(q) = \tilde{C}_{J/\psi\to\gamma\bar{N}N} + C_{J/\psi\to\gamma\bar{N}N} q^2, \qquad (6)$$

i.e., by two contact terms analogous to those that arise up to next-to-next-to-leading order ($\mathrm{N^2LO}$) in the treatment of the $\bar{N}N$ interaction within chiral EFT [27]. The quantity q in Eq. (5) is the center-of mass (c.m.) momentum in the $\bar{N}N$ system. We multiply the transition potentials in Eqs. (5) and (6) with a regulator (of exponential type) in the actual calculations. This is done consistently with the $\bar{N}N$ potentials in Ref. [27] where such a regulator is included. We also employ the same cutoff parameter as in the $\bar{N}N$ sector. Since the threshold for the $\eta'\pi\pi$ channel lies significantly below the one for $\bar{N}N$, the mesons carry—on average—already fairly high momenta. Thus, the dependence of the annihilation potential on those momenta should be small for energies around the $\bar{N}N$ threshold and it is, therefore, neglected [23]. The constants $\tilde{C}_\nu$ and C_ν can be determined by a fit to the $\bar{N}N \to \eta'\pi\pi$ cross section (and/or branching ratio) and the $J/\psi \to \gamma\bar{p}p$ invariant-mass spectrum, respectively. Note that those constants (and specifically the $\tilde{C}_\nu$'s) are basically normalization constants. The invariant-mass dependence of the spectrum is primarily determined by the FSI due to the $\bar{N}N$ interaction.

The term $A^0_{J/\psi\to\gamma\eta'\pi^+\pi^-}$ is likewise parametrized in the form (6), but as a function of the $\eta'\pi\pi$ invariant mass Q,

$$A^0_{J/\psi\to\gamma\eta'\pi\pi}(Q) = \tilde{C}_{J/\psi\to\gamma\eta'\pi\pi} + C_{J/\psi\to\gamma\eta'\pi\pi}Q. \quad (7)$$

The arguments for neglecting the dependence on the individual meson momenta are the same as above and they are valid again, of course, only for energies around the $\bar{N}N$ threshold. However, since in the $\eta'\pi\pi$ case this term represents a background amplitude rather than a transition potential we allow the corresponding constants to be complex valued, to be fixed by a fit to the $J/\psi \to \gamma\eta'\pi^+\pi^-$ event rate.

The explicit form of Eq. (2) reads

$$A_{\gamma\eta'\pi\pi,J/\psi}(X;Q) = A^0_{\gamma\eta'\pi\pi,J/\psi}(X;Q) + \int_0^\infty \frac{dqq^2}{(2\pi)^3} \times V_{\eta'\pi\pi,\bar{p}p}(X,q)\frac{1}{Q-2E_q+i0^+} \times A_{\gamma\bar{p}p,J/\psi}(q;Q), \quad (8)$$

written in matrix notation. The quantity X stands here symbolically for the momenta in the $\eta'\pi\pi$ system. But since we assumed that the transition potential does not depend on those momenta, cf. Eqs. (5) and (7), X does not enter anywhere into the actual calculation of the amplitudes. All amplitudes (and the potential) can be written and evaluated as functions of the c.m. momenta in the $\bar{N}N$ (q) system and of the invariant mass Q in the $\eta'\pi\pi$ system, where the latter is identical to the energy in the $\bar{N}N$ subsystem.

Since the amplitudes do not depend on X the integration over the three-meson phase space can be done separately when the cross section or the invariant-mass spectrum is calculated. In practice, it amounts only to a multiplicative factor and, moreover, to a factor that is the same for the $\bar{N}N \to \eta'\pi\pi$ cross section and the $J/\psi \to \gamma\eta'\pi\pi$ invariant-mass spectrum for a fixed value of Q. We perform this phase-space integration numerically.

Of course, ignoring the dependence of $A^0_{J/\psi\to\gamma\eta'\pi\pi}$ on the $\eta'\pi\pi$ momenta is only meaningful for energies around the $\bar{N}N$ threshold. We cannot extend our calculation down to the threshold of the $\eta'\pi\pi$ channel. However, one has to keep in mind that also the validity of our $\bar{N}N$ interaction is limited to energies not too far away from the $\bar{N}N$ threshold.

The differential decay rate for the processes $J/\psi \to \gamma\bar{p}p$ can be written in the form [23,30]

$$\frac{d\Gamma}{dQ} = \frac{\lambda^{1/2}(m^2_\psi, Q^2, m^2_x)\sqrt{Q^2-4m^2_p}}{2^7\pi^3 m^3_\psi}|\mathcal{M}_{J/\psi\to\gamma\bar{p}p}|^2, \quad (9)$$

after integrating over the angles. Here the Källén function λ is defined as $\lambda(x,y,z) = (x-y-z)^2 - 4yz$, $Q \equiv M_{\bar{p}p}$ is the invariant mass of the $\bar{p}p$ system, m_ψ, m_p, m_x are the masses of the J/ψ, the proton, and the meson (or photon) in the final state, in order, while $\mathcal{M}$ is the total Lorentz-invariant reaction amplitude. The relations between the A's in Eqs. (1) and (2) and the Lorentz-invariant amplitudes $\mathcal{M}$ for the various reactions are [31]

$$\begin{aligned}
\mathcal{M}_{\bar{N}N\to\bar{N}N} &= -8\pi^2 E_N^2 T_{\bar{N}N\to\bar{N}N},\\
\mathcal{M}_{J/\psi\to\gamma\bar{p}p} &= -8\pi^2 E_N\sqrt{E_\gamma E_{J/\psi}}A_{J/\psi\to\gamma\bar{p}p},\\
\mathcal{M}_{\bar{N}N\to\eta'\pi^+\pi^-} &= -32\sqrt{\pi^7}E_N\sqrt{E_{\eta'}E_{\pi^+}E_{\pi^-}}A_{\bar{N}N\to\eta'\pi^+\pi^-},\\
\mathcal{M}_{J/\psi\to\gamma\eta'\pi^+\pi^-} &= -32\sqrt{\pi^7}\sqrt{E_\gamma E_{J/\psi}E_{\eta'}E_{\pi^+}E_{\pi^-}}A_{J/\psi\to\gamma\eta'\pi^+\pi^-}.
\end{aligned} \quad (10)$$

The energies in the reactions $J/\psi \to \gamma\bar{p}p$, $\bar{N}N \to \eta'\pi^+\pi^-$, and $J/\psi \to \gamma\eta'\pi^+\pi^-$ are given by

$$\begin{aligned}
E_N &= Q/2,\\
E_{J/\psi} &= \frac{m^2_\psi + Q^2}{2Q},\\
E_\gamma &= \frac{m^2_\psi - Q^2}{2Q},\\
E_{\eta'} &= \frac{Q^2 - t_1 + m^2_{\eta'}}{2Q},\\
E_{\pi^+} &= \frac{Q^2 - t_2 + m^2_\pi}{2Q},\\
E_{\pi^-} &= \frac{t_1 + t_2 - m^2_\pi - m^2_{\eta'}}{2Q},
\end{aligned}$$

where Q is either the energy in the $\bar{N}N$ system or the invariant mass of the $\bar{p}p$ or $\eta'\pi^+\pi^-$ system ($M_{\bar{p}p}$ or $M_{\eta'\pi^+\pi^-}$), $t_1 = M^2_{\pi^+\pi^-}$, and $t_2 = M^2_{\pi^-\eta'}$.

In Eq. (9) it is assumed that averaging over the spin states has been already performed. Anyway, in the present manuscript we will consider only individual partial-wave amplitudes. The cross section for the reaction $\bar{p}p \to \eta'\pi^+\pi^-$ is given by

$$\sigma(\bar{p}p \to \eta'\pi^+\pi^-) = \int_{t_1^-}^{t_1^+} dt_1 \int_{t_2^-}^{t_2^+} \frac{dt_2|\mathcal{M}_{\bar{p}p\to\eta'\pi\pi}|^2}{1024\pi^3 Q^3\sqrt{Q^2-4m^2_p}}, \quad (11)$$

where

$$t_1^- = 4m_\pi^2,$$
$$t_1^+ = (Q - m_{\eta'})^2,$$
$$t_2^- = \frac{1}{4t_1}((Q^2 - m_{\eta'}^2)^2 - [\lambda^{1/2}(Q^2, t_1, m_{\eta'}^2) + \lambda^{1/2}(t_1, m_\pi^2, m_\pi^2)]^2),$$
$$t_2^+ = \frac{1}{4t_1}((Q^2 - m_{\eta'}^2)^2 - [\lambda^{1/2}(Q^2, t_1, m_{\eta'}^2) - \lambda^{1/2}(t_1, m_\pi^2, m_\pi^2)]^2). \quad (12)$$

The decay rate for $J/\psi \to \gamma\eta'\pi^+\pi^-$ is given by

$$\frac{d\Gamma}{dQ} = \int_{t_1^-}^{t_1^+} dt_1 \int_{t_2^-}^{t_2^+} dt_2 \frac{(m_\psi^2 - Q^2)|\mathcal{M}_{J/\psi\to\gamma\eta'\pi^+\pi^-}|^2}{6144\pi^5 m_\psi^3 Q}. \quad (13)$$

III. THE REACTION $J/\psi \to \gamma\bar{p}p$

Due to the unusually large enhancement observed in the near-threshold $\bar{p}p$ invariant-mass spectrum in the reaction $J/\psi \to \gamma\bar{p}p$ [7,8,32], it has been the topic of many studies and a variety of explanations for the strongly peaked spectrum have been suggested [11,12]. In scenarios like ours, were FSI effects in the $\bar{N}N$ channel are assumed to be responsible for the enhancement, one faces a challenging task. There are measurements for several other decay channels where the produced $\bar{N}N$ state must be in the very same partial wave, the $^1S_0$, at least near threshold, and accordingly, in principle, the same FSI effects should arise. This concerns the reactions $J/\psi \to \omega\bar{p}p$ [33] and $J/\psi \to \phi\bar{p}p$ [34], and also $\psi(2S) \to \gamma\bar{p}p$ [8]. No enhancements of a comparable magnitude were observed in experiments for any of these reactions. So far, a few suggestions for a way out of this dilemma have been made [14,23,26]. In our own work the emphasis was always on the isospin dependence. Already in our initial studies [21,22], still based on the Migdal-Watson approximation and on the Jülich meson-exchange $\bar{N}N$ potential [35,36], it was the isospin $I = 1$ amplitude that produced the large enhancement. Then there is no conflict with the rather moderate enhancements observed in the $J/\psi \to \omega\bar{p}p$ and $J/\psi \to \phi\bar{p}p$ channels, because in those cases the produced $\bar{p}p$ system has to be in $I = 0$ (assuming that isospin is conserved in this purely hadronic decay).

Indeed, in the decays $J/\psi \to \gamma\bar{p}p$ and $\psi(2S) \to \gamma\bar{p}p$ isospin is not conserved and, therefore, in principle, one can have any combination of the $I = 0$ and $I = 1$ amplitudes. This freedom was exploited in a recent and more refined study of J/ψ decays by our group [23]. In that work we not only treated the FSI effects within a DWBA approach, but we also employed an $\bar{N}N$ potential that was derived within the framework of chiral effective field theory up to N^{2}LO [29]. Utilizing the "standard" hadronic combination for the $\bar{p}p$ amplitude, namely $T = T_{\bar{p}p} = (T^{I=0} + T^{I=1})/2$, for J/ψ decay and one with a predominant $I = 0$ component, $T = (0.9T^0 + 0.1T^1)$ for $\psi(2S)$ decay allowed us to achieve a consistent description of the $\gamma\bar{p}p$ spectrum for both decays [23].

Nonetheless, it should be said that we had to depart slightly from the $I = 1$ $^1S_0$ $\bar{N}N$ amplitude as determined in the PWA of Zhou and Timmermans [28]. However, already a rather modest modification of the interaction in the $I = 1$ channel—subject to the constraint that the corresponding partial-wave cross sections for $\bar{p}p \to \bar{p}p$ and $\bar{p}p \to \bar{n}n$ remain practically unchanged at low energies—allowed us to reproduce the event distribution of the radiative J/ψ decay, and consistently all other decays [23].

In the present work we repeat this exercise, employing now the new $\bar{N}N$ interaction [27]. First of all, we want to see whether the same scenario holds for the improved $\bar{N}N$ potential that is based on a different regularization scheme and that is now calculated up to N^{3}LO. In addition we have to establish the $J/\psi \to \gamma\bar{p}p$ amplitude in the $I = 0$ channel that enters into the calculation of the two-step process; see Eq. (2). Results for the $\bar{N}N$ sector, i.e., the $I = 1$ $^1S_0$ amplitude, are shown in Fig. 1. The parameters of the fit are

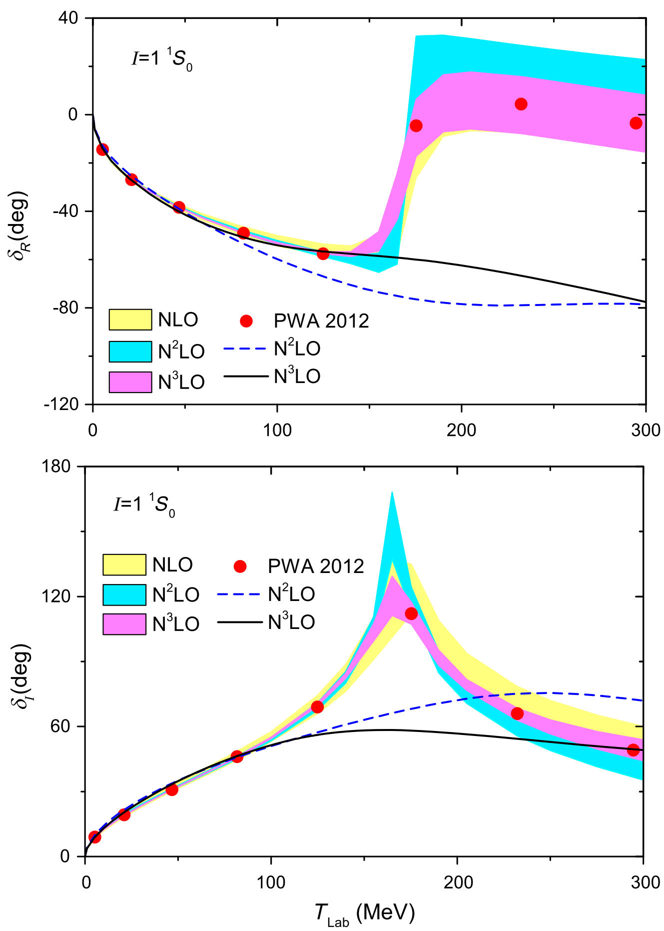

FIG. 1. Real and imaginary parts of the $^1S_0$ phase shift in the isospin $I = 1$ channel. The bands represent the fits to the PWA [28] (circles) at NLO, N^{2}LO, and N^{3}LO from Ref. [27]. The dashed and solid lines are refits at N^{2}LO and N^{3}LO, respectively, utilized in the present work.

TABLE I. Low-energy constants at N^2LO and N^3LO, for the $\bar{N}N$ interaction in the $I = 1$ $^1S_0$ partial wave. Note that all parameters are in units of 10^4; see Ref. [27] for details.

	N^2LO	N^3LO
$\tilde{C}_{3^1S_0}$ (GeV^{-2})	0.1935(14)	0.3155(15)
$C_{3^1S_0}$ (GeV^{-4})	−1.8160(52)	−3.5235(101)
$D^1_{3^1S_0}$ (GeV^{-6})	⋯	−8.0840(627)
$D^2_{3^1S_0}$ (GeV^{-6})	⋯	10.0000(286)
$\tilde{C}^a_{3^1S_0}$ (GeV^{-1})	0.1733(25)	0.0230(33)
$C^a_{3^1S_0}$ (GeV^{-3})	−4.1780(21)	−3.1759(100)

summarized in Table I. Corresponding results for the $\bar{p}p$ invariant-mass spectrum of the reaction $J/\psi \to \gamma\bar{p}p$ are displayed in Fig. 2. It is reassuring to see that the results are basically the same as those reported in Ref. [23] for the chiral N^2LO interaction. The presented results are for the combination $T = (0.4T^0 + 0.6T^1)$ that yields the lowest χ^2 value in the fit. Note, however, that those for weights of the isospin amplitudes differing by, say, ± 0.1 are very similar, even on a quantitative level.

Interestingly, the modified potential in Ref. [23] generates a bound state in the $I = 1$ $^1S_0$ partial wave which was not the case for the original interaction presented in Ref. [29]. For example, for the cutoff combination $\{\Lambda, \tilde{\Lambda}\} = \{450\ \mathrm{Mev}, 500\ \mathrm{Mev}\}$ the bound state is located at $E_B = (-36.9 - i47.2)$ MeV, where the real part denotes the energy with respect to the $\bar{N}N$ threshold. As noted in Ref. [23], this bound state is not very far away from the position of the $X(1835)$ resonance found by the BES Collaboration in the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$ [1,9,10]. However, the bound state in Ref. [23] is in the $I = 1$ channel *and not* in $I = 0$ as advocated in publications of the BES Collaboration [1] and of other authors [13,14]. The refit of the new $\bar{N}N$ potential [27] employed in the present study leads likewise to a bound state in the $I = 1$ $^1S_0$ partial wave. The binding energies are $E_B = (-50.8 - i40.9)$ MeV for the chiral N^3LO interaction and $E_B = (-2.1 - i94.0)$ MeV for the chiral N^2LO interaction. The former value is close to that found in our earlier work [23], while the latter differs drastically. Once again, this illustrates the warning remarks in Ref. [23] that, in general, any data above the reaction threshold, like the $\bar{p}p$ invariant-mass spectrum or even phase shifts, do not allow to pin down the binding energy reliably.

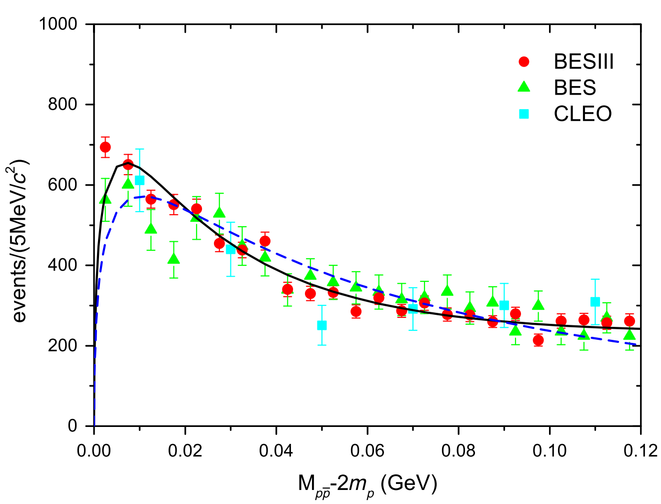

FIG. 2. $J/\psi \to \gamma\bar{p}p$ results with a refitted $I = 1$ $^1S_0$ amplitude, analogous to Ref. [23]. Data are from Ref. [8] (BESIII), Ref. [7] (BES), and Ref. [32] (CLEO). Note that the latter two are scaled to those by the BESIII Collaboration by eye.

IV. THE REACTION $J/\psi \to \gamma\eta'\pi^+\pi^-$

As already mentioned in the Introduction, in studies of e^+e^- annihilation to multipion states structures were observed around the $\bar{N}N$ threshold for several channels, specifically in $e^+e^- \to 3(\pi^+\pi^-)$, $e^+e^- \to 2(\pi^+\pi^-\pi^0)$, and $e^+e^- \to 2(\pi^+\pi^-)\pi^0)$ [15–18]. An analysis of those structures performed by us [19] and by others [20] suggested that they could be simply a result of a threshold effect due to the opening of the $\bar{N}N$ channel. In that work we could estimate the contribution of the two-step process $e^+e^- \to \bar{N}N \to$ multipions to the total reaction amplitude rather reliably because cross-section measurements for all involved processes were available in the literature. Specifically, the amplitude for $e^+e^- \to \bar{N}N$ could be constrained from near-threshold data on the $e^+e^- \to \bar{p}p$ cross section and the one for $\bar{N}N \to 5\pi, 6\pi$ could be fixed from available experimental information on the corresponding annihilation ratios [37]. It turned out that the resulting amplitude for $e^+e^- \to \bar{N}N \to$ multipions was large enough to play a role for the considered e^+e^- annihilation channels and that it is possible to reproduce the data quantitatively near the $\bar{N}N$ threshold in most of the considered reaction channels [19].

In the case of $J/\psi \to \gamma\eta'\pi^+\pi^-$ we are not in such an advantageous situation. While cross sections (or branching ratios) are available for $\bar{p}p \to \eta'\pi^+\pi^-$, so far only event rates have been published for $J/\psi \to \gamma\eta'\pi^+\pi^-$ itself and for $J/\psi \to \gamma\bar{p}p$. Thus, a reliable assessment of the magnitude of the two-step process $J/\psi \to \gamma\bar{p}p \to \gamma\eta'\pi^+\pi^-$ cannot be given at present. Nonetheless, in the following we provide a rough order-of-magnitude estimate and plausibility arguments for why we believe that the $\bar{N}N$ intermediate step should play an important role here. The main and most important support comes certainly from the $\gamma\eta'\pi^+\pi^-$ data itself, where a clear structure is seen near the $\bar{N}N$ threshold in the latest high-statistics measurement by the BESIII Collaboration [10]. In addition a comparison of the event rates for $J/\psi \to \gamma\bar{p}p$ and $J/\psi \to \gamma\eta'\pi^+\pi^-$ with the cross sections for $\bar{p}p \to \bar{p}p$ in the $^1S_0$ partial wave and for $\bar{p}p \to \eta'\pi^+\pi^-$ suggests that the two-step process in question should be of relevance.

Let us discuss the latter issue in more detail. With the central value of the branching ratio, $\mathrm{BR}(\bar{p}p \to \eta'\pi^+\pi^-) = 0.626\%$ [38], the resulting cross section at $p_{\mathrm{lab}} = 106\ \mathrm{MeV}/c$ is 2.23 mb, based on the total annihilation cross section given in Ref. [39]. Though the branching ratio is tiny, at first sight, one has to compare the resulting cross section with the relevant quantity, namely the $\bar{p}p$ elastic cross section in the $^1S_0$ partial wave. The latter is around 20 mb in our $\bar{N}N$ potential [27], but also in the PWA [28]. Thus, the annihilation cross section for $\bar{p}p \to \eta'\pi^+\pi^-$ is roughly a factor of 10 smaller than that for $\bar{p}p \to \bar{p}p$.

When comparing the event rates one has to consider that the number of J/ψ decay events used in the $\gamma\eta'\pi^+\pi^-$ analysis [10] is roughly a factor of 5 larger than that in the $\gamma\bar{p}p$ paper [8]. Moreover, the bin size is different. Combining those two aspects suggests a roughly 5 times larger rate for $\gamma\bar{p}p$, based on the data shown in Refs. [8,10], which mostly compensates for the factor of 10 reduction estimated above. Accordingly, in principle, the two-step process via an $\bar{N}N$ intermediate state could be responsible for as much as 50% of the total rate.

In the actual calculation we fix the constant $\tilde{C}_{\eta'\pi\pi}$ in the $\bar{N}N \to \eta'\pi\pi$ transition potential [cf. Eq. (5)] from the corresponding annihilation cross section discussed above. Since there is no experimental information on the energy dependence, we set the constant $C_{\eta'\pi\pi}$ to zero. For the amplitude $A_{J/\psi\to\gamma\bar{p}p}$ we employ the one described in Sec. III, with $\tilde{C}_{J/\psi\to\gamma\bar{N}N}$ fixed to the most recent BESIII data [10]. However, we allow for some variations of the overall magnitude because, as said above, only event rates are available in this case. The value for $C_{J/\psi\to\gamma\bar{N}N}$ obtained in the fit turned out to be very small so that we simply set it to zero.

Finally, the constants in the quantity $A^0_{J/\psi\to\gamma\eta'\pi^+\pi^-}$ [cf. Eq. (7)] are adjusted to the event rate for $J/\psi \to \gamma\eta'\pi^+\pi^-$. This term has to account for all other contributions to $J/\psi \to \gamma\eta'\pi^+\pi^-$, besides the one with an intermediate $\gamma\bar{N}N$ state. Thus, it can have a relative phase as compared to the contribution from the $\bar{N}N$ loop, i.e., the corresponding C's can be complex valued. However, it turns out that optimal results are already achieved for real values of $\tilde{C}_{J/\psi\to\gamma\eta'\pi\pi}$ and $C_{J/\psi\to\gamma\eta'\pi\pi}$. In the fit we consider data in the range $1800\ \mathrm{MeV} \le E \le 1950\ \mathrm{MeV}$, i.e., in a region that encompasses more or less symmetrically the $\bar{N}N$ threshold.

Our results for the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$ are presented in Figs. 3 and 4. They are based on the N^{2}LO and N^{3}LO EFT $\bar{N}N$ interactions with the cutoff $R = 0.9$ fm ($\Lambda = 438$ MeV), cf. Ref. [27] for details. Exploratory calculations for the other cutoffs considered in Ref. [27] turned out to be very similar. Like for $\bar{N}N$ scattering itself, much of the cutoff dependence is absorbed by the contact terms [$\tilde{C}_\nu$ and C_ν in Eqs. (5) and (6)] that are fitted to the data so that the variation of the results for energies of, say, ± 50 MeV around the $\bar{N}N$ threshold is rather small. For consistency the momentum-space regulator function as given in Eq. (3.1) (right side) in Ref. [27] is also attached to the transition potentials in Eqs. (5) and (6), i.e., to all quantities that depend on the $\bar{N}N$ momentum q.

In Fig. 3 a more detailed view on our calculation is presented, exemplary for the N^{3}LO interaction. Full results for the $\eta'\pi^+\pi^-$ invariant-mass spectrum (solid line) are shown, together with the individual contributions from the $J/\psi \to \gamma\bar{N}N \to \gamma\eta'\pi\pi$ transition (dotted line) and the

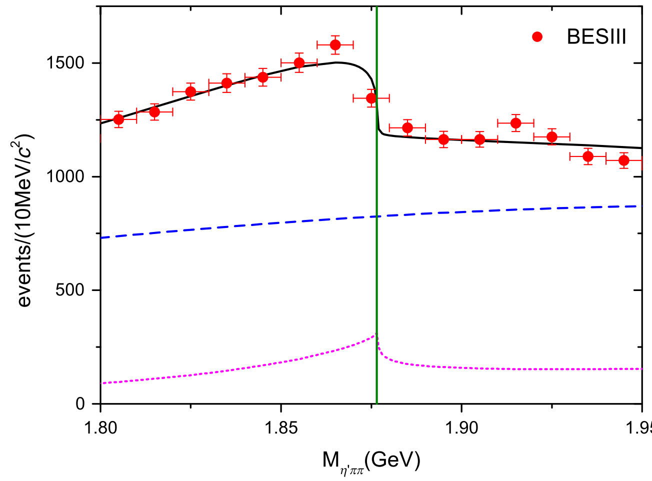

FIG. 3. The $\eta'\pi^+\pi^-$ invariant-mass spectrum in the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$. Results for the contribution from the $J/\psi \to \gamma\bar{N}N \to \gamma\eta'\pi^+\pi^-$ transition (dotted line) and the background term (dashed line) are shown, together with the full results (solid line). The N^{3}LO $\bar{N}N$ potential [27] is employed. Data are from the BESIII Collaboration [10]. The horizontal line indicates the $\bar{p}p$ threshold.

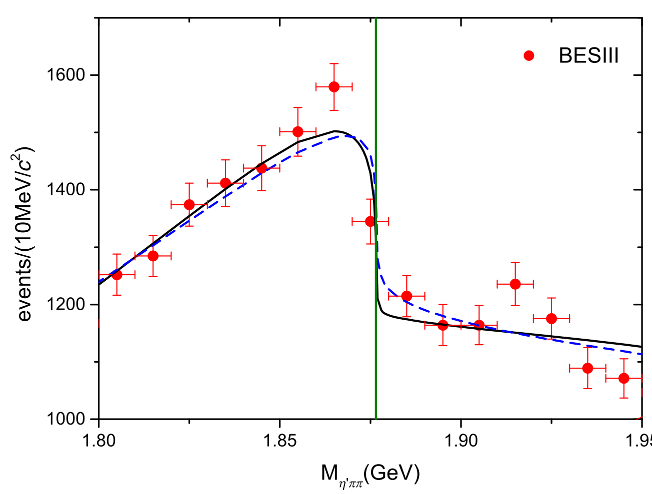

FIG. 4. Results for $J/\psi \to \gamma\eta'\pi^+\pi^-$ including the background term and $\bar{N}N \to \eta'\pi^+\pi^-$ transition amplitude for the N^{2}LO (dashed line) and N^{3}LO (solid line) $\bar{N}N$ interactions. Data are from the BESIII Collaboration [10]. The horizontal line indicates the $\bar{p}p$ threshold.

background term (dashed line) that simulates contributions which do not involve the $\bar{N}N$ intermediate state. By construction the background is a smooth function of the $\eta'\pi^+\pi^-$ invariant mass, whereas the contribution from the two-step process via the $\bar{N}N$ channel exhibits a pronounced cusp-like structure at the $\bar{N}N$ threshold. The (square of the) latter amplitude is roughly a factor of 3 smaller than the background contribution. However, there is a sizable interference between the two amplitudes so that the coherent sum reflects the opening of (coupling to) the $\bar{N}N$ channel and leads to results for the invariant-mass spectrum that are very close to the measurements of the BESIII Collaboration. The actual magnitude of the $\bar{N}N$ two-step process is about 20% of the total event rate, well in line with the rough estimate provided above, based on data for the $J/\psi \to \gamma\bar{p}p$ and $\bar{p}p \to \eta'\pi^+\pi^-$ reactions.

In Fig. 4 we present the complete results for the N^{2}LO and N^{3}LO interactions, on a scale similar to that in the BESIII publication [10], cf. the inserts in Figs. 3 and 4 of that reference. First we note that the $\eta'\pi^+\pi^-$ invariant mass spectrum based on the two $\bar{N}N$ interactions is very similar around the $\bar{N}N$ threshold. It is also very similar to the fit within the *first model* considered in Ref. [10] (cf. the corresponding Fig. 3). That model includes explicitly a $X(1835)$ resonance and simulates the effect of the $\bar{N}N$ channel via a Flatté formula [40]. Obviously, in our calculation the data can be described with the same quality, but without such a $X(1835)$ resonance. The more elaborate treatment of the coupling to the $\bar{N}N$ channel via Eq. (8) with the explicit inclusion of the $\bar{N}N$ interaction itself is already sufficient to generate an invariant-mass dependence in line with the data.

For completeness, let us mention that a second resonance has been introduced in Ref. [10] in the invariant-mass region covered by our study, namely a $X(1920)$, in order to reproduce a possible enhancement at the corresponding invariant mass suggested by two data points, cf. Fig. 4. Furthermore, a *second model* has been considered in Ref. [10] where instead of the coupling to the $\bar{N}N$ channel an additional and rather narrow resonance was included—the $X(1870)$. In that scenario a slightly better description of the data very close to the $\bar{N}N$ threshold could be achieved.

Now the key question is, of course, are those structures seen in the experiment a signal for a $\bar{N}N$ bound state? We did not find any near-threshold poles for our EFT $\bar{N}N$ interactions in the $^1S_0$ partial wave with $I = 0$, i.e., the one relevant for the $\gamma\eta'\pi^+\pi^-$ channel, neither for the N^{2}LO potential presented in Ref. [29] nor for the new N^{2}LO and N^{3}LO interactions [27] employed in the present calculation. As already discussed in the preceding section, there is only a pole in the $I = 1$ case in the versions established in the study of the reactions $J/\psi \to \gamma\bar{p}p$.

Thus, our results provide a clear indication that bound states are not necessarily required for achieving a quantitative description of the observed structure in the $\eta'\pi^+\pi^-$ invariant-mass spectrum near the $\bar{p}p$ threshold. This is in contrast to other investigations in the literature. For example, bound states in the $I = 0$ $^1S_0$ partial wave are present in the Paris $\bar{N}N$ potential [41] employed in Refs. [13,26] [$E_B = (-4.8 - i26)$ MeV] and also in the $\bar{N}N$ interaction constructed in Ref. [14] [$E_B = (22 - i33)$ MeV]. In the latter case, the positive sign of the real part of E_B indicates that the pole found is actually located above the $\bar{N}N$ threshold (in the energy plane). As discussed in Ref. [14], the pole moves below the threshold when the imaginary part of the potential is switched off and that is the reason why it is referred to as bound state.

In this context, it is worth mentioning that no bound states or resonances were found in a study of the $\eta' K\bar{K}$ system [42] in an attempt to explore if such states could be generated dynamically as $\eta' f_0(980)$- or $\eta' a_0(980)$-like configurations.

Past studies suggest that there is a distinct difference in the amplitude for $J/\psi \to \gamma +$ mesons due to the $\bar{N}N$ loop contribution depending on the absence/presence of a bound state. Its modulus exhibits specific features, namely either a genuine cusp at the $\bar{N}N$ threshold (cf. Fig. 3) or a rounded step and a maximum below the threshold. This was discussed in detail in Ref. [19] in the context of the reaction $e^+e^- \to$ multipions (cf. Fig. 4 in that reference) and also in Ref. [14]. However, in both studies the bound states in question belong to the special class discussed above, i.e., they are located above the $\bar{N}N$ threshold.

In order to illustrate what happens for the case of a "regular" bound state we present here an exemplary calculation based on the $I = 1$ $^1S_0$ partial wave of our N^{3}LO potential, where the binding energy is $(-50.8 - i40.9)$ MeV, cf. Sec. III. A J/ψ decay reaction where the corresponding $\bar{N}N$ loop could contribute is, for example, $J/\psi \to \gamma\omega\rho^0$. Pertinent predictions are shown in Fig. 5. Obviously, the invariant-mass dependence of the loop (dotted line) is fairly different from the one of the $I = 0$ amplitude, cf. the dotted line in Fig. 3. Specifically, there is a clear enhancement in the spectrum around 50 MeV below the $\bar{N}N$ threshold reflecting the presence of the $\bar{N}N$ bound state. Due to the fairly large width ($\Gamma = -2\mathrm{Im}E_B$) the structure is not very pronounced. Of course, the final signal will be strongly influenced and modified by the interference with the background amplitude, as testified by the results presented above for the $\eta'\pi^+\pi^-$ case. To demonstrate this we include also results for two different but arbitrary choices for the background term; see the dashed and solid lines in Fig. 5. Of course, if the $\bar{N}N$ bound state is more narrow then the signal will be certainly more pronounced. Note that the decay $J/\psi \to \gamma\omega\rho^0$ has been already measured by the BES Collaboration [43]. However, the statistics is simply too low to draw any conclusions. It would be definitely interesting to revisit this reaction in a future experiment.

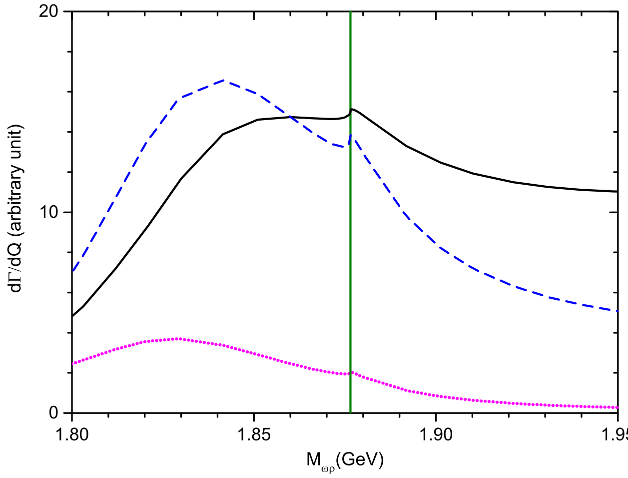

FIG. 5. Predicted $\omega\rho^0$ invariant-mass spectrum for $J/\psi \to \gamma\omega\rho^0$, based on the N^{3}LO $\bar{N}N$ interaction described in Sec. III. The contribution from the $J/\psi \to \gamma\bar{N}N \to \gamma\omega\rho^0$ transition alone (dotted line) and with two arbitrary choices for the background term included (dashed and solid lines) are shown. The horizontal line indicates the $\bar{p}p$ threshold.

V. CONCLUSIONS

We analyzed the origin of the structure associated with the $X(1835)$ resonance, observed in the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$. Specific emphasis was put on the $\eta'\pi^+\pi^-$ invariant mass spectrum around the $\bar{p}p$ threshold, where the most recent BESIII measurement [10] provided strong evidence for an interplay of the $\eta'\pi^+\pi^-$ and $\bar{p}p$ channels.

Motivated by this experimental observation, we evaluated the contribution of the two-step process $J/\psi \to \gamma\bar{p}p \to \gamma\eta'\pi^+\pi^-$ to the total reaction amplitude. The amplitude for $J/\psi \to \gamma\bar{p}p$ was constrained from corresponding data by the BESIII Collaboration, while for $\bar{N}N \to \eta'\pi\pi$ we took available branching ratios for $\bar{p}p \to \eta'\pi^+\pi^-$ as a guideline. Combining the contribution of this two-step process with a background amplitude, that simulates other transition processes which do not involve a $\gamma\bar{N}N$ intermediate state, allowed us to achieve a quantitative description of the invariant-mass dependence shown by the data near the $\bar{p}p$ threshold. In particular, the structure detected in the experiment emerges as a threshold effect. It results from an interference of the smooth background amplitude with the strongly energy-dependent two-step contribution, which itself exhibits a cusp-like behavior at the $\bar{N}N$ threshold.

The question of whether there is evidence for a $\bar{N}N$ bound state was discussed, but no firm conclusion could be made. While in our own calculation such states are not present, and are also not required to describe the data for the reaction $J/\psi \to \gamma\eta'\pi^+\pi^-$, contrary claims have been brought forth in the literature [14,26]. In any case, it should be said that the possibility that a genuine resonance is ultimately responsible for the structure observed in the experiment cannot be categorically excluded based on an analysis like ours. Yet, our calculation provides a strong indication for the important role played by the $\bar{N}N$ channel in the $J/\psi \to \gamma\eta'\pi^+\pi^-$ decay for energies around its threshold and we consider the fact that it yields a natural and quantitative description of the structure observed in the invariant-mass spectrum as rather convincing.

Data with improved resolution around the $\bar{p}p$ threshold could possibly help to shed further light on the relation of a possible $X(1835)$ with the $\bar{p}p$ channel. An absolute determination of the relevant invariant-mass spectra would certainly put stronger constraints on the question whether the intermediate $\bar{p}p$ state can play such an important role as suggested by the present study. In addition, we believe that an analogous measurement for channels like $J/\psi \to \gamma\eta\pi^+\pi^-$ could be very instructive. Indeed, this was already recommended around the time when the first evidence for the $X(1835)$ was reported [44]. The branching ratio for $\bar{p}p \to \eta\pi^+\pi^-$ is more than a factor of 2 larger than that for $\eta'\pi^+\pi^-$ [45] which would enhance the role played by the $\bar{p}p$ channel. Thus, if the count rate for $J/\psi \to \gamma\eta\pi^+\pi^-$ turns out to be similar to that for $\gamma\eta'\pi^+\pi^-$ [30,44] then the effect from the transition to $\bar{p}p$ could be fairly strong.

Finally, we want to mention that there are data on $J/\psi \to \omega\eta\pi^+\pi^-$ [46] and $J/\psi \to \phi\eta\pi^+\pi^-$ [47]. For the latter, $\eta\pi^+\pi^-$ invariant masses corresponding to the $\bar{p}p$ threshold are already close to the boundary of the available phase space and, therefore, no appreciable signal is expected. In the case of $J/\psi \to \omega\eta\pi^+\pi^-$ the BESIII Collaboration sees a resonance-like enhancement at $1877.3 \pm 6.3^{+3.4}_{-7.4}$ MeV [46] which coincides almost perfectly with the $\bar{p}p$ threshold. However, the invariant-mass resolution of those data is only 20 MeV/c^2. Moreover, it is our understanding that non-ω (background) events are not well separated in the data presented in Ref. [46]. These two issues handicap a dedicated analysis for the time being. Clearly, new measurements with higher statistics could be indeed rather useful for providing further information on the role that the (opening of the) $\bar{N}N$ channel plays for the reaction in question.

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