Effect of resonance for *CP* asymmetry of the decay process $\bar{B}_s \to P\pi^+\pi^-$ in perturbative QCD

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In the framework of the perturbative QCD (PQCD) approach we study the direct *CP* asymmetry for the decay channel $\bar{B}_s \to P\pi^+\pi^-$ around the resonance range via the $\rho - \omega$ mixing mechanism (where P refers to pseudoscalar meson). We find that the *CP* asymmetry can be enhanced by $\rho - \omega$ mixing when the masses of the $\pi^+\pi^-$ pairs are at the area of $\rho - \omega$ resonance, and the maximum *CP* asymmetry can reach 59% for the relevant decay channels.

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I. INTRODUCTION

The rich data from B meson factories make the study of B physics a very hot topic. A lot of research has been made, especially for *CP* asymmetry. *CP* asymmetry is an important area in testing the standard model (SM) and searching for new physics signals. The detection of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements plays an important role in the understanding of *CP* asymmetry. The nonleptonic decay of B meson is expected to be ideal decay process in searching *CP* asymmetry. Direct *CP* asymmetry in B meson decay channel arises from weak phase and strong phase differences. In SM, the weak phase is responsible for the *CP* asymmetry by CKM matrix [1,2]. Meanwhile, the large strong phase is needed for producing *CP* asymmetry which comes from QCD correction. Recently, the large *CP* asymmetry was found by the LHCb Collaboration in the three-body decay channels of $B^\pm \to \pi^\pm\pi^+\pi^-$ and $B^\pm \to K^\pm\pi^+\pi^-$ [3]. Hence, more attention about *CP* asymmetry has been focused on the three body decay channels of B meson.

Direct *CP* asymmetry arises from the weak phase difference and the strong phase difference. The weak phase difference is determined by the CKM matrix elements, while the strong phase can be produced by the hadronic matrix and interference between intermediate states. The vacuum polarization of photons are described by coupling the vector meson in the vector meson dominance (VMD) model. The strength of coupling of the ω meson to the photon is weak comparing with the ρ meson [4]. However, the strong interaction enhances the $\pi^+\pi^-$ pair production amplitudes in the ρ and ω resonance region. $\rho - \omega$ interference presents the large contribution for the process of $e^+e^- \to \pi^+\pi^-$ due to the isospin-breaking effects. Since the strong phase exists, the ρ and ω interference can affect the direct *CP* asymmetry and present the sizeable contribution.

The direct *CP* asymmetry is discussed via $\rho - \omega$ interference in B decays by the naive factorization approach [5]. But the method bases on the assumption of no strong rescattering, and cannot predict direct *CP* asymmetry effectively. Recently, the *CP* asymmetry of charmless three-body B-decay is presented in the leading term of QCD factorization by model dependent approach, where we focus on the local *CP* asymmetry [6]. The direct *CP* asymmetry of the quasi-two-body decay of $B \to P\rho \to P\pi\pi$ is calculated in perturbative QCD approach, where does not taking into account the resonance effects [7]. In our opinion, $B \to P\pi\pi$ have effectively three contributions around the ρ resonance: (a) $B \to P\rho \to P\pi\pi$, (b) $B \to P\omega \to P\rho \to P\pi\pi$, and (c) $B \to P\omega \to P\pi\pi$. Roughly speaking, the amplitudes of their contributions: $a > b > c$. We have absorbed (c) into (b) effectively, which is just the (effective) $\rho - \omega$ mixing parameter: $\tilde{\Pi}_{\rho\omega}$.

The hadronic matrix elements can be calculated by the factorization approach introducing the strong phase. Adding the QCD corrections, the different dynamic methods are given based on the leading power of $1/m_b$ (m_b is b quark mass). The non-leptonic weak decay amplitudes of B mesons can be calculated by the perturbative QCD (PQCD) approach taking into account transverse momenta [8–11]. In the PQCD approach, the hard interaction consisting of six quark operator dominants the decay amplitude from short distance. The nonperturbative dynamics are included in the meson

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wave function which can be extracted from experiment. Finally, we obtain new large strong phases by the phenomenological mechanism of $\rho-\omega$ mixing and the dynamics of the PQCD approach. The large CP asymmetry may be obtained by the resonant region due to the strong phase.

The remainder of this paper is organized as follows. In Sec. II we present the form of the effective Hamiltonian. In Sec. III we give the calculating formalism of CP asymmetry from $\rho-\omega$ mixing in $\bar{B}_s \to P\pi^+\pi^-$. Input parameters are presented in Sec. V. We present the numerical results in Sec. VI. Summary and discussion are included in Sec. VII. The related function defined in the text are given in the Appendix.

II. THE EFFECTIVE HAMILTONIAN

Based on the expansion of the operator product, the effective weak Hamiltonian can be written as [12]

$$\mathcal{H}_{\Delta B=1} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{ud}^*(c_1O_1^u + c_2O_2^u) - V_{tb}V_{td}^*\sum_{i=3}^{10}c_iO_i\right] + \mathrm{H.c.}, \tag{1}$$

where G_F represents the Fermi constant, c_i ($i=1,\ldots,10$) are the Wilson coefficients, V_{ub}, V_{ud}, V_{tb}, and V_{td} are the CKM matrix elements. The operators O_i have the following forms:

$$\begin{aligned}
O_1^u &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)u_\beta\bar{u}_\beta\gamma^\mu(1-\gamma_5)b_\alpha,\\
O_2^u &= \bar{d}\gamma_\mu(1-\gamma_5)u\bar{u}\gamma^\mu(1-\gamma_5)b,\\
O_3 &= \bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_4 &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,\\
O_5 &= \bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_6 &= \bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_7 &= \frac{3}{2}\bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q',\\
O_8 &= \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha,\\
O_9 &= \frac{3}{2}\bar{d}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q',\\
O_{10} &= \frac{3}{2}\bar{d}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha,
\end{aligned} \tag{2}$$

where α and β are color indices, and $q'=u$, d, or s quarks. In Eq. (3) O_1^u and O_2^u are tree operators, O_3–O_6 are QCD penguin operators, and O_7–O_{10} are the operators associated with electroweak penguin diagrams.

We can obtain numerical values of c_i. When $c_i(m_b)$ [11],

$$\begin{aligned}
c_1 &= -0.2703, & c_2 &= 1.1188,\\
c_3 &= 0.0126, & c_4 &= -0.0270,\\
c_5 &= 0.0085, & c_6 &= -0.0326,\\
c_7 &= 0.0011, & c_8 &= 0.0004,\\
c_9 &= -0.0090, & c_{10} &= 0.0022.
\end{aligned} \tag{3}$$

One can obtain numerical values of a_i including Wilson coefficients and the color index N_c [9]:

$$\begin{aligned}
a_1 &= C_2 + C_1/N_c, & a_2 &= C_1 + C_2/N_c,\\
a_3 &= C_3 + C_4/N_c, & a_4 &= C_4 + C_3/N_c,\\
a_5 &= C_5 + C_6/N_c, & a_6 &= C_6 + C_5/N_c,\\
a_7 &= C_7 + C_8/N_c, & a_8 &= C_8 + C_7/N_c,\\
a_9 &= C_9 + C_{10}/N_c, & a_{10} &= C_{10} + C_9/N_c.
\end{aligned} \tag{4}$$

III. CP ASYMMETRY IN $\bar{B}_s^0 \to \rho^0(\omega)P \to \pi^+\pi^-P$

A. Formalism

In the vector meson dominace model (VMD), photons are dressed by coupling to the vector mesons. Based on the same mechanism, $\rho-\omega$ mixing was proposed and later gradually applied to B meson physics [5,13–20]. Due to the effective Hamiltonian, the amplitude A ($\bar{A}$) for the decay process of $\bar{B}_s^0 \to \pi^+\pi^-P$ ($B_s^0 \to \pi^+\pi^-\bar{P}$) can be written as [13]:

$$A = \langle\pi^+\pi^-P|H^T|\bar{B}_s^0\rangle + \langle\pi^+\pi^-P|H^P|\bar{B}_s^0\rangle, \tag{5}$$

$$\bar{A} = \langle\pi^+\pi^-\bar{P}|H^T|B_s^0\rangle + \langle\pi^+\pi^-\bar{P}|H^P|B_s^0\rangle, \tag{6}$$

with H^T and H^P are the Hamiltonian of the tree and penguin operators, respectively.

The relative amplitudes and phases of H^T and H^P can be expressed as follows [13]:

$$A = \langle\pi^+\pi^-P|H^T|\bar{B}_s^0\rangle[1 + re^{i(\delta+\phi)}], \tag{7}$$

$$\bar{A} = \langle\pi^+\pi^-\bar{P}|H^T|B_s^0\rangle[1 + re^{i(\delta-\phi)}], \tag{8}$$

with δ and ϕ are strong and weak phases, respectively. ϕ is the weak phase in the CKM matrix that causes the CP asymmetry, which is $\arg[V_{tb}V_{tq}^*/(V_{ub}V_{uq}^*)](q=d,s)$. The parameter r represents the absolute value of the ratio of penguin and tree amplitudes:

$$r \equiv \left|\frac{\langle\pi^+\pi^-P|H^P|\bar{B}_s^0\rangle}{\langle\pi^+\pi^-P|H^T|\bar{B}_s^0\rangle}\right|. \tag{9}$$

The *CP* violating asymmetry, A_{CP}, can be written as

$$A_{CP} \equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} = \frac{-2r\sin\delta\sin\phi}{1 + 2r\cos\delta\cos\phi + r^2}. \quad (10)$$

From Eq. (10), it can be seen that the *CP* asymmetry depends on the weak phase difference and the strong phase difference. The weak phase is determined for a particular decay process. Hence, in order to obtain a large *CP* asymmetry, we need some mechanism to increase δ. It has been found that $\rho - \omega$ mixing can lead to a large strong phase difference [4,14–20]. Based on $\rho - \omega$ mixing and working to the first order of isospin violation, we have the following results [13]:

$$\langle \pi^+\pi^- P|H^T|\bar{B}_s^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega} t_\omega + \frac{g_\rho}{s_\rho} t_\rho, \quad (11)$$

$$\langle \pi^+\pi^- P|H^P|\bar{B}_s^0\rangle = \frac{g_\rho}{s_\rho s_\omega}\tilde{\Pi}_{\rho\omega} p_\omega + \frac{g_\rho}{s_\rho} p_\rho. \quad (12)$$

where $t_\rho(p_\rho)$ and $t_\omega(p_\omega)$ are the tree (penguin) amplitudes for $\bar{B}_s^0 \to \rho^0 P$ and $\bar{B}_s^0 \to \omega P$, respectively; g_ρ is the coupling constant of $\rho^0 \to \pi^+\pi^-$ decay process; $\tilde{\Pi}_{\rho\omega}$ is the effective $\rho - \omega$ mixing amplitude which also effectively absorbed into the direct coupling $\omega \to \pi^+\pi^-$. s_V, m_V, and Γ_V ($V = \rho$ or ω) represent the inverse propagator, mass and decay rate of the vector meson V, respectively.

$$s_V = s - m_V^2 + \mathrm{i} m_V \Gamma_V, \quad (13)$$

where $\sqrt{s}$ denotes the invariant mass of the $\pi^+\pi^-$ pairs [13].

The $\rho - \omega$ mixing paraments were recently determined precisely by Wolfe and Maltnan [21,22]

$$\mathfrak{Re}\Pi_{\rho\omega}(m_\rho^2) = -4470 \pm 250_{\text{model}} \pm 160_{\text{data}}\ \text{MeV}^2,$$
$$\mathfrak{Im}\Pi_{\rho\omega}(m_\rho^2) = -5800 \pm 2000_{\text{model}} \pm 1100_{\text{data}}\ \text{MeV}^2. \quad (14)$$

One can find that the mixing parameter is the momentum dependence including the nonresonant contribution that absorbs the direct decay $\omega \to \pi^+\pi^-$. We introduce the momentum dependence of the mixing parameter $\tilde{\Pi}_{\rho\omega}(s)$ for $\rho - \omega$ mixing, which leads to the explicit s dependence. It is reasonable to devote one's energies to search the mixing contribution at the region of ω mass where the two pions can be produced. We write $\tilde{\Pi}_{\rho\omega}(s) = \mathfrak{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) + \mathfrak{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2)$, and update the values as follows [23]:

$$\mathfrak{Re}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -4760 \pm 440\ \text{MeV}^2,$$
$$\mathfrak{Im}\tilde{\Pi}_{\rho\omega}(m_\omega^2) = -6180 \pm 3300\ \text{MeV}^2. \quad (15)$$

In fact, the contribution of the s dependence of $\tilde{\Pi}_{\rho\omega}$ is negligible. We can make the expansion $\tilde{\Pi}_{\rho\omega}(s) = \tilde{\Pi}_{\rho\omega}(m_\omega^2) + (s - m_\omega)\tilde{\Pi}'_{\rho\omega}(m_\omega^2)$. From Eqs. (5), (7), (11), and (12) one has

$$re^{i\delta}e^{i\phi} = \frac{\tilde{\Pi}_{\rho\omega} p_\omega + s_\omega p_\rho}{\tilde{\Pi}_{\rho\omega} t_\omega + s_\omega t_\rho}, \quad (16)$$

Defining

$$\frac{p_\omega}{t_\rho} \equiv r' e^{i(\delta_q + \phi)}, \qquad \frac{t_\omega}{t_\rho} \equiv \alpha e^{i\delta_\alpha}, \qquad \frac{p_\rho}{p_\omega} \equiv \beta e^{i\delta_\beta}, \quad (17)$$

with δ_α, δ_β, and δ_q are strong phases. It is available from Eqs. (16) and (17):

$$re^{i\delta} = r' e^{i\delta_q} \frac{\tilde{\Pi}_{\rho\omega} + \beta e^{i\delta_\beta} s_\omega}{\tilde{\Pi}_{\rho\omega}\alpha e^{i\delta_\alpha} + s_\omega}. \quad (18)$$

In order to obtain the *CP* violating asymmetry in Eq. (10), $\sin\phi$ and $\cos\phi$ are necessary. The weak phase ϕ is fixed by the CKM matrix elements. In the Wolfenstein parametrization [24], one has

$$\sin\phi = \frac{\eta}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}},$$
$$\cos\phi = \frac{\rho(1-\rho) - \eta^2}{\sqrt{[\rho(1-\rho) - \eta^2]^2 + \eta^2}}. \quad (19)$$

where the same result has been found for $b \to d$ transition from Λ_b decay process [14].

IV. CALCULATION

For the simplification, we take the decay process of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^- K^0$ as example for the study of the $\rho - \omega$ interference. The other decay channels can be obtained similarly. According to the Hamiltonian(1), based on CKM matrix elements of $V_{ub}V_{ud}^*$, $V_{tb}V_{td}^*$, the decay amplitude of $\bar{B}_s^0 \to \rho^0 K^0$ in perturbation QCD approach can be written as

$$\sqrt{2}M(\bar{B}_s^0 \to \rho^0 K^0) = V_{ub}V_{ud}^* t_\rho - V_{tb}V_{td}^* p_\rho \quad (20)$$

where t_ρ and p_ρ refer to the tree and penguin contributions respectively. We write:

$$t_\rho = f_\rho F_{B_s\to K}^{LL}[a_2] + M_{B_s\to K}^{LL}[C_2] \quad (21)$$

and

$$p_\rho = f_\rho F^{LL}_{B_s\to K}\left[-a_4+\frac{3}{2}a_7+\frac{1}{2}a_{10}+\frac{3}{2}a_9\right]+M^{LR}_{B_s\to K}\left[-C_5+\frac{1}{2}C_7\right]+M^{LL}_{B_s\to K}\left[-C_3+\frac{1}{2}C_9+\frac{3}{2}C_{10}\right]-M^{SP}_{B_s\to K}\left[\frac{3}{2}C_8\right]$$
$$+f_{B_s}F^{LL}_{ann}\left[-a_4+\frac{1}{2}a_{10}\right]+f_{B_s}F^{SP}_{ann}\left[-a_6+\frac{1}{2}a_8\right]+M^{LL}_{ann}\left[-C_3+\frac{1}{2}C_9\right]+M^{LR}_{ann}\left[-C_5+\frac{1}{2}C_7\right]. \quad (22)$$

The decay amplitude for $\bar{B}^0_s \to \omega K^0$ can be written as

$$\sqrt{2}M(\bar{B}^0_s \to \omega\pi^0) = V_{ub}V^*_{ud}t_\omega - V_{tb}V^*_{td}p_\omega. \quad (23)$$

One can also present the contributions of t_ω and p_ω as well.

$$t_\omega = f_\omega F^{LL}_{B_s\to K}[a_2] + M^{LL}_{B_s\to K}[C_2] \quad (24)$$

$$p_\omega = f_\omega F^{LL}_{B_s\to K}\left[2a_3+a_4+2a_5+\frac{1}{2}a_7+\frac{1}{2}a_9-\frac{1}{2}a_{10}\right]+M^{LL}_{B_s\to K}\left[C_3+2C_4-\frac{1}{2}C_9+\frac{1}{2}C_{10}\right]+M^{LR}_{B_s\to K}\left[C_5-\frac{1}{2}C_7\right]$$
$$-M^{SP}_{B_s\to K}\left[2C_6+\frac{1}{2}C_8\right]+f_{B_s}F^{LL}_{ann}\left[a_4-\frac{1}{2}a_{10}\right]+f_{B_s}F^{SP}_{ann}\left[a_6-\frac{1}{2}a_8\right]+M^{LL}_{ann}\left[C_3-\frac{1}{2}C_9\right]+M^{LR}_{ann}\left[C_5-\frac{1}{2}C_7\right]. \quad (25)$$

The function F and M are given in Sec. IX. The index LL, LR, and SP arise from the $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively.

$$\alpha e^{i\delta_\alpha} = \frac{t_\omega}{t_\rho}, \quad (26)$$

$$\beta e^{i\delta_\beta} = \frac{p_\rho}{p_\omega}, \quad (27)$$

$$r' e^{i\delta_q} = \frac{p_\omega}{t_\rho}\times\left|\frac{V_{tb}V^*_{td}}{V_{ub}V^*_{ud}}\right|, \quad (28)$$

where

$$\left|\frac{V_{tb}V^*_{td}}{V_{ub}V^*_{ud}}\right| = \frac{\sqrt{[\rho(1-\rho)-\eta^2]^2+\eta^2}}{(1-\lambda^2/2)(\rho^2+\eta^2)} \quad (29)$$

From the above equations, the new strong phases δ_α, δ_β, and δ_q are introduced by the interference of $\rho-\omega$ mesons. The strong phase δ are obtained by the Eqs. (17) and (18) in the framework of PQCD.

In a similar way, we can get the t_ρ, t_ω, p_ρ, and p_ω for the processes of $\bar{B}^0_s \to \rho^0(\omega)\eta$ and $\bar{B}^0_s \to \rho^0(\omega)\eta'$, respectively. The relevant CP asymmetry can also be produced in similar approach. In the calculation, η and η' mesons are introduced. The η and η' mixing depend on the quark flavor basis [25]. The mesons are consisted of $\bar{n}n = (\bar{u}u+\bar{d}d)/\sqrt{2}$ and $\bar{s}s$:

$$\begin{pmatrix}|\eta\rangle\\|\eta'\rangle\end{pmatrix} = U(\phi)\begin{pmatrix}|\eta_n\rangle\\|\eta_s\rangle\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}|\eta_n\rangle\\|\eta_s\rangle\end{pmatrix} \quad (30)$$

where the mixing angle $\phi = 39.3° \pm 1.0°$. Explicitly, only two decay constants are needed is the advantage here:

$$\langle 0|\bar{n}\gamma^\mu\gamma_5 n|\eta_n P^\mu\rangle = \frac{i}{\sqrt{2}}f_n P^\mu, \quad (31)$$

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_s P^\mu\rangle = i f_s P^\mu. \quad (32)$$

We use [26]

$$f_n = 139.1\pm 2.6\,\text{MeV}, \quad f_s = 174.2\pm 7.8\,\text{MeV}. \quad (33)$$

For the pure annihilation type decay process, one can also divides the amplitudes into t_ρ, t_ω, p_ρ, and p_ω depending on $V_{ub}V^*_{us}$ and $V_{tb}V^*_{ts}$. The amplitudes can be given as following for the channel $\bar{B}^0_s\to\pi^0\rho^0(\omega)$: $M(\bar{B}^0_s\to\rho^0\pi^0) = V_{ub}V^*_{us}t_\rho - V_{tb}V^*_{ts}p_\rho$ and $M(\bar{B}^0_s\to\omega\pi^0) = V_{ub}V^*_{us}t_\omega - V_{tb}V^*_{ts}p_\omega$.

V. INPUT PARAMETERS

The CKM matrix, which elements are determined from experiments, can be expressed in terms of the Wolfenstein parameters A, ρ, λ, and η [24]:

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-\mathrm{i}\eta)\\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2\\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1\end{pmatrix}, \quad (34)$$

where $\mathcal{O}(\lambda^4)$ corrections are neglected. The latest values for the parameters in the CKM matrix are [27]:

$$\lambda = 0.22506 \pm 0.00050, \qquad A = 0.811 \pm 0.026,$$
$$\bar{\rho} = 0.124^{+0.019}_{-0.018}, \qquad \bar{\eta} = 0.356 \pm 0.011, \tag{35}$$

where

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \qquad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \tag{36}$$

From Eqs. (35) and (36) we have

$$0.109 < \rho < 0.147, \qquad 0.354 < \eta < 0.377. \tag{37}$$

The other parameters are given as following [24,28,29]:

$$f_\pi = 0.131\ \text{GeV}, \qquad f_K = 0.160\ \text{GeV},$$
$$m_{B_s^0} = 5.36677\ \text{GeV}, \qquad \tau_{B_s^0} = 1.512 \times 10^{-12} s$$
$$m_{\rho^0(770)} = 0.77526\ \text{GeV}, \qquad \Gamma_{\rho^0(770)} = 0.1491\ \text{GeV},$$
$$m_{\omega(782)} = 0.78265\ \text{GeV}, \qquad \Gamma_{\omega(782)} = 8.49 \times 10^{-3}\ \text{GeV},$$
$$m_\pi = 0.13957\ \text{GeV}, \qquad m_W = 80.385\ \text{GeV},$$
$$f_\rho = 209 \pm 2\ \text{MeV}, \qquad f_\rho^T = 165 \pm 9\ \text{MeV},$$
$$f_\omega = 195.1 \pm 3\ \text{MeV}, \qquad f_\omega^T = 145 \pm 10\ \text{MeV}. \tag{38}$$

VI. NUMERICAL RESULTS

In the numerical results, we find the CP asymmetry can be enhanced when the masses of the $\pi^+\pi^-$ pairs are in the area around the $\rho - \omega$ resonance, and the maximum CP asymmetry for our considering the decay channels can reach 59%. We also discuss the numerical results from the case of tree and penguin dominated type decay and the case of pure annihilation type decay in the framework of Perturbative QCD. The CP violation is associated with the CKM matrix elements and $\sqrt{s}$. In our numerical calculations, we find that the CP asymmetry depend weakly on the variation of the CKM matrix elements. Hence, we let (ρ, η) vary between the central values $(\rho_{\text{central}}, \eta_{\text{central}})$.

A. The case of tree and penguin dominated type decay

We refer to the decay processes of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$, $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$, and $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$ as the case of tree and penguin dominated type decay. In Fig. 1, we show the plot of CP asymmetry as a function of $\sqrt{s}$. One can find the CP asymmetry varies sharply when the masses of the $\pi^+\pi^-$ pairs are in the area around the $\rho - \omega$ resonance range. For the decay process of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$, the maximum CP asymmetry can reach 40%. For the decay channels of $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$ and $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$, we obtain the maximum CP asymmetry is 59% and 21%, respectively. From Eq. (10), one can find the CP asymmetry is affected by the weak phase difference, the strong phase difference and r. The weak phase depends on the CKM matrix elements. Hence, the change of CP asymmetry is derived from the variation of strong phase δ and r except the CKM matrix. We take the central values from the parameters of $(\rho_{\text{central}}, \eta_{\text{central}})$. Taking into account of $\rho - \omega$ mixing, we can see that $\sin\delta$ oscillate considerably at the area of $\rho - \omega$ resonance from Fig. 2 for the considering decay processes. The plot of r as a function of $\sqrt{s}$ is presented in Fig. 3. One can see that the r change sharply for the process of $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$ and $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$.

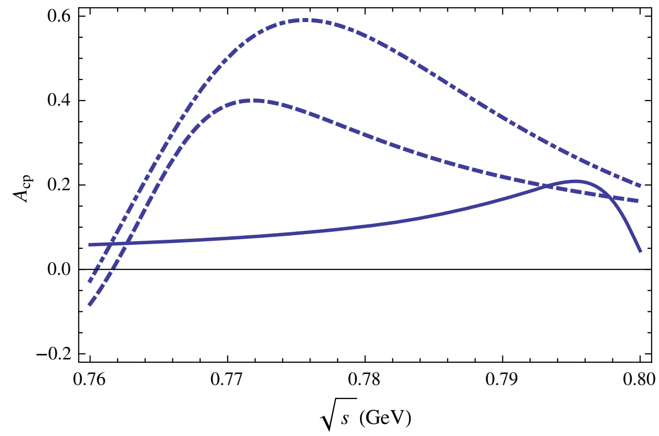

FIG. 1. Plot of A_{CP} as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements. The dashed line, dash-dotted, solid line refer to the decay channels of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$, $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$, and $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$, respectively.

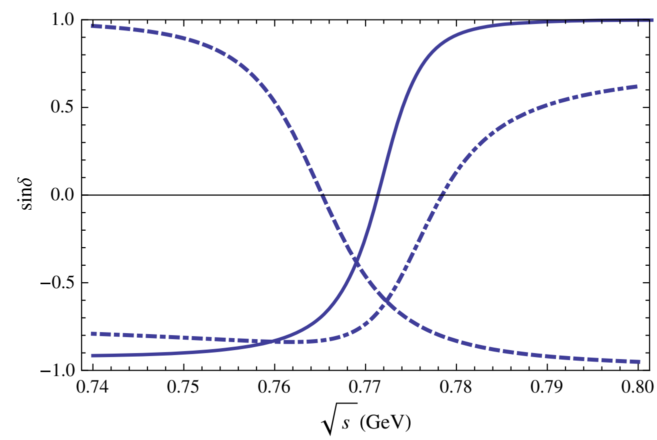

FIG. 2. Plot of $\sin\delta$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements. The dashed line, dash-dotted, solid line refer to the decay channels of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$, $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$, and $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$, respectively.)

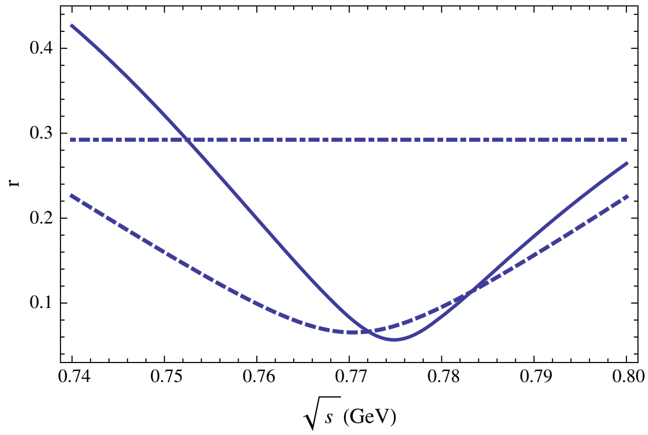

FIG. 3. Plot of r as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements. The dashed line, dash-dotted, solid line refer to the decay channels of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$, $\bar{B}_s^0 \to \rho^0(\omega)\eta' \to \pi^+\pi^-\eta'$ and $\bar{B}_s^0 \to \rho^0(\omega)\eta \to \pi^+\pi^-\eta$, respectively.)

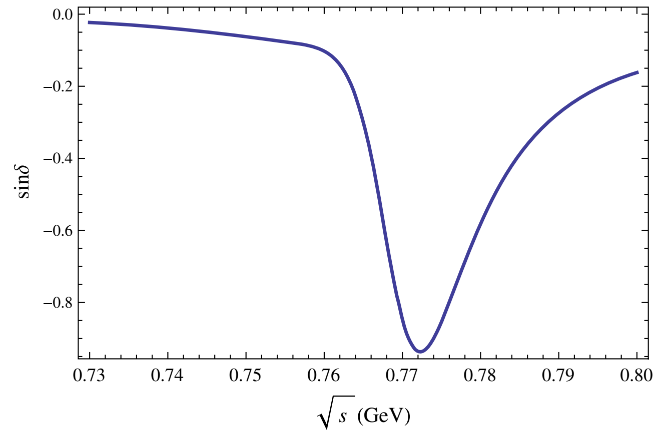

FIG. 5. Plot of $\sin\delta$ as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$.

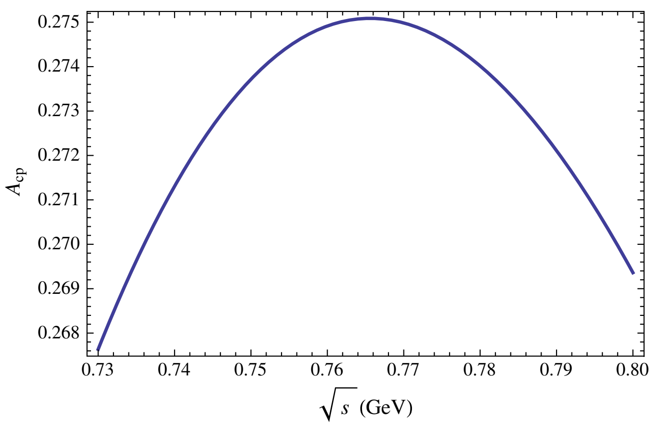

FIG. 4. Plot of A_{CP} as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$.

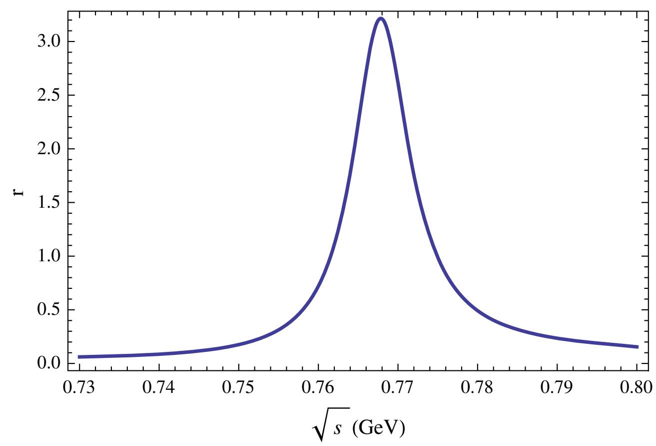

FIG. 6. Plot of r as a function of $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$.

B. The case of pure annihilation decay type

In Fig. 4, we present the plot of CP asymmetry parameter as a function $\sqrt{s}$ corresponding to central parameter values of CKM matrix elements for the pure annihilation decay type of $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$. One can find the maximum CP asymmetry reach 28% when the masses of the $\pi^+\pi^-$ pairs are in the area around the $\rho - \omega$ resonance range. The plots of $\sin\delta$ and r as a function of $\sqrt{s}$ are given in Fig. 5 and Fig. 6, respectively. We can see that $\sin\delta$ and r oscillate sharply taking into account $\rho - \omega$ resonance. Generally, the CP asymmetry is tiny in the case of pure annihilation decay. However, the maximum CP asymmetry can reach 28% at the area of $\rho - \omega$ resonance, which give us a chance to search CP asymmetry from the pure annihilation decay type.

VII. SUMMARY AND CONCLUSION

In this paper, we study the CP asymmetry for the decay process of $\bar{B}_s \to P\pi^+\pi^-$ in perturbative QCD. It has been found the CP asymmetry can be enhanced greatly at the area of $\rho - \omega$ resonance. The maximum CP asymmetry can reach 40% for the process of $\bar{B}_s^0 \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$. However, the paper has also discussed the CP asymmetry of the decay process of $\bar{B}_s \to \rho^0(\omega)K^0 \to \pi^+\pi^-K^0$ from $b \to d$ transition in QCD factorization. The maximum CP asymmetry reach 46% when the invariant mass of the $\pi^+\pi^-$ pair is in the vicinity of the ω resonance from QCD factorization [19]. The difference of CP asymmetry mainly comes from the strong phase difference between QCD factoriztion and perturbative QCD. The hadronic matrix elements can be calculated from first principles in the decays of B-meson. Due to the power expansion of $1/m_b$

(m_b is b quark mass), all of the theories of factorization are shown to deal with the hadronic matrix elements in the leading power of $1/m_b$. But these methods are different significantly due to the collinear degree or transverse momenta. The power counting is different from the hard kernels between QCDF and PQCD. It is important to extract the strong phase difference for *CP* violation. The more different feature of QCDF and PQCD is the strong interaction scale at which of PQCD is low, typically of order 1–2 GeV, the case of QCDF is order $O(m_b)$ for the Wilson coefficients.

Meanwhile, we find that the *CP* asymmetry associated with the case of pure annihilation type decay process of $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$ can be enhanced and the maximum value reach 28%. Hence, one can search for the large *CP* asymmetry at the area of $\rho-\omega$ resonance from pure annihilation type decay process of $\bar{B}_s^0 \to \rho^0(\omega)\pi^0 \to \pi^+\pi^-\pi^0$.

In this work, we have take the perturbative QCD approximation which add the QCD correction to the naive factorization which is based on the power expansion of $1/m_b$. The final state interaction is also neglected in this approximation which may give some uncertainties. There are some uncertainties from the input parameters, the hard scattering scale and CKM matrix elements. The theoretical results can be improved by high order correction from α_s and $1/m_b$.

ACKNOWLEDGMENTS

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APPENDIX: RELATED FUNCTIONS DEFINED IN THE TEXT

The functions associated with the tree and penguin contributions are presented for the factorization and non-factorization amplitudes in PQCD approach [10,11,30]. The functions of the case of tree and penguin dominated type decay are written as

(i) $(V-A)(V-A)$ operators:

$$\begin{aligned} f_{M_2}F^{LL}_{B_s\to M_3}(a_i) = {}& 8\pi C_F M_{B_s}^4 f_{M_2}\int_0^1 dx_1 dx_3\int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1) \\ & \times\{a_i(t_a)E_e(t_a)[(1+x_3)\phi_3^A(x_3)+r_3(1-2x_3)(\phi_3^P(x_3)+\phi_3^T(x_3))]h_e(x_1,x_3,b_1,b_3) \\ & + 2r_3\phi_3^P(x_3)a_i(t'_a)E_e(t'_a)h_e(x_3,x_1,b_3,b_1)\}, \end{aligned} \tag{A1}$$

(ii) $(V-A)(V+A)$ operators:

$$F^{LR}_{B_s\to M_3}(a_i) = -F^{LL}_{B_s\to M_3}(a_i), \tag{A2}$$

(iii) $(S-P)(S+P)$ operators:

$$\begin{aligned} f_{M_2}F^{SP}_{B_s\to M_3}(a_i) = {}& 16\pi r_2 C_F M_{B_s}^4 f_{M_2}\int_0^1 dx_1 dx_3\int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1,b_1) \\ & \times\{a_i(t_a)E_e(t_a)[\phi_3^A(x_3)+r_3(2+x_3)\phi_3^P(x_3)-r_3x_3\phi_3^T(x_3)]h_e(x_1,x_3,b_1,b_3) \\ & + 2r_3\phi_3^P(x_3)a_i(t'_a)E_e(t'_a)h_e(x_3,x_1,b_3,b_1)\}, \end{aligned} \tag{A3}$$

(iv) $(V-A)(V-A)$ operators:

$$\begin{aligned} M^{LL}_{B_s\to M_3}(a_i) = {}& 32\pi C_F M_{B_s}^4/\sqrt{6}\int_0^1 dx_1 dx_2 dx_3\int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1,b_1)\phi_2^A(x_2) \\ & \times\{[(1-x_2)\phi_3^A(x_3)-r_3x_3(\phi_3^P(x_3)-\phi_3^T(x_3))]a_i(t_b)E'_e(t_b) \\ & \times h_n(x_1,1-x_2,x_3,b_1,b_2)+h_n(x_1,x_2,x_3,b_1,b_2) \\ & \times[-(x_2+x_3)\phi_3^A(x_3)+r_3x_3(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t'_b)E'_e(t'_b)\}, \end{aligned} \tag{A4}$$

(v) $(V-A)(V+A)$ operators:

$$\begin{aligned}M^{LR}_{B_s\to M_3}(a_i) = {}& 32\pi C_F M^4_{B_s} r_2/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\\ &\times\{h_n(x_1,1-x_2,x_3,b_1,b_2)[(1-x_2)\phi_3^A(x_3)(\phi_2^P(x_2)+\phi_2^T(x_2))+r_3x_3(\phi_2^P(x_2)-\phi_2^T(x_2))\\ &\times(\phi_3^P(x_3)+\phi_3^T(x_3))+(1-x_2)r_3(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3))]a_i(t_b)E'_e(t_b)\\ &-h_n(x_1,x_2,x_3,b_1,b_2)[x_2\phi_3^A(x_3)(\phi_2^P(x_2)-\phi_2^T(x_2))+r_3x_2(\phi_2^P(x_2)-\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3))\\ &+r_3x_3(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t'_b)E'_e(t'_b)\},\end{aligned}\tag{A5}$$

(vi) $(S-P)(S+P)$ operators:

$$\begin{aligned}M^{SP}_{B_s\to M_3}(a_i) = {}& 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)\phi_2^A(x_2)\\ &\times\{[(x_2-x_3-1)\phi_3^A(x_3)+r_3x_3(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t_b)E'_e(t_b)h_n(x_1,1-x_2,x_3,b_1,b_2)\\ &+a_i(t'_b)E'_e(t'_b)[x_2\phi_3^A(x_3)+r_3x_3(\phi_3^T(x_3)-\phi_3^P(x_3))]h_n(x_1,x_2,x_3,b_1,b_2)\}.\end{aligned}\tag{A6}$$

The functions are associated with the annihilation type process as following:

(i) $(V-A)(V-A)$ operators:

$$\begin{aligned}f_{B_s}F^{LL}_{ann}(a_i) = {}& 8\pi C_F M^4_{B_s} f_{B_s}\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{a_i(t_c)E_a(t_c)\\ &\times[(x_3-1)\phi_2^A(x_2)\phi_3^A(x_3)-4r_2r_3\phi_2^P(x_2)\phi_3^P(x_3)+2r_2r_3x_3\phi_2^P(x_2)(\phi_3^P(x_3)-\phi_3^T(x_3))]h_a(x_2,1-x_3,b_2,b_3)\\ &+[x_2\phi_2^A(x_2)\phi_3^A(x_3)+2r_2r_3(\phi_2^P(x_2)-\phi_2^T(x_2))\phi_3^P(x_3)\\ &+2r_2r_3x_2(\phi_2^P(x_2)+\phi_2^T(x_2))\phi_3^P(x_3)]a_i(t'_c)E_a(t'_c)h_a(1-x_3,x_2,b_3,b_2)\}.\end{aligned}\tag{A7}$$

(ii) $(V-A)(V+A)$ operators:

$$F^{LR}_{ann}(a_i) = F^{LL}_{ann}(a_i),\tag{A8}$$

(iii) $(S-P)(S+P)$ operators:

$$\begin{aligned}f_{B_s}F^{SP}_{ann}(a_i) = {}& 16\pi C_F M^4_{B_s} f_{B_s}\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\\ &\times\{[2r_2\phi_2^P(x_2)\phi_3^A(x_3)+(1-x_3)r_3\phi_2^A(x_2)(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t_c)E_a(t_c)h_a(x_2,1-x_3,b_2,b_3)\\ &+[2r_3\phi_2^A(x_2)\phi_3^P(x_3)+r_2x_2(\phi_2^P(x_2)-\phi_2^T(x_2))\phi_3^A(x_3)]a_i(t'_c)E_a(t'_c)h_a(1-x_3,x_2,b_3,b_2)\}.\end{aligned}\tag{A9}$$

(iv) $(V-A)(V-A)$ operators:

$$\begin{aligned}M^{LL}_{ann}(a_i) = {}& 32\pi C_F M^4_{B_s}/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int_0^\infty b_1db_2b_2db_2\phi_{B_s}(x_1,b_1)\\ &\times\{h_{na}(x_1,x_2,x_3,b_1,b_2)[-x_2\phi_2^A(x_2)\phi_3^A(x_3)-4r_2r_3\phi_2^P(x_2)\phi_3^P(x_3)\\ &+r_2r_3(1-x_2)(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3))\\ &+r_2r_3x_3(\phi_2^P(x_2)-\phi_2^T(x_2))(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t_d)E'_a(t_d)\\ &+h'_{na}(x_1,x_2,x_3,b_1,b_2)[(1-x_3)\phi_2^A(x_2)\phi_3^A(x_3)+(1-x_3)r_2r_3(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3))\\ &+x_2r_2r_3(\phi_2^P(x_2)-\phi_2^T(x_2))(\phi_3^P(x_3)+\phi_3^T(x_3))]a_i(t'_d)E'_a(t'_d)\},\end{aligned}\tag{A10}$$

(v) $(V-A)(V+A)$ operators:

$$M_{ann}^{LR}(M_2,M_3,a_i) = 32\pi C_F M_{B_s}^4/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int^{\infty} b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)$$
$$\times\{h_{na}(x_1,x_2,x_3,b_1,b_2)[r_2(2-x_2)(\phi_2^P(x_2)+\phi_2^T(x_2))\phi_3^A(x_3)$$
$$-r_3(1+x_3)\phi_2^A(x_2)(\phi_3^P(x_3)-\phi_3^T(x_3))]a_i(t_d)E_a'(t_d)$$
$$+h_{na}'(x_1,x_2,x_3,b_1,b_2)[r_2x_2(\phi_2^P(x_2)+\phi_2^T(x_2))\phi_3^A(x_3)$$
$$+r_3(x_3-1)\phi_2^A(x_2)(\phi_3^P(x_3)-\phi_3^T(x_3))]a_i(t_d')E_a'(t_d')\}, \tag{A11}$$

(vi) $(S-P)(S+P)$ operators:

$$M_{ann}^{SP}(a_i) = 32\pi C_F M_{B_s}^4/\sqrt{6}\int_0^1 dx_1dx_2dx_3\int_0^{\infty} b_1db_1b_2db_2\phi_{B_s}(x_1,b_1)$$
$$\times\{a_i(t_d)E_a'(t_d)h_{na}(x_1,x_2,x_3,b_1,b_2)[(x_3-1)\phi_2^A(x_2)\phi_3^A(x_3)-4r_2r_3\phi_2^P(x_2)\phi_3^P(x_3)$$
$$+r_2r_3x_3(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3))+r_2r_3(1-x_2)(\phi_2^P(x_2)-\phi_2^T(x_2))(\phi_3^P(x_3)+\phi_3^T(x_3))]$$
$$+a_i(t_d')E_a'(t_d')h_{na}'(x_1,x_2,x_3,b_1,b_2)[x_2\phi_2^A(x_2)\phi_3^A(x_3)$$
$$+x_2r_2r_3(\phi_2^P(x_2)+\phi_2^T(x_2))(\phi_3^P(x_3)-\phi_3^T(x_3)))+r_2r_3(1-x_3)(\phi_2^P(x_2)-\phi_2^T(x_2))(\phi_3^P(x_3)+\phi_3^T(x_3))]\}. \tag{A12}$$

The hard scales are chosen as

$$t_a = \max\{\sqrt{x_3}M_{B_s},1/b_1,1/b_3\}, \tag{A13}$$

$$t_a' = \max\{\sqrt{x_1}M_{B_s},1/b_1,1/b_3\}, \tag{A14}$$

$$t_b = \max\{\sqrt{x_1x_3}M_{B_s},\sqrt{|1-x_1-x_2|x_3}M_{B_s},1/b_1,1/b_2\}, \tag{A15}$$

$$t_b' = \max\{\sqrt{x_1x_3}M_{B_s},\sqrt{|x_1-x_2|x_3}M_{B_s},1/b_1,1/b_2\}, \tag{A16}$$

$$t_c = \max\{\sqrt{1-x_3}M_{B_s},1/b_2,1/b_3\}, \tag{A17}$$

$$t_c' = \max\{\sqrt{x_2}M_{B_s},1/b_2,1/b_3\}, \tag{A18}$$

$$t_d = \max\{\sqrt{x_2(1-x_3)}M_{B_s},\sqrt{1-(1-x_1-x_2)x_3}M_{B_s},1/b_1,1/b_2\}, \tag{A19}$$

$$t_d' = \max\{\sqrt{x_2(1-x_3)}M_{B_s},\sqrt{|x_1-x_2|(1-x_3)}M_{B_s},1/b_1,1/b_2\}. \tag{A20}$$

The functions h in the decay amplitudes consist of two parts: one is the jet function $S_t(x_i)$ derived by the threshold resummation [31], the other is the propagator of virtual quark and gluon. They are defined by

$$h_e(x_1,x_3,b_1,b_3) = [\theta(b_1-b_3)I_0(\sqrt{x}_3M_{B_s}b_3)K_0(\sqrt{x}_3M_{B_s}b_1)+\theta(b_3-b_1)I_0(\sqrt{x}_3M_{B_s}b_1)K_0(\sqrt{x}_3M_{B_s}b_3)]$$
$$\times K_0(\sqrt{x_1x_3}M_{B_s}b_1)S_t(x_3), \tag{A21}$$

$$h_n(x_1,x_2,x_3,b_1,b_2) = [\theta(b_2-b_1)K_0(\sqrt{x_1x_3}M_{B_s}b_2)I_0(\sqrt{x_1x_3}M_{B_s}b_1)+\theta(b_1-b_2)K_0(\sqrt{x_1x_3}M_{B_s}b_1)I_0(\sqrt{x_1x_3}M_{B_s}b_2)]$$
$$\times\begin{cases}\frac{i\pi}{2}H_0^{(1)}(\sqrt{(x_2-x_1)x_3}M_{B_s}b_2), & x_1-x_2<0\\ K_0(\sqrt{(x_1-x_2)x_3}M_{B_s}b_2), & x_1-x_2>0\end{cases}, \tag{A22}$$

$$h_a(x_2,x_3,b_2,b_3) = \left(\frac{i\pi}{2}\right)^2 S_t(x_3)[\theta(b_2-b_3)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_2)J_0(\sqrt{x_3}M_{B_s}b_3)+\theta(b_3-b_2)H_0^{(1)}(\sqrt{x_3}M_{B_s}b_3)J_0(\sqrt{x_3}M_{B_s}b_2)] \times H_0^{(1)}(\sqrt{x_2x_3}M_{B_s}b_2), \tag{A23}$$

$$\begin{aligned} h_{na}(x_1,x_2,x_3,b_1,b_2) = \frac{i\pi}{2}\Big[&\theta(b_1-b_2)H_0^{(1)}\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_1\Big)J_0\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_2\Big) \\ &+\theta(b_2-b_1)H_0^{(1)}\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_2\Big)J_0\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_1\Big)\Big] \\ &\times K_0\Big(\sqrt{1-(1-x_1-x_2)x_3}M_{B_s}b_1\Big), \end{aligned} \tag{A24}$$

$$\begin{aligned} h'_{na}(x_1,x_2,x_3,b_1,b_2) = \frac{i\pi}{2}[&\theta(b_1-b_2)H_0^{(1)}\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_1\Big)J_0\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_2\Big) \\ &+\theta(b_2-b_1)H_0^{(1)}\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_2\Big)J_0\Big(\sqrt{x_2(1-x_3)}M_{B_s}b_1\Big)\Big] \\ &\times \begin{cases} \frac{i\pi}{2}H_0^{(1)}\Big(\sqrt{(x_2-x_1)(1-x_3)}M_{B_s}b_1\Big), & x_1-x_2<0 \\ K_0\Big(\sqrt{(x_1-x_2)(1-x_3)}M_{B_s}b_1\Big), & x_1-x_2>0 \end{cases}, \end{aligned} \tag{A25}$$

where $H_0^{(1)}(z) = \mathrm{J}_0(z) + i\mathrm{Y}_0(z)$.

The S_t resums the threshold logarithms $\ln^2 x$ appearing in the hard kernels to all orders and it has been parameterized as

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A26}$$

with $c = 0.4$. In the nonfactorizable contributions, $S_t(x)$ gives a very small numerical effect to the amplitude [32]. Therefore, we drop $S_t(x)$ in h_n and h_{na}.

The evolution factors $E_e^{(\prime)}$ and $E_a^{(\prime)}$ entering in the expressions for the matrix elements (see Sec. 3) are given by

$$E_e(t) = \alpha_s(t)\exp[-S_B(t)-S_3(t)], \qquad E'_e(t) = \alpha_s(t)\exp[-S_B(t)-S_2(t)-S_3(t)]|_{b_1=b_3}, \tag{A27}$$

$$E_a(t) = \alpha_s(t)\exp[-S_2(t)-S_3(t)], \qquad E'_a(t) = \alpha_s(t)\exp[-S_B(t)-S_2(t)-S_3(t)]|_{b_2=b_3}, \tag{A28}$$

in which the Sudakov exponents are defined as

$$S_B(t) = s\left(x_1\frac{M_{B_s}}{\sqrt{2}}, b_1\right) + \frac{5}{3}\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A29}$$

$$S_2(t) = s\left(x_2\frac{M_{B_s}}{\sqrt{2}}, b_2\right) + s\left((1-x_2)\frac{M_{B_s}}{\sqrt{2}}, b_2\right) + 2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu})), \tag{A30}$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. Replacing the kinematic variables of M_2 to M_3 in S_2, we can get the expression for S_3. The explicit form for the function $s(Q,b)$ is

$$s(Q,b)=\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b})+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{q})+1}{\hat{q}}-\frac{\ln(2\hat{b})+1}{\hat{b}}\right]+\frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q})-\ln^2(2\hat{b})], \quad (A31)$$

where the variables are defined by

$$\hat{q}\equiv\ln[Q/(\sqrt{2}\Lambda)], \qquad \hat{b}\equiv\ln[1/(b\Lambda)], \quad (A32)$$

and the coefficients $A^{(i)}$ and β_i are

$$\beta_1=\frac{33-2n_f}{12}, \qquad \beta_2=\frac{153-19n_f}{24},$$
$$A^{(1)}=\frac{4}{3}, \qquad A^{(2)}=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\left(\frac{1}{2}e^{\gamma_E}\right), \quad (A33)$$

n_f is the number of the quark flavors and γ_E is the Euler constant. We will use the one-loop running coupling constant, i.e., we pick up the four terms in the first line of the expression for the function $s(Q,b)$.

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