$\mathcal{N} = 3$ harmonic supersymmetric Wilson loop

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We study supersymmetric Wilson loops in $d = 3, \mathcal{N} = 3$ harmonic superspace, leading to a construction of a supersymmetrized generalization of the $\frac{1}{3}$-BPS Wilson loop for $\mathcal{N} = 3$ gauge theories. This also includes the generalization of the $\frac{1}{6}$-BPS loop for ABJM theory. We perform a "one-loop" computation of the vacuum expectation value of this operator directly in superspace and compare with the known $\mathcal{N} = 2$ localization results at large N. This comparison also lets us identify certain fermionic contributions that do not receive any subleading corrections.

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I. INTRODUCTION

The power of supersymmetry to simplify computations and gain insights cannot be overstated. It sheds light on hidden structures and illuminates relationships among seemingly different objects. A perfect example of this power is given by the Wilson loops/scattering amplitudes duality in $d = 4, \mathcal{N} = 4$ super-Yang Mills (SYM) theory. Even though evidence for such a duality existed [1–6], only after the construction of a supersymmetrized Wilson loop (WL) in superspace [7,8] has the duality been confirmed for all helicity sectors of the amplitude. In three dimensions, while similar evidence in the case of the four-point amplitude/four-gluon Wilson loop for $\mathcal{N} = 6$ ABJM theory [9] exists [10,11], extending beyond four points immediately forces us into the remaining sectors (in terms of R-symmetry instead of helicity) of the theory. This motivates us to construct supersymmetric Wilson loops in superspace.

After the introduction of ABJM theory, various Wilson loop operators with different amounts of preserved supersymmetry were studied extensively. Earlier efforts dealt with construction and perturbative computations of $\frac{1}{6}$-BPS WL [12–15]. Localization was applied to evaluate the vacuum expectation value (vev) of this WL in [16], and the results were found to match the perturbative calculations at the large N limit. $\frac{1}{2}$-BPS operators were constructed later in [17] and more calculations followed in [18,19] where even finite N contributions were computed. Being "cohomologically equivalent" to the $\frac{1}{6}$-BPS operator, the localization results do not differ for these two operators. In [20], a classification was given for Wilson loops preserving various amounts of supersymmetry in $\mathcal{N} = 2, \ldots, 6$ Chern-Simons (CS) matter theories. New Wilson loops in $\mathcal{N} = 4$ theories have been constructed recently in [21].

In this ever-expanding literature of construction, classification, and computation involving Wilson loops, we present here a supersymmetrization of the simplest WL operator in three-dimensional CS matter theories including ABJ(M) theories. Such an attempt has been made in [22] for the ABJM theory in the framework of "ordinary" $\mathcal{N} = 6$ superspace. It was also pointed out that there are at least three reasons why such a WL cannot be dual to the scattering amplitudes of ABJM theory. The main issue is the nonchiral nature of the superspace that leads to torsion, which does not allow a straightforward identification of the kinematics on the two sides of the duality [23]. So we content ourselves with the well-studied framework of $\mathcal{N} = 3$ harmonic superspace [24,25] to construct the supersymmetrized Wilson loop.[1] This is to have as much manifest (off-shell) supersymmetry as possible along with a notion of chirality (or "harmonic analyticity") built in.

In the next section, we consider a warm-up exercise of constructing a supersymmetrized WL in $\mathcal{N} = 2$ superspace and a sample localization computation. Then we review the $d = 3$, $\mathcal{N} = 3$ harmonic superspace in Sec. III before constructing the supersymmetrized $\frac{1}{3}$-BPS WL in Sec. IV. This leads to a generalization for $\frac{1}{6}$-BPS WL in ABJ(M)

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[1]Harmonic Superspace was originally constructed for $d = 4$, $\mathcal{N} = 2$ supersymmetric theories in [26].

theories. In Sec. V, we compute perturbatively the one-loop vev of this new WL operator directly in harmonic superspace. Finally, we compare the perturbative result with localization computation and comment on future outlook in Sec. VI.

II. WARM-UP

We construct here a supersymmetrized Wilson loop operator in $d=3$, $\mathcal{N}=2$ superspace with coordinates $\{x^{\mu}(x^{(\alpha\beta)}),\theta^{\alpha},\bar{\theta}^{\alpha}\}$, where the vector index $\mu=0,1,2$ and spinorial index $\alpha=1,2$ correspond to the $SO(2,1)\simeq SL(2,\mathbb{R})$ group[2] Though it is rather straightforward, we think this analysis has not appeared in the literature in this form so we discuss it as a warm-up exercise leading to the less trivial $\mathcal{N}=3$ superspace in the next section.

The $\mathcal{N}=2$ supersymmetry algebra has the following set of gauge-covariant superspace derivatives: $\{\mathcal{D}_{\mu}(\mathcal{D}_{(\alpha\beta)}),\mathcal{D}_{\alpha},\bar{\mathcal{D}}_{\alpha}\}$. These satisfy the following algebra:

$$\{\mathcal{D}_{\alpha},\bar{\mathcal{D}}_{\beta}\}=i\mathcal{D}_{\alpha\beta}+\varepsilon_{\alpha\beta}W;\qquad\{\mathcal{D}_{\alpha},\mathcal{D}_{\beta}\}=0$$
$$[\mathcal{D}_{\alpha},\mathcal{D}^{\beta\gamma}]=\delta_{\alpha}^{(\beta}\bar{W}^{\gamma)}$$
$$[\mathcal{D}_{\mu},\mathcal{D}_{\nu}]=i\mathcal{F}_{\mu\nu}. \tag{2.1}$$

The Jacobi identities give further relations among the field strengths W, W^{α}, and $\mathcal{F}_{\mu\nu}$. One such relation is $\mathcal{D}_{\alpha}W=-i\bar{W}_{\alpha}$ along with the chirality constraint $\mathcal{D}_{\alpha}\bar{W}_{\beta}=0$ [27,28].

The supersymmetrization of the familiar $\frac{1}{2}$-BPS Wilson loop in chiral superspace then looks like

$$\mathcal{W}(x,\theta,\bar{\theta})=\frac{1}{\dim R}\mathrm{tr}_{R}\mathcal{P}\exp\int d\tau\left[\frac{-i}{2}\dot{x}_{A}^{\alpha\beta}A_{\alpha\beta}+\dot{\theta}^{\alpha}A_{\alpha}+|\dot{x}_{A}|W\right]\equiv\frac{1}{\dim R}\mathrm{tr}_{R}\mathcal{P}e^{w}, \tag{2.2}$$

where $x_{A}^{\alpha\beta}=x^{\alpha\beta}+i\theta^{(\alpha}\bar{\theta}^{\beta)}$. We can do the component analysis of the connections and field strengths, leading to the fields of $\mathcal{N}=2$ vector multiplet $\{a_{\alpha\beta},\sigma,\lambda_{\alpha},\bar{\lambda}_{\alpha},D\}$ along with the field strength $f_{\alpha\beta}$:

$$W_{|}=\sigma;\qquad\mathcal{D}_{\alpha}W_{|}=\bar{\lambda}_{\alpha};\qquad\bar{\mathcal{D}}_{\alpha}W_{|}=\lambda_{\alpha};\qquad\mathcal{D}_{\alpha}\bar{\mathcal{D}}_{\beta}W_{|}=f_{\alpha\beta}+\varepsilon_{\alpha\beta}D$$
$$\bar{\mathcal{D}}_{(\alpha}A_{\beta)|}=a_{\alpha\beta};\qquad\bar{\mathcal{D}}\cdot A_{|}=\sigma;\qquad\bar{\mathcal{D}}^{2}A_{\alpha|}=\lambda_{\alpha};\qquad\mathcal{D}_{\alpha}\bar{\mathcal{D}}\cdot A_{|}=\bar{\lambda}_{\alpha};\qquad\mathcal{D}^{\alpha}\bar{\mathcal{D}}^{2}A_{\alpha|}=D$$
$$A_{\alpha\beta|}=a_{\alpha\beta};\qquad\mathcal{D}^{\alpha}A_{\alpha\beta|}=\bar{\lambda}_{\beta};\qquad\bar{\mathcal{D}}^{\alpha}A_{\alpha\beta|}=\lambda_{\beta};\qquad\mathcal{D}^{\alpha}\bar{\mathcal{D}}^{\beta}A_{\alpha\beta|}=D. \tag{2.3}$$

Here $_{|}$ denotes that all θ's are set to vanish. Also relevant is $\mathcal{D}_{(\alpha}\bar{\mathcal{D}}^{2}A_{\beta)|}=f_{\alpha\beta}$. It is now trivial to verify that the θ-independent piece of the exponent in (2.2) reduces to the well-known bosonic expression: $\int d\tau(i\dot{x}^{\mu}A_{\mu}+|\dot{x}|\sigma)$.

It can be easily checked that the $\mathcal{W}(x,\theta,\bar{\theta})$ preserves some supersymmetry:

$$\delta\mathcal{W}\equiv\epsilon^{\gamma}\mathcal{D}_{\gamma}\mathcal{W}(x,\theta,\bar{\theta})\sim\mathrm{tr}_{R}\mathcal{P}\left\{e^{w}\int d\tau\left(\epsilon^{\gamma}\mathcal{D}_{\gamma}\left[\frac{-i}{2}\dot{x}_{A}^{\alpha\beta}A_{\alpha\beta}+\dot{\theta}^{\alpha}A_{\alpha}+|\dot{x}_{A}|W\right]\right)\right\}$$
$$\sim\mathrm{tr}_{R}\mathcal{P}\left\{e^{w}\int d\tau(i\epsilon^{\alpha}(\dot{x}_{\alpha\beta}^{A}+|\dot{x}^{A}|\varepsilon_{\alpha\beta})\bar{W}^{\beta})\right\}. \tag{2.4}$$

In arriving at the last step, we have used the algebra (2.1) to convert covariant derivatives acting on connections into the corresponding field strengths, and terms that look like field-dependent gauge transformations of the connections, i.e., $\dot{x}^{A,\alpha\beta}\mathcal{D}_{\alpha\beta}(\epsilon^{\gamma}A_{\gamma})$, are dropped as $\mathcal{W}(x,\theta,\bar{\theta})$ is gauge invariant. The BPS condition for the purely bosonic WL requires $x^{\mu}(\tau)$ to be an infinite line in Minkowski space or a great circle on S^{3} and one can choose it to satisfy $|\dot{x}|=1$ [16,20]. Since (2.4) for the supersymmetrized case results in a similar equation, we will also consider $|\dot{x}^{A}|=1$. This does not determine $\theta(\tau)$ completely but only up to a function of τ: $\theta(\tau)=f(\tau)\theta_{0}$, $\bar{\theta}(\tau)=f^{-1}(\tau)\bar{\theta}_{0}$.[3] Hence, constant solutions for ϵ can still be found for these configurations, where the condition $\epsilon^{\alpha}(\dot{x}_{\alpha\beta}^{A}+|\dot{x}^{A}|\varepsilon_{\alpha\beta})=0$ projects half of the degrees of freedom, thus preserving two real degrees of freedom, i.e., $\frac{1}{2}$-BPS.

Given the Lagrangian and propagators of [27,28], one should be able to compute the vev of the WL (2.2) perturbatively in superspace as well as in components at

[2]The vector x^{μ} can be traded for a real second-rank symmetric tensor $x^{\alpha\beta}\equiv x^{\mu}(\gamma_{\mu})^{\alpha\beta}$ with the help of $d=3$ "gamma" matrices. We do not need the explicit basis but the relation $x^{\alpha\beta}x_{\alpha\beta}=-2x^{\mu}x_{\mu}\equiv-2|x|^{2}$ will be quite useful to know.

[3]It is most likely that one needs to consider superconformal transformations of the WL operator to fully determine the $\theta(\tau)$ profile consistent with the circular bosonic WL. We do not pursue this exercise here. Thus, we will not evaluate the τ integrals explicitly and leave all the τ dependence of the coordinates intact.

different θ orders. However, we will skip this analysis here and comment on the nonperturbative analysis instead. Using the localization results of [16] where a $\mathcal{N}=2$ theory on S^3 (of radius r) is considered, we can obtain an exact result for the vev of the supersymmetrized Wilson loop. Since the path integral is localized on the vector multiplet's scalar field $\sigma=$ constant and $D=-\frac{\sigma}{r}$, we have

$$\mathcal{W}(x,\theta,\bar{\theta})=\frac{1}{\dim R}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-i}{2}\dot{x}_A^{\alpha\beta}\theta_\alpha\bar{\theta}_\beta D + \dot{\theta}\cdot(\bar{\theta}\sigma+\theta\bar{\theta}^2 D)+|\dot{x}_A|(\sigma+\theta\cdot\bar{\theta}D)\right]. \tag{2.5}$$

Even though we do not know $f(\tau)$ explicitly, we can evaluate $\langle\mathcal{W}(x,\theta,\bar{\theta})\rangle$ formally. Let us denote everything in the exponent by $\Theta\sigma$, with $\Theta=\frac{1}{2\pi}\int d\tau(1+\frac{i}{2}\dot{x}_A^{\alpha\beta}\theta_\alpha\bar{\theta}_\beta+\dot{\theta}\cdot\bar{\theta}-\theta\cdot\bar{\theta}+\dot{\theta}\cdot\theta\bar{\theta}^2)$ (also set $r=1$). The path integral reduces to a matrix model in terms of eigenvalues λ_i of σ (we choose ABJM for concreteness, which has two $U(N)$'s as gauge groups and $\pm k$ as the two Chern-Simons levels),

$$\langle\mathcal{W}(x,\theta,\bar{\theta})\rangle=\frac{1}{N!NZ}\int d\lambda_i d\hat{\lambda}_i(e^{-\frac{N}{2\alpha}\lambda_i^2-\frac{N}{2\hat{\alpha}}\hat{\lambda}_i^2})\Delta(\lambda)^2 \times\Delta(\hat{\lambda})^2\left(\sum_i e^{\Theta\lambda_i}\right)\times Z_{\text{1-loop}}, \tag{2.6}$$

where $\alpha=-\hat{\alpha}=2\pi i\frac{N}{k}$. We refer the reader to [16] for the definitions of various factors in the above result as we are interested in its perturbative limit only. To obtain a perturbative α expansion, we can expand $\langle\mathcal{W}\rangle$ in λ and compute the vev using the orthogonal polynomials method (note that $\langle\lambda^{2k}\rangle=\mathcal{O}(\alpha^k)$):

$$\langle\mathcal{W}(x,\theta,\bar{\theta})\rangle=1+\frac{1}{2}\Theta^2\alpha -\left[\frac{1}{6}\left(1+\frac{1}{2N^2}\right)\Theta^2-\frac{1}{24}\left(2+\frac{1}{N^2}\right)\Theta^4\right]\alpha^2 +\mathcal{O}(\alpha^3). \tag{2.7}$$

Rewriting $\Theta=1+\frac{1}{2}\vartheta$, we get (note $\vartheta^3=0$)

$$\langle\mathcal{W}(x,\theta,\bar{\theta})\rangle=1+\frac{1}{2}\left[\vartheta+\frac{\vartheta^2}{4}\right]\alpha -\left[\frac{1}{24}\left(5+\frac{1}{N^2}\right)+\frac{1}{4}\vartheta-\frac{1}{6}\left(\frac{1}{2}+\frac{1}{N^2}\right)\frac{\vartheta^2}{4}\right]\alpha^2 +\mathcal{O}(\alpha^3). \tag{2.8}$$

In the above expression, we have removed the bosonic term at $\mathcal{O}(\alpha)$ by multiplying the result by an overall phase $e^{-\frac{1}{2}\alpha}$, which is necessary in matching the perturbative computation [16]. Note that we do not remove the whole ϑ-dependent term at $\mathcal{O}(\alpha)$, since as we will see later there are indeed fermionic contributions at $\mathcal{O}(\alpha)$ in perturbative computation. We will return back to this result in Sec. VI.

III. REVIEW OF $\mathcal{N}=3$ HARMONIC SUPERSPACE

Now, we turn to $\mathcal{N}=3$ supersymmetry. We collect here the necessary ingredients from three-dimensional $\mathcal{N}=3$ harmonic superspace literature along with a few explicitly worked out details that will be relevant for us in later sections.

A. $\mathcal{N}=3$ harmonic superspace

The ordinary $d=3$, $\mathcal{N}=3$ superspace with coordinates $\{x^{\alpha\beta},\theta_{ij}^\alpha\}$ has the following algebra of superspace derivatives:

$$\{D_\alpha^{ij},D_\beta^{kl}\}=i(\varepsilon^{ik}\varepsilon^{jl}+\varepsilon^{il}\varepsilon^{jk})\partial_{\alpha\beta}$$
$$D_\alpha^{ij}=\partial_\alpha^{ij}+i\theta^{ij}\partial_{\alpha\beta}. \tag{3.1}$$

To obtain constrained superfields in the form of $D_\alpha^{ij}\Phi=0$, it is useful to consider the case where D_α^{ij} is given by a simple partial derivative, indicating the independence of Φ on certain variables. The obstacle to having a representation of D_α^{ij} as a partial derivative is its anticommutator algebra. This can be overcome by the introduction of $SU(2)/U(1)$ harmonics $u_i^\pm$. These bosonic variables satisfy

$$u^{+i}u_i^-=1,\qquad u^{\pm i}u_i^\pm=0, \tag{3.2}$$

where the raising and lowering of the $SU(2)$ index i is done by contracting with the invariant tensor ϵ^{ij}. (The contracted i among u's will be suppressed most of the time.) These new variables are to be integrated away using the following rules:

$$\int du1=1,\qquad\int du u_{(i_1}^+\ldots u_{i_n)}^-=0. \tag{3.3}$$

In other words, only the $SU(2)$ invariant polynomial with vanishing $U(1)$ charge survives the integration. The harmonic variables allow us to linearly recombine the 3×2 fermionic coordinates into three new $SL(2,\mathbb{R})$ doublets $\theta^{\alpha,\pm\pm}\equiv u^{i\pm}u^{j\pm}\theta_{ij}^\alpha$, $\theta^{\alpha,0}\equiv u^{i+}u^{j-}\theta_{ij}^\alpha$. The upshot is that doing the same for the covariant derivatives, the supersymmetry algebra now reads,

$$\{D_\alpha^{++},D_\beta^{--}\}=2i\partial_{\alpha\beta},\qquad\{D_\alpha^0,D_\beta^0\}=-i\partial_{\alpha\beta},$$
$$\{D_\alpha^{\pm\pm},D_\beta^{\pm\pm}\}=0,\qquad\{D_\alpha^{\pm\pm},D_\beta^0\}=0, \tag{3.4}$$

where one finds that we can $SU(2)$ covariantly isolate a doublet of commuting fermionic derivatives, for example D_α^{++}. This implies that we can have a representation for the covariant derivatives where D_α^{++} is a simple partial derivative. This is referred to as the "analytic basis," and it is given as the following:

$$\partial_{\alpha\beta} \to \partial^A_{\alpha\beta}$$
$$D^{ij}_\alpha \to \begin{cases} D^{++}_\alpha = \partial^{++}_\alpha \\ D^{--}_\alpha = \partial^{--}_\alpha + 2i\theta^{--\beta}\partial^A_{\alpha\beta} \\ D^0_\alpha = -\frac{1}{2}\partial^0_\alpha + i\theta^{0\beta}\partial^A_{\alpha\beta}. \end{cases} \tag{3.5}$$

We defined $x^A_{\alpha\beta} = x_{\alpha\beta} + i\theta^{++}_{(\alpha}\theta^{--}_{\beta)}$. In the analytic basis, we obtain constrained superfields by imposing the 'analytic' constraint $D^{++}_\alpha\Phi = 0$, which now implies that Φ does not depend on θ^{--}_α:

$$D^{++}_\alpha\Phi = 0 \Rightarrow \Phi \equiv \Phi(x^A_{\alpha\beta}, \theta^{++}_\alpha, \theta^0_\alpha, u). \tag{3.6}$$

The introduction of harmonic variables also introduces R-symmetry covariant derivatives, and are given by[4]

$$D^{\pm\pm} \equiv \partial^{\pm\pm} = u^\pm_i \frac{\partial}{\partial u^\mp_i}, \qquad D^0 = [D^{++}, D^{--}]. \tag{3.7}$$

These have nontrivial commutator algebra with the fermionic derivatives[5]

$$[D^{\pm\pm}, D^{\mp\mp}_\alpha] = 2D^0_\alpha, \qquad [D^{\pm\pm}, D^0_\alpha] = D^{\pm\pm}_\alpha. \tag{3.8}$$

B. Chern-Simons matter theories

To study gauge theories, we gauge-covariantize the full superspace derivatives $D \to \mathcal{D} = D + A$, which define the relevant field strengths:

$$\{\mathcal{D}^{++}_\alpha, \mathcal{D}^{--}_\beta\} = 2i\mathcal{D}_{\alpha\beta} + 2\varepsilon_{\alpha\beta}W^0, \qquad \{\mathcal{D}^0_\alpha, \mathcal{D}^0_\beta\} = -i\mathcal{D}_{\alpha\beta}, \tag{3.9}$$

$$\{\mathcal{D}^{\pm\pm}_\alpha, \mathcal{D}^{\pm\pm}_\beta\} = 0, \qquad \{\mathcal{D}^{\pm\pm}_\alpha, \mathcal{D}^0_\beta\} = \pm\varepsilon_{\alpha\beta}W^{\pm\pm}. \tag{3.10}$$

The covariant derivatives, and the field-strengths, transform as $\mathcal{D} \to e^\tau\mathcal{D}e^{-\tau}$. Choosing a suitable gauge-frame (from $\tau \to \lambda$) such that $A^{++}_\alpha = 0$ allows us to define analytic super fields covariantly while maintaining its implication of independence on θ^{--}_α: $\mathcal{D}^{++}_\alpha\Phi = D^{++}_\alpha\Phi = 0$. Note that choosing such a gauge generates (new) harmonic connections $A^{\pm\pm}$, from which all other connections can be obtained through Bianchi identities. In particular, A^{++} turns out to be the unique analytic ($D^{++}_\alpha A^{++} = 0$) prepotential in this formalism. The prepotential transforms under a gauge variation as usual,

$$A^{++\prime} = e^\lambda\mathcal{D}^{++}e^{-\lambda} \Rightarrow \delta_\lambda A^{++} = -\mathcal{D}^{++}\lambda, \tag{3.11}$$

where λ is an analytic gauge parameter. A convenient gauge is the Wess-Zumino gauge in which the prepotential has the following component expansion [25]:

$$\begin{aligned} &\frac{1}{2}\mathcal{D}^0_\alpha\mathcal{D}^{--}_\beta A^{++}|= a_{\alpha\beta}; \qquad \frac{1}{2}(\mathcal{D}^0_\beta)^2\mathcal{D}^{--}_\alpha A^{++}| = 2\lambda_\alpha; \qquad \frac{1}{2}\mathcal{D}^0_\alpha(\mathcal{D}^{--}_\beta)^2A^{++}| = 3\chi^{--}_\alpha; \\ &\frac{1}{2}(\mathcal{D}^{--}_\alpha)^2A^{++}| = 3\phi^{--}; \qquad \frac{1}{2}(\mathcal{D}^0_\alpha)^2\frac{1}{2}(\mathcal{D}^{--}_\beta)^2A^{++}| = 3X^{--}. \end{aligned} \tag{3.12}$$

This is clearly the $\mathcal{N} = 3$ vector multiplet with fields $(a_\mu, \lambda_\alpha, \chi^{(ij)}_\alpha, \phi^{(ij)}, X^{(ij)})$. Of course, $\phi^{--} = u^-_iu^-_j\phi^{ij}$ and so on.

It is now possible to write every other connection and field strength in terms of the analytic prepotential A^{++}. We start with the following connections:

$$\begin{aligned} D^0 = [\mathcal{D}^{++}, \mathcal{D}^{--}] &\Rightarrow A^{--}(u) = \sum_{n=1}^\infty(-1)^n\int du_{1,\ldots,n}\frac{A^{++}_1\ldots A^{++}_n}{(u^+u^+_1)(u^+_1u^+_2)\ldots(u^+_nu^+)}. \\ 2\mathcal{D}^0_\alpha = [\mathcal{D}^{--}, D^{++}_\alpha] &\Rightarrow A^0_\alpha = -\frac{1}{2}D^{++}_\alpha A^{--}, \\ \mathcal{D}^{--}_\alpha = [\mathcal{D}^{--}, \mathcal{D}^0_\alpha] &\Rightarrow A^{--}_\alpha = D^{--}A^0_\alpha - D^0_\alpha A^{--} + [A^{--}, A^0_\alpha]. \\ -i\mathcal{D}_{\alpha\beta} = \{\mathcal{D}^0_\alpha, \mathcal{D}^0_\beta\} &\Rightarrow A_{\alpha\beta} = 2iD^0_{(\alpha}A^0_{\beta)} = -iD^0_{(\alpha}D^{++}_{\beta)}A^{--}. \end{aligned} \tag{3.13}$$

[4] D^0 is strictly speaking not a covariant derivative on $SU(2)/U(1)$. It should be treated as the subgroup generator that defines the $U(1)$ charge for a given operator or field, as in $D^0\Phi^{(q)} = q\Phi^{(q)}$.

[5] For completeness, their explicit forms in the analytic basis are given as

$$D^{\pm\pm} \to D^{\pm\pm} = \partial^{\pm\pm} \pm 2i\theta^{\pm\pm\alpha}\theta^{0\beta}\partial^A_{\alpha\beta} + 2\theta^{0\alpha}\partial^{\pm\pm}_\alpha + \theta^{\pm\pm\alpha}\partial^0_\alpha.$$

Then the covariant field strengths can be derived from the connections as follows:

$$D_\alpha^{++}W^{++}=\mathcal{D}^{++}W^{++}=0\Rightarrow W^{++}\text{ is analytic.}$$
$$W^{++}=\frac{1}{2}D_\alpha^{++}A^{0\alpha}=-\frac{1}{4}(D_\alpha^{++})^2A^{--}.$$
$$W^0=\frac{1}{2}\mathcal{D}^{--}W^{++}\quad\text{and}\quad W^{--}=\mathcal{D}^{--}W^0.\tag{3.14}$$

The $\mathcal{N}=3$ matter multiplet consists of two complex scalars f^i transforming as a doublet under $SU(2)$ and their fermionic partners ψ_α^i, which are encoded in the following hypermultiplet superfield,

$$q^+{}_|=f^+;\quad \mathcal{D}_\alpha^{--}q^+{}_|=\psi_\alpha^-;\quad \frac{1}{2}\mathcal{D}_\alpha^0\mathcal{D}_\beta^{--}q^+{}_|=-i\partial_{\alpha\beta}f^-,$$
$$\bar{q}^+{}_|=-\bar{f}^+;\quad \mathcal{D}_\alpha^{--}\bar{q}^+{}_|=\bar{\psi}_\alpha^-;\quad \frac{1}{2}\mathcal{D}_\alpha^0\mathcal{D}_\beta^{--}\bar{q}^+{}_|=i\partial_{\alpha\beta}\bar{f}^-,\tag{3.15}$$

where $f^\pm\equiv u_i^\pm f^i$, $\bar{f}^\pm\equiv u_i^\pm\bar{f}^i$, and similarly for the fermions.

For the ABJM theory, we have two sets of q^{+a}, with $a=1,2$. In this representation, the $SO(6)$ R-symmetry is broken: $SO(6)\to SU(2)_\mathrm{R}\times SU(2)_\mathrm{ext}$, and the ABJ(M) action for $U_L(N)\times U_R(M)$ theory:

$$\mathcal{S}=\mathcal{S}_{CS}[A_L^{++}]-\mathcal{S}_{CS}[A_R^{++}]+\mathrm{tr}\int d\zeta^{(-4)}\bar{q}_a^+\mathcal{D}^{++}q^{+a},\tag{3.16}$$

$$\mathcal{S}_{CS}[A^{++}]=\frac{ik}{4\pi}\mathrm{tr}\sum_{n=2}^{\infty}\frac{(-1)^n}{n}\int d^3xd^6\theta du_{1,\ldots,n}\times\frac{A_1^{++}\ldots A_n^{++}}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)},\tag{3.17}$$

$$(\mathcal{D}^{++}q^{+a})_A^{\bar{B}}=D^{++}(q^{+a})_A^{\bar{B}}+(A_L^{++})_A^B(q^{+a})_B^{\bar{B}}-(q^{+a})_A^{\bar{A}}(A_R^{++})_{\bar{A}}^{\bar{B}},\tag{3.18}$$

where $A\in U(N)$, $\bar{A}\in U(M)$, and $\bar{q}^{+a}$ have "opposite" gauge charges under the two gauge groups. From the action, one finds the following equations of motion,

$$\frac{\delta\mathcal{S}}{\mathcal{D}\bar{q}_a^+}=\nabla^{++}q^{+a}=0,\quad \frac{\delta\mathcal{S}}{\delta A^{++}}=W^{++}+\frac{4\pi i}{k}\bar{q}_a^+q^{+a}=0,\tag{3.19}$$

with proper ordering of $\bar{q}q$ to match the gauge indices of $W_{L,R}^{++}$. The latter equation of motion implies that scalars from the vector multiplet get equated to biscalars of the matter multiplet. One such relation will be relevant for later use:

$$W^0=\frac{1}{2}D^{--}W^{++}$$
$$\Rightarrow\phi^0=-\frac{2\pi i}{k}u_i^-\frac{\partial}{\partial u_i^+}((-u_j^+\bar{f}_a^j)(u_k^+f^{ka}))$$
$$=\frac{2\pi i}{k}(u_j^-u_k^++u_j^+u_k^-)\bar{f}_a^jf^{ka}.\tag{3.20}$$

This crucial relation is responsible for generating the well-known sextic potential involving f's once ϕ's are integrated out from the ABJM action.

The CS theories coupled to matter can be quantized directly in superspace [25], and the resulting propagators read

$$\langle\bar{q}_1^+q_2^+\rangle=\frac{1}{2\pi i}\frac{u_1^+u_2^+}{\sqrt{2\rho^2}},\tag{3.21}$$

$$\langle A_1^{++}A_2^{++}\rangle=\frac{i}{2\pi}\frac{1}{\sqrt{2\rho^2}}\delta^2(\theta_{12}^{++})\delta^{(-2,2)}(u_1,u_2),\tag{3.22}$$

where

$$\rho^{\alpha\beta}=x_{A1}^{\alpha\beta}-x_{A2}^{\alpha\beta}-2i\theta_1^{0(\alpha}\theta_2^{0\beta)}-\frac{2i}{u_1^+u_2^+}[(u_1^-u_2^-)\theta_1^{++(\alpha}\theta_2^{++\beta)}-(u_1^-u_2^+)\theta_1^{++(\alpha}\theta_2^{0\beta)}-(u_1^+u_2^-)\theta_1^{0(\alpha}\theta_2^{++\beta)}+(u_1^-u_2^+)\theta_1^{++(\alpha}\theta_1^{0\beta)}+(u_1^+u_2^-)\theta_2^{0(\alpha}\theta_2^{++\beta)}].\tag{3.23}$$

The $\rho^{\alpha\beta}$ has quite a complicated expression but in the presence of $\delta^2(\theta_{12}^{++})\delta(u_1,u_2)$, it simplifies in the vector propagator to the following:

$$\rho^{\alpha\beta}=(x_A^{\alpha\beta})_{12}-2i\theta_1^{0(\alpha}\theta_2^{0\beta)}$$
$$\Rightarrow\rho^2=-2|x_{12}^A|^2-4ix_{12}^A\cdot\theta_1^0\theta_2^0+4\theta_1^{02}\theta_2^{02}.\tag{3.24}$$

The vertices are easily read from the relevant actions.

IV. SUPER-WILSON LOOP

There are two main types of Wilson loop operators that can be considered for $d=3$ Chern-Simons theories [12,16,17,20]: GY type ($\frac{1}{\mathcal{N}}$-BPS for $\mathcal{N}=2,3,4,6$) and DT type (still $\frac{1}{\mathcal{N}}$-BPS for $\mathcal{N}=2,3$ but $\frac{1}{2}$-BPS for $\mathcal{N}=4,6$). We will focus only on the former case here. The $\frac{1}{3}$-BPS Wilson loop is usually written for $\mathcal{N}=3$ CS theory as follows,

$$\mathcal{W}_{1/3}(x)=\frac{1}{\dim R}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-i}{2}\dot{x}^{\alpha\beta}a_{\alpha\beta}+\frac{1}{2}\dot{y}_{ij}\phi^{ij}\right],\tag{4.1}$$

where $y_{ij} = y_{ji}$ are 3 $SU(2)$ coordinates. For this operator to locally preserve any supersymmetry, the SUSY parameter ϵ_α^{ij} needs to be a solution of

$$\dot{x}^{\alpha\beta}\epsilon_\beta^{ij} + \dot{y}^i{}_k\epsilon^{\alpha,kj} = 0, \tag{4.2}$$

provided that $|\dot{x}| = |\dot{y}|$. To incorporate the condition on $|\dot{y}|$, we can rewrite the scalar term in WL as $\int d\tau|\dot{x}|(u_i^+u_j^-)\phi^{(ij)}$ using the harmonic coordinates on $SU(2)$.

Now, we are ready to write down the most general supersymmetrized expression for a Wilson loop (such that (4.1) is its bosonic component):

$$\mathcal{W}(x,\theta^{\pm\pm},\theta^0) = \frac{1}{\dim R}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-1}{4}\dot{x}^{A,\alpha\beta}A_{\alpha\beta} + \dot{\theta}^{++\alpha}A_\alpha^{--} + \dot{\theta}^{0\alpha}A_\alpha^0 + \sum_\pm u^{\pm i}\dot{u}_i^\pm A^{\mp\mp} + \frac{1}{2}|\dot{x}^A|W^0\right]. \tag{4.3}$$

The usual BPS condition on the bosonic WL ($\epsilon^{(ij)\gamma}Q_\gamma^{(ij)}\mathcal{W}_{1/3}(x) = 0$), which results in (4.2), translates to $\epsilon^{(ij)\gamma}D_\gamma^{(ij)}\mathcal{W}_{1/3} = 0$ (along with $\dot{x}\to\dot{x}^A$) in superspace for obvious reasons (see [29] for an explicit proof). Let us see what that implies for (4.3),

$$\begin{aligned}\epsilon^{(ij)\gamma}D_\gamma^{(ij)}\mathcal{W}(x,\theta^{++},\theta^0) \propto \int d\tau\bigg[&\frac{-1}{4}\dot{x}^{A,\alpha\beta}\epsilon^{(ij)\gamma}\mathcal{F}_{\gamma,\alpha\beta}^{(ij)} + \dot{\theta}^{++\alpha}\epsilon^{(ij)\gamma}\mathcal{F}_{\gamma,\alpha}^{(ij),--} + \dot{\theta}^{0\alpha}\epsilon^{(ij)\gamma}\mathcal{F}_{\gamma,\alpha}^{(ij),0}\\ &+ \underbrace{0}_{\mathcal{F}_\gamma^{(ij),\mp\mp}\equiv 0} + \frac{1}{2}|\dot{x}^A|\epsilon^{(ij)\gamma}\mathcal{D}_\gamma^{(ij)}W^0\bigg],\end{aligned}$$

where we use $\mathcal{F}_{A,B}$ to represent the field strength arising from the (anti)commutator of $\{\mathcal{D}_A,\mathcal{D}_B]$. As we did for the case of $\mathcal{N}=2$ WL, we have ignored here terms that look like field-dependent gauge transformations. Since we know that only $\mathcal{F}_{\gamma,\alpha}^{0,0} = \mathcal{F}_{\gamma,\alpha}^{\pm\pm,\pm\pm} = 0$, we can have only one of the $\dot{\theta}$ terms above in the Wilson loop. This means either ϵ^{++} or ϵ^0 can be the only unbroken SUSY. However, choosing ϵ^0, we find that $\mathcal{F}_{\gamma,\alpha\beta}^0$ [25] contains not only the $\mathcal{D}_\alpha^0W^0$ term but also $\mathcal{D}_\alpha^{++}W^{--}$, so the above variation cannot vanish. Thus, we are left with ϵ^{++} and the remaining couple of terms do vanish in this case because

$$\mathcal{F}_{\gamma,\alpha\beta}^{--} = -i(\epsilon_{\gamma\alpha}\mathcal{D}_\beta^{--}W^0 + \epsilon_{\gamma\beta}\mathcal{D}_\alpha^{--}W^0), \tag{4.4}$$

which implies

$$\begin{aligned}-\frac{1}{2}\dot{x}^{A,\alpha\beta}\epsilon^{++\gamma}\mathcal{F}_{\gamma,\alpha\beta}^{--} + |\dot{x}^A|\epsilon^{++\gamma}\mathcal{D}_\gamma^{--}W^0 &= \epsilon^{++\gamma}(i\dot{x}^{A,\alpha\beta}\epsilon_{\gamma\alpha}\mathcal{D}_\beta^{--}W^0 + |\dot{x}^A|\mathcal{D}_\gamma^{--}W^0)\\ &= i\epsilon_\alpha^{++}(\dot{x}^{A,\alpha\beta} - i|\dot{x}^A|\epsilon^{\alpha\beta})\mathcal{D}_\beta^{--}W^0 = 0.\end{aligned} \tag{4.5}$$

This expression vanishes (for arbitrary W^0) in a way similar to the $\mathcal{N}=2$ case, and we preserve half of the complex spinor $\epsilon^{++\gamma}$. Thus, the final result for the supersymmetric generalization of the $\frac{1}{3}$-BPS Wilson loop is

$$\mathcal{W}_{1/3} = \frac{1}{\dim R}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-1}{4}\dot{x}^{A,\alpha\beta}A_{\alpha\beta} + \dot{\theta}^{++\alpha}A_\alpha^{--} + \sum_\pm u^{\pm i}\dot{u}_i^\pm A^{\mp\mp} + \frac{1}{2}|\dot{x}^A|W^0\right]. \tag{4.6}$$

To compare with the usual bosonic WL operator, we write the above in component fields:

$$\mathcal{W}_{1/3} \sim \mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-i}{2}\dot{x}^{\alpha\beta}a_{\alpha\beta} + \dot{\theta}^{++\alpha}\theta_\alpha^{--}\phi^0 + \frac{1}{2}|\dot{x}^A|(\phi^0 + \theta^0\cdot\chi^0) + \mathcal{O}(\theta^2)\right]. \tag{4.7}$$

The difference starts at terms of order θ containing fermionic fields (χ_α^{ij}) and at θ^2 order with bosonic fields (ϕ^{ij}). Higher-order terms will contain λ_α, X^{ij} fields too.

With this construction, we can readily give the supersymmetrized generalization of the $\frac{1}{6}$-BPS WL operator for $\mathcal{N}=6$ ABJM theory in $\mathcal{N}=3$ harmonic superspace:

$$\mathcal{W}_{1/6}=\frac{1}{\dim R}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\left(\frac{-1}{4}\dot{x}^{A,\alpha\beta}A_{\alpha\beta}+\dot{\theta}^{++\alpha}A^{--}_{\alpha}+\sum_{\pm}u^{\pm i}\dot{u}^{\pm}_{i}A^{\mp\mp}+\frac{1}{2}|\dot{x}^A|W^0\right)_L+(L\to R)\right]. \quad (4.8)$$

This operator reduces to the canonical bosonic operator in ABJM theory with the matter coupling term $M^J_I C_J\bar{C}^I$, where $M^J_I=\mathrm{diag}(-1,-1,1,1)$ (up to the factor $\frac{2\pi}{k}$) if the u matrix further satisfies $u^+_1=(u^-_2)^{-1}=u(\tau)$. To show this, we need to use the equation of motion for A^{++} (3.19) and (3.20) along with a change of notation from $f^i\to C_I$ as discussed in [24]. (Without the constraint on u, this operator has more content due to W^0 containing not only $\phi^{12}\equiv M^J_I C_J\bar{C}^I$ but also ϕ^{11} and ϕ^{22}.)

V. COMPUTATION

In this section, we will compute the one-loop vacuum expectation value of the Wilson loop $\mathcal{W}_{1/3}$. The constraint on u will also be imposed so the operator and the expected vev slightly simplify (with R being the fundamental representation of $U(N)$ gauge group):

$$\mathcal{W}_{1/3}=\frac{1}{N}\mathrm{tr}_R\mathcal{P}\exp\int d\tau\left[\frac{-1}{4}\dot{x}^{A,\alpha\beta}A_{\alpha\beta}+\dot{\theta}^{++\alpha}A^{--}_{\alpha}+\frac{1}{2}|\dot{x}^A|W^0\right] \quad (5.1)$$

$$\langle\mathcal{W}_{1/3}\rangle=1+\frac{1}{2N}\int d\tau_1 d\tau_2\left\langle\left(\frac{-1}{4}\dot{x}^A\cdot A+\dot{\theta}^{++}\cdot A^{--}+\frac{1}{2}|\dot{x}^A|W^0\right)_1(\cdots)_2\right\rangle+\cdots. \quad (5.2)$$

An important subtlety that occurs repeatedly in the computation is when $D^{++}_\alpha(u)=u^+_i u^+_j D^{ij}_\alpha$ acts on an analytic superfield which depends on another harmonic variable, say $(\theta^{++}_\alpha,\theta^0_\alpha,u')$. The result is not the naive zero since, using $(u^+u^-)=1$ and repeated Schouten identities, one can rewrite

$$\begin{aligned}D^{++}_\alpha(u)&=(u'^+u'^-)^2u^+_i u^+_j D^{ij}_\alpha\\&=[(u^+u'^-)^2D^{++}_\alpha(u')+(u^+u'^+)^2D^{--}_\alpha(u')-2(u^+u'^-)(u^+u'^+)D^0_\alpha(u')],\end{aligned} \quad (5.3)$$

where we have converted the harmonic dependence of the derivative from u to u'. Note that the charges match on both sides separately for u's and u''s. Thus, for any analytic superfield $\Phi(\theta^{++}_\alpha,\theta^0_\alpha,u')$, we have

$$D^{++}_\alpha(u)\Phi(u')=(u^+u'^+)^2[D^{--}_\alpha\Phi](u')-2(u^+u'^-)(u^+u'^+)[D^0_\alpha\Phi](u'). \quad (5.4)$$

Similar manipulations lead to the following list of identities:

$$\begin{aligned}(D^{++}(u))^2\Phi(u')&=(u^+u'^+)^2((u^+u'^+)^2[(D^{--}_\alpha)^2\Phi](u')-4(u^+u'^-)(u^+u'^+)[D^{--\alpha}D^0_\alpha\Phi](u')+4(u^+u'^-)^2[(D^0_\alpha)^2\Phi](u'))\\D^0_\alpha(u)\Phi(u')&=(u^+u'^+)(u^-u'^+)[D^{--}_\alpha\Phi](u')+(1-2(u^+u'^+)(u^-u'^-))[D^0_\alpha\Phi](u')\\D^{--}_\alpha(u)\Phi(u')&=(u^-u'^+)^2[D^{--}_\alpha\Phi](u')-2(u^-u'^+)(u^-u'^-)[D^0_\alpha\Phi](u').\end{aligned} \quad (5.5)$$

For the sake of convenience, we list generating expressions for component expansions of some connections and field strengths below [that is, keeping only a single A^{++} in (3.13) and (3.14)]:

$$\begin{aligned}A_{\alpha\beta}(u)&=-i[D^0_{(\alpha}D^{++}_{\beta)}A^{--}](u)=i\int du'[D^0_{(\alpha}D^{--}_{\beta)}A^{++}](u')\\A^{--}_\alpha(u)&=-\frac{1}{2}[(D^{--}D^{++}_\alpha+2D^0_\alpha)A^{--}](u)\\&=-\int du'\left[\frac{u^-u'^+}{u^+u'^+}[D^{--}_\alpha A^{++}](u')+2\frac{u^-u'^-}{u^+u'^+}[D^0_\alpha A^{++}](u')\right]\\W^{++}(u)&=-\frac{1}{4}[(D^{++}_\alpha)^2A^{--}](u)\\&=-\frac{1}{4}\int du'[(u^+u'^+)^2[(D^{--}_\alpha)^2A^{++}](u')-4(u^+u'^-)(u^+u'^+)[D^{--}\cdot D^0A^{++}](u')].\end{aligned} \quad (5.6)$$

Note that all the fields depend on the same θ coordinate. The components can be obtained from the above expressions by using (3.12) and performing not only the D-algebra but some harmonic algebra too. The simplest component to obtain is the vector: $A_{\alpha\beta|} = 2ia_{\alpha\beta}$. To get the scalars, we need to perform slightly more involved algebra:

$$\begin{aligned} W^{++}{}_| &= -\frac{1}{4}\int du'(u^+u'^+)^2[(D_\alpha^{--})^2A^{++}](u')_| = 3\int du'(u^+u'^+)^2\phi^{--}(u') = u_j^+u_k^+\phi^{(jk)} = \phi^{++}; \\ W^0{}_| &= \frac{1}{2}D^{--}W^{++}{}_| = \frac{1}{2}(u_j^-u_k^+ + u_j^+u_k^-)\phi^{(jk)} = \phi^0; \\ W^{--}{}_| &= D^{--}W^0{}_| = \phi^{--}. \end{aligned} \tag{5.7}$$

Other components can be similarly obtained, which we leave as an exercise and refer the reader to [30] for useful identities involving harmonic variables.

Now we turn to evaluating various contributions to $\langle \mathcal{W}_{1/3}\rangle$. First, let us consider the contribution from the vector connection. In general, we have from (3.13):

$$\begin{aligned} \dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\langle A_{1,\alpha\beta}A_{2,\gamma\delta}\rangle &= -\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\langle D_{1\alpha}^0D_{1\beta}^{++}A_1^{--}D_{2\gamma}^0D_{2\delta}^{++}A_2^{--}\rangle \\ &= -\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\left\langle\int du\frac{D_{1\alpha}^0(u_1)D_{1\beta}^{++}(u_1)A_1^{++}(u)}{(u_1^+u^+)^2}\int dv\frac{D_{2\gamma}^0(u_2)D_{2\delta}^{++}(u_2)A_2^{++}(v)}{(u_2^+v^+)^2}\right\rangle + \cdots \end{aligned} \tag{5.8}$$

Using (5.6), we find

$$\begin{aligned} \dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\langle A_{1,\alpha\beta}A_{2,\gamma\delta}\rangle^{(1)} &= -\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\int du[D_{1\alpha}^0D_{1\beta}^{--}\underbrace{A_1^{++}](u)\int dv[D_{2\gamma}^0D_{2\delta}^{--}A_2^{++}}](v) \\ &= \dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\int du D_{1\alpha}^0D_{2\gamma}^0D_{1\beta}^{--}D_{2\delta}^{--}\frac{i}{2\pi\sqrt{2\rho^2}}\delta^2(\theta_{12}^{++}) \\ &= -\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\frac{\varepsilon_{\beta\delta}(ix_{12}^A - \theta_1^0\theta_2^0)_{\alpha\gamma}}{2\pi(x_{12}^{A\,2})^{3/2}} = \dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\frac{\varepsilon_{\beta\delta}\theta_{1\alpha}^0\theta_{2\gamma}^0}{2\pi|x_{12}^A|^3}. \end{aligned} \tag{5.9}$$

We used here $D_\alpha^{--}D_\beta^{--} = \frac{1}{2}\varepsilon_{\alpha\beta}D^{--2}$, $D^{--2}\delta^2(\theta^{++}) = 4$, $\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\varepsilon_{\beta\delta}x_{12,\alpha\gamma}^A \sim \varepsilon_{mnp}\dot{x}_{A,1}^m\dot{x}_{A,2}^n x_{A,12}^p \to 0$, and expanded $\frac{1}{\rho^2}$ in powers of θ^0's. The next term (quadratic in A^{++}) in the expansion of A^{--} also contributes

$$\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\langle A_{1,\alpha\beta}A_{2,\gamma\delta}\rangle^{(2)} = \frac{\dot{x}_{A,1}\cdot\dot{x}_{A,2}(|x_{12}^A|^2 - ix_{12}^A\cdot\theta_1^0\theta_2^0 + \frac{1}{4}\theta_1^{02}\theta_2^{02})^2}{4\pi^2|x_{12}^A|^6}. \tag{5.10}$$

Let us sketch how we got this result. We require that all $\delta^2(\theta_{12}^{++})$ be canceled so higher orders of A^{++} cannot contribute as there are not enough D_α^{--} derivatives in $\langle A_{1,\alpha\beta}A_{2,\gamma\delta}\rangle$ to cancel more than two such δ functions. After expanding $D_{1\alpha}^0D_{1\beta}^{++}$ using the identities given above, doing two harmonic integrals using the harmonic δ functions in the two propagators and then hitting the two $\delta^2(\theta_{12}^{++})$ with correct D^{--}'s, we are left with

$$\begin{aligned}\langle A_{1,\alpha\beta}A_{2,\gamma\delta}\rangle^{(2)} &\sim \int dvdw \frac{\begin{matrix}(-(u_1^+v^+)^2(w^+u_1^+)(w^+u_1^-)+(u_1^+w^+)^2(v^+u_1^+)(v^+u_1^-))\\ \times(-(u_2^+v^+)^2(w^+u_2^+)(w^+u_2^-)\varepsilon_{\beta\delta}\varepsilon_{\alpha\gamma}+(v^+u_2^+)(v^+u_2^-)(w^+u_2^+)^2\varepsilon_{\beta\gamma}\varepsilon_{\alpha\delta})\end{matrix}}{(u_1^+v^+)(w^+u_1^+)(v^+w^+)^2(u_2^+v^+)(w^+u_2^+)\sqrt{2\rho^2}\sqrt{2\rho^2}}\\ &\sim \int dvdw \frac{\begin{matrix}-(u_1^+v^+)(w^+u_1^-)((u_2^+v^+)(w^+u_2^-)\varepsilon_{\beta\delta}\varepsilon_{\alpha\gamma}+(v^+u_2^-)(w^+u_2^+)\varepsilon_{\beta\gamma}\varepsilon_{\alpha\delta})\\ -(u_1^+w^+)(u_1^-v^+)((u_2^+v^+)(w^+u_2^-)\varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma}+(v^+u_2^-)(w^+u_2^+)\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta})\end{matrix}}{(v^+w^+)^2(2\rho^2)}\\ &\sim \int dvdw \frac{\begin{matrix}-((u_1^+w^+)(u_1^-v^+)+(v^+w^+))(v^+w^+)\varepsilon_{\beta\gamma}\varepsilon_{\alpha\delta}\\ +(u_1^+w^+)(u_1^-v^+)(v^+w^+)\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}\end{matrix}}{(v^+w^+)^2(2\rho^2)}\\ &\sim \frac{\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}}{\rho^2}=\frac{\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}(|x_{12}^A|^2-ix_{12}^A\cdot\theta_1^0\theta_2^0+\frac{1}{4}\theta_1^{02}\theta_2^{02})^2}{|x_{12}^A|^6}.\end{aligned}$$

Keeping track of various signs and numerical factors above, we get (5.10).

Let us now evaluate the second contribution to $\langle\mathcal{W}\rangle$ due to the charged fermionic connection. Using the fact that we need enough D_α^{--} to get relevant terms, we ignore terms with D_α^0 in the expansion of A^{--} in (5.6):

$$\begin{aligned}&\dot\theta_{1\alpha}^{++}\dot\theta_{2\beta}^{++}\langle A_1^{--\alpha}A_2^{--\beta}\rangle\\ &\quad=\dot\theta_{1\alpha}^{++}\dot\theta_{2\beta}^{++}\int du\left(\frac{u^+u_1^-}{u^+u_1^+}\right)D_1^{--\alpha}\left(\frac{u^+u_2^-}{u^+u_2^+}\right)D_2^{--\beta}\frac{i\delta^2(\theta_{12}^{++})}{2\pi\sqrt{2\rho^2}}\\ &\quad=\dot\theta_1^{++}\cdot\dot\theta_2^{++}\left(\frac{u_1^-u_2^-}{u_1^+u_2^+}\right)\frac{|x_{12}^A|^2-ix_{12}^A\cdot\theta_1^0\theta_2^0+\frac{1}{4}\theta_1^{02}\theta_2^{02}}{2\pi|x_{12}^A|^3}.\end{aligned}\tag{5.11}$$

The u factor in parentheses might look divergent upon imposing the constraint on the u matrix discussed in the previous section, but using an explicit parametrization, one can show that it instead limits to unity up to a $U(1)$ "charge factor." We will, however, leave this factor as it is to account for the correct $U(1)$ charges along with an understanding that there is no nontrivial u dependence.

The third contribution to $\langle\mathcal{W}\rangle$ due to a mixed contraction of the two connections vanishes:

$$\begin{aligned}\langle A_{1,\alpha\beta}A_2^{--\gamma}\rangle&=-i\int duD_{1(\alpha}^0D_{1\beta)}^{--}\left(\frac{u^+u_2^-}{u^+u_2^+}\right)D_2^{--\gamma}\frac{i\delta^2(\theta_{12}^{++})}{2\pi\sqrt{2\rho^2}}\\ &=\frac{-(x_{12,\sigma(\alpha}^A\theta_2^{0\sigma}+\frac{i}{2}\theta_{1(\alpha}^0\theta_2^{02})\delta_{\beta)}^\gamma}{2\pi|x_{12}^A|^3}\int du\left(\frac{u^+u_2^-}{u^+u_2^+}\right)\\ &=0.\end{aligned}\tag{5.12}$$

The fourth contribution to $\langle\mathcal{W}\rangle$ due to the scalar field strength is

$$\begin{aligned}&|\dot x_{A,1}||\dot x_{A,2}|\langle W_1^0W_2^0\rangle^{(1)}\\ &\quad=\frac{1}{64}|\dot x_{A,1}||\dot x_{A,2}|\mathcal{D}_1^{--}D_{1\alpha}^{++2}\underbrace{A_1^{--}\mathcal{D}_2^{--}D_{2\beta}^{++2}A_2^{--}}\\ &\quad=\frac{|\dot x_{A,1}||\dot x_{A,2}|(\theta_1^{02}+\theta_1^0\cdot\theta_2^0+\theta_2^{02})}{2\pi|x_{12}^A|^3}(1-2(u_1^+u_2^+)(u_1^-u_2^-)).\end{aligned}\tag{5.13}$$

This is a contribution from the linear term in A^{--} and is straightforward to compute. Like $\langle A_{\alpha\beta}A_{\gamma\delta}\rangle$, we get a second contribution from the contraction of quadratic terms in A^{--} here too:

$$\begin{aligned}&|\dot x_{A,1}||\dot x_{A,2}|\langle W_1^0W_2^0\rangle^{(2)}\\ &\quad=-\frac{|\dot x_{A,1}||\dot x_{A,2}|(|x_{12}^A|^2-ix_{12}^A\cdot\theta_1^0\theta_2^0+\frac{1}{4}\theta_1^{02}\theta_2^{02})^2}{4\pi^2|x_{12}^A|^6}\\ &\qquad\times(1-2(u_1^+u_2^+)(u_1^-u_2^-)).\end{aligned}\tag{5.14}$$

This computation proceeds very similarly to the case of the vector connection, but there are more terms; we sketch them below (again, various signs and numerical factors need to be tracked):

$$\begin{aligned}\langle W_1^0 W_2^0\rangle^{(2)} &\sim \int dv_{1,2}\frac{\mathcal{D}_1^{--}D_{1\alpha}^{++2}[A^{++}(v_1)A^{++}(v_2)]}{(u_1^+v_1^+)(v_1^+v_2^+)(v_2^+u_1^+)}\int dw_{1,2}\frac{\mathcal{D}_2^{--}D_{2\beta}^{++2}[A^{++}(w_1)A^{++}(w_2)]}{(u_2^+w_1^+)(w_1^+w_2^+)(w_2^+u_2^+)}\\
&\sim D_1^{--}D_2^{--}\int dv_{1,2}\frac{D_\alpha^{--}(v_1)^2D_\beta^{--}(v_2)^2\left[\frac{(u_1^+v_1^+)^4\cdots}{\sqrt{2\rho^2}}\delta^2(\theta_{12}^{++})\frac{(u_2^+v_2^+)^4\cdots}{\sqrt{2\rho^2}}\delta^2(\theta_{12}^{++})\right]}{(u_1^+v_1^+)(v_1^+v_2^+)(v_2^+u_1^+)(u_2^+v_1^+)(v_1^+v_2^+)(v_2^+u_2^+)}\\
&\sim D_1^{--}D_2^{--}\int dv_{1,2}\frac{(u_1^+v_1^+)^2(u_1^+v_2^+)^2(u_2^+v_1^+)^2(u_2^+v_2^+)^2}{(u_1^+v_1^+)(v_1^+v_2^+)(v_2^+u_1^+)(u_2^+v_1^+)(v_1^+v_2^+)(v_2^+u_2^+)\rho^2}\\
&\sim D_1^{--}D_2^{--}\int dv_{1,2}\left[\frac{(u_1^+v_1^+)(u_1^+v_2^+)(u_2^+v_1^+)(u_2^+v_2^+)}{(v_1^+v_2^+)^2}\right]\frac{1}{\rho^2}\\
&\sim (2-4(u_1^+u_2^+)(u_1^-u_2^-))\frac{1}{\rho^2}.\end{aligned}$$

Similarly, we can compute two more mixed contractions between the two connections and W^0, but only one is nonvanishing:

$$\langle A_{1\alpha}^{--}W_2^0\rangle = -\frac{(ix_{12}^A\cdot\theta_1^0-\frac{1}{2}\theta_2^0\theta_2^{02})_\alpha}{2\pi|x_{12}^A|^3}(u_1^-u_2^+)(u_1^-u_2^-). \tag{5.15}$$

One more contribution to $\langle\mathcal{W}\rangle$ needs to be considered (at the order being studied), and this one includes a 3-point vertex insertion:

$$\begin{aligned}&\left\langle\int d\tau_1\dot{x}_{A,1}\cdot A_1\int d\tau_2\dot{x}_{A,2}\cdot A_2\int d\tau_3\dot{x}_{A,3}\cdot A_3\right\rangle\\
&\quad= -i\int d\tau_{1,2,3}\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\dot{x}_{A,3}^{\kappa\lambda}\int dv_{1,2,3}(D_\alpha^0D_\beta^{--}A^{++})_1(D_\gamma^0D_\delta^{--}A^{++})_2(D_\kappa^0D_\lambda^{--}A^{++})_3\\
&\quad\sim\int d^3\tau\cdots\int d^3v(D_\alpha^0D_\beta^{--})_1(D_\gamma^0D_\delta^{--})_2(D_\kappa^0D_\lambda^{--})_3\int d^3x_0d^6\theta_0\frac{dw_{1,2,3}}{(w_1^+w_2^+)(w_2^+w_3^+)(w_3^+w_1^+)}\prod_{i=1}^3\frac{\delta^2(\theta_{0i}^{++})\delta(v_i,w_i)}{\sqrt{2\rho_{0i}^2}}\\
&\quad\sim\int d^3\tau\cdots\int d^3v(v_1^+v_2^+)(v_2^+v_3^+)(v_3^+v_1^+)\int d^3x_0(D_\alpha^0D_\beta^{--})_{1,2,3}\prod_{i=1}^3\frac{|x_{0i}|^2+i(x_{0i})\cdot(\theta_i^{++}\theta_i^{--})-\frac{1}{4}\theta_i^{++2}\theta_i^{--2}}{|x_{0i}|^3}\\
&\quad\sim\int d^3\tau\dot{x}_{A,1}^{\alpha\beta}\dot{x}_{A,2}^{\gamma\delta}\dot{x}_{A,3}^{\kappa\lambda}\int d^3v(v_1^+v_2^+)(v_2^+v_3^+)(v_3^+v_1^+)(v_1^-v_2^+)(v_2^-v_3^+)(v_3^-v_1^+)(v_1^-v_2^-)(v_2^-v_3^-)(v_3^-v_1^-)\\
&\qquad\times\int d^3x_0\left(\frac{(x_{01})_{\beta\gamma}(x_{02})_{\delta\kappa}(x_{03})_{\lambda\alpha}}{|x_{01}|^3|x_{02}|^3|x_{03}|^3}-\frac{i(x_{01})_{\beta\gamma}(x_{02})_{\delta\kappa}\theta_{3,\lambda}^{++}\theta_{3,\alpha}^{--}}{|x_{01}|^3|x_{02}|^3|x_{03}|^3}-\cdots+\frac{i\theta_{1,\beta}^{++}\theta_{1,\gamma}^{--}\theta_{2,\delta}^{++}\theta_{2,\kappa}^{--}\theta_{3,\lambda}^{++}\theta_{3,\alpha}^{--}}{|x_{01}|^3|x_{02}|^3|x_{03}|^3}\right).\end{aligned} \tag{5.16}$$

The second to last line is obtained after performing the $\int d^6\theta_0$ in the previous line with the help of three $\delta^2(\theta_{0i}^{++})$'s, canceling the divergent harmonic denominator. The last line then follows by converting $D_i^0\to D_{i+1}^{++}$ in cyclic order and acting on the numerator, thus picking out eight terms. The $\int d^3v$ integral produces only a numerical factor. Note that the first term in the integral $\int d^3x_0$ is the only "bosonic" piece given by the well-known integral (6.12) of [15].

Finally, collecting all the results at one-loop order (we suppress u-dependent factors from $\langle W^0W^0\rangle$ to keep the expression below manageable), we have

$$\begin{aligned}\langle\mathcal{W}_{1/3}(x,\theta^{\pm\pm},\theta^0)\rangle = 1 &+ \frac{1}{2}\frac{4\pi}{ik}\frac{N}{2}\int d\tau_1 d\tau_2\Bigg\{\frac{\dot{x}^{\alpha\beta}_{A,1}\dot{x}^{\gamma\delta}_{A,2}\varepsilon_{\beta\delta}\theta^0_{1\alpha}\theta^0_{2\gamma}+|\dot{x}_{A,1}||\dot{x}_{A,2}|(\theta_1^{02}+\theta_1^0\cdot\theta_2^0+\theta_2^{02})}{2\pi|x^A_{12}|^3}\\ &+\dot{\theta}_1^{++}\cdot\dot{\theta}_2^{++}\left(\frac{u_1^-u_2^-}{u_1^+u_2^+}\right)\frac{|x^A_{12}|^2-ix^A_{12}\cdot\theta^0_1\theta^0_2+\frac{1}{4}\theta_1^{02}\theta_2^{02}}{2\pi|x^A_{12}|^3}\\ &-\frac{\dot{\theta}_1^{++}\cdot(ix^A_{12}\cdot\theta^0_2-\frac{1}{2}\theta^0_1\theta_2^{02})|\dot{x}_{A,2}|+|\dot{x}_{A,1}|\dot{\theta}_2^{++}\cdot(ix^A_{12}\cdot\theta^0_1-\frac{1}{2}\theta_1^{02}\theta^0_2)}{2\pi|x^A_{12}|^3(u_1^-u_2^+)^{-1}(u_1^-u_2^-)^{-1}}\Bigg\}\\ &-\frac{1}{2}\frac{16\pi^2}{k^2}\frac{N^2}{2}\int d\tau_1 d\tau_2\Bigg\{\frac{\dot{x}_{A,1}\cdot\dot{x}_{A,2}-|\dot{x}_{A,1}||\dot{x}_{A,2}|}{4\pi^2|x^A_{12}|^2}\left(1-\frac{2ix^A_{12}\cdot\theta^0_1\theta^0_2}{|x^A_{12}|^2}\right)\\ &+\int d\tau_3\int d^3x_0\frac{\dot{x}^{\alpha\beta}_{A,1}\dot{x}^{\gamma\delta}_{A,2}\dot{x}^{\kappa\lambda}_{A,3}(x_{01})_{\beta\gamma}(x_{02})_{\delta\kappa}(x_{03})_{\lambda\alpha}-\mathcal{O}(\theta_i{}^2)}{4\times16\pi^3|x_{01}|^3|x_{02}|^3|x_{03}|^3}\Bigg\}.\end{aligned}\tag{5.17}$$

VI. COMMENTS

We have constructed a $\frac{1}{3}$-BPS supersymmetrized Wilson loop operator in $d=3$, $\mathcal{N}=3$ harmonic superspace for CS theories. This operator readily generalizes the $\frac{1}{6}$-BPS operator for ABJM theories. We were also able to use the power of harmonic superspace to compute the one-loop perturbative corrections directly in superspace.

Using the component expansion of $\mathcal{N}=3$ connections and field strengths, and just focusing on the localization locus discussed in Sec. II ($\sigma\equiv\phi^0$ and $D\equiv X^0=-\frac{\sigma}{r}$), we can see that the $\mathcal{W}_{1/3}$ given in (5.1) reduces to (2.5) once we identify $\theta,\bar{\theta}$ with θ^{++},θ^{--}. Then one can expect that

$$\begin{aligned}\langle\mathcal{W}_{1/3}\rangle = 1 &+ \frac{1}{2}\left[\vartheta+\frac{\vartheta^2}{4}\right]\alpha\\ &-\left[\frac{1}{24}\left(5+\frac{1}{N^2}\right)+\frac{1}{4}\vartheta-\frac{1}{6}\left(\frac{1}{2}+\frac{1}{N^2}\right)\frac{\vartheta^2}{4}\right]\alpha^2\\ &+\mathcal{O}(\alpha^3).\end{aligned}\tag{6.1}$$

At order α^2, we can directly compare the "bosonic" factor $-\frac{5\alpha^2}{24}\equiv\frac{5\pi^2N^2}{6k^2}$ above to the corresponding perturbative expression in (5.17). They exactly match once we perform the integrals in the latter case for a circular WL, i.e., $x^\mu=(0,\sin(\tau),\cos(\tau))$ as one might expect.[6]

Formally, both (5.17) and (6.1) have nonvanishing "fermionic" contributions at $\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$. However, without knowing the explicit profile functions of $\theta(\tau)$ and $u(\tau)$, we cannot proceed further. However, we can identify a contribution at $\mathcal{O}(\alpha^2)$ that does not receive any $\mathcal{O}(\frac{1}{N})$ corrections.[7] These are the $\mathcal{O}(\theta\bar{\theta})$ terms in the ϑ piece of (6.1) and comparing with (5.17), we can give an explicit expression for these terms:

$$\begin{aligned}\vartheta|_{\mathcal{O}(\theta\bar{\theta})} = &\frac{2i}{\pi^2}\int d\tau_1 d\tau_2\\ &\times\frac{(\dot{x}_1\cdot\dot{x}_2-|\dot{x}_1||\dot{x}_2|)x^{\alpha\beta}_{12}(\theta_{1,\alpha}\bar{\theta}_{1,\beta}-\theta_{2,\alpha}\bar{\theta}_{2,\beta})}{|x_{12}|^4}\\ &-\frac{i}{16\pi^3}\int d\tau_{1,2,3}\dot{x}_1^{\alpha\beta}\dot{x}_2^{\gamma\delta}\dot{x}_3^{\kappa\lambda}\\ &\times\int d^3x_0\frac{(x_{01})_{\beta\gamma}(x_{02})_{\delta\kappa}\theta_{3,\lambda}\bar{\theta}_{3,\alpha}+2\text{ more terms}}{|x_{01}|^3|x_{02}|^3|x_{03}|^3}.\end{aligned}\tag{6.2}$$

The fermionic pieces from the term proportional to $(\dot{x}_{A,1}\cdot\dot{x}_{A,2}-|\dot{x}_{A,1}||\dot{x}_{A,2}|)$ do not contribute above because the combination $\theta\bar{\theta}$ is independent of τ as discussed in Sec. II. Such fermionic contributions to the Wilson loop operators do not seem to have been considered in $d=3$, but similar terms have appeared in the $d=4$, $\mathcal{N}=4$ SYM literature, specifically in the study of supersymmetrized Maldacena-Wilson loops [29,31]. Thus, a careful study of the τ dependence of the θ and u coordinates that is consistent with the "bosonic" circular WL is required to understand how the general perturbative result (5.17)

[6] We refer the readers to [15] for evaluation of the relevant integrals.

[7] We do not have $\mathcal{O}(\frac{1}{N})$ terms at $\mathcal{O}(\alpha)$ either, but that could still be treated as a phase factor. The striking feature of the term at $\mathcal{O}(\alpha^2)$ is that it remains unchanged even after the removal of the $\mathcal{O}(\alpha)$ phase:

$$\langle\mathcal{W}_{1/3}\rangle = 1-\left[\frac{1}{24}\left(5+\frac{1}{N^2}\right)+\frac{1}{4}\vartheta+\frac{1}{6}\left(\frac{5}{2}-\frac{1}{N^2}\right)\frac{\vartheta^2}{4}\right]\alpha^2 +\mathcal{O}(\alpha^3).$$

reduces to the simpler localization result (6.1) at various θ orders.[8] We leave this exercise for future work.

One can also ask whether the construction of a $\frac{1}{2}$-BPS WL with "supermatrix" structure [20] is feasible in harmonic superspace. Our preliminary analysis suggests that the $U(1)$ charge structure of the supersymmetry parameters $(\varepsilon^{\pm\pm}, \varepsilon^0)$ and the matter superfield q^+ is an obstacle to constructing a straightforward generalization. As mentioned in the Introduction, a motivation to study such supersymmetrized WL operators is to probe the Wilson loops/scattering amplitudes duality in ABJM theory. The expectation is that polygonal WL operators with certain bifundamental vertex insertions would be dual to the ABJM scattering amplitudes. The matter superfield $(q^{+a})_A^B$ in bifundamental representation provides a natural candidate for such insertions. However, this leads to some superficial divergences that need to be tamed. Progress on these aspects will be reported elsewhere.

[8] This is the case when the conjectured equality (6.2) would hold, and we assume that the consistency would require $\theta^0(\tau)$ to vanish.

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[1] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, On planar gluon amplitudes/Wilson loops duality, Nucl. Phys. **B795**, 52 (2008).
[2] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, The hexagon Wilson loop and the BDS ansatz for the six-gluon amplitude, Phys. Lett. B **662**, 456 (2008).
[3] Z. Bern, L. J. Dixon, D. A. Kosower, R. Roiban, M. Spradlin, C. Vergu, and A. Volovich, The two-loop six-gluon MHV amplitude in maximally supersymmetric Yang-Mills theory, Phys. Rev. D **78**, 045007 (2008).
[4] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, Hexagon Wilson loop = six-gluon MHV amplitude, Nucl. Phys. **B815**, 142 (2009).
[5] J. M. Drummond, J. Henn, G. P. Korchemsky, and E. Sokatchev, Dual superconformal symmetry of scattering amplitudes in $\mathcal{N}=4$ super-Yang-Mills theory, Nucl. Phys. **B828**, 317 (2010).
[6] N. Berkovits and J. Maldacena, Fermionic T-duality, dual superconformal symmetry, and the amplitude/Wilson loop connection, J. High Energy Phys. 09 (2008) 062.
[7] L. J. Mason and D. Skinner, The complete planar S-matrix of $\mathcal{N}=4$ SYM as a Wilson loop in twistor space, J. High Energy Phys. 12 (2010) 018.
[8] S. Caron-Huot, Notes on the scattering amplitude/Wilson loop duality, J. High Energy Phys. 07 (2011) 058.
[9] O. Aharony, O. Bergman, D. L. Jafferis, and J. Maldacena, $\mathcal{N}=6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals, J. High Energy Phys. 10 (2008) 091.
[10] W. M. Chen and Y. t. Huang, Dualities for loop amplitudes of $\mathcal{N}=6$ Chern-Simons matter theory, J. High Energy Phys. 11 (2011) 057.
[11] M. S. Bianchi, M. Leoni, A. Mauri, S. Penati, and A. Santambrogio, Scattering amplitudes/Wilson loop duality in ABJM theory, J. High Energy Phys. 01 (2012) 056.
[12] D. Gaiotto and X. Yin, Notes on superconformal Chern-Simons-matter theories, J. High Energy Phys. 08 (2007) 056.
[13] N. Drukker, J. Plefka, and D. Young, Wilson loops in 3-dimensional $\mathcal{N}=6$ supersymmetric Chern-Simons theory and their string theory duals, J. High Energy Phys. 11 (2008) 019.
[14] B. Chen and J. B. Wu, Supersymmetric Wilson loops in $\mathcal{N}=6$ super Chern-Simons-matter theory, Nucl. Phys. **B825**, 38 (2010).
[15] S. J. Rey, T. Suyama, and S. Yamaguchi, Wilson loops in superconformal Chern-Simons theory and fundamental strings in anti-de Sitter supergravity dual, J. High Energy Phys. 03 (2009) 127.
[16] A. Kapustin, B. Willett, and I. Yaakov, Exact results for Wilson loops in superconformal Chern-Simons theories with matter, J. High Energy Phys. 03 (2010) 089.
[17] N. Drukker and D. Trancanelli, A supermatrix model for $\mathcal{N}=6$ super Chern-Simons-matter theory, J. High Energy Phys. 02 (2010) 058.
[18] M. S. Bianchi, G. Giribet, M. Leoni, and S. Penati, 1/2 BPS Wilson loop in $\mathcal{N}=6$ superconformal Chern-Simons theory at two loops, Phys. Rev. D **88**, 026009 (2013).
[19] M. S. Bianchi, M. Leoni, M. Leoni, A. Mauri, S. Penati, and A. Santambrogio, ABJM amplitudes and WL at finite N, J. High Energy Phys. 09 (**2013**) 114.
[20] H. Ouyang, J. B. Wu, and J. j. Zhang, Construction and classification of novel BPS Wilson loops in quiver Chern-Simons-matter theories, Nucl. Phys. **B910**, 496 (2016).
[21] A. Mauri, S. Penati, and J. j. Zhang, New BPS Wilson loops in $\mathcal{N}=4$ circular quiver Chern-Simons-matter theories, J. High Energy Phys. 11 (**2017**) 174.
[22] M. Rosso and C. Vergu, Wilson loops in $\mathcal{N}=6$ superspace for ABJM theory, J. High Energy Phys. 06 (2014) 176.

[23] N. Beisert and C. Vergu, On the geometry of null polygons in full $\mathcal{N} = 4$ superspace, Phys. Rev. D **86**, 026006 (2012).

[24] I. L. Buchbinder, E. A. Ivanov, O. Lechtenfeld, N. G. Pletnev, I. B. Samsonov, and B. M. Zupnik, ABJM models in $\mathcal{N} = 3$ harmonic superspace, J. High Energy Phys. 03 (2009) 096.

[25] I. L. Buchbinder, E. A. Ivanov, O. Lechtenfeld, N. G. Pletnev, I. B. Samsonov, and B. M. Zupnik, Quantum $\mathcal{N} = 3$, $d = 3$ Chern-Simons matter theories in harmonic superspace, J. High Energy Phys. 10 (2009) 075.

[26] A. Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, and E. Sokatchev, Unconstrained $\mathcal{N} = 2$ matter, Yang-Mills and supergravity theories in harmonic superspace, Classical Quantum Gravity **1**, 469 (1984); Erratum, Classical Quantum Gravity **2**, 127 (1985).

[27] H. Nishino and S. J. Gates, Jr., Chern-Simons theories with supersymmetries in three dimensions, Int. J. Mod. Phys. A **08**, 3371 (1993).

[28] S. J. Gates, Jr. and H. Nishino, Remarks on the $\mathcal{N} = 2$ supersymmetric Chern-Simons theories, Phys. Lett. B **281**, 72 (1992).

[29] N. Beisert, D. Müller, J. Plefka, and C. Vergu, Smooth Wilson loops in $\mathcal{N} = 4$ non-chiral superspace, J. High Energy Phys. 12 (2015) 140.

[30] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky, and E. S. Sokatchev, *Harmonic Superspace* (Cambridge University Press, Cambridge, England, 2001).

[31] D. Müller, H. Münkler, J. Plefka, J. Pollok, and K. Zarembo, Yangian symmetry of smooth Wilson loops in $\mathcal{N} = 4$ super Yang-Mills theory, J. High Energy Phys. 11 (2013) 081.