No static spherically symmetric wormholes in Horndeski theory

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We consider the Horndeski theory in four-dimensional space-time as the most general theory with a single scalar field and second order field equations. We show that this theory does not admit stable, static, spherically symmetric, asymptotically flat, Lorentzian wormholes.

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I. INTRODUCTION

Wormholes [1–6] and semiclosed worlds [7–10] are spatial configurations with a throat which connects either two flat spaces or a flat space and a closed world, respectively. They can be traversable in theories violating the null energy condition (NEC) only [1–3,11–13]. This condition states that the Einstein tensor $G_{\mu\nu}$ obeys

$$G_{\mu\nu}\eta^\mu\eta^\nu \geq 0, \tag{1}$$

for any null vector η^μ. This condition in case of minimal coupling to gravity writes $T_{\mu\nu}\eta^\mu\eta^\nu \geq 0$, where $T_{\mu\nu}$ is the energy-momentum tensor of matter. It is known, however, that this condition is very hard to violate. In the case of the scalar field theories whose Lagrangians contain only the first derivatives over time and spatial coordinates, it was shown that NEC violation inevitably causes ghost and/or gradient instabilities [14]. This raises interest in models containing second derivatives in the Lagrangian leading, however, to the second order field equations, as they appear to admit stable NEC-violating solutions [15–20]. The Horndeski theory (generalized Galileons plus gravity) is the most general one with such a property [21]. It was originally introduced in an unnoticed work by Horndeski in 1974 [21], reintroduced by D. B. Fairlie, J. Govaerts and A. Morozov in 1992 [22–24] and became popular quite recently [15,17–19,25–37].

This theory is described by the following action [15,21,28,38]:

$$S = \int d^4x \sum_{i=2}^{5} \mathcal{L}_i$$

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3], \tag{2}$$

where $X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$, $\Box\phi = \nabla_\mu\nabla^\mu\phi$, R is the scalar curvature and $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ is the Einstein tensor, metric signature is $(-,+,+,+)$.

It has been shown that both asymptotically flat, static, spherically symmetric wormholes [39,40] and semiclosed worlds [41] are unstable for all nonsingular configurations in $\mathcal{L}_3$ theories with minimal coupling to gravity, i.e., $G_4 = M_{\rm Pl}^2/2$, $G_5 = 0$.

The purpose of this paper is to extend this result to wormholes in the most general Horndeski theory (2). The proof of the analogous (by interchanging radial coordinate and time) no-go theorem for bouncing cosmologies within a subclass of Horndeski theories was given in [42] and generalized to the case of the interaction of the Galileon field with an extra scalar field in [43]. Afterwards, this proof was further extended to the full Horndeski theory in [44] and the multi-Galileon Horndeski theory in [45].

The paper is organized as follows. We give a brief review of some of the results obtained by T. Kobayashi, H. Motohashi and T. Suyama [46,47] in Sec. II, as they are essential for our argument. We prove the instability in Sec. III. Finally, we discuss our results in Sec. IV.

II. STABILITY CONDITIONS

We consider static, spherically symmetric, asymptotically flat Lorentzian wormholes. The general form of metric describing them is

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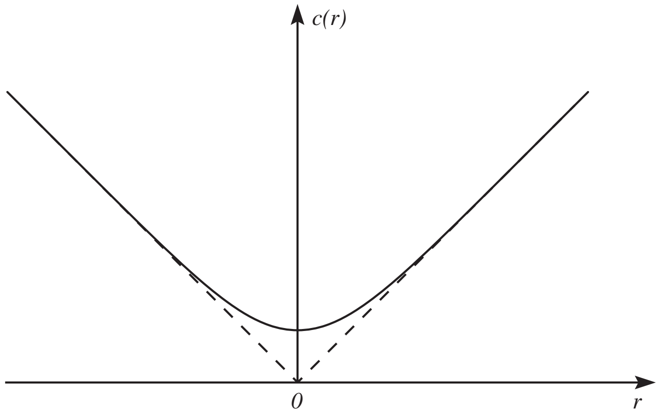

FIG. 1. Behavior of $c(r)$ for a wormhole.

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{b^2(r)} + c^2(r)(d\theta^2 + \sin^2\theta d\varphi^2). \quad (3)$$

With the convenient gauge choice

$$a(r) = b(r) \quad (4)$$

the metric becomes

$$ds^2 = -a^2(r)dt^2 + \frac{dr^2}{a^2(r)} + c^2(r)(d\theta^2 + \sin^2\theta d\varphi^2).$$

In the wormhole case, the functions $a(r)$ and $c(r)$ have the following asymptotic behavior as $r \to \pm\infty$:

$$a(r) \to 1, \qquad c(r) \to r,$$

and $c(r) > 0$ is bounded from below reaching its minimum at $r = 0$, see Fig. 1.

T. Kobayashi, H. Motohashi and T. Suyama in [46,47] obtained the stability conditions for perturbations about this background by performing the analysis in terms of spherical harmonics within the Regge-Wheeler approach [48–50] described below.

We consider metric perturbations $h_{\mu\nu}$ so the perturbed metric is

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}, \quad (5)$$

where $g^0_{\mu\nu}$ denotes the background metric: the interval ds^2 from (3) writes $ds^2 = g^0_{\mu\nu}dx^\mu dx^\nu$.

To provide the analysis in spherical harmonics we need to understand how different types of perturbations transform under rotations of 2D-sphere. Perturbations $h_{\mu\nu}$ consist of h_{tt}, h_{tr} and h_{rr} which are scalars under 2D-rotations on sphere, h_{ta} and h_{ra} which transform as vectors and h_{ab} transforming as a second-order tensor. Here and below a and b stand for either θ or φ. Scalar field ϕ is a scalar under 2D-rotations.

Any scalar s, vector V_a and second-order (symmetric) tensor T_{ab} can be decomposed in the following way:

$$s(t,r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} s_{lm}(t,r)Y_l^m(\theta,\varphi), \quad (6)$$

$$V_a(t,r,\theta,\varphi) = \overset{\gamma}{\nabla}_a\Phi_1(t,r,\theta,\varphi) + E_a^b\overset{\gamma}{\nabla}_b\Phi_2(t,r,\theta,\varphi), \quad (7)$$

$$\begin{aligned} T_{ab}(t,r,\theta,\varphi) = {}& \overset{\gamma}{\nabla}_a\overset{\gamma}{\nabla}_b\Psi_1(t,r,\theta,\varphi) + \gamma_{ab}\Psi_2(t,r,\theta,\varphi) \\ &+ \frac{1}{2}(E_a^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_b\Psi_3(t,r,\theta,\varphi) \\ &+ E_b^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_a\Psi_3(t,r,\theta,\varphi)), \end{aligned} \quad (8)$$

where γ_{ab} is the metric of a two-dimensional sphere, $\overset{\gamma}{\nabla}_a$ is the covariant derivative[1] in metric γ_{ab}, $E_{ab} = \sqrt{\det\gamma}\varepsilon_{ab}$, ε_{ab} is Levi-Civita symbol with $\varepsilon_{\theta\varphi} = 1$, Φ_1, Φ_2, Ψ_1, Ψ_2, Ψ_3 are scalar functions and $Y_l^m(\theta,\varphi)$ are spherical harmonics. The above allows one to rewrite any scalar, vector or second-order tensor in terms of spherical harmonics by applying the decomposition (6) to Φ_1, Φ_2, Ψ_1, Ψ_2, Ψ_3. From now on we reffer to the variables not containing E_{ab} as even-type variables and others as odd-type ones. Notice that even-type modes get a factor $(-1)^l$ under parity transformation $(\theta,\varphi) \to (\pi-\theta,\pi+\varphi)$ while odd-type modes get a factor $(-1)^{l+1}$ stipulating (at $l = 0$) the names of modes: they are also called even- and odd-parity modes.

A. Odd-parity sector

The odd-parity part of the perturbations is given by

$$\delta\phi = 0, \qquad h_{tt} = 0, \qquad h_{tr} = 0, \qquad h_{rr} = 0, \quad (9)$$

$$h_{ta} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} h_{0,lm}(t,r)E_{ab}\partial^b Y_l^m(\theta,\varphi), \quad (10)$$

$$h_{ra} = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} h_{1,lm}(t,r)E_{ab}\partial^b Y_l^m(\theta,\varphi), \quad (11)$$

$$\begin{aligned} h_{ab} = {}& \frac{1}{2}\sum_{l=2}^{\infty}\sum_{m=-l}^{l} h_{2,lm}(t,r)\Big[E_a^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_b Y_l^m(\theta,\varphi) \\ &+ E_b^c\overset{\gamma}{\nabla}_c\overset{\gamma}{\nabla}_a Y_l^m(\theta,\varphi)\Big]. \end{aligned} \quad (12)$$

[1] Though the Christoffel symbols for the metric γ_{ab} and for the angular part of metric $g_{\mu\nu}$ are the same, the difference between $\overset{\gamma}{\nabla}_a$ and ∇_a is significant, e.g., while acting on the angular part of a 4-vector (which is a vector under 2D-rotations as well):

$$\begin{aligned} \overset{\gamma}{\nabla}_a V_b &= \partial_a V_b - \Gamma^c_{ab}V_c, \\ \nabla_a V_b &= \partial_a V_b - \Gamma^\mu_{ab}V_\mu = \partial_a V_b - \Gamma^t_{ab}V_t - \Gamma^r_{ab}V_r - \Gamma^c_{ab}V_c \\ &= \overset{\gamma}{\nabla}_a V_b - \Gamma^t_{ab}V_t - \Gamma^r_{ab}V_r. \end{aligned}$$

Not all of these variables are physical due to the general covariance: we can use gauge transformation $x^\mu \to x^\mu + \xi^\mu$ to set some of them equal to zero. ξ^μ here is an infinitesimal function and in odd-parity sector it can be written as

$$\xi_t = 0, \qquad \xi_r = 0,$$
$$\xi_a = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \Lambda_{lm}(t,r) E_{ab}\partial^b Y_l^m(\theta,\varphi), \tag{13}$$

where $\Lambda_{lm}(t,r)$ are arbitrary functions. The metric perturbation transforms as

$$h_{\mu\nu} \to h_{\mu\nu} + \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu, \tag{14}$$

where ∇_μ is the covariant derivative for metric $g_{\mu\nu}$. For odd-type perturbations we find

$$h_{0,lm} \to h_{0,lm} + \dot{\Lambda}_{lm}(t,r), \tag{15}$$

$$h_{1,lm} \to h_{1,lm} + \Lambda'_{lm}(t,r) + 2\frac{c'}{c}\Lambda_{lm}(t,r), \tag{16}$$

$$h_{2,lm} \to h_{2,lm} + 2\Lambda_{lm}(t,r). \tag{17}$$

Here dot and prime denote the derivatives with respect to t and r, respectively. As $h_{2,lm}$ transformation does not contain derivatives, we can fix the gauge completely by implementing the condition $h_{2,lm} = 0$ for $l \geq 2$. This gauge fixing is called Regge-Wheeler gauge [48]. For $l = 1$ h_{ab} vanishes identically so we have to implement another gauge condition. This is described in detail in [46] but is irrelevant for our discussion as we are interested in obtaining particular stability conditions, and the high-momentum sector is sufficient for our purposes.

As we consider each set (l,m) separately and the corresponding perturbation modes do not mix we can omit hereafter all the indices l and m. The additional benefit of using the Regge-Wheeler gauge is that the field equations do not depend on m so we can fix $m = 0$ without loss of generality [46,48,51]. Then the spherical harmonics become Legendre polynomials:

$$Y_l^m(\theta,\varphi) = \sqrt{\frac{(2l+1)}{4\pi}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\varphi}$$
$$\underset{m=0}{\longrightarrow} \sqrt{\frac{(2l+1)}{4\pi}} P_l(\cos\theta). \tag{18}$$

The action expanded to the second order in the perturbations is

$$S^{(2)} = \int dt dr \mathcal{L}^{(2)}, \tag{19}$$

where we have integrated over angles θ and φ. $\mathcal{L}^{(2)}$ here is

$$\mathcal{L}^{(2)} = a_1 h_0^2 + a_2 h_1^2 + a_3\left(\dot{h}_1^2 + h_0'^2 - 2\dot{h}_1 h_0' + 4\frac{c'}{c}\dot{h}_1 h_0\right). \tag{20}$$

The coefficients in the Lagrangian (20) are given by

$$a_1 = \frac{l(l+1)}{2c^2}\left[\frac{d}{dr}(cc'\mathcal{H}) + \frac{(l-1)(l+2)}{2a^2}\mathcal{F}\right], \tag{21}$$

$$a_2 = -\frac{l(l+1)}{2}a^2\left[\frac{(l-1)(l+2)}{2c^2}\mathcal{G}\right], \tag{22}$$

$$a_3 = \frac{l(l+1)}{4}\mathcal{H}, \tag{23}$$

where background field equations are applied and

$$\mathcal{F} = 2\left(G_4 + \frac{a^2}{2}\phi' X' G_{5X} - X G_{5\phi}\right), \tag{24}$$

$$\mathcal{G} = 2[G_4 - 2XG_{4X} + X(aa'c'\phi' G_{5X} + G_{5\phi})], \tag{25}$$

$$\mathcal{H} = 2\left[G_4 - 2XG_{4X} + X\left(a^2\frac{c'}{c}\phi' G_{5X} + G_{5\phi}\right)\right]. \tag{26}$$

Following [52] we can rewrite the above Lagrangian as

$$\mathcal{L}^{(2)} = \left[a_1 - 2\frac{(cc'a_3)'}{c^2}\right]h_0^2 + a_2 h_1^2 + a_3\left(\dot{h}_1 - h_0' + 2\frac{c'}{c}h_0\right)^2, \tag{27}$$

or, introducing new auxiliary field q, as

$$\mathcal{L}^{(2)} = \left[a_1 - 2\frac{(cc'a_3)'}{c^2}\right]h_0^2 + a_2 h_1^2 + a_3\left[2q\left(\dot{h}_1 - h_0' + 2\frac{c'}{c}h_0\right) - q^2\right]. \tag{28}$$

We integrate by parts and put all derivatives on q leaving h_0 and h_1 as auxiliary fields. The variations with respect to h_0 and h_1 yield the algebraic expressions from which h_0 and h_1 can be found:

$$h_0 = -\frac{(c^2 a_3 q)'}{c^2 a_1 - 2(cc'a_3)'}, \qquad h_1 = \frac{a_3}{a_2}\dot{q}. \tag{29}$$

We now substitute these h_0 and h_1 into the action and obtain the Lagrangian in terms of a single variable q:

$$\mathcal{L}^{(2)} = \frac{l(l+1)}{4(l-1)(l+2)}[\mathcal{A}\dot{q}^2 - \mathcal{B}q'^2 - l(l+1)\mathcal{C}q^2 - V(r)q^2], \tag{30}$$

where

$$\mathcal{A} = \frac{c^2}{a^2}\frac{\mathcal{H}^2}{\mathcal{G}}, \qquad \mathcal{B} = a^2c^2\frac{\mathcal{H}^2}{\mathcal{F}}, \qquad \mathcal{C} = a^2\mathcal{H}, \tag{31}$$

and $V(r)$ is an effective potential, whose exact form is not essential for our purposes. The $\mathcal{C}$-part of the Lagrangian (30) represents wave propagation along the angular direction as $l(l+1)$ corresponds to the two-dimensional Laplacian.

Thus we obtain the following stability conditions from the odd-type sector:

$$\mathcal{F} > 0 \text{ to avoid gradient instabilities along the radial direction,} \tag{32}$$

$$\mathcal{G} > 0 \text{ to avoid ghost instabilities,} \tag{33}$$

$$\mathcal{H} > 0 \text{ to avoid gradient instabilities along angular directions.} \tag{34}$$

As we will see in Sec. III the essential ones for our proof are (32) and (34).

B. Even-parity sector

The even-type part of the perturbations is [48]:

$$\delta\phi = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\delta\phi_{lm}(t,r)Y_l^m(\theta,\varphi), \qquad h_{tt} = a^2\sum_{l=0}^{\infty}\sum_{m=-l}^{l}H_{0,lm}(t,r)Y_l^m(\theta,\varphi), \tag{35}$$

$$h_{tr} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}H_{1,lm}(t,r)Y_l^m(\theta,\varphi), \qquad h_{rr} = \frac{1}{a^2}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}H_{2,lm}(t,r)Y_l^m(\theta,\varphi), \tag{36}$$

$$h_{ta} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\beta_{lm}\partial_a Y_l^m(\theta,\varphi), \qquad h_{ra} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\alpha_{lm}\partial_a Y_l^m(\theta,\varphi), \tag{37}$$

$$h_{ab} = c^2\sum_{l=0}^{\infty}\sum_{m=-l}^{l}[K_{lm}(t,r)\gamma_{ab}Y_l^m(\theta,\varphi) + G_{lm}(t,r)\overset{\gamma}{\nabla}_a\overset{\gamma}{\nabla}_b Y_l^m(\theta,\varphi)]. \tag{38}$$

We shall note here that the scalar field is also perturbed in the even-parity sector. Like in the odd-parity sector, some of the perturbations are not physical and can be eliminated by using the gauge transformation $x^\mu \to x^\mu + \xi^\mu$. The three gauge functions are [48]:

$$\xi_0 = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}T_{lm}(t,r)Y_l^m(\theta,\varphi), \qquad \xi_r = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}R_{lm}(t,r)Y_l^m(\theta,\varphi), \qquad \xi_a = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\Theta_{lm}(t,r)\partial_a Y_l^m(\theta,\varphi), \tag{39}$$

where $T_{lm}(t,r)$, $R_{lm}(t,r)$ and $\Theta_{lm}(t,r)$ are arbitrary functions of t and r. Using (14) again we find the corresponding gauge transformations of perturbations:

$$H_{0,lm}(t,r) \to H_{0,lm}(t,r) + \frac{2}{a^2}\dot{T}_{lm}(t,r) - 2\frac{a'}{a}b^2R_{lm}(t,r), \tag{40}$$

$$H_{1,lm}(t,r) \to H_{1,lm}(t,r) + \dot{R}(t,r) + T'(t,r) - 2\frac{a'}{a}T(t,r), \tag{41}$$

$$H_{2,lm}(t,r) \to H_{2,lm}(t,r) + 2b^2R'_{lm}(t,r) - 2bb'R_{lm}(t,r), \tag{42}$$

$$\beta_{lm}(t,r) \to \beta_{lm}(t,r) + T_{lm}(t,r) + \dot{\Theta}_{lm}(t,r), \tag{43}$$

$$\alpha_{lm}(t,r) \to \alpha_{lm}(t,r) + R_{lm}(t,r) + \Theta'_{lm}(t,r) - 2\frac{c'}{c}\Theta_{lm}(t,r), \tag{44}$$

$$K_{lm}(t,r) \to K_{lm}(t,r) + 2b^2\frac{c'}{c}R_{lm}(t,r), \tag{45}$$

$$G_{lm}(t,r) \to G_{lm}(t,r) + \frac{2}{c^2}\Theta_{lm}(t,r). \tag{46}$$

Hereafter we again omit l and m indices as the corresponding perturbations do not mix with each other. For the purpose of generality we did not use the gauge (4) in this equations.

Now we can fix the gauge completely by setting $\beta(t,r)=0$, $K(t,r)=0$ and $G(t,r)=0$ and substitute field and metric perturbations into the action (2) to obtain the quadratic Lagrangian depending on $\delta\phi$, H_0, H_1, H_2 and α.

This approach applies, like in the odd-parity case, for $l\geq 2$ only: for $l=1$ the metric perturbation h_{ab} depends on the combination $K-G$ but not separately on K and G and thus we have a gauge freedom left which we can use to set $\delta\phi=0$. For $l=0$ α, β and G vanish making ξ_a useless. ξ_r is fixed by setting K equal to zero and ξ_t can be used to eliminate either H_0 or H_1. The whole procedure is described in detail in [47], we here just give a brief review.

H_0 and H_1 become auxiliary fields and can be excluded immediately leaving the Lagrangian depending on $\delta\phi$, H_2 and α and a constraint depending on $\delta\phi$, $\delta\phi'$, $\delta\phi''$, H_2, H_2', α and α'.

We use a new variable ψ and perform a field redefinition

$$H_0=-\frac{2}{2cc'\mathcal{H}+\Xi\phi'}\left(\frac{1}{a^2}\psi-c'\Xi\delta\phi'-l(l+1)c'\mathcal{H}\alpha\right), \tag{47}$$

where $\mathcal{H}$ is given by Eq. (26) and

$$\begin{aligned}\Xi=2c^2\bigg[&-XG_{3X}+2a^2\frac{c'}{c}\phi'\{G_{4X}+2XG_{4XX}-(XG_{5\phi})_X\}\\&+G_{4\phi}+2XG_{4\phi X}-\frac{1}{c^2}XG_{5X}\\&+a^2\frac{c'^2}{c^2}(3XG_{5X}+2X^2G_{5XX})\bigg].\end{aligned} \tag{48}$$

This change of variables allow us to exclude both second derivatives of $\delta\phi$ and first derivatives of α from the constraint and thus make it an algebraic equation for α. Finally, excluding α we come up with the Lagrangian for two variables (ψ and $\delta\phi$), which can be written in the following form:

$$\mathcal{L}=\frac{1}{2}\mathcal{K}_{ij}\dot{v}^i\dot{v}^j-\frac{1}{2}\mathcal{G}_{ij}v^{i\prime}v^{j\prime}-\frac{1}{2}\mathcal{Q}_{ij}v^iv^{j\prime}-\frac{1}{2}\mathcal{M}_{ij}v^iv^j, \tag{49}$$

where $v^1\equiv\psi$, $v^2\equiv\delta\phi$ and i, j run from 1 to 2. Expressions for the matrices[2] $\mathcal{G}_{ij}$, $\mathcal{Q}_{ij}$ and $\mathcal{M}_{ij}$ are very cumbersome and are not essential for our purposes. $\mathcal{K}_{ij}$ on the other hand gives us conditions for the absence of the ghost instabilities which will play a definitive role in the proof below. The ghost are absent when

$$\mathcal{K}_{11}>0,\qquad \det\mathcal{K}>0. \tag{50}$$

We will use the second one in our proof:

$$\det\mathcal{K}=\frac{4(l-1)(l+2)(2cc'\mathcal{H}+\Xi\phi')^2\mathcal{F}(2\mathcal{P}_1-\mathcal{F})}{l(l+1)a^4\mathcal{H}^2\phi'^2(2cc'\mathcal{H}+\Xi\phi')^2}>0, \tag{51}$$

where

$$\mathcal{P}_1=\frac{(2cc'\mathcal{H}+\Xi\phi')}{2c^2\mathcal{H}^2}\cdot\frac{d}{dr}\left[\frac{c^4\mathcal{H}^4}{(2cc'\mathcal{H}+\Xi\phi')^2}\right], \tag{52}$$

or, considering (32), simply

$$2\mathcal{P}_1-\mathcal{F}>0. \tag{53}$$

III. NO-GO THEOREM FOR HORNDESKI THEORY

The stability conditions relevant for our purposes are (32), (34), and (53). We define a variable

$$Q=\frac{2cc'\mathcal{H}+\Xi\phi'}{c^2\mathcal{H}^2},$$

and write (53) in the following form:

$$2\mathcal{P}_1-\mathcal{F}=-2\frac{Q'}{Q^2}-\mathcal{F}>0$$

or

$$\frac{Q'}{Q^2}<-\frac{1}{2}\mathcal{F}. \tag{54}$$

By integrating this relation from r to $r'>r$ we obtain (cf. [40])

$$Q^{-1}(r)-Q^{-1}(r')<-\frac{1}{2}\int_r^{r'}\mathcal{F}dr. \tag{55}$$

Now, let $Q^{-1}(r)$ be negative at some r. Then we write (55) as follows:

$$Q^{-1}(r')>Q^{-1}(r)+\frac{1}{2}\int_r^{r'}\mathcal{F}dr, \tag{56}$$

and notice that if the integral on the right side of the inequality (56) diverges as $r'\to+\infty$, then $Q^{-1}(r')$ has to become positive, meaning that $Q^{-1}(r^*)=0$ at some point r^* and Q is singular at this point.

Conversely, let $Q^{-1}(r')$ be positive at some r', then we write (55) as

[2] $\mathcal{G}_{ij}$ should not be confused with $\mathcal{G}$ from (25).

$$Q^{-1}(r) < Q^{-1}(r') - \frac{1}{2}\int_r^{r'} \mathcal{F} dr, \qquad (57)$$

and see that if the integral diverges as $r \to -\infty$ then $Q^{-1}(r)$ has to become negative, meaning again a singular Q at some point r^* where $Q^{-1}(r^*) = 0$. Considering (26) and the fact that c is bounded from below we conclude that either Ξ or ϕ' has to be singular, which leads to the singular Lagrangian.

Assuming general relativity is restored away from the wormhole throat

$$\begin{cases} G_4 \to M_{\rm Pl}^2/2 \\ G_5 \to 0 \end{cases} \quad \text{at } r \to \pm\infty. \qquad (58)$$

Equation (24) then leads to $\mathcal{F}(r) \to M_{\rm Pl}^2$ as $r \to \pm\infty$, so the integral in Eq. (55) diverges as $r' \to +\infty$ and $r \to -\infty$. This completes the argument.

IV. DISCUSSION

The argument given in this paper shows that the static spherically symmetric Lorentzian wormholes cease to exist in the Horndeski theory. This theorem is quite general besides the assumption on the divergent behavior of the integral

$$\int_r^{r'} \mathcal{F} dr$$

both at $r \to -\infty$ and $r \to +\infty$ which is natural if we want to obtain a flat Minkowski spacetime far enough from the wormhole in both worlds it connects.

To make contact with Ref. [40], we notice that in the cubic Galileon theory with $G_4 = M_{\rm Pl}^2/2$, $G_5 = 0$, we have

$$Q = \frac{\mathcal{Q}}{M_{\rm Pl}^2}, \qquad \mathcal{F} = M_{\rm Pl}^2,$$

where $\mathcal{Q}$ is the variable introduced by V. Rubakov in [40]. Thus, the inequality (55) coincides with that used in [40].

In spite of the generality of this no-go theorem, there is a chance that a stable wormhole solution can be constructed in beyond Horndeski theory, similarly to the cosmological bounce which can be stable throughout the whole evolution [53–56].

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