Tilted shear-free axially symmetric fluids

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We carry on a systematic study of the physical properties of axially symmetric fluid distributions, which appear to be geodesic, shearfree, irrotational, nondissipative, and purely electric, for the comoving congruence of observers, from the point of view of the tilted congruence. The vanishing of the magnetic part of the Weyl tensor for the comoving congruence of observers, suggests that no gravitational radiation is produced during the evolution of the system. Instead, the magnetic part of the Weyl tensor as measured by tilted observers is nonvanishing (as well as the shear, the four–acceleration, the vorticity and the dissipation), giving rise to a flux of gravitational radiation that can be characterized through the super–Poynting vector. This result strengthens further the relevance of the role of observers in the description of a physical system. An explanation of this dual interpretation in terms of the information theory, is provided.

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I. INTRODUCTION

In a recent paper [1] we have analyzed in some detail shearfree and geodesic dissipative fluids, using a general framework for studying axially symmetric dissipative fluids [2], based on the $1+3$ approach [3–6].

Such configurations (which have been previously considered in great detail by Coley and McManus [7,8]), are shown to be necessarily irrotational and purely electric (the magnetic part of the Weyl tensor vanishes). Therefore, these fluid distributions produce spacetimes which belong to what are known as silent spacetimes [9–11]. Strictly speaking the term "silent universe" includes additional restrictions, such as the gravitational field is sourced by dust and cosmological constant only. However here we shall use this term as implying only the vanishing of the magnetic part of the Weyl tensor and the vorticity.

On the other hand, the magnetic part of the Weyl tensor as well as the vorticity of the fluid lines, are described by tensors defined in terms of the four–velocity of the fluid. Accordingly it is pertinent to ask, if the above-mentioned properties (irrotational and purely electric) remain valid for a congruence of observers, tilted (Lorentz boosted) with respect to the congruence of comoving observers which, as is obvious, are described by a different four-velocity vector field.

This issue is related to the well known fact that there is an observer dependence in the description of the source (see [12–26] and references therein), related to the arbitrariness in the choice of the four velocity in terms of which the energy–momentum tensor is split, and the kinematical variables are defined.

Thus for example, it can be shown [19], that the usual interpretation of the Lemaitre–Tolman–Bondi spacetime [27–29], as geodesic and produced by a nondissipative dust, is valid for comoving observers exclusively. Tilted observers would detect real (entropy producing) dissipative processes in such spacetime, and the fluid congruence is no longer geodesic. An explanation for this particular duality in the interpretation of the physical properties of the fluid, in terms of the information theory, was given in [30].

It is the purpose of this work to analyze in detail the physical properties of axially symmetric fluid distributions, which appear to be geodesic and shearfree, for the comoving congruence of observers, from the point of view of the tilted congruence. To simplify the analysis we shall consider that the fluid distribution in the comoving frame is nondissipative. As expected from previous work (see [12–26] and references therein), the fluid distribution appears to be dissipative for the tilted observer.

The novelty in this work is, as we shall see, that unlike the comoving observers, the tilted ones will detect a flux of

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gravitational radiation associated to the magnetic part of the Weyl tensor, which for the tilted observers is non vanishing. This is a remarkable result, since the vanishing (or not) of the magnetic part of the Weyl tensor is very often invoked as a significant property of a given spacetime (see [31,32] and references therein). As in [30], an explanation for such a result is given in terms of the information theory. However, in this work we stress the fact that an argument similar to the one put forward by Bennet [33] to solve the Maxwell's demon paradox [34], may be used to explain the very different pictures of a given system, presented by different congruences of observers in general relativity.

Also, it is obtained that the fluid for the tilted congruence, appears to be shearing, nongeodesic and nonirrotational.

In order to avoid rewriting most of the equations, we shall very often refer to [1,2]. Thus, we suggest that the reader have at hand these references, when reading this manuscript.

II. THE SHEARFREE, GEODESIC, AXIALLY SYMMETRIC FLUID: THE COMOVING PICTURE

We shall consider axially and reflection symmetric, nondissipative fluid distributions (not necessarily bounded). For such a system the most general line element may be written in "Weyl spherical coordinates" as:

$$ds^2 = -A^2dt^2 + B^2(dr^2 + r^2d\theta^2) + C^2d\phi^2 + 2Gd\theta dt, \quad (1)$$

where A, B, C, G are positive functions of t, r and θ. We number the coordinates $x^0 = t, x^1 = r, x^2 = \theta, x^3 = \phi$.

The energy momentum tensor in the "canonical" form reads:

$$T_{\alpha\beta} = (\mu + P)V_\alpha V_\beta + Pg_{\alpha\beta} + \Pi_{\alpha\beta}, \quad (2)$$

where as usual, $\mu, P, \Pi_{\alpha\beta}, V_\beta$ denote the energy density, the isotropic pressure, the anisotropic stress tensor and the four velocity, respectively.

We emphasize that, so far, we are considering an Eckart (comoving) frame where fluid elements are at rest.

If we now impose the shearfree and the geodesic conditions, and assume that the fluid is nondissipative, the line element (1) becomes

$$ds^2 = -dt^2 + B^2(t)[dr^2 + r^2d\theta^2 + R^2(r,\theta)d\phi^2]. \quad (3)$$

From regularity conditions at the origin we must require $R(0,\theta) = R'(0,\theta) = R_{,\theta}(0,\theta) = R_{,\theta\theta}(0,\theta) = 0$, where prime denotes derivative with respect to r. Also it can be shown that all geodesic and shearfree fluids, are necessarily irrotational (see [1] for details). As mentioned in the Introduction, metrics of this type have been thoroughly investigated in [7,8], therefore we shall not enter into a detailed analysis of their properties here.

For our comoving observer the four-velocity vector reads

$$V^\alpha = (1,0,0,0); \qquad V_\alpha = (-1,0,0,0). \quad (4)$$

We shall next define a canonical orthonormal tetrad (say $e^{(a)}_\alpha$), by adding to the four-velocity vector $e^{(0)}_\alpha = V_\alpha$, three spacelike unitary vectors (these correspond to the vectors $\mathbf{K}, \mathbf{L}, \mathbf{S}$ in [2])

$$e^{(1)}_\alpha = K_\alpha = (0,B,0,0); \qquad e^{(2)}_\alpha = L_\alpha = (0,0,Br,0), \quad (5)$$

$$e^{(3)}_\alpha = S_\alpha = (0,0,0,BR), \quad (6)$$

with $a = 0, 1, 2, 3$ (latin indices within the round brackets labeling different vectors of the tetrad).

The dual vector tetrad $e^\alpha_{(a)}$ is easily computed from the condition

$$\eta_{(a)(b)} = g_{\alpha\beta}e^\alpha_{(a)}e^\beta_{(b)}, \qquad e^\alpha_{(a)}e^{(b)}_\alpha = \delta^{(b)}_{(a)}, \quad (7)$$

where $\eta_{(a)(b)}$ denotes the Minkowski metric.

In the above, the tetrad vector $e^\alpha_{(3)} = (1/BR)\delta^\alpha_\phi$ is parallel to the only admitted Killing vector (it is the unit tangent to the orbits of the group of 1-dimensional rotations that defines axial symmetry). The other two basis vectors $e^\alpha_{(1)}, e^\alpha_{(2)}$ define the two *unique* directions that are orthogonal to the 4-velocity and to the Killing vector.

For the energy density and the isotropic pressure, we have

$$\mu = T_{\alpha\beta}e^\alpha_{(0)}e^\beta_{(0)}, \qquad P = \frac{1}{3}h^{\alpha\beta}T_{\alpha\beta}, \quad (8)$$

where

$$h^\alpha_\beta = \delta^\alpha_\beta + V^\alpha V_\beta, \quad (9)$$

whereas the anisotropic tensor may be expressed through three scalar functions defined as (see [2], but notice the change of notation):

$$\Pi_{KL} = e^\alpha_{(2)}e^\beta_{(1)}T_{\alpha\beta}, \quad (10)$$

$$\Pi_I = (2e^\alpha_{(1)}e^\beta_{(1)} - e^\alpha_{(2)}e^\beta_{(2)} - e^\alpha_{(3)}e^\beta_{(3)})T_{\alpha\beta}, \quad (11)$$

$$\Pi_{II} = (2e^\alpha_{(2)}e^\beta_{(2)} - e^\alpha_{(3)}e^\beta_{(3)} - e^\alpha_{(1)}e^\beta_{(1)})T_{\alpha\beta}. \quad (12)$$

In [1] it was shown, that for the geodesic, shearfree nondissipative fluid, we have: $\Pi_{KL} = \Pi_I = \Pi_{II} = \Pi$, accordingly, the anisotropic tensor may be written in the form:

$$\Pi_{\alpha\beta} = \Pi\left(e_\alpha^{(1)}e_\beta^{(1)} + e_\alpha^{(2)}e_\beta^{(2)} + e_\alpha^{(2)}e_\beta^{(1)} + e_\alpha^{(1)}e_\beta^{(2)} - \frac{2h_{\alpha\beta}}{3}\right). \tag{13}$$

As mentioned before, for the comoving observer, and the line element (3), the four-acceleration, the shear and the vorticity vanish, whereas for the expansion we get:

$$\Theta = \frac{3\dot{B}}{B}, \tag{14}$$

where overdot denotes derivatives with respect to t.

III. THE ELECTRIC AND MAGNETIC PARTS OF THE WEYL TENSOR AND THE SUPER–POYNTING VECTOR

Let us now introduce the electric ($E_{\alpha\beta}$) and magnetic ($H_{\alpha\beta}$) parts of the Weyl tensor ($C_{\alpha\beta\gamma\delta}$), defined as usual by

$$E_{\alpha\beta} = C_{\alpha\nu\beta\delta}V^\nu V^\delta,$$
$$H_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\nu\epsilon\rho}C_{\beta\delta}{}^{\epsilon\rho}V^\nu V^\delta, \tag{15}$$

where $\eta_{\alpha\beta\mu\nu}$ denotes the Levi-Civita tensor.

In general, for the line element (1), the electric part of the Weyl tensor has only three independent nonvanishing components, whereas only two components define the magnetic part. However, in our case [comoving observers and line element (3)] the electric part is defined by a single scalar function $\mathcal{E}$, whereas the magnetic part vanishes. Thus we may write:

$$E_{\alpha\beta} = \mathcal{E}\left(e_\alpha^{(1)}e_\beta^{(1)} + e_\alpha^{(2)}e_\beta^{(2)} - \frac{2}{3}h_{\alpha\beta} + e_\alpha^{(1)}e_\beta^{(2)} + e_\beta^{(1)}e_\alpha^{(2)}\right), \tag{16}$$

and

$$H_{\alpha\beta} = 0. \tag{17}$$

Also, from the Riemann tensor we may define three tensors $Y_{\alpha\beta}$, $X_{\alpha\beta}$ and $Z_{\alpha\beta}$ as

$$Y_{\alpha\beta} = R_{\alpha\nu\beta\delta}V^\nu V^\delta, \tag{18}$$

$$X_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\nu}{}^{\epsilon\rho}R^\star_{\epsilon\rho\beta\delta}V^\nu V^\delta, \tag{19}$$

and

$$Z_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\epsilon\rho}R_{\delta\beta}{}^{\epsilon\rho}V^\delta, \tag{20}$$

where $R^\star_{\alpha\beta\nu\delta} = \frac{1}{2}\eta_{\epsilon\rho\nu\delta}R_{\alpha\beta}{}^{\epsilon\rho}$ and $\epsilon_{\alpha\beta\rho} = \eta_{\nu\alpha\beta\rho}V^\nu$.

From the above tensors, we may define the super-Poynting vector by

$$P_\alpha = \epsilon_{\alpha\beta\gamma}(Y_\delta^\gamma Z^{\beta\delta} - X_\delta^\gamma Z^{\delta\beta}). \tag{21}$$

In our case, we may write:

$$P_\alpha = P_{(1)}e_\alpha^{(1)} + P_{(2)}e_\alpha^{(2)}. \tag{22}$$

In the theory of the super-Poynting vector, a state of gravitational radiation is associated to a nonvanishing component of the latter (see [35–37]). This is in agreement with the established link between the super–Poynting vector and the news functions [38], in the context of the Bondi–Sachs approach [39,40].

For the comoving observer and the line element (3), the magnetic part of the Weyl tensor vanishes identically, implying at once that $P_{(1)} = P_{(2)} = 0$. In other words, no gravitational radiation is detected by the comoving observer.

We shall now proceed to apply a Lorentz boost to our comoving congruence, in order to obtain the tilted one.

IV. THE TILTED CONGRUENCE

In order to obtain the tilted congruence, we have to find the expression for the four-velocity corresponding to this congruence [in the same globally defined coordinate system as in (3)]. For doing that we shall proceed in three steps.

We shall first perform a (strictly locally defined) coordinate transformation to the locally Minkowskian frame (LMF).

Denoting by $\Lambda_\mu^{\bar{\nu}}$ the local coordinate transformation matrix, and by $\bar{V}^\alpha$ the components of the four-velocity in such LMF, where $\bar{x}^\alpha$ denotes the locally Minkowskian coordinates, we have:

$$\bar{V}^\mu = \Lambda_\nu^{\bar{\mu}}V^\nu, \tag{23}$$

where

$$\Lambda_0^{\bar{0}} = 1; \quad \Lambda_1^{\bar{1}} = B; \quad \Lambda_2^{\bar{2}} = Br; \quad \Lambda_3^{\bar{3}} = BR. \tag{24}$$

Next, let us apply a Lorentz boost to the LMF associated to $\bar{V}^\alpha$, in order to obtain the (tilted) LMF with respect to which a fluid element is moving with some nonvanishing three-velocity $\bar{v}_i$.

Thus the four-velocity in the tilted LMF is defined by:

$$\bar{\tilde{V}}_\beta = L_{\bar{\beta}}^{\bar{\alpha}}\bar{V}_\alpha, \tag{25}$$

where $L_{\bar{\beta}}^{\bar{\alpha}}$ denotes the corresponding Lorentz matrix.

The boost is applied along the two independent directions $(\bar{x}^1, \bar{x}^2)$, thus we have:

$$L^{\bar{0}}_{\bar{0}} = \Gamma; \qquad L^{\bar{0}}_{\bar{i}} = -\Gamma \bar{v}_i; \qquad L^{\bar{i}}_{\bar{j}} = \delta^i{}_j + \frac{(\Gamma - 1)\bar{v}_i \bar{v}_j}{\bar{v}^2}, \tag{26}$$

where latin indices i, j run from 1 to 3, $\Gamma \equiv \frac{1}{\sqrt{1-\bar{v}^2}}$, $\bar{v}^2 = \bar{v}_1^2 + \bar{v}_2^2$, and $\bar{v}_1$, $\bar{v}_2$ are the two components of the three-velocity of a fluid element as measured by the tilted observer.

Finally, we have to perform a transformation from the tilted LMF, back to the (global) frame associated to the line element (3). Such a transformation is defined by the inverse of $\Lambda^{\bar{\nu}}_{\mu}$, and produces the four-velocity of the tilted congruence in our globally defined coordinate system, say $\tilde{V}^\alpha$. This last operation produces:

$$\tilde{e}^{(0)}_\alpha = \tilde{V}_\alpha = (-\Gamma, B\Gamma v_1, Br\Gamma v_2, 0);$$
$$\tilde{V}^\alpha = \left(\Gamma, \frac{\Gamma v_1}{B}, \frac{\Gamma v_2}{Br}, 0\right). \tag{27}$$

We can also apply the above procedure to obtain the remaining vectors of the tilted tetrad, we find:

$$\tilde{e}^{(1)}_\alpha = \tilde{K}_\alpha = \left(-\Gamma v_1, B\left[1 + \frac{(\Gamma - 1)v_1^2}{v^2}\right], \frac{Br(\Gamma - 1)v_1 v_2}{v^2}, 0\right), \tag{28}$$

$$\tilde{e}^{(2)}_\alpha = \tilde{L}_\alpha = \left(-\Gamma v_2, \frac{B(\Gamma - 1)v_1 v_2}{v^2}, Br\left[1 + \frac{(\Gamma - 1)v_2^2}{v^2}\right], 0\right), \tag{29}$$

and

$$\tilde{e}^{(3)}_\alpha \equiv e^{(3)}_\alpha = \tilde{S}_\alpha = (0, 0, 0, BR), \tag{30}$$

where for simplicity we have omitted the bar over the components of the three velocity.

We can now calculate all the kinematical variables for the tilted congruence.

The four acceleration

$$\tilde{a}_\alpha = \tilde{V}^\beta \tilde{V}_{\alpha;\beta}, \tag{31}$$

may be expressed through two scalar functions as:

$$\tilde{a}_\alpha = \tilde{a}_{(1)} \tilde{e}^{(1)}_\alpha + \tilde{a}_{(2)} \tilde{e}^{(2)}_\alpha. \tag{32}$$

From (32) and (A1)–(A3), we can easily find the explicit expressions for the two scalars $\tilde{a}_{(1)}$ and $\tilde{a}_{(2)}$.

It is a simple matter to check that if we put $v = 0$ ($\Gamma = 1$), we obtain $\tilde{a}_\alpha = 0$, as expected.

Next, the shear tensor

$$\tilde{\sigma}_{\alpha\beta} = \tilde{\sigma}_{(a)(b)} e^{(a)}_\alpha e^{(b)}_\beta = \tilde{V}_{(\alpha;\beta)} + \tilde{a}_{(\alpha} \tilde{V}_{\beta)} - \frac{1}{3}\tilde{\Theta}\tilde{h}_{\alpha\beta}, \tag{33}$$

may be defined through two independent tetrad components (scalars) $\tilde{\sigma}_{(1)(1)}$ and $\tilde{\sigma}_{(2)(2)}$, defined by:

$$\tilde{\sigma}_I = 3\tilde{e}^\alpha_{(1)} \tilde{e}^\beta_{(1)} \tilde{\sigma}_{\alpha\beta}, \qquad \tilde{\sigma}_{II} = 3\tilde{e}^\alpha_{(2)} \tilde{e}^\beta_{(2)} \tilde{\sigma}_{\alpha\beta}. \tag{34}$$

These two scalars may be easily obtained from (34) and the expressions for the nonvanishing coordinate components of the shear tensor displayed in (A4)–(A10).

Again, if we go back to the comoving congruence by assuming $v = 0$ ($\Gamma = 1$), we get $\tilde{\sigma}_{\alpha\beta} = 0$.

For the vorticity vector defined as:

$$\tilde{\omega}_\alpha = \frac{1}{2}\eta_{\alpha\beta\mu\nu} \tilde{V}^{\beta;\mu} \tilde{V}^\nu = \frac{1}{2}\eta_{\alpha\beta\mu\nu} \tilde{\Omega}^{\beta\mu} \tilde{V}^\nu, \tag{35}$$

where $\tilde{\Omega}_{\alpha\beta} = \tilde{V}_{[\alpha;\beta]} + \tilde{a}_{[\alpha} \tilde{V}_{\beta]}$ denotes the vorticity tensor; we find a single component different from zero, producing:

$$\tilde{\Omega}_{\alpha\beta} = \tilde{\Omega}(\tilde{e}^{(2)}_\alpha \tilde{e}^{(1)}_\beta - \tilde{e}^{(2)}_\beta \tilde{e}^{(1)}_\alpha), \tag{36}$$

and

$$\tilde{\omega}_\alpha = -\tilde{\Omega}\tilde{e}^{(3)}_\alpha. \tag{37}$$

with the scalar function $\tilde{\Omega}$ given by

$$\tilde{\Omega} = -\frac{\Gamma^2}{2}\left(-\frac{v_2'}{B} - \frac{v_2}{Br} - v_1 \dot{v}_2 + v_2 \dot{v}_1 + \frac{v_{1,\theta}}{Br}\right). \tag{38}$$

Obviously in the limit when $v = 0$ the vorticity vanishes.

Finally, the expansion scalar, now reads:

$$\tilde{\Theta} = \dot{\Gamma} + \frac{3\dot{B}\Gamma}{B} + \frac{(\Gamma v_1)'}{B} + \left(\frac{1}{r} + \frac{R'}{R}\right)\frac{\Gamma v_1}{B} + \frac{\Gamma v_2 R_{,\theta}}{BRr} + \frac{(\Gamma v_2)_{,\theta}}{Br}, \tag{39}$$

which of course reduces to (14) if $v_1 = v_2 = 0$.

In the above equations and hereafter, primes and dots denote derivatives with respect to r and t respectively.

For the tilted observers the fluid distribution is described by the energy momentum tensor:

$$\tilde{T}_{\alpha\beta} = (\tilde{\mu}_+ \tilde{P})\tilde{V}_\alpha \tilde{V}_\beta + \tilde{P} g_{\alpha\beta} + \tilde{\Pi}_{\alpha\beta} + \tilde{q}_\alpha \tilde{V}_\beta + \tilde{q}_\beta \tilde{V}_\alpha. \tag{40}$$

It should be observed that for the tilted congruence the system may be dissipative, and the anisotropic tensor depends on three scalar functions. Thus we may write:

$$\tilde{\Pi}_{\alpha\beta} = \frac{1}{3}(2\tilde{\Pi}_I + \tilde{\Pi}_{II})\left(\tilde{e}^{(1)}_\alpha \tilde{e}^{(1)}_\beta - \frac{\tilde{h}_{\alpha\beta}}{3}\right) + \frac{1}{3}(2\tilde{\Pi}_{II} + \tilde{\Pi}_I)\left(\tilde{e}^{(2)}_\alpha \tilde{e}^{(2)}_\beta - \frac{\tilde{h}_{\alpha\beta}}{3}\right) + \tilde{\Pi}_{KL}(\tilde{e}^{(1)}_\alpha \tilde{e}^{(2)}_\beta + \tilde{e}^{(1)}_\beta \tilde{e}^{(2)}_\alpha), \quad (41)$$

with

$$\tilde{\Pi}_{KL} = \tilde{e}^\alpha_{(1)} \tilde{e}^\beta_{(2)} \tilde{T}_{\alpha\beta}, \quad (42)$$

$$\tilde{\Pi}_I = (2\tilde{e}^\alpha_{(1)} \tilde{e}^\beta_{(1)} - \tilde{e}^\alpha_{(2)} \tilde{e}^\beta_{(2)} - \tilde{e}^\alpha_{(3)} \tilde{e}^\beta_{(3)})\tilde{T}_{\alpha\beta}, \quad (43)$$

$$\tilde{\Pi}_{II} = (2\tilde{e}^\alpha_{(2)} e^\beta_{(2)} - \tilde{e}^\alpha_{(1)} \tilde{e}^\beta_{(1)} - \tilde{e}^\alpha_{(3)} \tilde{e}^\beta_{(3)})\tilde{T}_{\alpha\beta}. \quad (44)$$

Finally, we may write for the heat flux vector:

$$\tilde{q}_\mu = \tilde{q}_{(1)} \tilde{e}^{(1)}_\mu + \tilde{q}_{(2)} \tilde{e}^{(2)}_\mu. \quad (45)$$

Since, both congruences of observers are embedded within the same space-time (3), then it is obvious that the Einstein tensor is the same for both congruences, and therefore the energy-momentum tensors, although split differently, also must be the same.

Then equating (2) and (40), and projecting on all possible combinations of tetrad vectors (tilted and nontilted), we find expressions for the physical variables measured by comoving observers, in terms of the tilted ones, and vice versa. These are exhibited in the Appendix B.

For the tilted congruence, the nonvanishing components of the electric and magnetic parts of the Weyl tensor have been calculated and their expressions are given in the Appendix C. These tensors may be expressed through the five scalars ($\tilde{\mathcal{E}}_I$, $\tilde{\mathcal{E}}_{II}$, $\tilde{\mathcal{E}}_{KL}$, $\tilde{H}_1$, $\tilde{H}_2$), as follows:

$$\tilde{E}_{\alpha\beta} = \frac{1}{3}(2\tilde{\mathcal{E}}_I + \tilde{\mathcal{E}}_{II})\left(\tilde{e}^{(1)}_\alpha \tilde{e}^{(1)}_\beta - \frac{1}{3}\tilde{h}_{\alpha\beta}\right) + \frac{1}{3}(2\tilde{\mathcal{E}}_{II} + \tilde{\mathcal{E}}_I)\left(\tilde{e}^{(2)}_\alpha \tilde{e}^{(2)}_\beta - \frac{1}{3}\tilde{h}_{\alpha\beta}\right) + \tilde{\mathcal{E}}_{KL}(\tilde{e}^{(1)}_\alpha \tilde{e}^{(2)}_\beta + \tilde{e}^{(1)}_\beta \tilde{e}^{(2)}_\alpha), \quad (46)$$

and

$$\tilde{H}_{\alpha\beta} = \tilde{H}_1(\tilde{e}^{(1)}_\beta \tilde{e}^{(3)}_\alpha + \tilde{e}^{(1)}_\alpha \tilde{e}^{(3)}_\beta) + \tilde{H}_2(\tilde{e}^{(3)}_\alpha \tilde{e}^{(2)}_\beta + \tilde{e}^{(2)}_\alpha \tilde{e}^{(3)}_\beta), \quad (47)$$

where the above-mentioned scalars are expressed through the nonvanishing components of the electric and magnetic parts of the Weyl tensor, as indicated in the Appendix C.

The above expressions produce for the super-Poynting vector:

$$\tilde{P}_{(1)} = \frac{2\tilde{H}_2}{3}(2\tilde{\mathcal{E}}_{II} + \tilde{\mathcal{E}}_I) + 2\tilde{H}_1\tilde{\mathcal{E}}_{KL} + 32\pi^2\tilde{q}_{(1)}\left(\tilde{\mu} + \tilde{P} + \frac{\tilde{\Pi}_I}{3}\right) + 32\pi^2\tilde{q}_{(2)}\tilde{\Pi}_{KL},$$

$$\tilde{P}_{(2)} = -\frac{2\tilde{H}_1}{3}(2\tilde{\mathcal{E}}_I + \tilde{\mathcal{E}}_{II}) - 2\tilde{H}_2\tilde{\mathcal{E}}_{KL} + 32\pi^2\tilde{q}_{(2)}\left(\tilde{\mu} + \tilde{P} + \frac{\tilde{\Pi}_{II}}{3}\right) + 32\pi^2\tilde{q}_{(1)}\tilde{\Pi}_{KL}. \quad (48)$$

We can identify two different contributions in (48). On the one hand we have contributions from the heat transport process. These are in principle independent of the magnetic part of the Weyl tensor, which explains why they remain in the spherically symmetric limit. Next we have contributions related to the gravitational radiation. These contributions are described by the first two terms in $\tilde{P}_{(1)}$ and $\tilde{P}_{(2)}$. In order of these contributions to be different from zero we require that, both, the electric and the magnetic part of the Weyl tensor to be nonvanishing. More specifically, the sum of the first two terms in $\tilde{P}_{(1)}$ and $\tilde{P}_{(2)}$ should not vanish. This is in fact the case, as can be seen from (C12)–(C14) and (C19), (C20). Indeed, the vanishing of the above mentioned terms implies $R \sim r\cos\theta$, which produces the vanishing of the Weyl tensor (conformal flatness). Therefore, excluding the particular conformally flat case, the tilted observer detects a nonvanishing gravitational contribution of the super-Poynting vector, which as mentioned above indicates the presence of gravitational radiation.

V. CONCLUSIONS

Using the framework developed in [2] and the results obtained in [1], we have compared the physical properties of a physical system described by the line element (3), as observed by two different congruences of observers (comoving and tilted).

Thus, whereas the fluid is shearfree, geodesic, irrotational and nondissipative, from the point of view of the comoving observer, it appears nongeodesic, shearing, dissipative and endowed with vorticity, for the tilted congruence.

The fact that tilted observers detect dissipation in a system that appears nondissipative for comoving observers, is not new and was emphasized in [19]. To explain such difference in the description of a given system, as provided by different congruences of observers, it has been conjectured in [30] that the origin of this strange situation resides in the fact that passing from one of the congruences to the other we usually overlook the fact that both congruences of observers store a different amount of information.

This is in fact the clue to resolve the quandary about the presence or not of dissipative processes, depending on the congruence of observers, that carry out the analysis of the system.

However, in the present case the difference is still sharper since the tilted observer not only detect a dissipative process, but also gravitational radiation. Both phenomena are of course absent in the description of the comoving observer. This last point is relevant since the tilted observer also detects vorticity, and as pointed out in [38], vorticity and gravitational radiation are tightly associated.

The explanation for such a difference is basically the same as the one proposed for dissipative processes described by the heat flux vector (remember that gravitational radiation is a dissipative process too), and reminds us the resolution of the well-known paradox of the Maxwell's demon [34].

The Maxwell's demon (in one of its many, but equivalent presentations) is a small "being" living in a cylinder filled with a gas, and divided in two equal portions, by a partition with a small door. Then the demon may open the door when the molecules come from the right, while closing it when the molecules approach from the left. Doing so the demon is able to concentrate all the molecules on the left, reducing the entropy by $NK\ln 2$ (where N is the number of molecules, and K is the Boltzman constant), thereby violating the second law of thermodynamics. Brillouin [41] tried to solve the paradox by arguing that in the process of selection of molecules, the demon increases the entropy by an amount equal or larger than the decreasing of entropy achieved by concentrating all molecules on one side. However, soon after, different researchers were able to propose different ways by means of which the demon could select the molecules in a reversible way (i.e., without entropy production). It has been necessary to wait for more than a century, until Bennet [33] gave a satisfactory resolution of this paradox.

Roughly speaking, Bennet showed that the irreversible act which prevents the violation of the second law, is not the selection of molecules to put all of them in one side of the cylinder, but the restauration of the measuring apparatus by means of which the selection is achieved, to the standard state previous to the state where the demon knows from which side comes any molecule. The erasure of such information, according to the Landauer's principle [42], entails dissipation. In other words, to get the demon's mind back to its initial state, generates dissipation. A somehow similar picture appears when we go from comoving (which assign zero value to the three-velocity of any fluid element) to tilted observers, for whom the three-velocity represents another degree of freedom. The erasure of the information stored by comoving observers (vanishing three velocity), when going to the tilted observers, explains the presence of dissipative processes (included gravitational radiation) observed by the latter. The above comments provide full significance to the statement by Max Born: *"Irreversibility is a consequence of the explicit introduction of ignorance into the fundamental laws"* [43].

Finally, it is worth mentioning that the effect described here (the detection of gravitational radiation by tilted observers), somehow reminds us the Unruh effect [44,45], according to which an accelerating observer (Rindler) in a Minkowski vacuum state will observe a thermal spectrum of particles, thereby indicating that two different sets of observers (inertial and Rindler) describe the same state in very different terms.

Of course the Unruh effect is of quantum nature, whereas our results belong to the classical realm. However the main moral emerging from both results, points to the same direction, namely: the description of a physical system may heavily rely on the nature of the observer carrying on the analysis of the system.

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APPENDIX A: KINEMATICAL VARIABLES

The nonvanishing coordinate components of the four-acceleration for the tilted congruence are

$$\tilde{a}_0 = -\Gamma\left[\dot{\Gamma} + \frac{\Gamma' v_1}{B} + \frac{\Gamma_{,\theta} v_2}{Br} + \frac{\Gamma v^2 \dot{B}}{B}\right], \tag{A1}$$

$$\tilde{a}_1 = \Gamma B\left[(\Gamma v_1)^{\cdot} + \frac{(\Gamma v_1)' v_1}{B} + \frac{(\Gamma v_1)_{,\theta} v_2}{Br} - \frac{\Gamma v_2^2}{Br} + \frac{\Gamma v_1 \dot{B}}{B}\right], \tag{A2}$$

$$\tilde{a}_2 = \Gamma Br\left[(\Gamma v_2)^{\cdot} + \frac{(\Gamma v_2)' v_1}{B} + \frac{(\Gamma v_2)_{,\theta} v_2}{Br} + \frac{\Gamma v_2 \dot{B}}{B} + \frac{\Gamma v_2 v_1}{Br}\right]. \tag{A3}$$

The nonvanishing coordinate components of the shear tensor are

$$\begin{aligned}\tilde{\sigma}_{00} = &-\frac{2\dot{\Gamma}(1-\Gamma^2)}{3} + \frac{\Gamma^2\Gamma' v_1}{B} \\ &+ \frac{1-\Gamma^2}{3B}\left[(\Gamma v_1)' + \Gamma v_1\left(\frac{1}{r} + \frac{R'}{R}\right)\right] \\ &+ \frac{\Gamma^2\Gamma_{,\theta} v_2}{Br} + \frac{1-\Gamma^2}{3Br}\left[(\Gamma v_2)_{,\theta} + \frac{\Gamma v_2 R_{,\theta}}{R}\right],\end{aligned} \tag{A4}$$

$$\tilde{\sigma}_{01} = \frac{B}{2}\left[(\Gamma v_1)^{\cdot}(1-\Gamma^2) - \frac{\Gamma^2\dot{\Gamma}v_1}{3} - \frac{\Gamma'(1+\Gamma^2 v_1^2)}{B} - \frac{\Gamma^2 v_1(\Gamma v_1)'}{3B} + \frac{\Gamma^3 v_2^2}{Br} + \frac{2\Gamma^3 v_1^2}{3B}\left(\frac{1}{r}+\frac{R'}{R}\right) - \frac{\Gamma^2 v_1 v_2 \Gamma_{,\theta}}{Br} - \frac{\Gamma^2 v_2 (\Gamma v_1)_{,\theta}}{Br} + \frac{2\Gamma^2 v_1(\Gamma v_2)_{,\theta}}{3Br} + \frac{2\Gamma^3 v_1 v_2 R_{,\theta}}{3BrR}\right], \tag{A5}$$

$$\tilde{\sigma}_{02} = \frac{Br}{2}\left[(\Gamma v_2)^{\cdot}(1-\Gamma^2) - \frac{\Gamma^2\dot{\Gamma}v_2}{3} - \frac{\Gamma'\Gamma^2 v_1 v_2}{B} + \frac{2\Gamma^2 v_2(\Gamma v_1)'}{3B} - \frac{\Gamma^3 v_1 v_2}{3B}\left(\frac{1}{r}-\frac{2R'}{R}\right) - \frac{\Gamma^2 v_2^2 \Gamma_{,\theta}}{Br} - \frac{\Gamma^2 v_2(\Gamma v_2)_{,\theta}}{3Br} - \frac{\Gamma^2 v_1(\Gamma v_2)'}{B} - \frac{\Gamma_{,\theta}}{Br} + \frac{2\Gamma^3 v_2^2 R_{,\theta}}{3BrR}\right], \tag{A6}$$

$$\tilde{\sigma}_{11} = B^2\left\{-\frac{\dot{\Gamma}(1+\Gamma^2 v_1^2)}{3} + \Gamma^2 v_1(\Gamma v_1)^{\cdot} + \frac{(1+\Gamma^2 v_1^2)}{3B}\left[2(\Gamma v_1)' - \Gamma v_1\left(\frac{1}{r}+\frac{R'}{R}\right)\right] - \frac{\Gamma^3 v_2^2 v_1}{Br} + \frac{\Gamma^2 v_2 v_1 (\Gamma v_1)_{,\theta}}{Br} - \frac{(1+\Gamma^2 v_1^2)}{3Br}\left[(\Gamma v_2)_{,\theta} + \frac{\Gamma v_2 R_{,\theta}}{R}\right]\right\}, \tag{A7}$$

$$\tilde{\sigma}_{12} = \frac{B^2 r}{2}\left\{-\frac{2\dot{\Gamma}\Gamma^2 v_1 v_2}{3} + \Gamma^2 v_2(\Gamma v_1)^{\cdot} + \Gamma^2 v_1(\Gamma v_2)^{\cdot} + \frac{\Gamma^2 v_1 v_2}{3B}\left[(\Gamma v_1)' + \Gamma v_1\left(\frac{1}{r}-\frac{2R'}{R}\right)\right] + \frac{(1+\Gamma^2 v_2^2)(\Gamma v_1)_{,\theta}}{Br} + \frac{1+\Gamma^2 v_1^2}{B}\left[(\Gamma v_2)' - \frac{\Gamma v_2}{r}\right] + \frac{\Gamma^2 v_1 v_2}{3Br}\left[(\Gamma v_2)_{,\theta} - \frac{2\Gamma v_2 R_{,\theta}}{R}\right]\right\}, \tag{A8}$$

$$\tilde{\sigma}_{22} = B^2 r^2\left\{-\frac{\dot{\Gamma}(1+\Gamma^2 v_2^2)}{3} + \Gamma^2 v_2(\Gamma v_2)^{\cdot} + \frac{\Gamma^2 v_1 v_2}{B}(\Gamma v_2)' + \frac{(1+\Gamma^2 v_2^2)}{3B}\left[-(\Gamma v_1)' + \Gamma v_1\left(\frac{2}{r}-\frac{R'}{R}\right)\right] + \frac{1+\Gamma^2 v_2^2}{3Br}\left[2(\Gamma v_2)_{,\theta} - \frac{\Gamma v_2 R_{,\theta}}{R}\right]\right\}, \tag{A9}$$

$$\tilde{\sigma}_{33} = \frac{B^2 R^2}{3}\left\{-\dot{\Gamma} - \frac{1}{B}\left[(\Gamma v_1)' + \Gamma v_1\left(\frac{1}{r}-\frac{2R'}{R}\right)\right] + \frac{1}{Br}\left[-(\Gamma v_2)_{,\theta} + \frac{2\Gamma v_2 R_{,\theta}}{R}\right]\right\}. \tag{A10}$$

APPENDIX B: RELATIONSHIPS BETWEEN TILTED AND NONTILTED PHYSICAL VARIABLES

Proceeding as indicated in Sec. IV, we get for the tilted variables:

$$\tilde{\mu} = \Gamma^2\left[\mu + Pv^2 + \Pi\left(\frac{v^2}{3} + 2v_1 v_2\right)\right], \tag{B1}$$

$$\tilde{P} = P + \frac{\Gamma^2}{3}\left[(\mu+P)v^2 + \Pi\left(\frac{v^2}{3} + 2v_1 v_2\right)\right], \tag{B2}$$

$$\tilde{\Pi}_I = \Pi + \Gamma^2\left(\mu + P + \frac{\Pi}{3}\right)(3v_1^2 - v^2) + \frac{2\Pi(\Gamma-1)v_1 v_2}{v^2} \times \left[2 - \Gamma + \frac{3(\Gamma-1)v_1^2}{v^2}\right], \tag{B3}$$

$$\tilde{\Pi}_{II} = \Pi + \Gamma^2\left(\mu + P + \frac{\Pi}{3}\right)(3v_2^2 - v^2) + \frac{2\Pi(\Gamma-1)v_1 v_2}{v^2} \times \left[2 - \Gamma + \frac{3(\Gamma-1)v_2^2}{v^2}\right], \tag{B4}$$

$$\tilde{\Pi}_{KL} = \Gamma^2 v_1 v_2(\mu+P) + \Pi\left[\Gamma + \frac{\Gamma^2 v_1 v_2}{3} + \frac{2(\Gamma-1)^2 v_1^2 v_2^2}{v^4}\right], \tag{B5}$$

$$-\tilde{q}_{(1)} = \Gamma^2 v_1(\mu+P) + \Pi\Gamma v_1\left[\frac{\Gamma}{3} + \frac{2(\Gamma-1)v_1 v_2}{v^2}\right] + \Pi\Gamma v_2, \tag{B6}$$

$$-\tilde{q}_{(2)} = \Gamma^2 v_2(\mu+P) + \Pi\Gamma v_2\left[\frac{\Gamma}{3} + \frac{2(\Gamma-1)v_1 v_2}{v^2}\right] + \Pi\Gamma v_1. \tag{B7}$$

Obviously, if in the above we put $v = v_1 = v_2 = 0$, $\Gamma = 1$, we obtain at once $\tilde{\mu} = \mu; \tilde{P} = P; \tilde{\Pi}_I = \tilde{\Pi}_{II} = \tilde{\Pi}_{KL} = \Pi$, and $\tilde{q}_{(1)} = \tilde{q}_{(2)} = 0$, as it must be.

Inversely, we may obtain by the same way, expressions for the physical variables associated to comoving observers, in terms of the variables corresponding to tilted observers, thus we find:

$$\mu\Gamma = \tilde{\mu}\Gamma + \tilde{q}_{(1)}\Gamma v_1 + \tilde{q}_{(2)}\Gamma v_2, \tag{B8}$$

$$3P = \tilde{\mu}\Gamma^2 v^2 + \tilde{P}(3 + \Gamma^2 v^2) + 2\tilde{q}_{(1)}\Gamma^2 v_1 + 2\tilde{q}_{(2)}\Gamma^2 v_2 + \frac{\tilde{\Pi}_I \Gamma^2 v_1^2}{3} + \frac{\tilde{\Pi}_{II}\Gamma^2 v_2^2}{3} + 2\tilde{\Pi}_{KL}\Gamma^2 v_1 v_2, \tag{B9}$$

$$\Pi = \Gamma^2(\tilde{\mu} + \tilde{P})(v^2 - v_1 v_2) + \frac{\tilde{\Pi}_I}{3}\left\{3 + \Gamma^2 v_1^2 - \frac{(\Gamma-1)v_1 v_2}{v^2}\left[1 + \frac{(\Gamma-1)v_1^2}{v^2}\right]\right\} + \frac{\tilde{\Pi}_{II}}{3}\left\{3 + \Gamma^2 v_2^2 - \frac{(\Gamma-1)v_1 v_2}{v^2}\left[1 + \frac{(\Gamma-1)v_2^2}{v^2}\right]\right\} + \tilde{\Pi}_{KL}\left[-\Gamma + 2\Gamma^2 v_1 v_2 + \frac{2(\Gamma^2-1)v_1^2 v_2^2}{v^4}\right] + \tilde{q}_{(1)}\left\{2\Gamma^2 v_1 - \Gamma v_2\left[1 + \frac{2(\Gamma-1)v_1^2}{v^2}\right]\right\} + \tilde{q}_{(2)}\left\{2\Gamma^2 v_2 - \Gamma v_1\left[1 + \frac{2(\Gamma-1)v_2^2}{v^2}\right]\right\}. \quad \text{(B10)}$$

In the limit when $v \to 0$ the above equations become identities.

APPENDIX C: THE MAGNETIC AND THE ELECTRIC PARTS OF THE WEYL TENSOR FOR THE TILTED CONGRUENCE

Using MAPLE we have calculated the nonvanishing components of the electric and magnetic part of the Weyl tensor. For the former we found:

$$\tilde{E}_{00} = \frac{1}{6B^2 r^2 R(v^2-1)}[v_1^2(R'' r^2 - 2R' r - 2R_{,\theta\theta}) + v_2^2(-2R'' r^2 + R' r + R_{,\theta\theta}) + 6v_1 v_2(R'_{,\theta} r - R_{,\theta})], \quad \text{(C1)}$$

$$\tilde{E}_{01} = -\frac{1}{6Br^2 R(v^2-1)}[v_1(R'' r^2 - 2R' r - 2R_{,\theta\theta}) + 3v_2(R'_{,\theta} r - R_{,\theta})], \quad \text{(C2)}$$

$$\tilde{E}_{02} = -\frac{1}{6BrR(v^2-1)}[v_2(-2R'' r^2 + R' r + R_{,\theta\theta}) + 3v_1(R'_{,\theta} r - R_{,\theta})], \quad \text{(C3)}$$

$$\tilde{E}_{11} = -\frac{1}{6r^2 R(v^2-1)}[v_2^2(R'' r^2 + R' r + R_{,\theta\theta}) + (-R'' r^2 + 2R' r + 2R_{,\theta\theta})], \quad \text{(C4)}$$

$$\tilde{E}_{12} = \frac{1}{6rR(v^2-1)}[v_1 v_2(R'' r^2 + R' r + R_{,\theta\theta}) + 3(R'_{,\theta} r - R_{,\theta})], \quad \text{(C5)}$$

$$\tilde{E}_{22} = -\frac{1}{6R(v^2-1)}[v_1^2(R'' r^2 + R' r + R_{,\theta\theta}) + (2R'' r^2 - R' r - R_{,\theta\theta})], \quad \text{(C6)}$$

$$\tilde{E}_{33} = \frac{R}{6r^2(v^2-1)}[v_1^2(2R'' r^2 - R' r - R_{,\theta\theta}) + v_2^2(-R'' r^2 + 2R' r + 2R_{,\theta\theta}) + 6v_1 v_2(R'_{,\theta} r - R_{,\theta}) + R'' r^2 + R' r + R_{,\theta\theta}]. \quad \text{(C7)}$$

These seven components are related by the following four relationships, which allow us to write the electric part of the Weyl tensor in terms of three independent scalar functions:

$$Br\tilde{E}_{00} + r\tilde{E}_{01} v_1 + \tilde{E}_{02} v_2 = 0, \quad \text{(C8)}$$

$$B^2 r^2 \tilde{E}_{00} - \frac{r^2}{R^2}\tilde{E}_{33} - r^2\tilde{E}_{11} - \tilde{E}_{22} = 0, \quad \text{(C9)}$$

$$\frac{r^2}{R^2}\tilde{E}_{33} + Br^2\tilde{E}_{01} v_1 + Br\tilde{E}_{02} v_2 + r^2\tilde{E}_{11} + \tilde{E}_{22} = 0, \quad \text{(C10)}$$

$$v_1 v_2(r^2\tilde{E}_{11} + \tilde{E}_{22}) + Br(r\tilde{E}_{01} v_2 + \tilde{E}_{02} v_1) + r\tilde{E}_{12} v^2 = 0. \quad \text{(C11)}$$

Thus we may express the electric part of the Weyl tensor, in terms of the three scalars $\mathcal{E}_I, \mathcal{E}_{II}, \mathcal{E}_{KL}$, given by:

$$\frac{2\tilde{\mathcal{E}}_I + \tilde{\mathcal{E}}_{II}}{3} = \Gamma^2 v_1^2 \tilde{E}_{00} + \frac{\tilde{E}_{11}}{B^2}\left[1 + \frac{(\Gamma-1)v_1^2}{v^2}\right]^2 + \frac{(\Gamma-1)^2 v_1^2 v_2^2 \tilde{E}_{22}}{B^2 r^2 v^4} - \frac{\tilde{E}_{33}}{B^2 R^2} + \frac{2\Gamma v_1 \tilde{E}_{01}}{B}\left[1 + \frac{(\Gamma-1)v_1^2}{v^2}\right] + \frac{2\Gamma v_1^2 v_2(\Gamma-1)\tilde{E}_{02}}{Brv^2} + \frac{2(\Gamma-1)v_1 v_2 \tilde{E}_{12}}{B^2 v^2 r}\left[1 + \frac{(\Gamma-1)v_1^2}{v^2}\right], \quad \text{(C12)}$$

$$\frac{2\tilde{\mathcal{E}}_{II} + \tilde{\mathcal{E}}_I}{3} = \Gamma^2 v_2^2 \tilde{E}_{00} + \frac{\tilde{E}_{22}}{B^2 r^2}\left[1 + \frac{(\Gamma-1)v_2^2}{v^2}\right]^2 + \frac{(\Gamma-1)^2 v_1^2 v_2^2 \tilde{E}_{11}}{B^2 v^4} - \frac{\tilde{E}_{33}}{B^2 R^2} + \frac{2\Gamma(\Gamma-1)v_2^2 v_1 \tilde{E}_{01}}{Bv^2} + \frac{2\Gamma v_2 \tilde{E}_{02}}{Br}\left[1 + \frac{(\Gamma-1)v_2^2}{v^2}\right] + \frac{2(\Gamma-1)v_1 v_2 \tilde{E}_{12}}{B^2 v^2 r}\left[1 + \frac{(\Gamma-1)v_2^2}{v^2}\right], \quad \text{(C13)}$$

$$\tilde{\mathcal{E}}_{KL} = \Gamma^2 v_1 v_2 \tilde{E}_{00} + \frac{\tilde{E}_{22}(\Gamma-1)v_1v_2}{B^2r^2v^2}\left[1+\frac{(\Gamma-1)v_2^2}{v^2}\right] + \frac{(\Gamma-1)v_1v_2\tilde{E}_{11}}{B^2v^2}\left[1+\frac{(\Gamma-1)v_1^2}{v^2}\right]$$
$$+ \tilde{E}_{01}\left\{\frac{\Gamma(\Gamma-1)v_2v_1^2}{Bv^2} + \frac{\Gamma v_2}{B}\left[1+\frac{(\Gamma-1)v_1^2}{v^2}\right]\right\} + \tilde{E}_{02}\left\{\frac{\Gamma(\Gamma-1)v_2^2v_1}{Brv^2} + \frac{\Gamma v_1}{Br}\left[1+\frac{(\Gamma-1)v_2^2}{v^2}\right]\right\}$$
$$+ \tilde{E}_{12}\left\{\frac{(\Gamma-1)^2v_2^2v_1^2}{B^2rv^4} + \frac{1}{B^2r}\left[1+\frac{(\Gamma-1)v_2^2}{v^2}\right]\left[1+\frac{(\Gamma-1)v_1^2}{v^2}\right]\right\}. \quad \text{(C14)}$$

Whereas for the magnetic part we obtain the following expressions:

$$\tilde{H}_{03} = \frac{1}{2Br^2(v^2-1)}[v_1v_2(R''r^2 - R'r - R_{,\theta\theta}) + (v_2^2 - v_1^2)(R'_{,\theta}r - R_{,\theta})], \quad \text{(C15)}$$

$$\tilde{H}_{13} = \frac{1}{2r^2(v^2-1)}[v_1(R'_{,\theta}r - R_{,\theta}) + v_2(R_{,\theta\theta} + R'r)], \quad \text{(C16)}$$

$$\tilde{H}_{23} = -\frac{1}{2r(v^2-1)}[v_1R''r^2 + v_2(R'_{,\theta}r - R_{,\theta}]. \quad \text{(C17)}$$

These three components are not independent since they satisfy the relationship:

$$\tilde{H}_{03} = -\frac{\tilde{H}_{13}v_1}{B} - \frac{\tilde{H}_{23}v_2}{Br}. \quad \text{(C18)}$$

Thus we may express the magnetic part of the Weyl tensor in terms of the two scalars $(\tilde{H}_1, \tilde{H}_2)$, given by:

$$\tilde{H}_1 = \frac{\tilde{H}_{03}(\Gamma-1)v_1}{\Gamma BRv^2} + \frac{\tilde{H}_{13}}{B^2R}, \quad \text{(C19)}$$

$$\tilde{H}_2 = \frac{\tilde{H}_{03}(\Gamma-1)v_2}{\Gamma BRv^2} + \frac{\tilde{H}_{23}}{B^2Rr}. \quad \text{(C20)}$$

As is obvious from the above expressions, the magnetic part of the Weyl tensor vanishes if we put $v=0$, as it should be since for the comoving congruence the field is purely electric.

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