Solution of the dark energy and its coincidence problem based on local antigravity sources without fine-tuning or new scales

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A novel idea is proposed for a natural solution of the dark energy and its cosmic coincidence problem. The existence of local antigravity sources, associated with astrophysical matter configurations distributed throughout the Universe, can lead to a recent cosmic acceleration effect. Various physical theories can be compatible with this idea, but here, in order to test our proposal, we focus on quantum originated spherically symmetric metrics matched with the cosmological evolution through the simplest Swiss cheese model. In the context of asymptotically safe gravity, we have explained the observed amount of dark energy using Newton's constant, the galaxy or cluster length scales, and dimensionless order-one parameters predicted by the theory, without fine-tuning or extra unproven energy scales. The interior modified Schwarzschild–de Sitter metric allows us to approximately interpret this result as that the standard cosmological constant is a composite quantity made of the above parameters, instead of a fundamental one.

I. INTRODUCTION

The so-called cosmological constant problem is nothing more than the simple observation, due to Zeldovich, that the quantum vacuum energy density should unavoidably contribute to the energy-momentum of the Einstein equations in the form of a cosmological constant. The recent discovery of a Higgs-like particle at the CERN Large Hadron Collider provides the experimental verification of the existence of the electroweak vacuum energy, and thus of the reality of the cosmological constant problem. The obvious absence of such a huge vacuum energy in the Universe has led to theoretical and phenomenological attempts to cancel out this vacuum energy, which are all so far subject to fine-tuning problems [1–4].

On the other hand, the observational evidence of the accelerated expansion of the Universe [5–11] has introduced the notion of dark energy. One option is that the dark energy is due to a cosmological constant Λ in the ΛCDM model, where in this case the explanation of the huge discrepancy between this observed Λ and the expected quantum vacuum energy is rather more pertinent. Even if the dark energy has nothing to do with the quantum vacuum energy, but is due to quite different phenomena, the cosmological constant problem remains as a hard problem in physics. In any case, the existence of dark energy is associated with another cosmological puzzle, the so-called cosmic coincidence problem. The latter refers to the need for an explanation of the recent passage from a deceleration era to present acceleration cosmic phase.

The aim of the present work is twofold: First, to propose a novel idea for a natural solution to the dark energy issue and its associated cosmic coincidence problem of recent acceleration. Second, to implement this idea through an interesting and concrete scenario, among others, which explains the correct amount of dark energy without the introduction of new and arbitrary scales or fine-tuning.

The proposed solution is based on the simple idea that the acceleration/dark energy can be due to infrared modifications of gravity at intermediate astrophysical scales which effectively generate local antigravity effects. The cosmological consequence of all these homogeneously distributed local antigravity sources is an overall cosmic acceleration through the matching between the local and the cosmic patches. Before the appearance of astrophysical structures (galaxies, clusters of galaxies), such antigravity effects do not exist, and therefore, the recent emergence of dark energy is not a coincidence but an outcome of the recent formation of structure. Before the appearance of structure and the emergence of sufficient repulsive effects, the conventional deceleration scenario is expected.

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Various physical theories (alternative gravities, extradimensional gravities, quantum gravities, etc.) can be implemented and be compatible with the previous general idea, providing intermediate distance infrared modifications which act as local antigravity sources. It is worth to notice that when the same physical theories are applied directly at the far infrared cosmic scales do not necessarily give comparable or significant cosmological effects. So, a dark energy of local origin in the Universe is not an equivalent or alternative description, but can be a necessity in order to reveal the relevant phenomena at intermediate scales. Quantum theories of gravity, in particular, provide types of models where local repulsive effects are naturally expected (for example, a quantum gravity origin of negative pressure can be formed in the interior of astrophysical black holes). Asymptotically safe (AS) gravity [12] is one of the promising quantum gravity frameworks that we will elaborate more thoroughly in the following sections in relation to the previous ideas. We will show in our most successful scenario that the observed dark energy can be explained from the Newton's constant, the galaxy or cluster length scales, and dimensionless order-one parameters predicted by AS theory, without fine-tuning or introduction of new scales. This can approximately be interpreted as that the observed cosmological constant Λ is not a fundamental parameter, but it is composite and naturally arises from other fundamental quantities.

In order to study the effect of all local sources of antigravity in the cosmic evolution, we adopt in the present work a simple Swiss cheese model by matching a homogeneously and isotropic spacetime with the appropriate local spherically symmetric metrics [13], and this formulation is presented in Sec. II. As an introductory step to set up the Swiss cheese evolution equations, we work out in Sec. III the classical Schwarzschild metric. In Sec. IV, we provide some general thoughts on the relation between a possible locally originated dark energy and the coincidence problem. In Sec. V, the Schwarzschild–de Sitter black hole is discussed with respect to the above scenario. In Sec. VI, quantum improved Schwarzschild–de Sitter metrics are considered; quantum gravity effects may indeed introduce an explicit or effective cosmological constant which arises from ultraviolet or infrared modifications of gravity [14–18]. Finally, Sec. VII is the largest and most important one, where the AS theory is applied in the context of our ideas. The first subsection VII A discusses the running of the cosmological constant close to the Gaussian fixed point of the AS evolution and the resulting cosmology is practically indistinguishable from the ΛCDM scenario with the same fine-tuning problems. The last subsection VII B discusses in detail, for the running of the cosmological constant close to the infrared (IR) fixed point of the AS evolution, the quite interesting emergence of the dark energy out of known physical scales and parameters predicted by the theory, and provides a natural explanation to the recent cosmic acceleration without

obvious observational conflicts with internal dynamics of galaxies or clusters. We finish with the conclusions in Sec. VIII.

It is worth mentioning that attempts to explain acceleration without a dark energy component, or also to produce dark energy, all due to structure formation, have already appeared in the literature (e.g., [19,20,21] and references therein). The existence of structure formation in the Universe implies a nonlinear local evolution, while the distribution of the nonlinear regions is homogeneous and isotropic above a current homogeneity scale of the order 100 Mpc. The apparent recent cosmic acceleration could be the effect of inhomogeneities and/or anisotropies on the average expansion rate, broadly referred as backreaction. This approach can potentially solve the coincidence problem too. However, it is fair to say that our viewpoint in the present work is different. In the averaging procedure, the matter is treated as a usual pressureless ideal fluid in the context of general relativity, gravity has the standard attractive behavior inside the structure, and cosmic acceleration arises due to the nontrivial complexity of the considered solution; there are no explicit antigravity forces and repulsive effects come only through averaging. Here, on the contrary, the acceleration and the dark energy come from the existence of antigravity sources related to the astrophysical structures in a as simple as possible spacetime and no averaging is performed; in the present work, these repulsive forces are basically of quantum origin as AS suggests, although in general, they can be of some other geometric nature generated by some modified gravity theory with IR gravity modifications. In our approach, this simple spacetime is described as a first step by the homogeneous Swiss cheese model with its known Schucking matching surface, although a better approximation would be to use inhomogeneous Swiss cheese models (e.g., some analogues of Lemaitre-Tolman-Bondi or Szekeres); averaging processes in this case are expected to enhance the cosmic acceleration found here. A different scenario, where structure is responsible for acceleration, was presented in [22]; in a five-dimensional setup, a brane-bulk energy exchange in the interior of galactic core black holes produces a sufficient negative dark pressure to play the role of dark energy.

II. SWISS CHEESE MODELS

The Swiss cheese cosmological model, first introduced by Einstein and Strauss [13], is solution of general relativity that globally respects homogeneity and isotropy, while locally describes a spherically symmetric solution. Other more general Swiss cheese models refer to inhomogeneous solutions. A Swiss cheese model with spherical symmetry overcomes the difficulty of how to glue a static solution of the theory at hand within a larger time-dependent homogeneous and isotropic spacetime. The idea is to assume a very large number of local objects homogeneously and

isotropically distributed in the Universe. The matching of a spatially homogeneous metric as the exterior spacetime to a local interior solution has to be realized across a spherical boundary that stays at a fixed coordinate radius in the cosmological frame while evolves in the interior frame.

Let us consider a four-dimensional manifold M with metric $g_{\mu\nu}$ and a timelike hypersurface Σ which splits the spacetime M into two parts. The spacetime coordinates are denoted by x^{μ} ($\mu, \nu, \ldots$ are four-dimensional coordinate indices) and can be different between the two regions. The coordinates on Σ are denoted by χ^{i} ($i, j, \ldots$ are three-dimensional coordinate indices on Σ). The embedding of Σ in M is given by some functions $x^{\mu}(\chi^{i})$. The unit normal vector n^{μ} to Σ points inwards the two regions. The first relevant quantity characterizing Σ is the induced metric $h_{\mu\nu} = g_{\mu\nu} - n_{\mu}n_{\nu}$ coming from the spacetime in which it is embedded. The second quantity is the extrinsic curvature $K_{\mu\nu} = h^{\kappa}_{\mu}h^{\lambda}_{\nu}n_{\kappa;\lambda}$, where a ; denotes covariant differentiation with respect to $g_{\mu\nu}$. In the adapted frame where x^{μ}, say $\bar{x}^{\mu}$, contains $\bar{x}^{i}$ with $\bar{x}^{i}|_{\Sigma} = \chi^{i}$ and some extra transverse coordinate, it is $h_{ij} = g_{ij}$. However, this quantity can be expressed in terms of arbitrary spacetime coordinates x^{μ} as

$$h_{ij} = g_{\mu\nu}\frac{\partial x^{\mu}}{\partial \chi^{i}}\frac{\partial x^{\nu}}{\partial \chi^{j}}. \tag{2.1}$$

Similarly, for the extrinsic curvature it is $K_{ij} = n_{i;j}$, and can be expressed as

$$K_{ij} = \left(\frac{\partial n_{\mu}}{\partial \chi^{j}} - \Gamma^{\lambda}_{\mu\nu}n_{\lambda}\frac{\partial x^{\nu}}{\partial \chi^{j}}\right)\frac{\partial x^{\mu}}{\partial \chi^{i}} \tag{2.2}$$

$$= -n_{\lambda}\left(\frac{\partial^{2}x^{\lambda}}{\partial \chi^{i}\partial \chi^{j}} + \Gamma^{\lambda}_{\mu\nu}\frac{\partial x^{\mu}}{\partial \chi^{i}}\frac{\partial x^{\nu}}{\partial \chi^{j}}\right), \tag{2.3}$$

where $\Gamma^{\kappa}_{\mu\nu}$ are the Christoffel symbols of $g_{\mu\nu}$.

Continuity of the spacetime across the hypersurface Σ implies that h_{ij} is continuous on Σ, which means that h_{ij} is the same when computed on either side of Σ. If we consider Einstein gravity with a regular spacetime matter content and vanishing distributional energy-momentum tensor on Σ, then the Israel-Darmois matching conditions [23] imply that the sum of the two extrinsic curvatures computed on the two sides of Σ is zero.

The model of Einstein-Strauss refers to the embedding of a Schwarzschild mass into FRW cosmology. Here, we shall assume a general static spherically symmetric metric which matches smoothly to a homogeneous and isotropic cosmological metric. In spherical coordinates. the cosmological metric takes the form

$$ds^{2} = -dt^{2} + a^{2}(t)\left[\frac{dr^{2}}{1 - \kappa r^{2}} + r^{2}(d\theta^{2} + \sin^{2}\theta d\varphi^{2})\right], \tag{2.4}$$

where $a(t)$ is the scale factor and $\kappa = 0, \pm 1$ characterizes the spatial curvature. In these coordinates, a "spherical" boundary is defined to have a fixed coordinate radius $r = r_{\Sigma}$, with r_{Σ} constant. Of course, this boundary is seen by a cosmological observer to expand, following the universal expansion. If $\bar{x}^{\mu} = (t, r, \theta, \varphi)$ are the coordinates of the metric (2.4), then the hypersurface Σ is determined by the function $\bar{f}(\bar{x}^{\mu}) = r - r_{\Sigma} = 0$ and the cosmological metric occurs for $r \geq r_{\Sigma}$. From the coordinates $\bar{x}^{\mu}$ one can parametrize Σ by the coordinates $\chi^{i} = \bar{x}^{i}|_{\Sigma} = (t, \theta, \varphi)$, and therefore, on Σ, it is $\bar{x}^{\mu}(\chi^{i}) = (t, r_{\Sigma}, \theta, \varphi)$. The unit normal vector can be calculated from

$$\bar{n}_{\mu} = \frac{\bar{f}_{,\mu}}{\sqrt{|\bar{g}^{\kappa\lambda}\bar{f}_{,\kappa}\bar{f}_{,\lambda}|}}, \tag{2.5}$$

where a comma means differentiation with respect to $\bar{x}^{\mu}$. Obviously, the plus sign in (2.5) makes certain that $\bar{n}^{\mu}$ is inward of the cosmological region (to the direction of increasing r). Thus,

$$\bar{n}_{\mu} = \left(0, \frac{a}{\sqrt{1 - \kappa r_{\Sigma}^{2}}}, 0, 0\right). \tag{2.6}$$

Note that $\bar{n}^{\mu}$ is spacelike, $\bar{n}^{\mu}\bar{n}_{\mu} = 1$, as expected.

The interior region $r \leq r_{\Sigma}$ is replaced by another metric which has the following form,

$$ds^{2} = -J(R)F(R)dT^{2} + \frac{dR^{2}}{F(R)} + R^{2}(d\theta^{2} + \sin^{2}\theta d\varphi^{2}), \tag{2.7}$$

where $J, F > 0$. This metric represents a static spherically symmetric spacetime in Schwarzschild-like coordinates. The functions $F(R)$ and $J(R)$ are given by the specific metric in use. Since the two-dimensional sphere (θ, φ) is the common fiber for both metrics (2.4), (2.7), the position of Σ in the spacetime described by (2.7) does not depend on θ, φ and is given by the functions $T = T_{S}(t)$, $R = R_{S}(t)$. The subscript S refers to Schucking, R_{S} is called Schucking radius and it is time dependent. Therefore, the spherical boundary does not remain in constant radial coordinate distance in the Schwarzschild-like patch as the Universe expands. The coordinates $\hat{x}^{\mu} = (T, R, \theta, \varphi)$ of the metric (2.7) take on Σ the form $\hat{x}^{\mu}(\chi^{i}) = (T_{S}(t), R_{S}(t), \theta, \varphi)$. The unit normal vector $\hat{n}^{\mu}$ cannot be calculated now directly from a formula as (2.5), since the function $\hat{f}(\hat{x}^{\mu})$ of the matching surface is now unknown. However, due to the symmetry, it is expected that $\hat{n}_{\theta} = \hat{n}_{\varphi} = 0$; therefore, the orthonormality of $\hat{n}^{\mu}$ will provide two conditions for $\hat{n}_{T}, \hat{n}_{R}$. Indeed, since the three vectors $\frac{\partial \hat{x}^{\mu}}{\partial \chi^{i}}$ are tangent to Σ, the condition $\hat{n}_{\mu}\frac{\partial \hat{x}^{\mu}}{\partial \chi^{i}} = 0$ implies $\hat{n}_{\theta} = \hat{n}_{\varphi} = 0$ and

$$\frac{dT_S}{dt}\hat{n}_T + \frac{dR_S}{dt}\hat{n}_R = 0. \qquad (2.8)$$

Furthermore, from $\hat{n}^\mu \hat{n}_\mu = 1$, one obtains

$$\frac{1}{JF}\hat{n}_T^2 - F\hat{n}_R^2 = -1, \qquad (2.9)$$

where J, F are located at R_S.

So far, we have established the geometrical setting on the two sides of the boundary hypersurface. The junction of the two regions on Σ demands $\bar{h}_{ij} = \hat{h}_{ij}$, which provides through (2.1) the conditions

$$JF\left(\frac{dT_S}{dt}\right)^2 - \frac{1}{F}\left(\frac{dR_S}{dt}\right)^2 = 1 \qquad (2.10)$$

and

$$R_S = ar_\Sigma. \qquad (2.11)$$

The two Eqs. (2.10) and (2.11) can also arise more easily from the two expressions for the induced metric on Σ coming from (2.4), (2.7):

$$ds_\Sigma^2 = -dt^2 + a^2 r_\Sigma^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (2.12)$$

$$= -\left[JF\left(\frac{dT_S}{dt}\right)^2 - \frac{1}{F}\left(\frac{dR_S}{dt}\right)^2\right]dt^2$$
$$+ R_S^2(d\theta^2 + \sin^2\theta d\varphi^2). \qquad (2.13)$$

Solving (2.8), (2.9) for $\hat{n}_T$, $\hat{n}_R$ and using (2.10), we find the normal vector $\hat{n}^\mu$ from

$$\hat{n}_\mu = \left(\epsilon\sqrt{J}\frac{dR_S}{dt}, -\epsilon\sqrt{J}\frac{dT_S}{dt}, 0, 0\right), \qquad (2.14)$$

where $\epsilon = \pm 1$. The demand that $\hat{n}^\mu$ is inward the central void (to the direction of decreasing R) implies $\hat{n}_R < 0$. Additionally, the forms of $\bar{n}_\mu$, $\hat{n}_\mu$ show that the directions defined by the coordinate axes T, R are different than those of t, r; however, the centers of the two coordinate systems coincide.

What remains is the matching of the two extrinsic curvatures on Σ, i.e., the demand $\bar{K}_{ij} + \hat{K}_{ij} = 0$. Due to the simple form of $\bar{n}_\mu$, the extrinsic curvature $\bar{K}_{ij}$ in the cosmological region can be easily computed from either Eq. (2.2) or (2.3) as

$$\bar{K}_{ij} = -\bar{n}_r\bar{\Gamma}_{ij}^r = \frac{1}{2}\bar{n}_r\bar{g}^{rr}\bar{g}_{ij,r} \qquad (2.15)$$

and finally

$$(\bar{K}_{tt}, \bar{K}_{\theta\theta}, \bar{K}_{\varphi\varphi}) = \sqrt{1 - \kappa r_\Sigma^2}ar_\Sigma(0, 1, \sin^2\theta). \qquad (2.16)$$

In the interior region, the computation is more involved and it is slightly more convenient to use the expression (2.3) to compute $\hat{K}_{ij}$. The corresponding nonvanishing Christoffel symbols are $\hat{\Gamma}_{TT}^R = JF^2\hat{\Gamma}_{TR}^T = \frac{F}{2}(JF' + FJ')$, $\hat{\Gamma}_{RR}^R = -\frac{F'}{2F}$, $\hat{\Gamma}_{\varphi\varphi}^R = \sin^2\theta\hat{\Gamma}_{\theta\theta}^R = -RF\sin^2\theta$, $\hat{\Gamma}_{R\theta}^\theta = \hat{\Gamma}_{R\varphi}^\varphi = \frac{1}{R}$, $\hat{\Gamma}_{\varphi\varphi}^\theta = -\sin\theta\cos\theta$, $\hat{\Gamma}_{\theta\varphi}^\varphi = \cot\theta$, where a prime denotes differentiation with respect to R. Then, it arises that all $\hat{K}_{ij} = 0$ for $i \neq j$, while $\hat{K}_{\varphi\varphi} = \sin^2\theta\hat{K}_{\theta\theta}$,

$$\hat{K}_{\theta\theta} = -\hat{n}_R\hat{\Gamma}_{\theta\theta}^R = R_S F\hat{n}_R \qquad (2.17)$$

$$\hat{K}_{tt} = -\hat{n}_T\frac{d^2 T_S}{dt^2} - \hat{n}_R\frac{d^2 R_S}{dt^2} - \hat{n}_R\hat{\Gamma}_{TT}^R\left(\frac{dT_S}{dt}\right)^2$$
$$- \hat{n}_R\hat{\Gamma}_{RR}^R\left(\frac{dR_S}{dt}\right)^2 - 2\hat{n}_T\hat{\Gamma}_{TR}^T\frac{dT_S}{dt}\frac{dR_S}{dt}. \qquad (2.18)$$

Finally, the condition $\bar{K}_{\theta\theta} + \hat{K}_{\theta\theta} = 0$ (or equivalently for φ) gives the consistency equation

$$\frac{dT_S}{dt} = \frac{\epsilon\sqrt{1 - \kappa r_\Sigma^2}ar_\Sigma}{R_S F\sqrt{J}}, \qquad (2.19)$$

which, with the use of (2.11), takes the form

$$\frac{dT_S}{dt} = \frac{\epsilon\sqrt{1 - \kappa r_\Sigma^2}}{F\sqrt{J}}. \qquad (2.20)$$

It then follows from (2.10) that

$$\left(\frac{dR_S}{dt}\right)^2 = 1 - \kappa r_\Sigma^2 - F(R_S). \qquad (2.21)$$

Therefore, Eqs. (2.20) and (2.21) determine the position of Σ in the space (T, R). From (2.20) it is obvious that indeed it is $\hat{n}_R < 0$.

The final task is the examination of the matching condition $\bar{K}_{tt} + \hat{K}_{tt} = 0$, i.e., $\hat{K}_{tt} = 0$. This equation contains the second time derivatives of T_S, R_S that we need to calculate. From Eqs. (2.20) and (2.21), it arises

$$\frac{d^2 T_S}{dt^2} = -\epsilon\sqrt{1 - \kappa r_\Sigma^2}\frac{(F\sqrt{J})'}{F^2 J}\frac{dR_S}{dt} \qquad (2.22)$$

$$\frac{d^2 R_S}{dt^2} = -\frac{F'}{2}. \qquad (2.23)$$

Using all the previous expressions in (2.18), it turns out that $J' = 0$, which means $J'(R_S) = 0$. This relation, due to (2.11), implies in general an algebraic equation for $a(t)$, which will be inconsistent with Eq. (2.21). There are, however, various functions $J(R)$ which satisfy this equation.

For example, a consistent choice is that $J(R)$ is constant throughout (as happens in Schwarzschild metric), and in this case without loss of generality we can rescale T so that this constant is one. Another consistent case would be $J(R)$ to be a power series of the form $J(R) = J(R_S) + c_1(R - R_S)^2 + ...$ The successful matching has proved that the choice of the matching surface Σ was the appropriate one.

III. GENERAL RELATIVITY BLACK HOLES AND THE ENSUING FRW COSMOLOGY

If the black hole is described by the classical Schwarzschild solution, i.e.,

$$F(R) = 1 - \frac{2G_N M}{R}, \qquad J(R) = 1, \qquad (3.1)$$

Eq. (2.21) provides through (2.11) the cosmic evolution of the scale factor a. Namely, we take

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} - \frac{\kappa}{a^2}, \qquad (3.2)$$

where a dot denotes differentiation with respect to cosmic time t. This equation is qualitatively similar to the standard FRW evolution with dust (zero pressure) as its cosmic fluid and a possible curvature term. Of course, in order for this solution to be physically realistic and represent a spatially homogeneous universe, not just a single sphere of comoving radius r_Σ should be present, but a number of such spheres are uniformly distributed throughout the space. Otherwise, there would exist a preferred position in the Universe. Each such sphere can be physically realized by an astrophysical object, such as a galaxy (with its extended spherical halo) or a cluster of galaxies, which we assume that it has a typical mean mass M. It will be seen that the Schuching radius lies outside the real border of the astrophysical object; therefore, $F(R_S)$ in (2.21) is provided by the value of the expression (3.1) (otherwise, we would meet the inconvenient situation to consider an interior Oppenheimer-Volkof type of solution or some other more realistic matter profile). This means that for the value $F(R_S)$, which is our only interest in order to make the matching and derive the cosmological metric, it is like if all the mass M is gathered at the center of the spherical symmetry. The same is true for other spherically symmetric metrics, modifications of Schwarzschild solution, to be discussed later. Furthermore, Eq. (2.23) gives

$$\frac{\ddot{a}}{a} = -\frac{G_N M}{r_\Sigma^3 a^3}, \qquad (3.3)$$

which indicates a decelerated expansion.

In order for Eqs. (3.2) and (3.3) to describe precisely a standard matter-dominated universe, the matter dilution term in (3.2) should be $\frac{8\pi G_N}{3}\rho$, where ρ is the cosmic matter energy density, and the term in (3.3) should be $-\frac{4\pi G_N}{3}\rho$. Therefore, we make the standard assumption of Swiss cheese models that the matching radius r_Σ is such that when its interior region is filled with energy density equal to the cosmic matter density ρ, the interior energy equals M. Namely, we set

$$\rho = \frac{M}{\frac{4\pi}{3}R_S^3} = \frac{3M}{4\pi r_\Sigma^3 a^3}. \qquad (3.4)$$

This condition can also equivalently be interpreted that the mass M of the object is uniformly stretched up to the radius r_Σ. Since in any case the mass M can be considered that is located at the center of spherical symmetry, the above definition of r_Σ offers a simple way to determine the spheres where the matching with the cosmological metric occurs. Although this definition is certainly *ad hoc* and uses the mass M of the object and the cosmic density ρ, it has the merit that it avoids using other details of the structure, such as the size of the object and the distance between similar structures. However, still in the Swiss cheese model, the cosmic evolution remains exactly the same as in the cosmological picture.

It is now clear that Eqs. (3.2) and (3.3) become

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{\kappa}{a^2} \qquad (3.5)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\rho. \qquad (3.6)$$

If we define the matter density parameter in the conventional way

$$\Omega_m = \frac{8\pi G_N \rho}{3H^2}, \qquad (3.7)$$

it is found

$$r_\Sigma = \left(\frac{2G_N M}{\Omega_{m0} a_0^3 H_0^2}\right)^{\frac{1}{3}}, \qquad (3.8)$$

where a subscript 0 denotes the current value.

Let us suppose we want to model a universe consisting of two types of dust with different densities, ρ_1 and ρ_2 (e.g., dark matter and stars or black holes). In order to avoid unnecessary technical complexity arising from inhomogeneous placement of dusts, it would be a fair approximation to describe the cosmic evolution assuming that the Universe is filled with a homogeneous distribution of spherical configurations that consist of two spherical objects that have different masses M_1 and M_2 within the Schucking radius R_S. The quantities ρ_1, M_1 satisfy Eq. (3.4) and similarly for the other ingredient. Since the total cosmic energy density ρ is the sum of the two energy

densities ρ_1, ρ_2, it is implied that the matching is performed as before with the difference that now the mass M of the central object is the sum of the two masses, i.e., $M = M_1 + M_2$. In this case, the same Eqs. (3.2) and (3.3) apply, with M this total mass.

Let us finish with a few numerics. The present value of the Hubble parameter will be taken as $H_0 = 0.72 \times 10^{-10}$ yr^{-1}. In the present work, we are not going to perform fittings to real data, where H_0 could also be considered as a fitted parameter. We will use as a typical mass for a galaxy $M = 10^{11} M_\odot$ and for a cluster of galaxies $M = 10^{15} M_\odot$, (with $M_\odot$ the solar mass) in order to give some estimates. If we ignore the term of spatial curvature in (3.5), then $\Omega_m = 1$, and Eq. (3.8) gives for the galaxy $r_\Sigma = 0.56$ Mpc and for the cluster $r_\Sigma = 12$ Mpc (for a realistic Ω_{m0} these distances become larger). Since the typical radius of a spiral galaxy (including its dark matter halo) is $R_b \approx 0.15$ Mpc, it is obvious that r_Σ is a few times larger than the galactic radius. Moreover, the mean distance between galaxies is a few Mpc, thus, the Schucking radii of two neighboring galaxies do not overlap. As for clusters, they have radii from $R_b \approx 0.5$ Mpc to $R_b \approx 5$ Mpc, and therefore, r_Σ is again outside the cluster. If a mean distance between the borders of two adjacent clusters is something like 20 Mpc, the two Schucking radii still do not intersect.

IV. A PERSPECTIVE FOR THE COINCIDENCE PROBLEM

In the ΛCDM model, Λ has been found observationally to be of the order H_0^2 (more precisely, $\Lambda = 3\Omega_{\Lambda 0}H_0^2$, $\Omega_{\Lambda 0} \approx 0.7$). This means that the energy scale defined by $\sqrt{\Lambda}$ is extremely small compared to the Planck mass scale $M_{\rm Pl}$ (which is a typical scale of gravity), and also its energy density $\rho_\Lambda = \frac{\Lambda}{8\pi G_N} \approx 2.8 \times 10^{-11}$ eV$^4 \sim (10^{-3}$ eV$)^4$ is many orders of magnitude smaller than the theoretical vacuum energy value $\rho_{\rm vac}$ estimated by quantum corrections of quantum field theory with any sensible cutoff. This discrepancy is called the cosmological constant problem, which is the most severe hierarchy problem in modern physics. It is also called the fine-tuning problem since adding a bare cosmological constant of opposite sign in the action to cancel $\rho_{\rm vac}$, this should be tuned to extreme accuracy in order to give the effective value 10^{-3} eV above (if supersymmetry is restored in the high energy, extreme and unnatural fine-tuning is still needed). Even if the present dark energy in the Universe has nothing to do with a cosmological constant, the question of understanding why the estimated quantum vacuum energy cancels out and does not contribute, still remains and may need quantum gravity or other physics to be discovered.

Beyond the previous problem, why Λ is so extraordinarily small, there is an extra question named coincidence problem, related to the specific value of $\Lambda \sim H_0^2$. Since the energy density ρ falls like $\rho \sim a^{-3}$ starting from a huge

(if not infinite) value, why does it happen today to be $8\pi G_N \rho_0 \sim \Lambda$ (actually $8\pi G_N \rho_0 \approx 0.4\Lambda$), and not a very big or a very small proportionality factor to be present? Why are dark matter and dark energy of the same order today, $\rho_0 \sim \rho_\Lambda$? Moreover, since $\sqrt{\Lambda} \sim H_0$, the time scale $t_\Lambda \sim 1/\sqrt{\Lambda}$ is of the same order as the age of the Universe H_0^{-1}, something that did not need necessarily to be the case. There are three unrelated quantities ρ_0, Λ, G_N and there is no obvious reason why they should be related like that. To be more precise, the same relation $8\pi G_N \rho \sim \Lambda$ holds recently, for $0 \leq z \lesssim \mathcal{O}(1)$, which means for a few billion years (taking into account the time, it may be thought that the problem is not so sharp). On the contrary, such a relation between $G_N \rho$ and Λ could have happened in the very past, at even larger redshifts (which is probably precluded by anthropic arguments), and this implies that today we would have a universe full of cosmological constant and negligible matter contribution. Or, finally, such a relation could occur in the very future, and today we would observe matter domination with negligible Λ. It is the same to say that although the Hubble parameter started in the past from huge values (if not infinite), recently it is $H^2 \sim \Lambda \sim 8\pi G_N \rho$, and not $H^2 \approx \Lambda/3$ or $H^2 \approx 8\pi G_N \rho/3$. To realize better the clear sensitivity of the recent coincidence on the value of Λ, let us assume that the cosmological constant was just one hundred times larger than the observed one. Then its coincidence with the matter would have occurred at a redshift of almost 5, and today the dark matter would be less than just one percent of the dark energy. Or at the other end, if Λ was one hundred times smaller than the observed value, its coincidence with the matter would occur at a redshift of almost -0.7, and today the dark energy would be almost two percent of the dark matter. Therefore, the coincidence problem appears because Λ takes a value inside a very narrow range of the ρ values. In terms of the flatness parameters the coincidence problem is stated by a relation of the form $\Omega_{m0} \sim \Omega_{\Lambda 0}$, and not $\Omega_{m0} \ll \Omega_{\Lambda 0}$ or $\Omega_{m0} \gg \Omega_{\Lambda 0}$. Ignoring κ, it holds $\Omega_m \approx 1$, $\Omega_\Lambda \approx 0$ for a broad range of redshifts in the past until recently, where it is $\Omega_m \sim \Omega_\Lambda$. In the far future it will be $\Omega_m \approx 0$, $\Omega_\Lambda \approx 1$. Acceleration exists as long as $2\Omega_\Lambda > \Omega_m$.

In general dark energy models, the coincidence problem is formulated through the observational current acceleration along with the relation $\Omega_{m0} \sim \Omega_{\rm DE,0}$, while in the past it is strongly believed, due to structure formation reasons, that it was $\Omega_m \approx 1$, $\Omega_{\rm DE} \approx 0$. The dark energy density $\rho_{\rm DE}$ defined by $8\pi G_N \rho_{\rm DE} = 3\Omega_{\rm DE}H^2$ obeys $\rho_0 \sim \rho_{\rm DE,0}$. Depending on the particular dark energy model, the quantity $\rho_{\rm DE,0}$ contains integration constants reflecting initial conditions of possible fields involved (e.g., $\phi_0, \dot{\phi}_0$ for a scalar field, or ρ_0 itself in a geometrical modification of gravity), dimensionful or dimensionless couplings/parameters of the theory, and probably other quantities (e.g., of astrophysical nature). The previous coincidence relation between the

energy densities provides an equation between all these quantities which have to be appropriately adjusted. Usually, in dark energy models the scale defined by Λ is exchanged by another scale of the same order describing some new physics. It looks like the coincidence problem is a question of naturalness between integration constants and other parameters, and analyzing naturalness is not an issue easily quantified.

A proposal as a solution of the coincidence problem will consist of one more dark energy model, which may be more or less natural, may introduce new physics or not, may introduce new scales or not, but its verification will come not out of concept but out of experimental evidence of the particular model. And finally, when the origin of dark energy has been apprehended, it will become obvious what the independent scales and initial conditions created by nature are. Even a model which contains new scales, that at present are unrelated from the rest of physics, may be very close to reality. This is why analyzing a dark energy model, the values of Ω_{m0}, $\Omega_{DE,0}$, as well as the rest of the parameters/initial conditions, are in general extracted after fittings to the observational data and there is no special concern about a deeper understanding which would mean to express these values in terms of other more fundamental ones. Of course, if such an explanation for the recent emergence of dark energy through the coincidence relation $\rho \sim \rho_{DE}$ can be provided in terms of quantities that already play a role in nature and/or other quantities theoretically predicted in the context of a theory, this would possess extra naturalness and might render the particular dark energy model promising (this is the case for one of the models to be presented in the present work). Another idea that alleviates the coincidence problem comes through the realization of the current state of the Universe close to a global fixed point (saddle or preferably attractor) of the cosmic evolution, since then, the coincidence problem becomes an issue of only the parameters of the model and not also of the initial conditions [24]. In order for this to be possible with the present acceleration and a scaling behavior between Ω_m, Ω_{DE}, the violation of the standard energy-momentum conservation of the matter is necessary [25].

The proposal introduced in the present paper is that the dark energy observed recently in the Universe may be the result of local gravity effects occurring in the interior of astrophysical objects, such as massive structures (galaxies, clusters) or even black holes, and these effects will directly determine the cosmic evolution. These local effects can arise from an arbitrary gravitational theory (alternative/modified gravity, extradimensional gravity, quantum gravity, etc.). The main point is that the specific gravitational theory is not applied directly to cosmology in the conventional way, with the matter described as a usual perfect fluid, in order to obtain time-dependent differential equations for the geometry (e.g., scale factor) and the other ingredients; the reason for this is that it is not clear how the

cosmic effective energy-momentum tensor can be quantified taking into account the extra contributions of local origin. So, even if the theory is managed to be applied directly to cosmology, the result will in general be different that the one arising from the process described here because, depending on the scales of the theory, the dark energy can be suppressed in one of the two derived cosmologies and be considerable in the other. Thus, if a gravity effect becomes substantial only at an intermediate infrared scale (astrophysical one), it cannot be revealed at the far infrared cosmological scale. In addition, integration constants emanating from possible integrations (due to extra fields or geometrical effects) in the local metric will be of quite different nature than the cosmological ones; these constants might be specified or at least estimated from quantities characterizing the astrophysical object itself or from regularity arguments at the center of the structure, contrary to the specification of the integration constants of a cosmological quantity which needs a quite different treatment. Applying the gravitational theory first inside the structure means to find the gravitational and the other possible fields in the interior of the object. Because the astrophysical structures are not point-like but they are extended (the galaxies have a luminous profile which is surrounded by the dark matter halo and the clusters contain a distribution of galaxies and dark matter) or may be described by a collapsing phase, this task can be complicated. However, depending on the ansatz for the local and the cosmological metrics and how these are interrelated, it may be enough to find the static spherically symmetric solution of the theory (as happens in the Swiss cheese model described previously), or other more complete solutions may be needed to describe an inhomogeneous universe with more realistic structures. In the present work the Swiss cheese model will be adopted as the simplest but not necessarily the most realistic construction, and therefore, the ensuing cosmologies will arise through the matching of the interior metric with the exterior FRW metric on the Schucking surface. It is obvious that such a local gravity effect should not contradict with observations at the relevant astrophysical scale.

In the context where the dark energy owes its origin to the presence of structure, either due to the reasons elaborated here or due to averaging process (as e.g., in [19–21]), since the various structures are formed during the cosmic evolution recently at small redshifts, dark energy also appears recently not as a coincidence but as an emerging effect of the structure. With this in mind, that the coincidence problem can be a guiding line for studying cosmology, it becomes tempting to see what are the new scales and integration constants introduced by some gravitational theory in the above context, so that the corresponding cosmology confronts (if possible) with the acceleration and other data, and especially with the relation $\rho \sim \rho_{DE}$. In the following sections, we will implement the previous ideas

to derive a few dark energy models, where in the last subsection our most promising model will appear.

V. BLACK HOLES WITH COSMOLOGICAL CONSTANT

When a constant cosmological term is added to the Schwarzschild metric, the well-known Schwarzschild–de Sitter metric arises

$$ds^2 = -\left(1 - \frac{2G_N M}{R} - \frac{1}{3}\Lambda R^2\right)dT^2 + \frac{dR^2}{1 - \frac{2G_N M}{R} - \frac{1}{3}\Lambda R^2}$$
$$+ R^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{5.1}$$

It is apparent that it is $J(R) = 1$. Equation (2.21) provides through (2.11) the cosmic evolution of the scale factor

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \frac{\Lambda}{3}. \tag{5.2}$$

Furthermore, Eq. (2.23) gives

$$\frac{\ddot{a}}{a} = -\frac{G_N M}{r_\Sigma^3 a^3} + \frac{\Lambda}{3}, \tag{5.3}$$

which indicates the well-known late-times accelerated expansion when the two terms on the rhs of (5.2) become comparable. Using the Swiss cheese condition (3.4), Eqs. (5.2) and (5.3) are also written as

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}\rho + \frac{\Lambda}{3} \tag{5.4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}\rho + \frac{\Lambda}{3}. \tag{5.5}$$

Since Ω_{m0} is close to 0.30 according to the most recent constraints [26], Eq. (3.8) gives for a galaxy with $M = 10^{11} M_\odot$ that $r_\Sigma = 0.83$ Mpc and for a cluster with $M = 10^{15} M_\odot$ that $r_\Sigma = 18$ Mpc.

Equations (5.4) and (5.5) are the standard cosmological equations of ΛCDM model. In this model, the cosmological constant Λ is considered as a universal constant related to the vacuum energy. However, in the context of the present work, Λ arises from the interior black hole solution (5.1) and has a quite different origin and meaning. This Λ is of astrophysical origin and is the total cosmological constant coming from the sum of all antigravity sources inside the Schucking radius of the galaxy or cluster. It is expected that, through some concrete quantum gravity theory, matter is related to the generation of an explicit or effective cosmological constant. For example, in the centers of astrophysical black holes, the avoidance of singularity could be achieved due to the presence of a repulsive pressure of quantum origin balancing the attraction of gravity. Another important difference with this Λ is that since, according to our proposal, the antigravity sources are connected to either massive structures (galaxies, clusters) or astrophysical black holes; therefore, before the appearance of all these objects, the total Λ is zero. As a result, this Λ becomes a function of cosmic time, suppressed at larger redshifts where the antigravity effect is weaker. A constant Λ is expected to be only an approximation at late times.

The metric (5.1) contains the Newtonian term $\frac{2G_N M}{R}$ and the cosmological constant term $\frac{1}{3}\Lambda R^2$. For distances R close to the border with coordinate distance R_b, the matter can be considered as being gathered at the origin, as mentioned above. We will give an estimate of the corresponding values of the potential and the force due to the cosmological constant. In the weak field limit the force corresponding to the Newtonian term is $-\frac{G_N M}{R^2}$, while the cosmological constant force is $\frac{1}{3}\Lambda R$ and is repulsive. The ratio of the magnitudes of the cosmological constant force to the Newtonian force is $\frac{2\Omega_{\Lambda 0}}{\Omega_{m0}}\left(\frac{R}{r_\Sigma}\right)^3$; therefore, the significance of the cosmological constant increases with distance. At the border of a typical galaxy with mass $10^{11} M_\odot$ and radius 0.15 Mpc the Newtonian term has a value approximately 6×10^{-8}, while the cosmological constant term is almost 9×10^{-10}, which is therefore 2 orders of magnitude smaller than the former term. Of course, the two potentials at the Schucking radius are of the same order since dark matter and dark energy today are of the same order. Similarly, the repulsive force at the border is also almost 2 orders of magnitude smaller that the Newtonian force. Therefore, the Λ term is ignorable at the galaxy level and the galaxy dynamics is not disturbed by this antigravity effect.

For the clusters there is a larger variability of the range of their radii and the corresponding masses. At the Schucking radius still the two potentials are of the same order. For a mass $10^{15} M_\odot$ and radius 0.5 Mpc the Newtonian term is 2×10^{-4} at the border, while the cosmological constant term is 10^{-8}. As a result, the repulsive force is 4 orders of magnitude smaller that the attractive force. For a radius of 5 Mpc, the Λ force is still smaller than the Newtonian force, but just 1 order of magnitude. For a mass $10^{14} M_\odot$ and radius 5 Mpc, the two forces become equal in magnitude. If the cosmological constant is indeed generated at the cluster scales, then this constant should be present in all clusters. Therefore, more investigation is needed at particular clusters that could show off some abnormal dynamics and if this can be explained through a constant Λ.

The previous discussion shows that Λ could be generated inside astrophysical objects, either without affecting their dynamics or signaling some observable deviations in this dynamics, and at the same time to create the standard ΛCDM cosmology. Of course, this constant Λ does not offer any alleviation to the coincidence puzzle.

VI. BLACK HOLES WITH VARYING COSMOLOGICAL CONSTANT OF QUANTUM ORIGIN

The Schwarzschild–de Sitter metric can be progressed to a quantum improved Schwarzschild–de Sitter metric describing the astrophysical object. This metric has the form

$$ds^2 = -\left(1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2\right)dT^2 + \frac{dR^2}{1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2}$$
$$+ R^2(d\theta^2 + \sin^2\theta d\varphi^2), \tag{6.1}$$

where the quantities G_k, Λ_k are functions of a characteristic energy scale k and $F(R) = 1 - \frac{2G_kM}{R} - \frac{1}{3}\Lambda_k R^2$. The functional behavior of G_k, Λ_k is determined by the underlying quantum theory of gravity. This energy scale k is related to the distance from the center of the object and the exact dependence arises from the particular quantum corrections. Therefore, G_k, Λ_k are also related to the distance. In the Swiss cheese analysis, however, only the front value R_S of the distance at the matching surface influences the cosmic evolution, thus only the corresponding energy value k_S will be relevant. As mentioned before, in a real galaxy or cluster the total mass consists of either stars, dark matter, or black holes (classical or quantum modified) that we collectively denote M. Although the various objects are distributed throughout, in our approach it is sufficient to consider that these materials are gathered together at the center of spherical symmetry.

As is known, the Israel matching conditions are only applicable in Einstein gravity with some regular energy-momentum tensor. In an alternative/modified gravity, either containing extra fields or not, the corresponding matching conditions are in general modified. One might wonder if the Israel conditions are still applicable in our case, with a metric of the form (6.1). The answer is positive and we will explain this in the following. A quantum originated spherically symmetric metric, as the one described above, does not in general arise as a solution of some classical field equations for the metric, but is obtained by considering some quantum corrections beyond the classical Einstein term. For example, in AS gravity, the solution of the renormalization group (RG) flow equations gives G_k, Λ_k. Therefore, a metric, such as (6.1), is quite reasonable and necessary for our Swiss cheese approach to be interpreted as a solution of a coupled gravity-matter system satisfying Einstein equations $G_{\mu\nu} = 8\pi G_N T_{\mu\nu}^{(\text{tot})}$. This $T_{\mu\nu}^{(\text{tot})} = T_{\mu\nu} + T_{\mu\nu}^{(\text{eff})}$ contains, apart from a possible real matter energy-momentum tensor $T_{\mu\nu}$ (which for us is zero since the mass is just an integration constant), an effective energy-momentum tensor $T_{\mu\nu}^{(\text{eff})}$ of gravitational origin which takes into account the quantum corrections (for an interpretation of such a $T_{\mu\nu}^{(\text{eff})}$ in terms of fluid variables see [27]). Since (6.1)

expresses the quantum corrections of the classical Schwarzschild metric, the tensor $T_{\mu\nu}^{(\text{eff})}$ appears as the correction beyond the Einstein equations of motion and not beyond some other modified classical equations of motion. To find this $T_{\mu\nu}^{(\text{eff})}$, we need to compute the Einstein tensor $G_{\mu\nu}$ of the metric (6.1). This could lead to a nontrivial situation, where the Israel matching conditions are satisfied or not, depending on the form of this effective energy-momentum tensor. However, for the whole analysis of the present paper, just the Israel conditions arise. Indeed, the Einstein tensor G_ν^μ which is constructed from the metric (6.1) has the following nonvanishing components:

$$G_T^T = G_R^R = \frac{1}{R^2}(RF' + F - 1),$$
$$G_\theta^\theta = G_\varphi^\varphi = \frac{1}{2R}(RF'' + 2F'). \tag{6.2}$$

Since the Schwarzschild metric with $F_{\text{Sch}} = 1 - \frac{2G_NM}{R}$ satisfies the vacuum Einstein equations, we get

$$G_T^T = G_R^R = \frac{1}{R^2}(R\mathcal{Q}' + \mathcal{Q}), G_\theta^\theta = G_\varphi^\varphi = \frac{1}{2R}(R\mathcal{Q}'' + 2\mathcal{Q}'), \tag{6.3}$$

where the quantity

$$\mathcal{Q} = \frac{2(G_N - G_k)M}{R} - \frac{1}{3}\Lambda_k R^2 \tag{6.4}$$

is defined by $\mathcal{Q} \equiv F - (1 - \frac{2G_NM}{R})$; i.e., it is the deviation of the metric component F from $1 - \frac{2G_NM}{R}$. The quantity $\mathcal{Q}$ will be seen that is well defined, depending on the assumptions of the quantum theory. Therefore, in the interior regime with the metric (6.1), the Einstein equations $G_\nu^\mu = 8\pi G_N T_\nu^{\mu(\text{eff})}$ acquire a well-defined $T_\nu^{\mu(\text{eff})}$, which is given by the right-hand sides of (6.3) and parametrized by the quantity $\mathcal{Q}$,

$$T_T^{T(\text{eff})} = T_R^{R(\text{eff})} = \frac{1}{8\pi G_N R^2}(R\mathcal{Q}' + \mathcal{Q}),$$
$$T_\theta^{\theta(\text{eff})} = T_\varphi^{\varphi(\text{eff})} = \frac{1}{16\pi G_N R}(R\mathcal{Q}'' + 2\mathcal{Q}'). \tag{6.5}$$

Moreover, equating the right-hand sides of the expressions (6.2), (6.3), we find the ordinary differential equations of F,

$$RF' + F - 1 = R\mathcal{Q}' + \mathcal{Q}, \qquad RF'' + 2F' = R\mathcal{Q}'' + 2\mathcal{Q}'. \tag{6.6}$$

Differentiating the first equation of (6.6), we get the second equation. We are not particularly interested in the precise function $F(R)$ since for cosmology the most important is

only the value of F at the Schucking radius. We discern now the following cases.

If G_k, Λ_k are functions of R in an explicit algebraic form (as happens in the subsection VII B 1), then the metric component F, the quantity $\mathcal{Q}$ of (6.4) and $T_\nu^{\mu(\text{eff})}$ of (6.5) become also such functions of R. The second (as well as the first) derivatives of the metric component F in the Einstein equations (6.6) are contained only on their left-hand sides, which come from G_ν^μ, while the $\mathcal{Q}$ terms on the right-hand sides, which come from $T_\nu^{\mu(\text{eff})}$, act as a sort of given potentials that modify $F(R)$ from the Schwarzschild metric. Although, this picture is rather trivial due to that the evaluation is performed on-shell, on the explicit metric component $F(R)$; however, it is still meaningful since the function $F(R)$ still satisfies the differential equations (6.6). As is well known, the matching conditions are extracted from the Einstein equations $G_\nu^\mu = 8\pi G_N T_\nu^{\mu(\text{eff})}$ focusing only to the second derivatives of the metric components. Since these derivatives are only inside the Einstein tensor G_ν^μ and not inside $T_\nu^{\mu(\text{eff})}$, the Israel matching conditions naturally arise in this case.

In the subsections VII A and VII B 2, we will discuss another situation, which is nontrivial in the sense that F is not a known function of R. At the same time, this situation is actually more promising in its cosmological results and also more favorable theoretically. Here, Λ_k is not an explicit function of R, but it contains an integral of $F(R)$. Since $F(R)$ is not known, but is the metric component to be found by solving (6.6), this integral cannot be performed. This means that the quantity $\mathcal{Q}$ contains an independent geometrical field $D(R)$ with its own equation of motion. The important point is that this equation of motion is only of first order; i.e., it contains D' and not D'', and therefore, there is no discontinuity of D' at the matching surface. As a result, $T_\nu^{\mu(\text{eff})}$ in (6.5) becomes a function of R, D, F, F', while no F'' is present in $T_\nu^{\mu(\text{eff})}$. Therefore, again F'' is only contained in the tensor G_ν^μ of the Einstein equations $G_\nu^\mu = 8\pi G_N T_\nu^{\mu(\text{eff})}$ and the Israel conditions remain intact. We will provide more precise explanations about this issue at the appropriate point later. The metric component $F(R)$ can be found by solving for F, D the coupled system of the first equation in (6.6), i.e., equation $RF' + F - 1 = R\mathcal{Q}'(R, D, F) + \mathcal{Q}(R, D)$, together with the first-order differential equation for D.

Finally, if we assume, merely as a mathematical extension of the above, the more complicated situation where G_k, Λ_k contain double integrations of F, F', this implies that the equation of motion of the D field is of second order. Then, a discontinuity of D' could be present at the matching surface and the Israel matching conditions might be modified. We only mention such a possibility, without regarding it in our analysis, to show how the Israel conditions could be violated in extreme and artificial situations, where no clear physical motivation justifies such constructions.

To summarize, we have proved that the Israel matching conditions are still applicable for the metric (6.1) in the context of our analysis. Therefore, in the Swiss cheese approach, Eqs. (2.21) and (2.23) can be used to derive the cosmological evolution.

If G_k has the constant observed value G_N and the underlying theory provides only ultraviolet (UV) corrections to Λ, it is not possible to get cosmic acceleration since Λ is suppressed at large distances and $\Lambda(R_S)$ almost vanishes. Therefore, infrared corrections of Λ are necessary. For example, a well-known phenomenological description of a quantum corrected nonsingular black hole is provided by the Hayward metric [15] where

$$
\begin{aligned}
F(R) &= 1 - \frac{2G_N M R^2}{R^3 + 2G_N M L^2} \\
&= 1 - \frac{2G_N M}{R} - 2G_N M \left(\frac{1}{R^3 + 2G_N M L^2} - \frac{1}{R^3} \right) R^2.
\end{aligned}
\tag{6.7}
$$

The length scale L controls the ultraviolet correction close to the origin and smoothes out the singularity. In this metric the effective cosmological constant becomes a function of R, namely $\Lambda(R) = 6G_N M (\frac{1}{R^3 + 2G_N M L^2} - \frac{1}{R^3})$. At distances where the Newtonian potential is very weak and $R \gtrsim L$, it arises that the potential of this cosmological "constant" is negligible compared to the Newtonian potential and its corresponding weak-field force (which is repulsive) is well suppressed compared to the Newtonian force. Equation (2.23) for the acceleration gives

$$
\frac{\ddot{a}}{a} = G_N M \frac{4G_N M L^2 - r_\Sigma^3 a^3}{(2G_N M L^2 + r_\Sigma^3 a^3)^2}.
\tag{6.8}
$$

This metric does not provide a recent cosmic acceleration since at late times deceleration emerges. Similarly, the solution presented in [14] fails for the same reason.

VII. THE CASE OF ASYMPTOTICALLY SAFE GRAVITY

A concrete realization of the functions G_k, Λ_k is provided by the asymptotically safe scenario of quantum gravity. In Appendix A, we present a few basic elements of the theory and of its application in cosmology. According to the AS program, both Newton's constant G_k and cosmological constant Λ_k are energy dependent, $G_k = G(k) = g_k k^{-2}$, $\Lambda_k = \Lambda(k) = \lambda_k k^2$, where k is an energy measure of the system and g_k, λ_k are the dimensionless running couplings governed by some RG flow equations. The exact RG flow of the couplings from the Planck regime down to the present epoch is not yet known. So, it is not clear what is the real trajectory in the space g_k, λ_k followed by the Universe and how the classical general relativity

regime with a constant G_N and negligible Λ can be obtained. However, even if it was possible to predict the flow of the decrease of Λ_k all the way down to the current cold cosmic energy scale and a very small value of Λ could be realized, the coincidence problem, having to do with the precise order of magnitude of Λ, would still be manifest and profound. Furthermore, in the picture with a constant G and a Λ_k monotonically decreasing with time, a recent passage from a deceleration to an acceleration phase would not be possible. The far infrared limit of g_k, λ_k certainly covers the late-times cosmological scales. However, even if a late-times behavior with an increasing G does exists, it will describe the future and not the present Universe which possesses rather an antiscreening instead of a screening behavior. Additionally, there is the possibility that the far infrared corrections of the cosmological constant at cosmic scales are too small to affect the present Universe evolution and drive into acceleration.

Anyway, the cosmic scale corrections of G, Λ are not of interest in our approach. Our interest is focused on the intermediate infrared corrections occurring at the astrophysical structures scales. The hope is that these intermediate scale quantum corrections will be significant enough to have direct influence on the current cosmology and on the observed dark energy component, but at the same time they will not conflict in an obvious way with the local dynamics. Indeed, we will show until the end of the paper that the recent cosmic acceleration can be the result of such quantum corrections of the cosmological constant at the galactic or cluster of galaxies scale.

As before, the Universe will be described by the Swiss cheese model and the matching will take place on the surface between a cosmological metric and a quantum modified spherically symmetric metric. Several interesting approaches of RG improved black hole metrics appear in literature. In our analysis, we will work with the quantum improved Schwarzschild–de Sitter metric (6.1), as e.g., presented in [28]. Of course, the precise form of the metric will arise from the forms adopted for G_k, Λ_k, as these are predicted or motivated by AS. A different treatment would be to substitute the functions G_k, Λ_k inside some consistent quantum corrected gravitational equations of motion or inside some consistent action, and derive first the quantum corrected spherically symmetric solution. As mentioned, here we will follow for simplicity the method of obtaining a quantum corrected metric by starting from a classical solution and promoting G_N, Λ to energy-dependent quantities according to the AS program.

In order to proceed further, the energy measure k has to be connected with a length scale L, i.e., $k = \xi/L$, where ξ is a dimensionless parameter which is expected to be of order one. As we approach the center of spherical symmetry, the mean energy increases and k is a measure of the energy scale that is encoded in renormalization group approaches to quantum gravity. A simple option is to set as L the

coordinate distance R of the spherically symmetric metric. Then, the value of the cosmological constant Λ_k, which provides a value for the local vacuum energy density, depends explicitly on the distance from the center, $\Lambda_k = \Lambda(R)$. The same is true for $G_k = G(R)$, and so, the metric component F of the metric (6.1) becomes an explicit function of R, $F = F(R)$. Let us remind that the function $F(R)$ is precise when the mass M is located at the origin. In our approach, where we want to describe real astrophysical objects with a mass profile, $F(R)$ in the interior should be treated with caution.

A more natural option is to set as L the proper distance $D > 0$ [29]. This case is more involved physically and technically. Following a radial curve defined by $dT = d\theta = d\varphi = 0$ to reach a point with coordinate R, we have

$$D(R) = \int_{R_1}^{R} \frac{d\mathcal{R}}{\sqrt{F(\mathcal{R})}}. \tag{7.1}$$

This is a formal expression until one realizes what is its meaning and the meaning of R_1. Now, the function F is not an explicit function of R since D is also contained in F by construction, so $F(\mathcal{R})$ in (7.1) basically means $F(\mathcal{R}, D(\mathcal{R}))$. So, (7.1) is not a simple integral but it is an integral equation which can be converted to the more useful differential equation

$$D'(R) = \frac{1}{\sqrt{F(R)}}. \tag{7.2}$$

This is a well-defined, but complicated, differential equation for D since F contains again $D(R)$ (or in different words, F is given by (6.6)). Integration of Eq. (7.2) will provide an integration constant σ, so the solution of (7.2) is $D(R; \sigma)$. Plugging this D in F will provide $F(R; \sigma)$. For a specific σ, a value $R_1(\sigma)$ should exist that makes Eq. (7.1) meaningful for $R > R_1$. The positiveness of D may also provide some restrictions on R. Even if a minimum horizon distance R_H exists where $F(R_H; \sigma) = 0$, R_1 does not necessarily coincide with R_H, since after $D(R; \sigma)$ has been found, it is possible that the arising function $1/\sqrt{F(R; \sigma)}$ is nonintegrable around R_H. So, the variable $D(R)$ is a proper distance, but not in the conventional sense of an integration in a prefixed background. It can rather be considered as a new dynamical field of geometrical nature with its own equation of motion (7.2), where the spacetime metric is determined through $D(R; \sigma)$. The role of the function $D(R)$ will be crucial in our analysis. The integration constant σ could be determined by some assumption, for example if R_H exists, it could be set $D_H = 0$ or $D_H = R_H$. In our case of Swiss cheese models, σ will be determined from the demand of having the correct amount of dark energy today. However, the interesting thing, especially in relation to the coincidence problem, is that the corresponding $D(R)$ will have throughout natural values of the order of the length of

the astrophysical object, while at the same time it will provide the correct estimate of dark energy.

Let us now complete the discussion started in Sec. VI about the validity of the Israel matching conditions when the choice (7.1) is made. The cosmological constant becomes a function of D; i.e., it is $\Lambda_k = \Lambda_k(D)$. If $G_k = G_N$ (as we will assume in our analysis), the quantity $\mathcal{Q}$ in (6.4) becomes $\mathcal{Q} = -\frac{1}{3}\Lambda_k R^2$, so it is a function of R, D, i.e., $\mathcal{Q} = \mathcal{Q}(R, D)$. Making use of (7.2), we can easily compute $\mathcal{Q}' = -\frac{2}{3}\Lambda_k R + \frac{\xi R^2}{3D^2\sqrt{F}}\frac{\partial\Lambda_k}{\partial k} = \mathcal{Q}'(R, D, F)$, $\mathcal{Q}'' = -\frac{2}{3}\Lambda_k + \frac{\xi R}{6D^3 F^{3/2}}(8DF - 4R\sqrt{F} - RDF')\frac{\partial\Lambda_k}{\partial k} - \frac{\xi^2 R^2}{3D^4 F}\frac{\partial^2\Lambda_k}{\partial k^2} = \mathcal{Q}''(R, D, F, F')$. It is now clear, as also mentioned above, that the right-hand sides of the Einstein equations (6.6), i.e., the components of $T_\nu^{\mu(\text{eff})}$, are only functions of R, D, F, F' and not of F''. Since F'' is only contained in G_ν^μ on the left-hand sides of (6.6), and the evolution of D is governed by the first-order differential equation (7.2), the Israel matching conditions arise.

In our Swiss cheese approach of cosmology, the matching between the interior and the exterior metric occurs at the Schucking radius which only enters the cosmological evolution. The front value k_S at the Schucking radius is inversely proportional to a characteristic length of the metric. For the choice $L = R$, we get

$$k_S = \frac{\xi}{R_S}. \qquad (7.3)$$

For $L = D$, it is

$$k_S = \frac{\xi}{D_S}, \qquad (7.4)$$

where

$$D_S = \int_{R_1}^{R_S} \frac{dR}{\sqrt{F(R)}} \qquad (7.5)$$

is the proper distance of the Schucking radius. Therefore, the front values $\Lambda(R_S)$, $G(R_S)$ at the Schucking radius are the ones where the matching occurs and determine the cosmological evolution from (2.21), (2.11) as

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G(R_S)M}{r_\Sigma^3 a^3} + \frac{1}{3}\Lambda(R_S). \qquad (7.6)$$

We finish with a comment which doesn't have a special significance. The dependence of Λ, G on the distance inside the object does not seem to affect our cosmology. However, there is an indirect influence which affects the parameters. Indeed, the total cosmic energy of the cosmological portion that is excised from the Swiss cheese should be equal to the energy provided by the various masses inside the astrophysical object plus the vacuum energy due to the

cosmological constant. Since the vacuum energy of a cosmological constant is $\frac{\Lambda}{8\pi G}$, we have approximately the equation

$$\frac{4\pi}{3}R_S^3\rho_{\text{tot}} = M + \int_0^{R_S} \frac{\Lambda(R)}{8\pi G(R)} 4\pi R^2 dR, \qquad (7.7)$$

where $\rho_{\text{tot}} = \rho + \rho_{\text{DE}}$ is the total cosmic energy density (dark matter plus dark energy) [30]. Equation (7.7) evaluated at today's values, according to (2.11), sets a restriction between the various parameters. Unfortunately, however, we cannot make use of this equation since we do not know the precise functions $\Lambda(k)$, $G(k)$ from the UV with $k = \infty$ up to $k_S \sim R_S^{-1}$. The difficulty is rather basically due to $\Lambda(k)$ since $G(k)$ rapidly evolves to its constant value G_N. So, we will have one more parameter left free in our analysis.

A. First RG flow behavior: Close to the Gaussian fixed point

There is a fixed point of the RG flow equations, the Gaussian fixed point (GFP) [31], which is saddle and is located at $g = \lambda = 0$. A appropriate class of trajectories in the Einstein-Hilbert truncation of the RG flow can be linearized about the GFP, where the dimensionless couplings are pretty small. These trajectories possess interesting qualitative properties such as a long classical regime (long $\ln k$ "time" due to the vanishing of beta functions) and a small positive cosmological constant in the infrared, features that seem relevant to the description of gravitational phenomena in the real Universe. The analysis is fairly clear and in the vicinity of the GFP it arises that Λ has a running k^4 and G has an approximately constant value which is interpreted as G_N. Therefore,

$$G_k = G_N, \qquad \Lambda_k = \alpha + \beta k^4, \qquad (7.8)$$

where α, β are positive constants. Moreover it is $\beta = \nu G_N$, where $\nu = \mathcal{O}(1)$. In terms of the dimensionless couplings it is $\lambda_k = \alpha k^{-2} + \beta k^2$, $g_k = G_N k^2$. These equations are valid if $\lambda_k \ll 1$, $g_k \ll 1$. While the above segment which lies inside the linear regime of the GFP can be continued with the flow equation into the UV fixed point, this approximation breaks down in the IR where λ_k approaches the value $1/2$. Therefore, as our first choice, we will assume that within a certain range of k-values encountered in an astrophysical object, the RG trajectory is approximated by (7.8).

For the choice (7.4), Eq. (7.6) provides the cosmic evolution of the scale factor a as

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \frac{\alpha}{3} + \frac{\beta\xi^4}{3D_S^4}. \qquad (7.9)$$

The choice (7.3) is not interesting, since the last term in Eq. (7.9) would be a radiation term a^{-4}, and (7.9) would

lead to ΛCDM model with a radiation term. From (7.5), it can be easily found that the time evolution of D_S is given by the equation

$$\dot{D}_S = \frac{r_\Sigma a H}{\sqrt{1 - \frac{2G_N M}{r_\Sigma a} - \frac{\alpha}{3} r_\Sigma^2 a^2 - \frac{\beta \xi^4 r_\Sigma^2 a^2}{3 D_S^4}}}. \tag{7.10}$$

Equations (7.9) and (7.10) form a system of two coupled differential equations for a, D_S. We can bring this system in a more standard form defining

$$\chi = \frac{\alpha}{3} + \frac{\beta \xi^4}{3 D_S^4}, \tag{7.11}$$

and then

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \chi \tag{7.12}$$

$$\dot{\chi} = -\frac{4 \cdot 3^{\frac{1}{4}} r_\Sigma a H (\chi - \frac{\alpha}{3})^{\frac{5}{4}}}{\xi \beta^{\frac{1}{4}} \sqrt{1 - \frac{2G_N M}{r_\Sigma a} - r_\Sigma^2 a^2 \chi}}. \tag{7.13}$$

Note, from (7.11), that $\chi - \frac{\alpha}{3} > 0$. From (7.12), it is seen that the quantity χ plays the role of dark energy. Namely, it is

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3} (\rho + \rho_{\text{DE}}), \tag{7.14}$$

where ρ is given by (3.4) and $\rho_{\text{DE}} = \frac{3}{8\pi G_N} \chi$.

Note in passing that Eq. (7.9), combined with Eq. (7.14), gives

$$\frac{4\pi}{3} R_S^3 \rho_{\text{tot}} = M + \frac{\Lambda(R_S) R_S^3}{6 G_N}. \tag{7.15}$$

Comparing this equation with (7.7), it arises that the total energy due to the cosmological constant is given by two expressions, first by the integral in (7.7) and second by the last term in (7.15). There is no contradiction with that, since the equality of these two expressions at the current values simply provides the additional constraint on the parameters mentioned above.

The density parameters are defined in the standard way:

$$\Omega_m = \frac{8\pi G_N \rho}{3 H^2}, \qquad \Omega_{\text{DE}} = \frac{8\pi G_N \rho_{\text{DE}}}{3 H^2}. \tag{7.16}$$

From the first of Eq. (7.16), using the current values of the variables, a relation between the parameters M and r_Σ can be found

$$r_\Sigma = \left(\frac{2G_N M}{\Omega_{m0} a_0^3 H_0^2} \right)^{\frac{1}{3}}. \tag{7.17}$$

For the typical masses we use, it was found above that $r_\Sigma = 0.83$ Mpc for a galaxy, and $r_\Sigma = 18$ Mpc for a cluster of galaxies.

It is more convenient to work with the redshift $z = \frac{a_0}{a} - 1$, where the current value a_0 of the scale factor can be set to unity. From (7.13), (7.17), the evolution of $\chi(z)$ is found from

$$\frac{d\chi}{dz} = \frac{4 \cdot 3^{\frac{1}{4}} (\chi - \frac{\alpha}{3})^{\frac{5}{4}}}{\xi \beta^{\frac{1}{4}} (1+z)^2 \sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0} H_0^2 (1+z) - \frac{\chi}{(1+z)^2}}}. \tag{7.18}$$

After having solved (7.18), the evolution of the Hubble parameter as a function of z is given by the expression

$$H^2 = \Omega_{m0} H_0^2 (1+z)^3 + \chi - \frac{\kappa}{a_0^2} (1+z)^2. \tag{7.19}$$

Using (7.14), (7.17), the quantity Ω_m can also be found as a function of the redshift

$$\Omega_m = \left[1 + \frac{1}{\Omega_{m0} H_0^2} \frac{\chi}{(1+z)^3} - \frac{\kappa}{a_0^2 \Omega_{m0} H_0^2} \frac{1}{1+z} \right]^{-1}. \tag{7.20}$$

For the numerical investigation of the system, we will need the current value χ_0 of χ. From (7.12) or (7.19), we find

$$\chi_0 = \Omega_{\text{DE},0} H_0^2. \tag{7.21}$$

The differential equation (7.18) contains the parameters $\xi, \beta = \nu G_N, r_\Sigma, \alpha$. The parameters ξ and ν are of order unity. The parameter r_Σ was found from (7.17). Finally, the parameter α is free in order to try to achieve the correct phenomenology. With these parameters and χ_0 given in (7.21), we can solve numerically (7.18) and find $\chi(z)$. Then we can plot $\Omega_m(z)$ from (7.20).

From (7.12), (7.13) one can obtain a Raychaudhuri type of equation. Differentiating (7.12), and using (7.13) and (7.12) itself, we get

$$\frac{\ddot{a}}{a} = -\frac{1}{2} \Omega_{m0} H_0^2 (1+z)^3$$
$$+ \chi - \frac{2 \cdot 3^{\frac{1}{4}} (\chi - \frac{\alpha}{3})^{\frac{5}{4}}}{\xi \beta^{\frac{1}{4}} (1+z) \sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0} H_0^2 (1+z) - \frac{\chi}{(1+z)^2}}}. \tag{7.22}$$

The same equation also arises from (2.23). The deceleration parameter is given by $q = -H^{-2} \frac{\ddot{a}}{a}$. From (7.22), for $\kappa = 0$, and using (7.21), the condition for acceleration today, $\ddot{a}|_0 > 0$, is written as

$$\frac{1}{r_\Sigma^2 a_0^2 H_0^2} > \frac{4\cdot\sqrt{3}}{\xi^2\sqrt{\nu}}\frac{(1-\Omega_{m0}-\frac{\alpha}{3H_0^2})^{\frac{5}{2}}}{(1-\frac{3}{2}\Omega_{m0})^2}\frac{1}{H_0\sqrt{G_N}}+1, \quad (7.23)$$

which is approximated by

$$0 < 1-\Omega_{m0}-\frac{\alpha}{3H_0^2} < \left(\frac{\xi^2\sqrt{\nu}}{4\sqrt{3}}\right)^{\frac{2}{5}}\left(1-\frac{3}{2}\Omega_{m0}\right)^{\frac{4}{5}}\left(\frac{H_0\sqrt{G_N}}{r_\Sigma^2 a_0^2 H_0^2}\right)^{\frac{2}{5}}. \quad (7.24)$$

It is found from (7.24) that $1-\Omega_{m0}-\frac{\alpha}{3H_0^2} \lesssim 10^{-22}$, from where it is obvious that $\alpha > 0$. For such values of α, the conditions $\lambda_k \ll 1$, $g_k \ll 1$ are seen from (7.11), (7.24) that are easily satisfied. Of course, such values form an extreme fine tuning for α since α has to be extremely close to the value $3(1-\Omega_{m0})H_0^2$, which is also the value of the cosmological constant. Additionally, for such values of α it can be seen that $\dot{\Omega}_m|_0 < 0$, which assures a local increase of Ω_m in the past. There is also the option that $1-\Omega_{m0}-\frac{\alpha}{3H_0^2}$ is of order unity, and then from (7.24), it turns out that $\nu > 10^{106}$; however, such values are not predicted by the quantum theory.

Finally, since the pressure of the matter component vanishes, $p = 0$, Eq. (7.22) can be brought into the form

$$2\dot{H} + 3H^2 + \frac{\kappa}{a^2} = -8\pi G_N(p+p_{DE}), \quad (7.25)$$

where the dark energy pressure is given by

$$-8\pi G_N p_{DE}$$
$$= 3\chi - \frac{4\cdot 3^{\frac{1}{4}}(\chi-\frac{\alpha}{3})^{\frac{5}{4}}}{\xi\beta^{\frac{1}{4}}(1+z)\sqrt{\frac{1}{r_\Sigma^2 a_0^2}-\Omega_{m0}H_0^2(1+z)-\frac{\chi}{(1+z)^2}}}. \quad (7.26)$$

A significant parameter for the study of the late-times cosmology is the equation-of-state parameter for the effective dark energy sector $w_{DE} = \frac{p_{DE}}{\rho_{DE}}$, which is found from (7.26) to be

$$w_{DE} = -1 + \frac{4\cdot 3^{-\frac{3}{4}}(\chi-\frac{\alpha}{3})^{\frac{5}{4}}}{\xi\beta^{\frac{1}{4}}(1+z)\chi\sqrt{\frac{1}{r_\Sigma^2 a_0^2}-\Omega_{m0}H_0^2(1+z)-\frac{\chi}{(1+z)^2}}}. \quad (7.27)$$

Therefore, w_{DE} cannot take phantom values smaller than -1. Its current value becomes for $\kappa = 0$

$$w_{DE,0} \approx -1 + \frac{4\cdot 3^{-\frac{3}{4}}}{\xi\nu^{\frac{1}{4}}(1-\Omega_{m0})}\left(1-\Omega_{m0}-\frac{\alpha}{3H_0^2}\right)^{\frac{5}{4}}\left(\frac{r_\Sigma^2 a_0^2 H_0^2}{H_0\sqrt{G_N}}\right)^{\frac{1}{2}}. \quad (7.28)$$

and, according to (7.24), it can take any value $-1 < w_{DE,0} < -1 + (\frac{2}{3}-\Omega_{m0})(1-\Omega_{m0})^{-1} \approx -0.48$.

Because of the above extreme fine-tuning in α, numerical study of the equations is not possible and we need to perform an analytical study in order to understand the behavior of the system. This analysis is shown in the Appendix B for $\kappa = 0$. We summarize here the results of this analysis. Concerning the evolution of the Hubble parameter and the parameter Ω_m, the ΛCDM behavior arises practically in all the regime of applicability of the model all the way down to the present epoch, with an exception in the very first steps of the AS evolution. As for the deceleration parameter and the dark energy equation-of-state parameter, there are intervals where these parameters have a behavior different from that of ΛCDM. This discrepancy between the behavior of the parameters $H(z)$, $\Omega_m(z)$ and the behavior of the parameters $\ddot{a}(z)$, $w_{DE}(z)$ is peculiar and is due to the presence of the higher time derivatives contained in the acceleration, which can lead to significant contribution from terms which are negligible in H, Ω_m. However, for the numerical values of the various constants of astrophysical and cosmological origin describing our model, the final result is that in all physically interesting cases the acceleration properties of the model cannot be practically discerned from the ΛCDM scenario; therefore, our model is indistinguishable from ΛCDM. There is a possibility, as always in cosmology, that the perturbations of the model evolve in a distinct way from ΛCDM; however, the previous analysis of the background, with the particular values encountered, leaves little hope for observational evidence of the deviations from ΛCDM.

B. Second RG flow behavior: Close to the infrared

There are encouraging indications that for $k \to 0$ the cosmological constant runs proportional to k^2, so $\Lambda_k = \lambda_*^{IR} k^2$, where $\lambda_*^{IR} > 0$ is the infrared fixed point of the λ-evolution. At the same time, it seems that G_k increases a lot, for example g_k could converge to an IR value $g_*^{IR} > 0$ or even diverge. This postulated fixed point [32] can be considered as the IR counterpart of the UV non-Gaussian fixed point (NGFP) [33]. This nontrivial IR running can be assumed that is due to quantum fluctuations with very small momenta, corresponding to distances larger than the largest localized structures in the Universe. Since we are interested in generating the dark energy at astrophysical scales, the exact realization of the above IR fixed point at cosmological scales is not our purpose. There are works, not concerning cosmology, which consider the possibility that such a fixed point can act already at astrophysical scales [34,35]. Our approach will be different. We will consider, as it is pretty reasonable, that the above IR fixed point has not yet been reached at the intermediate astrophysical scales, and therefore, some deviations from the above functions G_k, Λ_k should be present inside the objects of our interest.

Concerning the gravitational constant G_k, since at moderate scales, well beyond the NGFP, G_k is approximately constant, we will assume that it has the constant value G_N over a broad range of scales ranging from the submillimeter up to the galaxy or cluster scale. Actually, we will not make use of the submillimeter lower bound, so we can just restrict ourselves above some observable macroscopic distances. There is the possibility that at large astrophysical scales, G acquires an IR correction beyond G_N; however, we will not be concerned about this in our introductory treatment here, since we do not want to assume arbitrary functional forms. Already the relatively successful explanation of the galactic or cluster dynamics using the standard Newton's law makes our adoption $G_k = G_N$ legal enough.

We will be restricted on the single effect of the variability of the cosmological constant Λ_k. Since the functional form of the deviation of Λ_k from its IR form k^2 is not known, we will assume that the running of Λ_k is described by a power law dependence on the energy, $\Lambda_k \sim k^b$. Of course, this is a simple *ad hoc* parametrization, and the analysis will show how far one can go with the single parameter b. However, if Λ_k differs slightly from its IR form k^2, what is actually not unexpected since astrophysical structures are already "large" enough, then b will be close to the value 2 and the power-law dependence k^b will be a very good approximation of the running behavior. Moreover, moving with the deformed law k^b from the IR cosmological law k^2 down to the astrophysical scales, it seems more probable for b to be slightly larger than 2, instead of smaller than 2. This is due to that since k increases at smaller length scales, the parameter b should also increase in order to have a significant astrophysical Λ_k; otherwise, k^b would be neutralized and Λ_k would be suppressed.

We summarize by writing our ansatz,

$$G_k = G_N, \qquad \Lambda_k = \gamma k^b, \qquad (7.29)$$

where $\gamma > 0, b$ are constants. The parameter γ has dimensions mass to the power $2 - b$ and will be parametrized as $\gamma = \tilde{\gamma} G_N^{\frac{b}{2}-1}$ with $\tilde{\gamma}$ dimensionless. If an order $\mathcal{O}(1)$ value for $\tilde{\gamma}$ arises, this will mean that no new mass scale is needed (no new physics is needed for the explanation of dark energy other than AS gravity and the knowledge of structure) and the coincidence problem might be resolved from already controllable physics without fine-tuning or new scales. Indeed, it will be shown that in the most faithful case, discussed in the last subsection VII B 2, it turns out that b is a little higher than 2 and $\tilde{\gamma} = \mathcal{O}(1)$, which means that our model with the dark energy originated from IR quantum corrections of the cosmological constant at the astrophysical level can be pretty successful. With the assumption (7.29), the metric (6.1) contains the Newtonian term $\frac{2G_N M}{R}$ and the nontrivial cosmological constant term $\frac{1}{3}\Lambda_k R^2$ which becomes $\frac{\xi^b \tilde{\gamma}}{3}(\frac{R}{L})^b(\frac{\sqrt{G_N}}{R})^{b-2}$.

For $L = R$ we will see that current acceleration requires that $b < 1.57$ (actually b should be smaller than 1 in order to have a reasonable w_{DE} today); therefore, we cannot be in the most interesting IR range of b a little larger than 2 and the law k^b loses its theoretical significance. Then, it will arise that $\tilde{\gamma}$ has to be various decades of orders of magnitude smaller than unity in order to arrange the amount of dark energy, which means that new scales are introduced through $\tilde{\gamma}$. All this situation, which will be examined in the next subsection VII B 1, although not an obvious fail, certainly it cannot be considered as a big success in relation to the coincidence problem. If such values are acceptable, then the internal dynamics presents some measurable deviations from the standard Newtonian dynamics which need more investigation to check their consistency with observations. As for the force corresponding to the new cosmological constant term, this is $\frac{\tilde{\gamma}\xi^b(2-b)}{6\sqrt{G_N}}(\frac{\sqrt{G_N}}{R})^{b-1}$, and therefore it is repulsive. The ratio of the new cosmological constant term to the Newtonian term turns out to be $\frac{\Omega_{DE,0}}{\Omega_{m0}}(\frac{R}{r_\Sigma})^{3-b}$, while the corresponding ratio of the forces gets an extra factor $b - 2$. Thus, for large clusters these ratios at the border are larger than 0.1 for any b and the ratios can reach up to 0.35 for the largest b. For small clusters the ratios are suppressed to less than one per cent. Finally, for galaxies the ratios are almost 0.2 for the largest b and decrease considerable for smaller b. Certainly, inside the structures, the matter profiles have to be taken into account to obtain accurate results.

What it will turn out to be the most exciting case is $L = D$, which will be examined in the last subsection VII B 2. In this case, the dark energy, along with its acceleration, will be explained with b a little larger than 2 and $\tilde{\gamma}$ some number of order unity. More precisely, the quantity D at the Schucking radius today, $D_{S,0}$, which can be considered as the integration constant of the differential equation for $D(R)$, has to be arranged to provide the correct amount of dark energy. This implies that $D_{S,0}$ is of order of r_Σ, and more generally it will be shown numerically that the whole function $D(R)$ turns out to be of order of r_Σ. This is an extra naturalness for our model, since D, which has some meaning of proper distance, is of the length of the astrophysical object, and $D_{S,0}$ does not introduce a new scale. Therefore, the new cosmological constant term becomes $\frac{1}{3}\Lambda_k R^2 = \frac{\xi^b \tilde{\gamma}}{3}[\frac{1}{G_N}(\frac{\sqrt{G_N}}{r_\Sigma})^b](\frac{r_\Sigma}{D})^b R^2$, where we can set $D \sim r_\Sigma$ in order to make an estimate. Of course, the deviations of the new cosmological constant term from the R^2 law and the precise functional form of $D(R)$ are very important and will provide a cosmology distinctive from ΛCDM. However, we already observe that the quantity $\frac{1}{G_N}(\frac{\sqrt{G_N}}{r_\Sigma})^b$ has dimensions of cosmological constant, and for b close to 2.1 it is very close to the order of magnitude of the standard cosmological constant $\Lambda = 4.7 \times 10^{-84}$ GeV^2 of ΛCDM. The factor $(\frac{r_\Sigma}{D})^b$ offers a small distance-dependent deformation of the constant, which however, as mentioned,

is important for the derived cosmology. Therefore, our model will provide a natural deformation of the ΛCDM model, without introducing arbitrary scales or fine-tuning. Similarly, the constant prefactor $\xi^b \tilde{\gamma}$ can contribute to 1 order of magnitude. As a result, beyond the general idea, motivated by the coincidence problem, of generating the dark energy from local antigravity sources, the most important thing to emerge from the present work is the introduction of the quantity

$$\frac{1}{G_N} \left(\frac{\sqrt{G_N}}{r_\Sigma} \right)^b, \qquad (7.30)$$

which arises in the context of infrared AS gravity and plays a role similar to the standard cosmological constant Λ. This quantity, with b a little larger (for galaxies) or a little smaller (for clusters) than 2.1, has the quite interesting property that it has the same order of magnitude as the standard Λ. For example, for galaxies with $b = 2.13$ the quantity (7.30) is 2.1×10^{-84} GeV^2, while for clusters with $b = 2.08$ it becomes 2.6×10^{-84} GeV^2. It remains to AS gravity to confirm or not if such values of b are predicted at the astrophysical scales. Therefore, in some loose rephrasing, it can be said that the standard cosmological constant is not any longer an arbitrary independent quantity, as in ΛCDM, but it is constructed out of G_N, r_Σ, b.

As a direct consequence of the above discussion for $L = D$, the new cosmological constant term itself, $\frac{1}{3}\Lambda_k R^2$, becomes at the matching surface $R = r_\Sigma$ of the order of $(\frac{\sqrt{G_N}}{r_\Sigma})^{b-2}$. For galaxies with $b = 2.13$ and typical mass $M = 10^{11} M_\odot$, this term is 4×10^{-8}, while the Newtonian term is 10^{-8}. For clusters with $b = 2.08$ and mass $M = 10^{15} M_\odot$, this term is 2×10^{-5}, while the Newtonian term is 5×10^{-6}. Therefore, the two gravitational potentials are comparable at the Schucking radius. We can see that approximately a change in b less than five per cent from the above values, gives a change in this cosmological constant term less than 1 order of magnitude. This shows clearly a sensitivity on the value of b, which however cannot be considered as fine-tuning or at least extreme fine-tuning. Additionally, the fact that b acquires smaller values at cluster scales relatively to galaxy scales is consistent with the IR value $b = 2$ at cosmological scales or beyond. At an extreme limit of the solar scale, a even larger value of b is expected. For a solar distance $R = 1$ A.U. the quantity $\sqrt{G_N}/R$ takes the value 10^{-46}, while the Newtonian potential of the sun at this distance is 10^{-8}. Already for $b = 2.25$ the quantity $(\frac{\sqrt{G_N}}{R})^{b-2}$, which gives an estimation of the new term, becomes 3 orders of magnitude smaller than the Newtonian potential and this discrepancy becomes huge for larger b. Therefore, it is reasonable to assume that the stringent solar-system, as well as the laboratory tests of gravity, remain unaffected by the new

term. Finally, as for the force of the new term, this is $\frac{\xi^b \tilde{\gamma}}{6}(2 - b\frac{R}{D\sqrt{F}})[\frac{1}{G_N}(\frac{\sqrt{G_N}}{r_\Sigma})^b](\frac{r_\Sigma}{D})^b R$, where $F(R) = 1 - \frac{2G_N M}{R} - \frac{1}{3}\Lambda_k R^2$. The study of the new potential and its force at the border and inside the object is more complicated due to the presence of $D(R)$, so in order to get relatively reliable results, we will need a more detailed treatment. A preliminary analysis is given in the last subsection, where the result is that there is no obvious inconsistency with the internal dynamics.

We continue with the implication in cosmology of the previous new cosmological constant term for $L = D$. While the Newtonian term is certainly responsible for the dust term $\Omega_{m0} H_0^2/a^3$ on the right-hand side of (7.6), the above running cosmological constant term is responsible for the dark energy term $\frac{\xi^b \tilde{\gamma}}{3} \frac{1}{G_N}(\frac{\sqrt{G_N}}{D_S})^b$. Therefore, $\Omega_{DE,0}$ is equal to $\frac{\xi^b \tilde{\gamma}}{3}(\frac{r_\Sigma}{D_{S,0}})^b \frac{1}{G_N H_0^2}(\frac{\sqrt{G_N}}{r_\Sigma})^b$. Since $D_{S,0} \sim r_\Sigma$ and $\tilde{\gamma}$ is taken to be of order one, it is implied from the discussion on the value of the quantity (7.30) that $\Omega_{DE,0} \sim 1$. This can be considered as a natural explanation of the coincidence problem. The hard coincidence of the value of the standard cosmological constant $\Lambda \sim H_0^2$ is exchanged to a mild coincidence of the index b close to 2.1, which happens to satisfy

$$\frac{1}{G_N} \left(\frac{\sqrt{G_N}}{r_\Sigma} \right)^b \sim H_0^2. \qquad (7.31)$$

Strictly speaking, it is the current value $\Lambda(R_{S,0})$ of the time-dependent function $\Lambda(R_S)$ that has a value of the order of H_0^2.

As discussed in Sec. IV, the form (7.29) for the running cosmological constant can be applied directly at conventional cosmologies without the Swiss cheese description to define a different cosmological evolution. In this cosmology, the dark energy term is $\frac{\xi^b \tilde{\gamma}}{3} \frac{1}{G_N}(\frac{\sqrt{G_N}}{L})^b$. The quantity L is a cosmological distance scale instead of an astrophysical one used in the previous paragraph and this creates a huge difference. It is a common and sensible approach to assume $L = H^{-1}$ in this case, and the current dark energy term becomes $\frac{\tilde{\gamma}\xi^b}{3}(\sqrt{G_N}H_0)^{b-2}H_0^2$. This quantity acquires the correct order of magnitude, H_0^2, if $|b - 2| < 0.01$, which means that b should be very close to the IR value 2. It is not that the two approaches, the one defined by (7.31) and the one discussed here, differ only quantitatively. They can also differ qualitatively, in the sense that one approach can be relevant for the explanation of dark energy and the other not. For example, if AS predicts for the index b at the current cosmological scales the value 2.04, this automatically means that the cosmological interpretation of (7.29) is irrelevant today, and the IR value of b approximately 2 will be useful for the future Universe evolution. In our work, we consider the values of b which imply the satisfaction of Eq. (7.31).

1. Energy-coordinate distance scale relation

We first choose the simpler case where the energy scale k is inversely proportional to the coordinate radius R; therefore, for the front value of k at the Schucking radius, we have the relation (7.3). From Eq. (7.6), the Hubble equation becomes

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \frac{\gamma \xi^b}{3 r_\Sigma^b a^b}. \tag{7.32}$$

This is a simple equation for the evolution of the scale factor, which is also written as

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}(\rho + \rho_{\mathrm{DE}}), \tag{7.33}$$

where ρ is given by (3.4) and $\rho_{\mathrm{DE}} = \frac{\gamma \xi^b}{8\pi G_N r_\Sigma^b a^b}$.

The density parameters are defined as in (7.16) and the value of r_Σ for a galaxy or a cluster is found from (7.17) as before. The evolution of the Hubble parameter as a function of z is

$$H^2 = \Omega_{m0} H_0^2 (1+z)^3 + \frac{\gamma \xi^b}{3 r_\Sigma^b a_0^b}(1+z)^b - \frac{\kappa}{a_0^2}(1+z)^2, \tag{7.34}$$

where a_0 is set to unity. Similarly, the matter density abundance Ω_m becomes

$$\Omega_m = \left[1 + \frac{\gamma \xi^b}{3 r_\Sigma^b a_0^b \Omega_{m0} H_0^2}\frac{1}{(1+z)^{3-b}} - \frac{\kappa}{a_0^2 \Omega_{m0} H_0^2}\frac{1}{1+z} \right]^{-1}. \tag{7.35}$$

These equations contain the parameters ξ (which is of order one), r_Σ (which is known from (7.17) for the typical masses we use), and γ, b. For $\kappa = 0$, in order to have Ω_m an increasing function of z in agreement with observations, it should be $b < 3$.

Now, from (7.35) we take the condition for the correct amount of current dark energy,

$$\xi^b \tilde{\gamma} = 3\Omega_{\mathrm{DE},0} G_N H_0^2 \left(\frac{r_\Sigma a_0}{\sqrt{G_N}} \right)^b. \tag{7.36}$$

Equation (7.36) is analogous to the relation $\Lambda = 3\Omega_{\Lambda 0}H_0^2$. As was also explained above, here it is the varying cosmological constant that creates the dark energy based on the parameters $\tilde{\gamma}$, b characterizing Λ_k, and the astrophysical scale r_Σ. For arbitrary values of b, the parameter $\tilde{\gamma}$ will be very large or very small, introducing in this way a new massive scale $\gamma = \tilde{\gamma} G_N^{\frac{b}{2}-1}$. The cosmological constant $\Lambda_k = \gamma k^b$ should be generated inside the structure due to quantum corrections, so the parameters $\tilde{\gamma}$, b should be given

by the AS theory at the astrophysical scales. Therefore, Eq. (7.36), as an equation between orders of magnitude, forms a coincidence similar to that of Λ (actually it can be considered as increased coincidence) among these two parameters $\tilde{\gamma}$, b, the astrophysical value r_Σ and the cosmological parameter H_0. What we find particularly interesting in relation to the coincidence problem is the situation with $\tilde{\gamma} \sim 1$, because then, no new scale is introduced for the explanation of dark energy, other than the astrophysical scale. In this favorite case of ours, the relation (7.31) is valid with the index b having values close to 2.1 as explained before, and the hard coincidence of (7.36) reduces and is just rendered to the mild coincidence of the value of b. Unfortunately, we will see below that such b are not allowed here due to acceleration reasons. We should note additionally that the situation with Eq. (7.36) is different than the picture where the dark energy is due to some extra field having its own equation of motion. In that case, the corresponding of Eq. (7.36) would form a coincidence relation of dark matter-dark energy where some integration constants of the extra field would be involved, while the parameters of the theory would remain free to accommodate some other observation. This will actually be the case of the next subsection.

The acceleration is found to be

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\Omega_{m0}H_0^2(1+z)^3 + \frac{(2-b)\gamma\xi^b}{6 r_\Sigma^b a_0^b}(1+z)^b. \tag{7.37}$$

The condition for current acceleration, $\ddot{a}|_0 > 0$, becomes

$$\frac{(2-b)\gamma\xi^b}{3 r_\Sigma^b a_0^b \Omega_{m0}H_0^2} > 1, \tag{7.38}$$

which implies that a necessary condition is $b < 2$. Equation (7.37) implies that we have the correct behavior with a past deceleration and a current acceleration. Due to (7.36), the inequality (7.38) takes the form

$$b < 2 - \frac{\Omega_{m0}}{\Omega_{\mathrm{DE},0}}, \tag{7.39}$$

which means that $b < 1.57$. Then, it arises from (7.36) that $\tilde{\gamma} \lesssim 10^{-30}$ (and even smaller for galaxies), so $\tilde{\gamma}$ is various decades of orders of magnitude smaller than unity, and finally, no alleviation is offered to the coincidence problem. We continue in the following with a little more investigation of the present case, but it has already become obvious that the new massive scale $\gamma = \tilde{\gamma} G_N^{\frac{b}{2}-1}$, very different than the one provided by $G_N^{\frac{b}{2}-1}$, is necessarily introduced. And this is due to the values of b implied by the subtle issue of acceleration. Note that, for the standard Λ, it is $\Lambda = 3.1 \times 10^{-122} G_N^{-1}$.
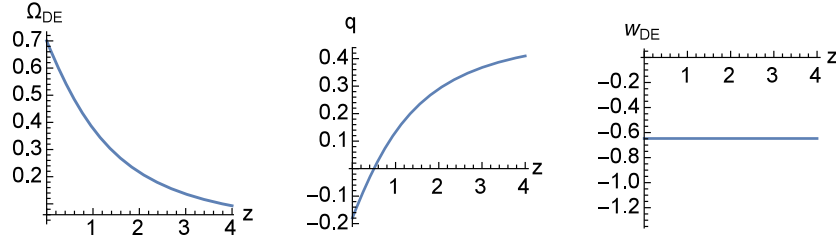
FIG. 1.   The late-times cosmological evolution for a spatially flat universe, for the parameter choice $b = 1.06$, $\xi = 1$, $\tilde{\gamma} = 2.8 \times 10^{-60}$, $M = 10^{15} \, M_{\odot}$ and $\Omega_{m0} = 0.3$. In the left graph we depict the evolution of the dark energy density parameter as a function of the redshift z. In the middle graph, we present the evolution of the deceleration parameter. Finally, the dark energy equation-of-state parameter is depicted in the right graph.

We have also to assure that the transition point from deceleration to acceleration is recent. Combining Eqs. (7.35) and (7.37), we find that this transition occurs at redshift z_t which satisfies the equation

$$\frac{(1 + z_t)^{3-b}}{2 - b} = \frac{1}{\Omega_{m0}} - 1 + \frac{\kappa}{a_0^2 \Omega_{m0} H_0^2}. \qquad (7.40)$$

So, z_t is basically determined from the parameter b (for example, for the value $z_t \approx 0.5$ it is $b \approx 1$). Ignoring κ, it is found from (7.40) that $z_t \lesssim 0.67$. The condition (7.39) is equivalent to $z_t > 0$. The deceleration parameter becomes today,

$$q_0 = \frac{1}{2}\Omega_{m0} - \frac{2 - b}{2}\Omega_{DE,0}. \qquad (7.41)$$

So, depending on b, it is $-0.55 < q_0 < 0$. The lowest value $q_0 = -0.55$ characterizes the ΛCDM model and here is attached to $b \to 0$ with $z_t = 0.67$. The upper value $q_0 = 0$ is associated to the maximum value $b = 1.57$ with $z_t \to 0$ (for the intermediate example with $b \approx 1$ it is $q_0 \approx -0.2$). If we are to insist to be close to the familiar value $q_0 = -0.55$, then b should be close to zero and the model is just a slight variation of the ΛCDM model, since there are no extra parameters to create some degeneracy. For such b close to zero, the dark energy term has a slow evolution instead of the constancy of the standard Λ and it is something like $\tilde{\gamma} \lesssim 10^{-100}$. We finish with writing the pressure of dark energy for a general b as

$$8\pi G_N p_{DE} = -\frac{(3 - b)\gamma \xi^b}{3 r_\Sigma^b a_0^b}(1 + z)^b, \qquad (7.42)$$

where the equation of state of dark energy is

$$w_{DE} = -1 + \frac{b}{3}, \qquad (7.43)$$

which is constant and is not compatible with a seemingly varying w_{DE}.

We present in Fig. 1 the cosmological evolution as a function of z for a spatially flat universe, with the quantum cosmological constant originated at the cluster level with $M = 10^{15} \, M_{\odot}$, $\Omega_{m0} = 0.3$, and for the parameter choice $b = 1.06$, $\xi = 1$, $\tilde{\gamma} = 2.8 \times 10^{-60}$, which provide $z_t = 0.5$. We depict in the left graph the evolution of the dark energy density parameter Ω_{DE} from Eq. (7.35), where it appears a typical decreasing behavior for larger redshifts. In the middle graph the evolution of the deceleration parameter q is shown, where a passage from deceleration to acceleration at late times can be seen. The dark energy equation-of-state parameter w_{DE} remains constant in the right graph. It is obvious that in order to have a $w_{DE,0}$ close to -1, according to observations, the value of the parameter b has to be close to zero, and thus, the model is finally very close to the ΛCDM model.

Although the present model is not taken seriously as it is not theoretically favorable in relation to the coincidence problem, and also it should be very close to the ΛCDM model (by choosing b close to zero) in order to have some compatibility with simple observational tests, we find interesting to leave it a little space to breath by noticing two points. First, the parameter b is not exactly constant, but more accurately, it is z dependent since at different cosmic epochs the running couplings move at different points of the AS renormalization group phase portrait. Therefore, in general, w_{DE} becomes time dependent. This point is also related to the specific model of structure growth one has to assume for a typical galaxy or cluster in order to make a more realistic implementation of the model. Since, during the collapsing phase of a structure (larger length scales), the index b is expected to be smaller than the current value of the formed object, the mean value of b is smaller, and thus the real w_{DE} is reduced. Second, in view of the inhomogeneous/anisotropic models discussed in the Introduction ([19–21]), the description of a inhomogeneous universe with more realistic structures would provide through averaging processes enhancement of cosmic acceleration, and therefore, w_{DE} would become even smaller. In any case, the scenario discussed here will be improved in various aspects in the next subsection where the energy scale will be determined from the length scale $L = D$.

2. Energy-proper distance scale relation

Here we use the proper distance as measure of the energy scale, which seems to be more realistic, so we assume the law (7.4). From Eq. (7.6), the Hubble evolution is given by

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \frac{\gamma \xi^b}{3 D_S^b}, \tag{7.44}$$

where

$$\dot{D}_S = \frac{r_\Sigma a H}{\sqrt{1 - \frac{2G_N M}{r_\Sigma a} - \frac{\gamma \xi^b r_\Sigma^2 a^2}{3 D_S^b}}}. \tag{7.45}$$

The quantity D_S, which expresses the proper distance of the matching surface, acts as a new cosmological field of geometrical nature with its own equation of motion (7.45). Equations (7.44) and (7.45) form a system of two coupled differential equations for a, D_S. Defining

$$\psi = \frac{\gamma \xi^b}{3 D_S^b}, \tag{7.46}$$

which should be positive, we bring the system to the more standard form,

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{2G_N M}{r_\Sigma^3 a^3} + \psi \tag{7.47}$$

$$\dot{\psi} = -\frac{3^{\frac{1}{b}} b r_\Sigma a H \psi^{1+\frac{1}{b}}}{\xi \gamma^{\frac{1}{b}} \sqrt{1 - \frac{2G_N M}{r_\Sigma a} - r_\Sigma^2 a^2 \psi}}, \tag{7.48}$$

where ψ plays the role of dark energy. We also have

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi G_N}{3}(\rho + \rho_{DE}), \tag{7.49}$$

where ρ is given by (3.4) and $\rho_{DE} = \frac{3}{8\pi G_N}\psi$.

The density parameters are defined as in (7.16) and the value of r_Σ for a galaxy or a cluster is found from (7.17) as before. It is more convenient to work with the redshift z and the evolution of $\psi(z)$ is given by

$$\frac{d\psi}{dz} = \frac{3^{\frac{1}{b}} b \psi^{1+\frac{1}{b}}}{\xi \gamma^{\frac{1}{b}} (1+z)^2 \sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0} H_0^2 (1+z) - \frac{\psi}{(1+z)^2}}}. \tag{7.50}$$

After having solved (7.50), the evolution of the Hubble parameter as a function of z is

$$H^2 = \Omega_{m0} H_0^2 (1+z)^3 + \psi - \frac{\kappa}{a_0^2}(1+z)^2, \tag{7.51}$$

while, the matter density abundance Ω_m becomes

$$\Omega_m = \left[1 + \frac{1}{\Omega_{m0} H_0^2}\frac{\psi}{(1+z)^3} - \frac{\kappa}{a_0^2 \Omega_{m0} H_0^2}\frac{1}{1+z}\right]^{-1}. \tag{7.52}$$

For the numerical investigation of the system, we will need the current value ψ_0 of ψ. In terms of the other parameters, it is

$$\psi_0 = \Omega_{DE,0} H_0^2. \tag{7.53}$$

The differential equation (7.50) contains the parameters ξ (which is of order one), r_Σ (which is known from (7.17) for the typical masses we use), and γ, b. With these parameters and ψ_0 given in (7.53), we can solve numerically (7.50) and find $\psi(z)$. Then, we can plot $\Omega_m(z)$ from (7.52).

It is illuminating to define the variable $\tilde{\psi} = \psi/H_0^2$, and then, Eqs. (7.50)–(7.52) become, respectively,

$$\frac{d\tilde{\psi}}{dz} = \frac{3^{\frac{1}{b}} b (G_N H_0^2)^{\frac{1}{b}-\frac{1}{2}} \tilde{\psi}^{1+\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}} (1+z)^2 \sqrt{\frac{1}{r_\Sigma^2 a_0^2 H_0^2} - \Omega_{m0}(1+z) - \frac{\tilde{\psi}}{(1+z)^2}}} \tag{7.54}$$

$$\frac{H^2}{H_0^2} = \Omega_{m0}(1+z)^3 + \tilde{\psi} + \Omega_{\kappa 0}(1+z)^2 \tag{7.55}$$

$$\Omega_m = \left[1 + \frac{1}{\Omega_{m0}}\frac{\tilde{\psi}}{(1+z)^3} + \frac{\Omega_{\kappa 0}}{\Omega_{m0}}\frac{1}{1+z}\right]^{-1}, \tag{7.56}$$

where $\tilde{\psi}_0 = \Omega_{DE,0}$ and $\Omega_{\kappa 0} = -\kappa/(a_0^2 H_0^2)$. Since $\Omega_{\kappa 0} \ll 1$, $\Omega_{m0} \approx 0.3$ and Ω_m should basically increase in the past, it arises from (7.56) that for recent redshifts, where our scenario makes sense, it should be $\tilde{\psi}/(1+z)^3 \lesssim 1$; otherwise, Ω_m would drop to unacceptably small values in the past. As a result, $\tilde{\psi}/(1+z)^2 \lesssim 10$, which is actually even smaller. On the other hand, the quantity $1/(r_\Sigma^2 H_0^2)$ in the square root of (7.54) is approximately 2×10^7 for a galaxy and 5×10^4 for a cluster; thus, only this term remains in the square root to very high accuracy (better than 0.02% for clusters and better than 0.00005% for galaxies). Therefore, the differential equation (7.54) for any practical reason is approximated by the simple equation

$$\frac{d\tilde{\psi}}{dz} = \frac{3^{\frac{1}{b}} b}{\xi \tilde{\gamma}^{\frac{1}{b}}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}\frac{\tilde{\psi}^{1+\frac{1}{b}}}{(1+z)^2}. \tag{7.57}$$

Integration of (7.57) gives

$$\tilde{\psi} = \left[\frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}\frac{1}{1+z} + c\right]^{-b}, \tag{7.58}$$

where c is the integration constant. From the value of $\tilde{\psi}_0$, we find c, and finally we have for the evolution of dark energy,

$$\tilde{\psi} = \left[\Omega_{\mathrm{DE},0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{z}{1+z} \right]^{-b}. \qquad (7.59)$$

The positiveness of ψ implies that

$$z^{-1} > \left(\frac{3\Omega_{\mathrm{DE},0}}{\xi^b \tilde{\gamma}} \right)^{\frac{1}{b}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} - 1. \qquad (7.60)$$

As a result of the inequality (7.60), we get that

$$\xi^b \tilde{\gamma} > \frac{3\Omega_{\mathrm{DE},0}}{(1 + z_{\max}^{-1})^b} G_N H_0^2 \left(\frac{r_\Sigma a_0}{\sqrt{G_N}} \right)^b, \qquad (7.61)$$

where $z_{\max} = \mathcal{O}(1)$ is a redshift such that in the interval $(0, z_{\max})$ the model should definitely make sense.

For concreteness, we write explicitly the Hubble evolution $H(z)$,

$$\frac{H^2}{H_0^2} = \Omega_{m0}(1+z)^3 + \left[\Omega_{\mathrm{DE},0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{z}{1+z} \right]^{-b}$$
$$+ \Omega_{\kappa 0}(1+z)^2, \qquad (7.62)$$

which is extremely accurate for all relevant recent redshifts where our model is applicable. The only unknown quantities in Eq. (7.62) are $\xi^b \tilde{\gamma}$ and b. Equation (7.62) is a new Hubble evolution which can be tested against observations at the background level. We note that the dark energy term in (7.62) is quite different than the one of Eq. (7.34). It is obvious from (7.62) that $\Omega_{m0} + \Omega_{\mathrm{DE},0} + \Omega_{\kappa 0} = 1$. When $\xi^b \tilde{\gamma}$ is much larger than the right-hand side of (7.61), the dark energy of Eq. (7.62) is approximately a cosmological constant. The most interesting case is certainly when $\xi^b \tilde{\gamma} \sim G_N H_0^2 (\frac{r_\Sigma}{\sqrt{G_N}})^b$, and then, all the terms in (7.62)—with the exception of the spatial curvature Ω_κ—are equally important and give a nontrivial dark energy evolution. In this case, various combinations of $\tilde{\gamma}, b$ are allowed such that (7.61) is satisfied, introducing in general new scales. Moreover, due to the presence of the integration constant c which arranges $\Omega_{\mathrm{DE},0}$, the terse relation (7.36) has now been replaced by the loose inequality (7.61), and therefore, the precise value of the quantity $\xi^b \tilde{\gamma}$ can be used in order to accommodate some other observation, e.g., the acceleration. For a general b, although the dark energy may sufficiently be attributed to the varying cosmological constant, the coincidence problem is not particularly alleviated. However, what we find particularly interesting for the explanation of the coincidence problem, as was discussed above, is the situation with $\tilde{\gamma} \sim 1$, because then, no new mass scale is introduced for the explanation of the dark energy, other than the astrophysical scales. This is our favorite case, where the relation (7.31) is valid with the index b having values close to 2.1, as explained. Such values of b are also theoretically interesting since they are close to the IR fixed-point value $b = 2$ of AS theory and it

remains to AS to find if such values are predicted at the astrophysical scales. For example, for a galaxy with $b = 2.13$, inequality (7.61) provides that $\xi^b \tilde{\gamma} > 2.2$ $(1 + z_{\max}^{-1})^{-b}$, while for a cluster with $b = 2.08$ it is $\xi^b \tilde{\gamma} > 1.8(1 + z_{\max}^{-1})^{-b}$, relations which can easily be satisfied with suitable $\tilde{\gamma} \sim 1$.

Combining Eqs. (7.46) and (7.53), we obtain

$$G_N H_0^2 \left(\frac{D_{S,0}}{\sqrt{G_N}} \right)^b = \frac{\xi^b \tilde{\gamma}}{3\Omega_{\mathrm{DE},0}}. \qquad (7.63)$$

For $\tilde{\gamma}$ such that inequality (7.61) is saturated, which means that the dark energy in (7.62) is nontrivial (different than a cosmological constant), we conclude from (7.63) that the value of D at the current Schucking surface is $D_{S,0} \sim r_\Sigma$. This is true, either for our favorite b or more generally. Note that $D_{S,0}$ is of the order of r_Σ and not approximately equal to r_Σ, as might be guessed initially. It is obvious that the relation (7.36) between the parameters has now disappeared in (7.63) due to the freedom introduced by the presence of $D_{S,0}$. The precise value of $D_{S,0}$ is given by (7.63) and depends on $\Omega_{\mathrm{DE},0}$. The quantity D_S acts as an extra independent cosmological field in the system (7.44), (7.45), whose initial condition $D_{S,0}$ is set today in agreement with the amount of measured dark energy. This initial condition measures the proper distance of the current matching surface of the Swiss cheese model and the extra interesting thing, which also sheds light to the coincidence problem, is that it is of the order of the radius of the astrophysical structure. If it was not, then it would be just the integration constant of an extra field that should be selected appropriately to create the measured dark energy, and therefore, introduce another scale. We can also find from (7.46), (7.59) the expression of $D_S(z)$ to high accuracy

$$\frac{D_S}{r_\Sigma a_0} = \frac{\xi \tilde{\gamma}^{\frac{1}{b}}}{3^{\frac{1}{b}} \Omega_{\mathrm{DE},0}^{\frac{1}{b}}} \frac{1}{(G_N H_0^2)^{\frac{1}{b}}} \frac{\sqrt{G_N}}{r_\Sigma a_0} - \frac{z}{1+z}. \qquad (7.64)$$

Equation (7.64) reduced to (7.63) for $z = 0$. For a nontrivial dark energy evolution in (7.62), the function D_S remains for all relevant z of the order of r_Σ.

In the interior of the object, the proper distance D is a function of the position R; i.e., it is $D(R)$, and the equation governing this evolution is (7.2). Since $D_{S,0}$ is known, the initial condition of Eq. (7.2) is set at $R = R_{S,0} = r_\Sigma$ as $D(r_\Sigma) = D_{S,0}$. In terms of the normalized variables $\hat{D} = D/r_\Sigma$ and $\hat{R} = R/r_\Sigma$, we have

$$\frac{d\hat{D}}{d\hat{R}} = \left\{ 1 - \frac{\xi^b \tilde{\gamma}}{3} \left(\frac{\sqrt{G_N}}{r_\Sigma} \right)^{b-2} \frac{1}{\hat{D}_{S,0}^b} \left[\frac{a_0^3 \Omega_{m0}}{\Omega_{\mathrm{DE},0}} \frac{1}{\hat{R}} \right. \right.$$
$$\left. \left. + \left(\frac{\hat{D}_{S,0}}{\hat{D}} \right)^b \hat{R}^2 \right] \right\}^{-\frac{1}{2}} \qquad (7.65)$$

with the initial condition set at $\hat{R} = 1$ as $\hat{D} = \hat{D}_{S,0} = (\frac{\xi^b \tilde{\gamma}}{3\Omega_{\text{DE},0}})^{\frac{1}{b}}(G_N H_0^2)^{-\frac{1}{b}}\frac{\sqrt{G_N}}{r_\Sigma}$. We note that the fact that D_S changes in time does not mean that the interior solution is time dependent. The interior solution is static and simply $\hat{D}_{S,0}$ is used as initial condition for the integration of (7.65), since this is known from the current amount of dark energy. Since for our favorite values of $\tilde{\gamma}$, b which validate Eq. (7.31), the factor $(\frac{\sqrt{G_N}}{r_\Sigma})^{b-2}$ is various orders of magnitude smaller than one and $\hat{D}_{S,0} \sim 1$, the initial value of $\frac{d\hat{D}}{d\hat{R}}$ at the Schucking surface is one to high accuracy. Therefore, close to r_Σ, the function $D(R) - R$ remains constant. We will see numerically that this is approximately true more generally and find the integration constant.

Concerning the acceleration we find

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\Omega_{m0}H_0^2(1+z)^3$$
$$+ \psi - \frac{3^{\frac{1}{b}}b\psi^{1+\frac{1}{b}}}{2\xi\gamma^{\frac{1}{b}}(1+z)\sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0}H_0^2(1+z) - \frac{\psi}{(1+z)^2}}}.$$
(7.66)

In terms of $\tilde{\psi}$ we have

$$\frac{\ddot{a}}{H_0^2 a} = -\frac{1}{2}\Omega_{m0}(1+z)^3$$
$$+ \tilde{\psi} - \frac{3^{\frac{1}{b}}b(G_N H_0^2)^{\frac{1}{b}-\frac{1}{2}}\tilde{\psi}^{1+\frac{1}{b}}}{2\xi\tilde{\gamma}^{\frac{1}{b}}(1+z)\sqrt{\frac{1}{r_\Sigma^2 a_0^2 H_0^2} - \Omega_{m0}(1+z) - \frac{\tilde{\psi}}{(1+z)^2}}},$$
(7.67)

which is very well approximated, as before, by

$$\frac{\ddot{a}}{H_0^2 a} = -\frac{1}{2}\Omega_{m0}(1+z)^3 + \tilde{\psi} - \frac{3^{\frac{1}{b}}b}{2\xi\tilde{\gamma}^{\frac{1}{b}}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}\frac{\tilde{\psi}^{1+\frac{1}{b}}}{1+z},$$
(7.68)

or explicitly in terms of z as

$$\frac{\ddot{a}}{H_0^2 a} = -\frac{1}{2}\Omega_{m0}(1+z)^3$$
$$+ \left[\Omega_{\text{DE},0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{2\xi\tilde{\gamma}^{\frac{1}{b}}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}\frac{b+2z}{1+z}\right]$$
$$\times \left[\Omega_{\text{DE},0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{\xi\tilde{\gamma}^{\frac{1}{b}}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}\frac{z}{1+z}\right]^{-1-b}. \quad (7.69)$$

The transition redshift z_t from deceleration to acceleration is found from (7.69) setting $\ddot{a} = 0$. The current deceleration parameter takes the following very accurate expression if we use Eq. (7.69),

$$q_0 = \frac{1}{2}\Omega_{m0} - \Omega_{\text{DE},0} + \frac{3^{\frac{1}{b}}b}{2\xi\tilde{\gamma}^{\frac{1}{b}}}\Omega_{\text{DE},0}^{1+\frac{1}{b}}(G_N H_0^2)^{\frac{1}{b}}\frac{r_\Sigma a_0}{\sqrt{G_N}}, \quad (7.70)$$

and therefore, q_0 takes values larger than the ΛCDM value -0.55. From (7.66) or (7.67), the condition $\ddot{a}|_0 > 0$ is written as

$$\xi^b\gamma > \frac{3b^b\Omega_{\text{DE},0}^{1+b}}{(2\Omega_{\text{DE},0} - \Omega_{m0})^b}H_0^{2-b}\left(\frac{1}{r_\Sigma^2 a_0^2 H_0^2} - 1 + \Omega_{\kappa 0}\right)^{-\frac{b}{2}}$$
(7.71)

and is very well approximated by

$$\xi^b\tilde{\gamma} > \frac{3b^b\Omega_{\text{DE},0}^{1+b}}{(2\Omega_{\text{DE},0} - \Omega_{m0})^b}G_N H_0^2\left(\frac{r_\Sigma a_0}{\sqrt{G_N}}\right)^b, \quad (7.72)$$

which also arises from (7.70). The inequality (7.72) can be seen that is sufficient in order to have the significant condition $\dot{\Omega}_m|_0 < 0$. For our favorite values of $\tilde{\gamma}$, b which validate Eq. (7.31), the right-hand side of (7.72) is of order one and $\xi^b\tilde{\gamma}$ should be chosen accordingly, still being of order unity. For example, for galaxies with $b = 2.13$ we take from (7.72) that $\xi^b\tilde{\gamma} > 4.2$, while for clusters with $b = 2.08$ it should be $\xi^b\tilde{\gamma} > 3.2$. These conditions are stronger that those implied by (7.61) for any $z_{\max}$, so (7.61) can be forgotten. For these values of b we provide now some indicative numerical results for q_0 from Eq. (7.70) and z_t. For galaxies with $b = 2.13$, we have the following: for $\xi = 9$, $\tilde{\gamma} = 5$ it is $z_t = 0.68$ and $q_0 = -0.49$; for $\xi = 3$, $\tilde{\gamma} = 2$ it is $z_t = 0.69$ and $q_0 = -0.29$; for $\xi = 1$, $\tilde{\gamma} = 6$ it is $z_t = 0.51$ and $q_0 = -0.09$. For clusters with $b = 2.08$, we have the following: for $\xi = 9$, $\tilde{\gamma} = 5$ it is $z_t = 0.68$ and $q_0 = -0.50$; for $\xi = 3$, $\tilde{\gamma} = 2$ it is $z_t = 0.69$ and $q_0 = -0.32$; for $\xi = 1$, $\tilde{\gamma} = 6$ it is $z_t = 0.60$ and $q_0 = -0.14$. In all these cases, the inequality (7.60) does not provide any restriction on z. If b increases more than 2% relative to the above indicative values, then the quantity $\xi^b\tilde{\gamma}$ moves to higher orders of magnitude to assure acceleration, while if b gets values smaller than the above indicative, the inequality (7.72) is easily satisfied since its right-hand side becomes suppressed.

Finally, the dark energy pressure and its equation-of-state parameter are

$$8\pi G_N p_{\text{DE}}$$
$$= -3\psi + \frac{3^{\frac{1}{b}}b\psi^{1+\frac{1}{b}}}{\xi\gamma^{\frac{1}{b}}(1+z)\sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0}H_0^2(1+z) - \frac{\psi}{(1+z)^2}}}$$
(7.73)

$$w_{\text{DE}} = -1 + \frac{3^{\frac{1}{b}-1}b\psi^{\frac{1}{b}}}{\xi\gamma^{\frac{1}{b}}(1+z)\sqrt{\frac{1}{r_\Sigma^2 a_0^2} - \Omega_{m0}H_0^2(1+z) - \frac{\psi}{(1+z)^2}}}.$$
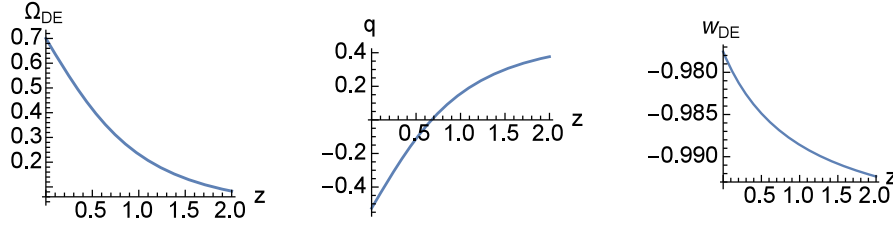(7.74)

FIG. 2. The late-times cosmological evolution for a spatially flat universe, for the parameter choice $b = 2.06$, $\xi = 5$, $\tilde{\gamma} = 5$, $M = 10^{15} M_\odot$ and $\Omega_{m0} = 0.3$. In the left graph we depict the evolution of the dark energy density parameter as a function of the redshift z. In the middle graph we present the evolution of the deceleration parameter. Finally, the dark energy equation-of-state parameter is depicted in the right graph.

In terms of $\tilde{\psi}$, we have

$$\frac{8\pi G_N}{H_0^2} p_{DE}$$

$$= -3\tilde{\psi} + \frac{3^{\frac{1}{b}} b (G_N H_0^2)^{\frac{1}{b} - \frac{1}{2}} \tilde{\psi}^{1 + \frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}} (1 + z) \sqrt{\frac{1}{r_\Sigma^2 a_0^2 H_0^2} - \Omega_{m0}(1 + z) - \frac{\tilde{\psi}}{(1+z)^2}}}$$

$$\tag{7.75}$$

$$w_{DE} = -1 + \frac{3^{\frac{1}{b} - 1} b (G_N H_0^2)^{\frac{1}{b} - \frac{1}{2}} \tilde{\psi}^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}} (1 + z) \sqrt{\frac{1}{r_\Sigma^2 a_0^2 H_0^2} - \Omega_{m0}(1 + z) - \frac{\tilde{\psi}}{(1+z)^2}}},$$

$$\tag{7.76}$$

which are very well approximated, as before, by

$$\frac{8\pi G_N}{H_0^2} p_{DE} = -3\tilde{\psi} + \frac{3^{\frac{1}{b}} b}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{\tilde{\psi}^{1 + \frac{1}{b}}}{1 + z} \tag{7.77}$$

$$w_{DE} = -1 + \frac{3^{\frac{1}{b} - 1} b}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{\tilde{\psi}^{\frac{1}{b}}}{1 + z}, \tag{7.78}$$

or explicitly in terms of z as

$$\frac{8\pi G_N}{H_0^2} p_{DE} = \left[\frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{b + 3z}{1 + z} - 3\Omega_{DE,0}^{-\frac{1}{b}} \right]$$

$$\times \left[\Omega_{DE,0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{z}{1 + z} \right]^{-1 - b}$$

$$\tag{7.79}$$

$$w_{DE} = \left[\frac{3^{\frac{1}{b} - 1}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{b + 3z}{1 + z} - \Omega_{DE,0}^{-\frac{1}{b}} \right]$$

$$\times \left[\Omega_{DE,0}^{-\frac{1}{b}} - \frac{3^{\frac{1}{b}}}{\xi \tilde{\gamma}^{\frac{1}{b}}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}} \frac{z}{1 + z} \right]^{-1}. \tag{7.80}$$

The present-day dark energy equation-of-state parameter takes the following very accurate expression if we use Eq. (7.80),

$$w_{DE,0} = -1 + \frac{3^{\frac{1}{b} - 1} b}{\xi \tilde{\gamma}^{\frac{1}{b}}} \Omega_{DE,0}^{\frac{1}{b}} (G_N H_0^2)^{\frac{1}{b}} \frac{r_\Sigma a_0}{\sqrt{G_N}}, \tag{7.81}$$

and therefore, $w_{DE,0}$ takes values larger than the ΛCDM value -1 (phantom values smaller than -1 can only be obtained if $b < 0$). According to (7.72), it can take any value $w_{DE,0} < -\frac{1}{3}(1 + \frac{\Omega_{m0}}{\Omega_{DE,0}}) \approx -0.48$. In our typical example, using $b = 2.13$ at a galaxy structure, for $\xi = 9$, $\tilde{\gamma} = 5$ it is $w_{DE,0} = -0.95$, for $\xi = 3$, $\tilde{\gamma} = 2$ it is $w_{DE,0} = -0.75$, while for $\xi = 1$, $\tilde{\gamma} = 6$ it is $w_{DE,0} = -0.56$. Similarly, using $b = 2.08$ at a cluster structure, for $\xi = 9$, $\tilde{\gamma} = 5$ it is $w_{DE,0} = -0.95$, for $\xi = 3$, $\tilde{\gamma} = 2$ it is $w_{DE,0} = -0.78$, while for $\xi = 1$, $\tilde{\gamma} = 6$ it is $w_{DE,0} = -0.61$.

To capture the results of this case, we present in Fig. 2 the cosmological evolution as a function of z for a spatially flat universe, with the quantum cosmological constant originated at the cluster level with $M = 10^{15} M_\odot$, $\Omega_{m0} = 0.3$, and for the parameter choice $b = 2.06$, $\xi = 5$, $\tilde{\gamma} = 5$, which provide $z_t = 0.67$, $q_0 = -0.52$, $w_{DE,0} = -0.978$. Of course, these are just indicative values of the parameters, which however look successful at least at first glance (in a forthcoming paper [36], we will perform a numerical analysis at the background level with data from SNIa, $H(z)$ measurements, BAO, etc., where we can say in advance that the fittings show excellent accuracy). We depict in the left graph the evolution of the dark energy density parameter Ω_{DE} from Eq. (7.56), where a typical decreasing behavior for larger redshifts is shown. In the middle graph the evolution of the deceleration parameter q is shown, where the passage from deceleration to acceleration at late times can be seen. The dark energy equation-of-state parameter w_{DE} in the right graph shows a nonconstant evolution with a current value $w_{DE,0}$ close to -1. We also note that if we solve numerically the differential equation (7.50), instead of using the analytical expression (7.59), the results are exactly the same due to the high degree of accuracy of our analytical expressions. Notice that the two reasons mentioned in the end of subsection VII B 1, about the time variability of b and the further decrease of w_{DE}, are also valid here. Namely, a more realistic study of the RG flow inside the structure,

together with a model of structure formation on one side, and the study of more realistic inhomogeneous/anisotropic cosmological models through averaging processes on the other side, would reduce w_{DE}, possibly even to phantom values.

Because we are taking our scenario seriously, we would like to finish with an estimate of the potentials and the forces inside and at the border of an astrophysical object. As for the border, the situation is clear and precise. Since the mass of the structure can be considered as being gathered at the origin, the Schwarzschild term is exact and provides the Newtonian force at the border. If this force and its potential are dominant compared to the new cosmological constant term at the border (which will indeed be the case), this will already be an indication that inside the object the interior dynamics will also not be severely disturbed. More precisely, in the interior of the object, either galaxy or cluster, there is a profile of the matter distribution (luminous, gas, dark matter) which will give some deviations from the central $1/R$ potential. We will first study what is the situation in Milky Way, which is a well-studied galaxy. A very good fit of dark matter halo profile (which is the dominant matter component) to available data for Milky Way is performed [37] using the Universal Rotation Curve (Burkert) profile with matter density $\varrho(R) = \varrho_c(1 + \frac{R}{R_c})^{-1}[1 + (\frac{R}{R_c})^2]^{-1}$, where the density scale $\varrho_c \sim 4 \times 10^7 \, M_\odot/\text{kpc}^3$ and the radius scale $R_c \sim 10$ kpc. The use of a matter profile is necessary in order to get precise values of the Newtonian forces and velocities inside the galaxy ($R < R_b$). It can be easily seen that in the inner regions of the galaxy ($R \lesssim 0.4R_b$), the real amount of matter is much larger than the matter predicted by a constant energy density profile, so this constant profile gives a poor underestimation of the Newtonian force. On the other hand, an interesting result is that away from the very center of the galaxy ($R \gtrsim 0.1R_b$) the real Newtonian force is of the same order as the Newtonian force due to the idealized picture with all the galaxy mass gathered at the origin. This result of Milky Way indicates that, although we will not enter the complicated discussion to study the precise Newtonian force inside other galaxies or clusters, this force will be estimated by the central $1/R^2$ force. This way, we will be in position to compare the varying cosmological constant force relatively to the Newtonian force, and see how long the new force does not give obvious inconsistencies with internal dynamics. Of course, a more detailed study and comparison to existing or upcoming data is necessary.

With the explanations of the previous paragraph, we assume that the gravitational field given by the modified Schwarzschild metric (6.1) in the interior of the object (far from its very center, e.g., for $R \gtrsim 0.1R_b$) provides a sufficient estimate to anticipate what are the interior Newtonian and cosmological constant forces. We use the indicative parameters $b = 2.08$, $\xi = 1$, $\tilde{\gamma} = 6$ to

numerically integrate Eq. (7.65). The solution of this equation gives the function $D(R)$, which turns out to be an increasing function of R, as it should be. The first observation is that the function $D(R)$ in all the relevant distances, throughout the interior of the astrophysical object up to the current Schucking radius $R_{S,0} = r_\Sigma$, has values of the order of r_Σ. Therefore, not only $D_S(z)$ remains of order r_Σ, but also the whole $D(R)$ is so, and provides a variable with values natural to the dimension of the object, without acquiring unnatural very large or very small values. In general, at distances up to a few decades of r_Σ, a reliable approximation for the solution, as it is seen numerically, is $\hat{D} \approx \hat{R} + 0.79$, thus D has approximately a constant difference from R. Of course, this expression of $\hat{D}$ can be used to obtain some intuition, but the detailed structure of $\hat{D}$ could also be significant elsewhere. The next important thing is to study the ratio of the potential term due to the varying cosmological constant to the Newtonian potential, as they arise from Eq. (7.65). For small clusters, this ratio is less than 1% either at the border or inside. For the largest possible clusters, the ratio becomes 15% at the border and less inside. Therefore, for the latest clusters, we have a non-negligible contribution to the pure Newtonian potential with possible observable signatures, still without an obvious inconsistency. Since the Newtonian potential is negligible relatively to unity for $R \gtrsim 0.1R_b$ up to the border, thus both potentials are very weak and F is approximately one to high accuracy. For all clusters, with any diameter, the two potentials become of the same order at the Schucking radius and this was the reason above for the successful explanation of dark energy. At even larger distances ($R \gtrsim 3r_\Sigma$) the cosmological constant becomes the dominant term (although in the Swiss-cheese model, at such distances the cosmological patch is present instead of the static one). Finally, the ratio of the cosmological constant force to the Newtonian force is $\frac{\Omega_{DE,0}}{a_0^3 \Omega_{m0}}(b\frac{\hat{R}}{\hat{D}\sqrt{F}} - 2)(\frac{\hat{D}_{S,0}}{\hat{D}})^b\hat{R}^3$. It turns out that this ratio is negative, which means that the new force is repulsive, as expected. For small clusters, this ratio is again less than 1% either at the border or inside. For the largest possible clusters, the ratio becomes 20% at the border and less inside. Again, for such clusters, a non-negligible contribution to the Newtonian force arises, which should be studied more thoroughly in comparison with real data. Similar results with all the above occur for clusters with different values of the parameters ξ, $\tilde{\gamma}$, where most usually the extra force and potential are further suppressed. As for galaxies, it can be seen that the force or the potential of the extra term is always restricted to a contribution of a few percent, independently of the parameters.

Some remarks are in order about what may happen with the formation of large scale structures of the Universe in the context of our model. It is well known that modifications of gravity that are almost ΛCDM at the background level may have very different evolution of structures. In our present

understanding of the theory, we cannot analyze the whole structure evolution because the scenario starts somehow suddenly with the appearance of the structures. The present work provides a study only at small redshifts. If in the future we manage to understand better the behavior of the RG flow at different energies and scales, we will able to quantify more accurately the antigravity effects during the formation and the evolution of structures. This knowledge will require from AS theory the correct RG flow of gravity with matter at the infrared energy scales and also the exact relation of the amount of energy/mass that is associated with the value of the varying cosmological constant. However, we foresee that general relativity will be preserved somehow accurately during structure formation because, in the initial clouds that collapse, the antigravity quantum effects are not expected to be very important and they become significant recently where structures are denser and smaller. But this, of course, must be studied with an exact RG flow.

VIII. DISCUSSION AND CONCLUSIONS

We have proposed that the dark energy and the recent cosmic acceleration can be the result of the existence of local antigravity sources associated with astrophysical matter configurations distributed throughout the Universe. This is a tempting proposal in relation to the coincidence problem since in that case the dark energy naturally emanates from the recent formation of structure.

The cosmic evolution can arise through some interrelation between the local and the cosmic patches. In the present work we have assumed a Swiss cheese model to derive the cosmological equations, where the interior spherically symmetric metric matches smoothly to an FRW exterior across a spherical boundary. This Schucking surface has a fixed coordinate radius in the cosmic frame, but expands with time in the local frame.

Various gravitational theories can be implemented in the above context and see if the corresponding intermediate distance infrared phenomena can provide the necessary cosmic acceleration. This is not always an easy task since the appropriate spherically symmetric solutions should be used along with the correct matching conditions. Our main concern in this work, in order to test our proposal, is to consider quantum modified spherically symmetric metrics, and more precisely, quantum improved Schwarzschild–de Sitter metrics, which are used for modeling the metric of galaxies or clusters of galaxies.

Asymptotically safe (AS) gravity provides specific forms for the quantum corrections of the cosmological and Newton's constants depending on the energy scale. In the far infrared (IR) regime of AS evolution, which certainly corresponds to the cosmological scales, there are encouraging indications for the existence of a fixed point of a specific form. In the intermediate infrared scales of our astrophysical objects, it is therefore quite reasonable

that some small deviations from this IR law occur, and on this behavior our most successful model was built containing the appropriate antigravity effect. This model uses dimensionless order-one parameters of AS, the Newton's constant and the astrophysical length scale (which enters through the Schucking radius of matching) and provides a recent dark energy comparable to dark matter. At the same time, sufficient cosmic acceleration emerges at small redshifts, while the freedom of the order one parameters has to be constrained by observational data in the future. To the best of our knowledge, this is the first solution of the dark energy problem without using fine-tuning or introducing add-hoc energy scales. Although this cosmology, given by Eq. (7.62), has a quite different functional form than the ΛCDM cosmology, the modified Schwarzschild–de Sitter interior metric allows us to interpret the encountered quantity (7.30) as giving approximately the correct order of magnitude of the standard cosmological constant Λ, which can thus be considered as a composite quantity.

As a more technical point concerning the above cosmology, there appears, from the relation of the AS energy scale with a length scale, a coupled geometrical field with its own equation of motion, which is identified as the proper distance. The integration constant of this field, which is the proper distance at the current Schucking radius, arranges the precise amount of dark energy, and since its value is of the order of the length of the astrophysical object, yet no new scale is introduced from this stage. Finally, we have presented a crude estimation of the antigravity effects in the interior of the structure and it appears that they stay at sufficiently small values, so that not to create obvious conflicts with the local dynamics of the object. This is an interesting issue that deserves a more thorough investigation.

As a future work, it is worth investigating the same scenario with inhomogeneous/anisotropic Swiss cheese models. The present work uses the simplest Swiss cheese model as a first simple approach. It is necessary to begin with it in order to isolate the magnitude of the effects of antigravity sources. Inhomogeneous Swiss cheese type models are certainly more realistic, and we expect that they will enhance the produced effective amount of acceleration. Moreover, the evolution of structure will add a more refined picture of the passage from the deceleration to the acceleration regime.

ACKNOWLEDGMENTS

APPENDIX A: ASYMPTOTICALLY SAFE GRAVITY

1. General theory

The elusive theory of quantum gravity is associated not only with mathematical challenges but also with many

conceptual problems, including a measurements scheme and several epistemological issues. This is something expected since quantum gravity will most probably provide the Theory of Everything, a model that most certainly will include revolutionary new mathematical and physical concepts. Nevertheless, some part of the scientific community last decades has been focused, as a first, on attempts to propose solutions to the well-known result that the quantization of the Einstein-Hilbert action leads to a quantum field theory which is perturbatively nonrenormalizable [38].

In general, a mathematical modeling of a system is greatly simplified if one allows for more parameters, more dimensions or more symmetries. Remarkably, there is one serious attempt of quantum gravity that works in four dimensions using only the symmetries of conventional quantum field theory and of general relativity. An effective quantum field theory of general relativity can give answers about the calculations of amplitudes at energy scales below the Planck scale. This is a result of the fact that higher-derivative terms are suppressed by powers of the Planck mass. However, for energies close to or larger than Planckian scales, the effective theory requires a fixing of an infinite number of free coupling constants from experimental input. This equivalently means that at every loop more experiments must be performed and this finally leads to loss of predictability.

Asymptotic safety (AS) exists in the space of theories that includes the corresponding effective field theory. The AS program recovers "predictivity" by imposing the demand/principle that the physically accepted quantum theory is located within the ultraviolet (UV) critical hypersurface of a renormalization group (RG) fixed point that is called the non-Gaussian fixed point (NGFP). The existence of the latter point guarantees that the UV description of the theory furnishes all dimensionless coupling constants to be finite. Now, determining the trajectory uniquely (which means to pick up a specific universe RG flow) requires a number of experimental input parameters equal to the dimensionality of the hypersurface. This has been proved, under some simplified approximations, that it is indeed possible and there is the NGFP where a trajectory begins and generates general relativity at low energy [39–45]. Approximations of the gravitational RG flow can be carried out with the help of the functional renormalization group equation (FRGE) [39]

$$\partial_k \Gamma_k[g, \bar{g}] = \frac{1}{2} \text{Tr}[(\Gamma_k^{(2)} + \mathcal{R}_k)^{-1} \partial_k \mathcal{R}_k], \qquad \text{(A1)}$$

regarding the effective average gravity action Γ_k, where $\Gamma_k^{(2)}$, $\bar{g}_{\mu\nu}$ and $\mathcal{R}_k$ are defined in the context of the background field formalism. This methodology splits the metric $g_{\mu\nu}$ into a fixed background $\bar{g}_{\mu\nu}$ and fluctuations $h_{\mu\nu}$. The quantity $\Gamma_k^{(2)}$ is the second order functional derivative of Γ_k with respect to the fluctuation field $h_{\mu\nu}$ and $\mathcal{R}_k$ gives a scale-dependent mass term for fluctuations with momenta $p^2 \ll k^2$, where the RG scale k is constructed from the background metric. This RG equation implements Wilson's idea which suggests integrating out momenta $p^2 \ll k^2$, i.e., small fluctuations. In this way, Γ_k provides an effective description of the system for scales k^2. Remarkably, this is a background independent method [46].

The simplest estimation of the RG flow, concerning gravity field, arises after projecting the FRGE onto the gravity action approximated by the following Γ_k

$$\Gamma_k = \frac{1}{16\pi G_k} \int d^4x \sqrt{|g|}(-R + 2\Lambda_k), \qquad \text{(A2)}$$

where gauge fixing and ghost terms are of course included. This approximation includes two energy-dependent couplings, the Newton's constant G_k and the cosmological constant Λ_k. For convenience, we define their dimensionless counterparts

$$g_k \equiv k^2 G_k, \qquad \lambda_k \equiv k^{-2}\Lambda_k, \qquad \text{(A3)}$$

which should respect the beta functions.

In the absence of knowledge of the real functional scale dependence of g_k, λ_k, it is not clear what is the correct trajectory in the space of g_k, λ_k that was followed by the Universe. In other words, we do not really know yet the detailed path along which the classical general relativity regime at the present-day epoch with a constant G_N and negligible Λ can be obtained. In the trans-Planckian regime the NGFP is present [33] and the behavior of the couplings near this point is given by constant values, $g_k = g_*$, $\lambda_k = \lambda_*$, so in the deep ultraviolet ($k \to \infty$), G approaches zero and Λ diverges. There is another fixed point, the Gaussian fixed point (GFP) [31], which is saddle and is located at $g = \lambda = 0$. In the linear regime of the GFP, where the dimensionless couplings are pretty small, the analysis predicts that G is approximately constant, while Λ displays a running proportional to k^4. To the other edge of the far infrared limit ($k \to 0$) [32], the behavior of the RG flow trajectories with positive G, Λ is not so well understood since the approximation breaks down (divergence of beta functions) when λ_k approaches $1/2$ at a nonzero k, where an unbounded growth of G appears together with a vanishingly small Λ (interestingly enough, this happens near $k = H_0$). The exact value of the current Λ is unknown due to this break down.

2. Cosmology

The framework of AS in principle describes a modified gravitational force at all length scales, something that makes the cosmological model building feasible [29,32], [47–67] (see also review [45]). Consequently, phenomenological studies that use cosmological data can constrain the various free parameters appearing in AS, including the

values of the cosmological constant and Newton's constant. The various research studies that appear in the literature incorporate the AS property of energy-dependent couplings in two ways.

In the first approach, the scale laws of the couplings are taken from the rigor RG computation of AS close to the NGFP or GFP, or at some infrared range. Then, either these laws are incorporated in general relativity solutions or they are included in properly modified Einstein equations that respect Bianchi identities. In this approach, there is the advantage of using rigorous and trusted results from RG studies of AS. However, this approach is supposed to concentrate on the study of a relatively restricted energy scale range, i.e., for the big bang regime or the infrared regime, and typically is not used for the description of the whole cosmological evolution.

In the second approach, RG improved techniques are used either in the equations of motion or in the machinery of the effective average action. The models are not implemented at the same level of rigor as the full RG flow studies forming the core of AS. However, they allow for the construction of interesting cosmological scenarios with extended cosmological evolution.

It is common in the AS literature to set G and Λ as functions of the energy k in the existing solutions of Einstein equations in order to improve their behavior. The simple input of $G(k)$ and $\Lambda(k)$ into the classical vacuum equations results to violation of the Bianchi identities, while this same input into a classical solution creates a metric which is not solution of a well-defined theory. In [68], the formalism of obtaining RG improved solutions that respect Bianchi identities was presented at the action level. In [69], an alternative and mathematically more solvable approach was developed at the level of equations of motion, consistent with Bianchi identities, where the appropriate covariant kinetic terms that support an arbitrary source field $\Lambda(k)$ was included without any symmetry assumption.

Many AS cosmological studies have analyzed the early cosmological evolution or the dark energy problem and it was even possible to propose solutions to the cosmic entropy issue [67]. Of particular interest are studies where "RG improved" cosmologies admit exponential or power-law

inflationary solutions [70]. The initial vacuum state of cosmos is characterized by an energy-dependent cosmological constant, and subsequently, Einstein equations, modified according to AS, include a nonzero matter energy-momentum tensor with an energy-dependent Newton's constant (matter is expected to appear due to energy transfer from vacuum to matter fields). Both these Λ and G respect the energy dependence that is predicted in the context of AS at the NGFP. In [71], extending the formalism presented in [69] beyond the vacuum case to also include matter, quantum gravity inspired modified Einstein equations were realized, capable to describe both absence of matter cases and configurations with matter contributions. There are also studies discussing the singularity problem [72] or the assumption that the Universe had a quantum vacuum birth [69].

An important question is the association of the RG scale parameter k to the cosmological time or proper length, in order for the model to be reasonable. First works have chosen the RG scale to be inversely proportional to cosmological time [29], while later, the more popular connection with the Hubble scale was developed. In some other works, the RG scale is linked with the plasma temperature or with the fourth root of the energy density [73], the cosmological event/particle horizons [74], or curvature invariants like Ricci scalar [63–65].

APPENDIX B: ANALYSIS CLOSE TO THE GAUSSIAN FIXED POINT

Here we perform an analytical study of Sec. VII A concerning the Gaussian fixed point. Because of the extreme fine-tuning in α, numerical study of the equations of Sec. VII A is not possible. For $\kappa = 0$, we define the quantities

$$\tilde{\chi} = \frac{\chi - (1 - \Omega_{m0})H_0^2}{(1 - \Omega_{m0})H_0^2 - \frac{\alpha}{3}}, \qquad \tilde{\alpha} = \frac{\alpha}{H_0^2} > 0. \qquad \text{(B1)}$$

Note that $\tilde{\chi} + 1 > 0$. From (7.24), the dimensionless quantity $\frac{\tilde{\alpha}}{3}$ is fine-tuned very close to $1 - \Omega_{m0}$, i.e., $0 < 1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3} \lesssim 10^{-22}$. From (7.21) it is $\tilde{\chi}_0 = 0$, while Eq. (7.18) becomes

$$\frac{d\tilde{\chi}}{dz} = \frac{4\zeta(\tilde{\chi} + 1)^{\frac{5}{4}}}{(1 + z)^2 \sqrt{1 - [\frac{\tilde{\alpha}}{3} + \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi} + 1)]\frac{r_\Sigma^2 a_0^2 H_0^2}{(1+z)^2}}}, \qquad \text{(B2)}$$

where

$$\zeta \equiv \frac{3^{\frac{1}{4}}}{\xi \nu^{\frac{1}{4}}} \left(\frac{r_\Sigma^2 a_0^2 H_0^2}{H_0 \sqrt{G_N}}\right)^{\frac{1}{2}} \left(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3}\right)^{\frac{1}{4}}. \qquad \text{(B3)}$$

Due to (7.24), it is $\zeta \lesssim 10^{23}$.

Equation (7.20) takes the form

$$\Omega_m = \frac{\Omega_{m0}(1+z)^3}{\frac{\tilde{\alpha}}{3} + \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1)}, \quad (B4)$$

where the first $\frac{\tilde{\alpha}}{3}$ in the denominator can be practically replaced by $1 - \Omega_{m0}$. Then, since $\tilde{\chi}_0 = 0$, Eq. (B4) is consistent today. For a recent range of redsifts z it is $(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1) \ll 10^5$. This condition actually defines this recent range of z. This is a very weak condition which is also physically reasonable. Indeed, in the opposite case, Ω_m from (B4) would become extremely small for recent z, which is unacceptable, since the Universe would be practically empty of matter. Therefore, this condition is expected to be valid for all relevant recent z. Thus, it arises that $(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})r_\Sigma^2 a_0^2 H_0^2(\tilde{\chi}+1) \ll 1$ and (B2) is well approximated by

$$\frac{d\tilde{\chi}}{dz} = \frac{4\zeta(\tilde{\chi}+1)^{\frac{5}{4}}}{(1+z)^2}, \quad (B5)$$

with general solution

$$\tilde{\chi} = \left(\tilde{c} + \frac{\zeta}{1+z}\right)^{-4} - 1, \quad (B6)$$

where $\tilde{c}$ is integration constant. From the integration of (B5), it arises that it should be $\tilde{c} + \frac{\zeta}{1+z} > 0$. Since $\tilde{\chi}_0 = 0$, the solution takes the form

$$\tilde{\chi} = \left[\frac{1+z}{1 - (\zeta-1)z}\right]^4 - 1, \quad (B7)$$

under the condition $(\zeta-1)z < 1$.

For $\zeta \leq 1$ the condition $(\zeta-1)z < 1$ is satisfied for any z. From (B7) it arises that $\tilde{\chi}+1 = \mathcal{O}(1)$, thus $(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1) \lesssim 10^{-22}$. Therefore, the previous inequality $(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1) \ll 10^5$ is indeed satisfied, and moreover, (B4) takes the form

$$\Omega_m = \frac{\Omega_{m0}(1+z)^3}{1 - \Omega_{m0} + \Omega_{m0}(1+z)^3}, \quad (B8)$$

which is the ΛCDM behavior. Therefore, in this case, the behavior of $\Omega_m(z)$ cannot be discerned from the ΛCDM behavior.

For $\zeta > 1$, the condition $(\zeta-1)z < 1$ is satisfied for $z < z_\zeta$, where $z_\zeta = (\zeta-1)^{-1}$. Therefore, for $\zeta > 1$, the model is valid only for $z < z_\zeta$. It is obvious that as ζ increases, z_ζ decreases and z is only meaningful for a short range around $z = 0$. Thus, physically the only reasonable values of ζ are those which are of order one. Furthermore, for any $\zeta > 1$, the redshift z should not be extremely close to z_ζ; otherwise, $\tilde{\chi}+1$ would become very large, and as stated above, Ω_m would become extremely suppressed, which is not acceptable. To be more precise, let us define the quantity

$$\varepsilon = \left(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3}\right)^{\frac{1}{4}}, \quad (B9)$$

where $\varepsilon \lesssim 10^{-5.5}$. Thus, it should be $z \lesssim (1-\varepsilon)z_\zeta$, which is the regime of applicability of the model, and then $(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1) \lesssim 1$. This means that very close to the higher value of z, i.e., very close to $(1-\varepsilon)z_\zeta$, there is a deviation from ΛCDM, while shortly later, as z reduces, the Ω_m behavior is not discerned from that of ΛCDM. When we say shortly later, we mean that for $z \lesssim (1-5\varepsilon)z_\zeta$ ΛCDM is established. In the initial era $(1-5\varepsilon)z_\zeta \lesssim z \lesssim (1-\varepsilon)z_\zeta$ the full Eq. (B4) is valid.

The evolution of the Hubble parameter is found from (7.19) to be

$$\frac{H^2}{H_0^2} = \frac{\tilde{\alpha}}{3} + \Omega_{m0}(1+z)^3 + \left(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3}\right)(\tilde{\chi}+1). \quad (B10)$$

The first term $\frac{\tilde{\alpha}}{3}$ on the rhs of (B10) can be practically replaced by $1 - \Omega_{m0}$. As above, for $\zeta \leq 1$ the ΛCDM expression arises

$$\frac{H^2}{H_0^2} = 1 - \Omega_{m0} + \Omega_{m0}(1+z)^3, \quad (B11)$$

while for $\zeta > 1$ the full expression (B10) is kept, which however reduces to (B11) when $z \lesssim (1-5\varepsilon)z_\zeta$.

In order to study the acceleration properties of the model, Eq. (7.22) is written as

$$\frac{\ddot{a}}{H_0^2 a} = \frac{\tilde{\alpha}}{3} - \frac{\Omega_{m0}}{2}(1+z)^3 + \left(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3}\right)(\tilde{\chi}+1)\left[1 - \frac{2\zeta(\tilde{\chi}+1)^{\frac{1}{4}}}{(1+z)\sqrt{1 - [\frac{\tilde{\alpha}}{3} + \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi}+1)]\frac{r_\Sigma^2 a_0^2 H_0^2}{(1+z)^2}}}\right], \quad (B12)$$

while Eq. (7.27) becomes

$$w_{DE} = -1 + \frac{4\zeta(1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi} + 1)^{\frac{5}{4}}}{3(1 + z)[\frac{\tilde{\alpha}}{3} + (1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi} + 1)]\sqrt{1 - [\frac{\tilde{\alpha}}{3} + \Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0} - \frac{\tilde{\alpha}}{3})(\tilde{\chi} + 1)]\frac{r_\Sigma^2 a_0^2 H_0^2}{(1+z)^2}}}. \tag{B13}$$

According to the previous results, the long square root in (B12) can be set to unity, while the term $\frac{\tilde{\alpha}}{3}$ in the beginning of the rhs can be replaced by $1 - \Omega_{m0}$. Similar simplifications occur also in (B13). If we define for convenience the quantity

$$\mu = \frac{3^{\frac{1}{4}}}{\xi\nu^{\frac{1}{4}}}\left(\frac{r_\Sigma^2 a_0^2 H_0^2}{H_0\sqrt{G_N}}\right)^{\frac{1}{2}}, \tag{B14}$$

then $\zeta = \mu\varepsilon$. For galaxies it is $\mu \sim 10^{27}$, while for clusters $\mu \sim 10^{28}$.

For $\zeta \leq 1 \Leftrightarrow \varepsilon < 10^{-28}$, Eq. (B12) obtains the ΛCDM behavior

$$\frac{\ddot{a}}{H_0^2 a} = 1 - \Omega_{m0} - \frac{\Omega_{m0}}{2}(1 + z)^3. \tag{B15}$$

Additionally, Eq. (B13) gives $w_{DE} = -1$. Therefore, for $\zeta \leq 1$ the acceleration properties of the model coincide with those of ΛCDM.

For $\zeta > 1 \Leftrightarrow \varepsilon > 10^{-28}$, Eqs. (B12) and (B13) become

$$\frac{\ddot{a}}{H_0^2 a} = 1 - \Omega_{m0} - \frac{\Omega_{m0}}{2}(1 + z)^3 + \varepsilon^4(\tilde{\chi} + 1)$$

$$\times \left[1 - \frac{2\mu\varepsilon(\tilde{\chi} + 1)^{\frac{1}{4}}}{1 + z}\right], \tag{B16}$$

$$w_{DE} = -1 + \frac{4\mu\varepsilon^5(\tilde{\chi} + 1)^{\frac{5}{4}}}{3(1 + z)[1 - \Omega_{m0} + \varepsilon^4(\tilde{\chi} + 1)]}. \tag{B17}$$

In these equations, there are some characteristic intervals of z with the following hierarchy: $(1 - \mu^{\frac{1}{5}}\varepsilon)z_\zeta < (1 - 0.5\mu^{\frac{1}{5}}\varepsilon)z_\zeta < (1 - 5\varepsilon)z_\zeta < (1 - \varepsilon)z_\zeta$. In the initial regime $(1 - 0.5\mu^{\frac{1}{5}}\varepsilon)z_\zeta \lesssim z \lesssim (1 - \varepsilon)z_\zeta$, Eqs. (B16) and (B17) can well be approximated by only their very last terms, which means that in this regime a deceleration is present. Especially for $(1 - 5\varepsilon)z_\zeta \lesssim z \lesssim (1 - \varepsilon)z_\zeta$ this deceleration is large and is basically controlled by the parameter μ. Progressively, as z reduces towards the value $(1 - \mu^{\frac{1}{5}}\varepsilon)z_\zeta$, these last terms become smaller, towards some values of order one, and thus, these terms are comparable to the conventional ΛCDM terms of (B16), (B17). Therefore, in the redshift interval around $z = (1 - \mu^{\frac{1}{5}}\varepsilon)z_\zeta$, w_{DE} gets a positive, order one

correction of the ΛCDM value -1. As we can see, for the astrophysical and cosmological values encountered in our model, the term with the unit 1 in the bracket of (B16) can always be omitted and also the quantity $\varepsilon^4(\tilde{\chi} + 1)$ in the denominator of (B17) is only significant for $z \sim (1 - \varepsilon)z_\zeta$. Finally, for $z = 0$ we get $\frac{\ddot{a}_0}{H_0^2 a_0} = 1 - \frac{3}{2}\Omega_{m0} - 2\mu\varepsilon^5$, $w_{DE,0} = -1 + \frac{4\mu\varepsilon^5}{3(1 - \Omega_{m0})}$. Depending on the numerical value of the quantity $\mu\varepsilon^5$, the values $\ddot{a}_0$, $w_{DE,0}$ coincide or not with the ΛCDM ones. Of course, in order to have acceleration today, it should be $\frac{4\mu\varepsilon^5}{3(1 - \Omega_{m0})} < 1$. Therefore, from the beginning of the AS effect, the functions $\ddot{a}(z)$, $w_{DE}(z)$ evolve in a non-ΛCDM way up to $z \sim (1 - \mu^{\frac{1}{5}}\varepsilon)z_\zeta$ or up to $z = 0$, while on the contrary, as seen above, the functions $H(z)$, $\Omega(z)$ have already passed into the ΛCDM behavior for $z \lesssim (1 - 5\varepsilon)z_\zeta$. This peculiar phenomenon is due to the presence of higher time derivatives contained in the acceleration, which can lead to significant contribution from terms which are negligible in H, Ω_m. For the most interesting case with $\zeta \sim 1$, the current values of $\ddot{a}_0$, $w_{DE,0}$ are the same with the ΛCDM ones, which means that after passing the era with $z \sim (1 - \mu^{\frac{1}{5}}\varepsilon)z_\zeta$ the model reduces to ΛCDM. Therefore, the behavior in the evolution of the model around the passage from deceleration to acceleration differs from the ΛCDM one and could in principle be discerned using precise observational data. However, this is not the case since it is $\mu^{\frac{1}{5}}\varepsilon = \zeta\mu^{-\frac{4}{5}} \ll 1$ for the numerical values of the astrophysical and cosmological quantities we are interested in. Only in the case that the quantity ν is (B14) is substantially enlarged, which is not predicted by the theory, could μ be reduced essentially. Therefore, the previous eras of deviation from ΛCDM cannot be observed and the model is practically identical to ΛCDM in all the range of its validity. For $1 \ll \zeta \ll 10^{22}$, it is still $\mu^{\frac{1}{5}}\varepsilon = \zeta\mu^{-\frac{4}{5}} \ll 1$, which means that the model is indistinguishable from ΛCDM, beyond the fact that z_ζ is already unphysically small. Finally, for $\zeta \sim 10^{22}$, which means $\varepsilon \sim 10^{-5}$, it is $\mu^{\frac{1}{5}}\varepsilon = \zeta\mu^{-\frac{4}{5}} \sim 1$. Therefore, in this case, the current value of w_{DE} is different from -1, and the model is always different from ΛCDM. However, the model in this case is meaningless since z_ζ in extraordinarily small. As a result, we can summarize saying that in the physically meaningful case with $\zeta \sim 1$, the acceleration properties of the model cannot be discerned from ΛCDM.

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